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A Solution to the Supersymmetric Fine-Tuning Problem within the MSSM

Ryuichiro Kitano[a] and Yasunori Nomura[b,c]

[a] *Stanford Linear Accelerator Center, Stanford University, Stanford, CA 94309*
[b] *Department of Physics, University of California, Berkeley, CA 94720*
[c] *Theoretical Physics Group, Lawrence Berkeley National Laboratory, Berkeley, CA 94720*

Abstract

Weak scale supersymmetry has a generic problem of fine-tuning in reproducing the correct scale for electroweak symmetry breaking. The problem is particularly severe in the minimal supersymmetric extension of the standard model (MSSM). We present a solution to this problem that does not require an extension of the MSSM at the weak scale. Superparticle masses are generated by a comparable mixture of moduli and anomaly mediated contributions, and the messenger scale of supersymmetry breaking is effectively lowered to the TeV region. Crucial elements for the solution are a large A term for the top squarks and a small B term for the Higgs doublets. Requiring no fine-tuning worse than 20%, we obtain rather sharp predictions on the spectrum. The gaugino masses are almost universal at the weak scale with the mass between 450 and 900 GeV. The squark and slepton masses are also nearly universal at the weak scale with the mass a factor of $\sqrt{2}$ smaller than that of the gauginos. The only exception is the top squarks whose masses split from the other squark masses by about $m_t/\sqrt{2}$. The lightest Higgs boson mass is smaller than 120 GeV, while the ratio of the vacuum expectation values for the two Higgs doublets, $\tan\beta$, is larger than about 5. The lightest superparticle is the neutral Higgsino of the mass below 190 GeV, which can be dark matter of the universe. The mass of the lighter top squark can be smaller than 300 GeV, which may be relevant for Run II at the Tevatron.

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1 Introduction and Summary

One of the primary reasons for looking for physics beyond the standard model comes from the concept of naturalness. In the standard model, the Higgs mass-squared parameter receives radiative corrections of order the cutoff scale squared, so unless there is an unnatural cancellation we expect that the cutoff of the standard model is not much larger than the scale of electroweak symmetry breaking. Weak scale supersymmetry provides an excellent candidate for physics above this scale, in which the role of the cutoff for the Higgs mass-squared parameter is played essentially by superparticle masses. The theory can then be extrapolated up to very high energies, giving the successful prediction for gauge coupling unification [1].

The negative result for the Higgs boson search at the LEP II, however, brings some doubt on this picture, at least the one based on the minimal supersymmetric extension of the standard model (MSSM). In the MSSM, the mass of the physical Higgs boson, $M_{\rm Higgs}$, is smaller than the Z boson mass at tree level. The mass can be pushed up to larger values by a top-stop loop contribution [2], but the LEP II bound of $M_{\rm Higgs} \gtrsim 114.4$ GeV [3] requires this contribution to be rather large. For a reasonably small value for the stop mixing parameter, such a large contribution arises only for rather large top squark masses, $m_{\tilde{t}} \gtrsim (800 \sim 1200)$ GeV [4]. This is a problem, because the (up-type) Higgs boson mass-squared parameter, $m_{H_u}^2$, then receives a large contribution

$$\delta m_{H_u}^2 \simeq -\frac{3y_t^2}{4\pi^2}\, m_{\tilde{t}}^2\, \ln\left(\frac{M_{\rm mess}}{m_{\tilde{t}}}\right), \tag{1}$$

where $M_{\rm mess}$ is the scale at which the squark and slepton masses are generated. For $M_{\rm mess}$ of order the unification scale, for example, this gives $|\delta m_{H_u}^2|^{1/2}$ larger than a TeV. Since the size of the Higgs quartic coupling is essentially determined by supersymmetry, we then find that this contribution must be canceled with some other contribution to $m_{H_u}^2$ at a level of one percent or even worse, to reproduce the correct scale for electroweak symmetry breaking, $v \simeq 174$ GeV. In fact, the problem becomes even keener if the top quark is relatively light as suggested by the latest experimental data: $m_t = 172.7 \pm 2.9$ GeV [5]. Since the correction to the Higgs boson mass is roughly proportional to $m_t^4 \ln(m_{\tilde{t}}/m_t)$, such a light top quark pushes up the lowest value of $m_{\tilde{t}}$ giving $M_{\rm Higgs} \gtrsim 114.4$ GeV quite high, e.g. $m_{\tilde{t}} \gtrsim 1$ TeV.

A simple way to avoid the above problem, called the supersymmetric fine-tuning problem, is to make both $m_{\tilde{t}}^2$ and $\ln(M_{\rm mess}/m_{\tilde{t}})$ small. For $\ln(M_{\rm mess}/m_{\tilde{t}})$ as small as a factor of a few, the fine-tuning can be avoided for $m_{\tilde{t}} \lesssim (400 \sim 510)(M_{\rm Higgs}/150\ {\rm GeV})(3.5/\ln(M_{\rm mess}/m_{\tilde{t}}))^{1/2}$ [6, 7], where the numbers correspond to making fine-tuning better than $(20 \sim 30)\%$. Such light top squarks can easily be accommodated if we introduce an additional contribution to the Higgs boson mass. (For theories giving such a contribution, see e.g. [8 – 15, 6].) This, however, necessarily requires a deviation from the MSSM at the weak scale.

In this paper we study the question: is it possible to solve the supersymmetric fine-tuning problem without extending the MSSM at the weak scale? The difficulty of doing that is numerically clear. The amount of fine-tuning, Δ^{-1}, is measured by the ratio of the left-hand-side to the largest contribution to the right-hand-side in the equation determining the weak scale, $M_{\rm Higgs}^2/2 \simeq -m_{H_u}^2 - |\mu|^2$ [16]. This gives

$$\Delta^{-1} \approx \frac{M_{\rm Higgs}^2}{2\,\delta m_{H_u}^2}, \tag{2}$$

where $\delta m_{H_u}^2$ is the top-stop contribution of Eq. (1). Suppose now that the superparticle masses are generated at one loop through standard model gauge interactions. In this case, $M_{\rm mess}$ can be as small as $(12\pi^2/g_3^2)^{1/2} m_{\tilde{t}} \simeq 8$ TeV, where g_3 is the $SU(3)_C$ gauge coupling at $M_{\rm mess}$, so that $\ln(M_{\rm mess}/m_{\tilde{t}})$ can be as small as $\simeq 2.3$. Even with this small logarithm, Eq. (2) still gives fine-tuning as bad as $\Delta^{-1} \simeq 6\%$ for $m_{\tilde{t}} \simeq 800$ GeV (and $M_{\rm Higgs} \simeq 114.4$ GeV). The fine-tuning becomes even worse, $\Delta^{-1} \simeq 4\%$, if we use $m_{\tilde{t}} \simeq 1$ TeV.

There are two directions one can consider. One is to make $M_{\rm mess}$ really close to $m_{\tilde{t}}$. Such a situation may arise if we introduce non-renormalizable physics at the TeV scale (e.g. [17]), but then we lose the supersymmetric desert and physics associated with it, such as gauge coupling unification. The reduction of fine-tuning is also mild, and a cancellation of order 10% is still required. The other is to introduce a rather large stop mixing parameter A_t, which has implicitly been assumed to be small in the above analysis. This allows $M_{\rm Higgs} \gtrsim 114.4$ GeV even with $m_{\tilde{t}}$ as small as $\simeq (400 \sim 500)$ GeV. We must, however, then generate such a large A_t, keeping $M_{\rm mess}$ small and without introducing the supersymmetric flavor problem. The implication for the fine-tuning is also not obvious, because the large A_t gives an additional negative contribution to $m_{H_u}^2$ so that the entire problem should be reanalyzed.

Recently, it has been observed [18] that the mediation scale of supersymmetry breaking, $M_{\rm mess}$, can be effectively lowered in a scenario in which the moduli [19] and anomaly mediated [20, 21] contributions to supersymmetry breaking are comparable, which occurs naturally in a low energy limit of certain string-motivated setup [22 – 25]. The point is that, while $M_{\rm mess}$ can be effectively lowered to the TeV region, there is no physical threshold associated with it; the lowering occurs because of an interplay between the moduli and anomaly mediated contributions in the renormalization group evolutions.[1] A remarkable consequence of such a low effective messenger scale is that the superparticle masses can be unified at the TeV scale. An interesting recent suggestion is that this can be used to address the issue of fine-tuning by making $M_{\rm mess}$ close to $m_{\tilde{t}}$ and thus by producing a little hierarchy between the Higgs boson and the other scalar squared masses [29]. This, however, can hardly be the only essence for the solution to the

[1]For alternative possibilities for reducing $M_{\rm mess}$, or canceling the top-stop contribution, see e.g. [26 – 28].

supersymmetric fine-tuning problem, which we define such that the amount of cancellation can be made smaller than 1 in 5. Since the bound on $m_{\tilde{t}}$ is still strong even for the unit logarithm, $\ln(M_{\rm mess}/m_{\tilde{t}}) = 1$, it is not clear that the tight tension with $M_{\rm Higgs}$ really allows $\Delta^{-1} \geq 20\%$.

In this paper we show that it is possible to obtain $\Delta^{-1} \geq 20\%$ along this direction. We find that a crucial element for the reduction of fine-tuning is, besides a very small $M_{\rm mess}$, a rather large value for A_t, which necessarily arises as a consequence of lowering the effective messenger scale, $M_{\rm mess}$. The result is not trivial because the large A_t gives a sizable contribution to the right-hand-side of Eq. (1), giving a stronger bound on naturalness than the case with small A_t. In fact, we obtain $|\delta m^2_{H_u,{\rm top}}|^{1/2}/m_{\tilde{t}} \simeq 0.4 \gg (1/8\pi^2)^{1/2}$ even for the unit logarithm, so that there is no real hierarchy between $m_{\tilde{t}}$ and the Higgs soft mass. We find that $\Delta^{-1} \geq 20\%$ can be obtained only if the superparticle masses are in certain restricted ranges. This in turn provides a set of rather sharp predictions on the superparticle masses. A somewhat surprising result is that one of the top squarks can be rather light; even $m_{\tilde{t}_1} \lesssim 300$ GeV is allowed due to large A_t. This may be relevant for Run II at the Tevatron.

In the simplest setup, we obtain the following predictions on the spectrum. The gauginos are almost universal at the weak scale, as well as the squarks and sleptons:

$$m_{\tilde{b}} \simeq m_{\tilde{w}} \simeq m_{\tilde{g}} \simeq M_0, \tag{3}$$

$$m_{\tilde{q}} \simeq m_{\tilde{u}} \simeq m_{\tilde{d}} \simeq m_{\tilde{l}} \simeq m_{\tilde{e}} \simeq \frac{M_0}{\sqrt{2}}, \tag{4}$$

where the parameter M_0 is in the range

$$450 \text{ GeV} \lesssim M_0 \lesssim 900 \text{ GeV}. \tag{5}$$

The upper bound on M_0 arises from requiring $\Delta^{-1} \geq 20\%$. The top squark masses have appreciable splittings from $M_0/\sqrt{2}$:

$$m_{\tilde{t}_{1,2}} \simeq \frac{M_0 \mp m_t}{\sqrt{2}}. \tag{6}$$

The lightest Higgs boson mass is bound by

$$M_{\rm Higgs} \lesssim 120 \text{ GeV}, \tag{7}$$

and the ratio of the vacuum expectation values (VEVs) for the two Higgs doublets, $\tan\beta \equiv \langle H_u \rangle / \langle H_d \rangle$, satisfies

$$\tan\beta \gtrsim 5. \tag{8}$$

The lightest superparticle is the neutral Higgsino of the mass

$$m_{\tilde{h}^0} \lesssim 190 \text{ GeV}, \tag{9}$$

which is nearly degenerate with the charged Higgsino:

$$m_{\tilde{h}^\pm} - m_{\tilde{h}^0} = \frac{m_Z^2}{2M_0}(1+\epsilon), \tag{10}$$

where $|\epsilon| \lesssim 0.2$ in the relevant parameter region. The bounds in Eqs. (7, 9) follow from $\Delta^{-1} \geq$ 20%. The upper bounds in Eq. (5) and Eq. (9) can be relaxed to 1300 GeV and 270 GeV, respectively, if we allow Δ^{-1} as small as 10%. The bound in Eq. (8) is also relaxed to $\tan\beta \gtrsim 4$.

In the next section we describe the framework. The analysis of fine-tuning is given in section 3, together with the resulting predictions on the spectrum. The importance of a small B parameter is pointed out, and a possible mechanism to realize it is presented. Cosmological implications are also discussed.

2 Framework

Following Refs. [22, 23], we consider that the dominant source of supersymmetry breaking in the MSSM sector comes from the F-term VEVs of the chiral compensator superfield C and a single moduli field T. The effective supergravity action is given by

$$\begin{aligned} S \;\; = \;\; & \int d^4x \sqrt{-g} \Bigg[\int d^4\theta \, C^\dagger C \mathcal{F} - \theta^2 \bar{\theta}^2 C^{\dagger 2} C^2 \mathcal{P}_{\rm lift} \\ & + \left\{ \int d^2\theta \left(\frac{1}{4} f_a \mathcal{W}^{a\alpha} \mathcal{W}^a_\alpha + C^3 W \right) + {\rm h.c.} \right\} \Bigg], \end{aligned} \tag{11}$$

where $g_{\mu\nu}$ is the metric in the superconformal frame. The function $\mathcal{F}$, which is related to the Kähler potential K by $\mathcal{F} = -3\exp(-K/3)$, the superpotential W, and the gauge kinetic functions f_a take the form

$$\mathcal{F} = \mathcal{F}_0(T+T^\dagger) + (T+T^\dagger)^{r_i} \Phi_i^\dagger \Phi_i, \tag{12}$$

$$W = W_0(T) + W_{\rm Yukawa}, \tag{13}$$

$$f_a = T, \tag{14}$$

where $W_{\rm Yukawa}$ is the MSSM Yukawa couplings, and we have assumed that the nonlinear transformation acting on ImT is (approximately) preserved in $\mathcal{F}$. The second term in Eq. (11) is introduced to allow the cancellation of the cosmological constant at the minimum of the potential. This term is supposed to take the form

$$\mathcal{P}_{\rm lift} = d(T+T^\dagger)^{n_P}, \tag{15}$$

which should arise from supersymmetry breaking in some sector that does not give direct contributions to supersymmetry breaking in the MSSM sector. The functions $\mathcal{F}_0$ and W_0 are taken to be

$$\begin{aligned} \mathcal{F}_0(T+T^\dagger) &= -3(T+T^\dagger)^{n_0/3} + \cdots, & (16) \\ W_0(T) &= w_0 - Ae^{-aT}, & (17) \end{aligned}$$

where the dots in $\mathcal{F}_0$ represent possible higher order threshold corrections at the compactification and/or string scales. The parameters w_0 and A are assumed to be of order $m_{3/2}M_*^2 \ll M_*^3$ and M_*^3, respectively, where $m_{3/2}$ is the gravitino mass and M_* the fundamental scale of order the Planck scale.[2] These parameters can be taken real under the presence of an approximate shift symmetry for ImT. The parameter a is a real constant of order $8\pi^2$: $a = 8\pi^2/N$, where N is the size of the gauge group generating the nonperturbative superpotential for T through gaugino condensation. An interesting candidate for T is the moduli parameterizing the volume of the n compact extra dimensions: $T \propto R^n$, where R is the length scale for the compact space. In this case $n_0 = 3$, and $r_i = \tilde{n}_i/n$ if Φ_i propagates in $(4+\tilde{n}_i)$-dimensional subspace.

The superpotential of Eq. (17) stabilizes the modulus T and produces the F-term VEVs for C and T. At the leading order in $1/\ln(A/w_0) \sim 1/\ln(M_{\rm Pl}/m_{3/2})$,

$$\begin{aligned} aT &= \ln\left(\frac{M_{\rm Pl}}{m_{3/2}}\right), & (18) \\ \frac{m_{3/2}}{M_0} &= \frac{2}{3}\frac{\partial_T K_0}{\partial_T \ln(V_{\rm lift})} \ln\left(\frac{M_{\rm Pl}}{m_{3/2}}\right), & (19) \end{aligned}$$

where $K_0 = -3\ln(-\mathcal{F}_0/3)$, $M_{\rm Pl}$ is the reduced Planck scale, $m_{3/2} = e^{K_0/2}W_0$ is the gravitino mass, which is taken to be $m_{3/2} \approx (10 \sim 100)$ TeV, and $V_{\rm lift} = e^{2K_0/3}\mathcal{P}_{\rm lift}$ is the up-lifting potential. The quantity M_0 is defined by

$$M_0 = \frac{F_T}{T+T^\dagger}. \tag{20}$$

To obtain the simple relation of Eq. (19), it is essential that T is stabilized by a single exponential factor in W_0.

An interesting property of this setup is that it allows a reduction of the effective messenger scale, $M_{\rm mess}$, due to an interplay between the moduli and anomaly mediated contributions [18]. Suppose that only the appreciable Yukawa couplings, $W = (\lambda_{ijk}/6)\Phi_i\Phi_j\Phi_k$ with $y_{ijk} \equiv \lambda_{ijk}/(Z_iZ_jZ_k)^{1/2} \gtrsim \sqrt{8\pi^2}$, are among the fields satisfying $r_i + r_j + r_k = 1$, where Z_i's are the wavefunction renormalization factors, $Z_i = (T+T^\dagger)^{r_i}$. Suppose also that r_i's satisfy

[2] Realizing $w_0 \ll M_*^3$ in the context of particular string theory may require fine-tuning [22].

$\sum_i r_i Y_i = 0$, where Y_i is the hypercharge of Φ_i. In this case the low-energy soft supersymmetry breaking parameters, defined by

$$\mathcal{L}_{\rm soft} = -\frac{1}{2} M_a \lambda^a \lambda^a - m_i^2 |\phi_i|^2 - \frac{1}{6}(A_{ijk} y_{ijk} \phi_i \phi_i \phi_k + {\rm h.c.}), \tag{21}$$

are given by

$$M_a(\mu_R) = M_0 \left[1 - \frac{b_a}{8\pi^2} g_a^2(\mu_R) \ln\left(\frac{M_{\rm mess}}{\mu_R}\right) \right], \tag{22}$$

$$A_{ijk}(\mu_R) = M_0 \left[(r_i + r_j + r_k) - 2 \Big\{ \gamma_i(\mu_R) + \gamma_j(\mu_R) + \gamma_k(\mu_R) \Big\} \ln\left(\frac{M_{\rm mess}}{\mu_R}\right) \right], \tag{23}$$

$$m_i^2(\mu_R) = M_0^2 \left[r_i - 4 \left\{ \gamma_i(\mu_R) - \frac{1}{2} \frac{d\gamma_i(\mu_R)}{d \ln \mu_R} \ln\left(\frac{M_{\rm mess}}{\mu_R}\right) \right\} \ln\left(\frac{M_{\rm mess}}{\mu_R}\right) \right], \tag{24}$$

including one-loop renormalization group effects. Here, $g_a^2(\mu_R)$ are the running gauge couplings at a scale μ_R, b_a and $\gamma(\mu_R)$ are the beta-function coefficients and the anomalous dimensions defined by $d(1/g_a^2)/d\ln\mu_R = -b_a/8\pi^2$ and $d\ln Z_i/d\ln\mu_R = -2\gamma_i$, respectively. The parameter $M_{\rm mess}$ is given by

$$M_{\rm mess} = \frac{M_{\rm GUT}}{(M_{\rm Pl}/m_{3/2})^{\alpha/2}}, \tag{25}$$

where $M_{\rm GUT}$ is the scale at which the effective action of Eq. (11) is given, which is supposed to be the unification (compactification) scale, and α parameterizes the ratio of the anomaly-mediated to the moduli contributions:

$$\alpha \equiv \frac{m_{3/2}}{M_0 \ln(M_{\rm Pl}/m_{3/2})} = \frac{2n_0}{2n_0 - 3n_P} + \cdots. \tag{26}$$

The last equality of Eq. (26) follows from Eq. (19) with Eqs. (15, 16). Equations (22 – 24) show that the low energy values for the soft supersymmetry breaking parameters are obtained simply by setting the moduli-dominated supersymmetry breaking boundary conditions at the scale $M_{\rm mess}$ and then evolving them to the scale μ_R. In this sense, $M_{\rm mess}$ effectively plays a role of the messenger scale.

The idea suggested in Ref. [29] is to use this feature to make a little hierarchy between the Higgs boson and the other scalar squared masses. For this purpose, $M_{\rm mess}$ must be close to the TeV scale, requiring $\alpha \simeq 2$ to a high degree. A simple way to achieve this is to choose $n_0 = 3$ and $n_P = 1$ (see Eq. (26)). This is, however, not entirely enough; we also have to check that higher order terms of $\mathcal{F}_0$ in Eq. (16), given by powers of $1/(T + T^\dagger)$, do not give significant corrections to α. Since $\langle T + T^\dagger \rangle \simeq 2/g_{\rm GUT}^2$ is not so large, this requires the coefficients of these terms to be somewhat suppressed. From the effective field theory point of view, this is technically natural because nothing is strongly coupled below the scale $\approx M_{\rm GUT}$. In the string theory context, this may require both the compactification volume and the string coupling to be taken somewhat large. In any case, having seen that $\alpha \simeq 2$ can be obtained without fine-tuning parameters, we simply treat $M_{\rm mess}$ as a free parameter of order the TeV scale. The sensitivity of the weak scale to α, or $M_{\rm mess}$, will be included in our analysis of the supersymmetric fine-tuning problem.

The moduli couplings to the matter and Higgs fields, r_i, must also be chosen such that the renormalization group properties of Eqs. (22 – 24) are preserved. Here we impose the following conditions to determine the values of r_i. (i) Motivated by the successful b/τ Yukawa unification, we assume that the third generation matter is embedded into $SU(5)$ representations. (ii) To avoid the supersymmetric flavor problem, r_i must be assigned flavor universal. These two requirements lead to the following assignment for r_i:

$$r_Q = r_U = r_E = \frac{1}{2}, \qquad r_D = r_L, \qquad r_{H_u} = r_{H_d} = 0. \tag{27}$$

Note that the $SU(5)$ requirement of (i), together with $\sum_i r_i Y_i = 0$, necessarily leads to $r_{H_u} = r_{H_d}$. In the simplest case that all the matter fields have a common r_i (propagate in the same spacetime dimensions), we obtain

$$r_Q = r_U = r_D = r_L = r_E = \frac{1}{2}, \qquad r_{H_u} = r_{H_d} = 0. \tag{28}$$

This can be obtained in the string theory construction if the matter fields live on intersections of $D7$ branes while the Higgs fields on $D3$ branes.[3] Alternatively, it may arise in a six dimensional theory if matter and Higgs fields propagate in five and four dimensions, respectively ($n = 2$, $\tilde{n}_{\rm matter} = 1$ and $\tilde{n}_{\rm Higgs} = 0$). It is interesting that the choice of r_i required by low energy phenomenology, Eq. (28), is so simple that it can be accommodated into the string theory (or extra dimensional) setup without much difficulties. In our analysis for the fine-tuning, we only consider the universal matter model of Eq. (28). More general cases of Eq. (27), however, could also work if $\tan\beta$ is small, e.g. $\tan\beta \lesssim 10$, so that the renormalization group contribution to $m_{H_d}^2$ from the bottom Yukawa coupling is sufficiently small.

The assignment of Eq. (28) leads to the following flavor universal soft supersymmetry breaking parameters

$$M_1 = M_2 = M_3 = M_0, \tag{29}$$

$$m_{\tilde{q}}^2 = m_{\tilde{u}}^2 = m_{\tilde{d}}^2 = m_{\tilde{l}}^2 = m_{\tilde{e}}^2 = \frac{M_0^2}{2}, \tag{30}$$

$$A_u = A_d = A_e = M_0, \tag{31}$$

[3] In the string theory realization based on [25], the modulus T is related to the compactification radius R by $T \propto R^4$. Thus, the coefficient of $\Phi_i^\dagger \Phi_i$ in Eq. (12) is proportional to R^4 (R^0) if Φ_i propagates on a $D7$ ($D3$) brane: $r_i = 1$ ($r_i = 0$). If Φ_i lives on an intersection of $D7$ branes, the coefficient is $\propto R^2$: $r_i = 1/2$.

at the effective messenger scale $M_{\rm mess}$, which is very close to the TeV scale (see the analysis in the next section). Here, A_u, A_d, and A_e represent the trilinear scalar couplings for the up-type squarks, down-type squarks and charged sleptons, respectively. As we will see in the next section, a particularly important feature of Eqs. (29 – 31) in solving the supersymmetric fine-tuning problem is rather large values for the A parameters, specifically that for the top squark, A_t, which necessarily arise as a consequence of lowering $M_{\rm mess}$. This allows us to evade the LEP II bound on the Higgs boson mass with relatively small top squark masses. The situation, however, is not trivial because the large A_t also gives an additional negative radiative correction to $m_{H_u}^2$. We thus have to study the dynamics of electroweak symmetry breaking more carefully, to see that the supersymmetric fine-tuning problem can actually be solved.

3 Electroweak Symmetry Breaking without Fine-Tuning

As discussed in the introduction, the amount of fine-tuning is roughly measured by the ratio of the left-hand-side to the largest contribution to the right-hand-side of the equation determining the weak scale, $M_{\rm Higgs}^2/2 \simeq -m_{H_u}^2 - |\mu|^2$. For the model of Eq. (28), the correction to the Higgs mass-squared parameter is given by

$$\delta m_{H_u}^2 \simeq -\frac{3y_t^2}{4\pi^2} M_0^2 \ln\left(\frac{M_{\rm mess}}{m_{\tilde{t}}}\right) + \frac{15g_2^2 + 3g_1^2}{40\pi^2} M_0^2 \ln\left(\frac{M_{\rm mess}}{m_\lambda}\right), \tag{32}$$

where the first and second terms represent contributions from the top Yukawa coupling and the gauge couplings, respectively, and $m_{\tilde{t}} \simeq M_0/\sqrt{2}$ and $m_\lambda \simeq M_0$. Here, the first term includes the effects from both $m_{\tilde{t}}$ and A_t. The fine-tuning is then given by the ratio of $M_{\rm Higgs}^2/2$ to the largest of the two terms in Eq. (32).

One might naively think that the fine-tuning from the top Yukawa contribution may be entirely eliminated by choosing $M_{\rm mess}$ very close to the value with which the logarithm of the first term almost vanishes. This, however, leads to another fine-tuning of the parameter $M_{\rm mess}$. In general, the amount of fine-tuning for the parameter a_i can be obtained by studying the logarithmic sensitivity of v^2 to a_i using appropriate measures [16]. In our context, this leads to the following definition for the fine-tuning parameter

$$\Delta^{-1} = \frac{M_{\rm Higgs}^2}{2} \Bigg/ \max\left\{ \left| \frac{3y_t^2}{4\pi^2} M_0^2 \ln\left(\frac{M_{\rm mess}}{m_{\tilde{t}}}\right) \right|, \; \left| \left(\frac{3y_t^2}{4\pi^2} - \frac{15g_2^2 + 3g_1^2}{40\pi^2} \right) M_0^2 \right| \right\}. \tag{33}$$

The first and second factors in the denominator measure the sensitivity of the weak scale to the top contribution (the top Yukawa coupling) and the effective messenger scale (the ratio of the moduli to anomaly mediated contributions), respectively.

Equation (33) tells us that the fine-tuning becomes better if we lower M_0, since the sensitivity of $M_{\rm Higgs}$ to M_0 is much weaker than that of the denominator. There is, however, a lower bound on M_0, coming from the bound on the Higgs boson mass, $M_{\rm Higgs} \gtrsim 114.4$ GeV. The question is whether there is a parameter region in which $\Delta^{-1} \geq 20\%$, still satisfying $M_{\rm Higgs} \gtrsim 114.4$ GeV.

There are four parameters μ, B, $m_{H_u}^2$ and $m_{H_d}^2$ in the Higgs potential which need to be fixed to calculate $M_{\rm Higgs}$ and Δ^{-1}. For our choice of $r_{H_u} = r_{H_d} = 0$, the Higgs soft mass-squared parameters $m_{H_u}^2$ and $m_{H_d}^2$ vanish at $M_{\rm mess}$, at the leading order in $1/aT \approx 1/8\pi^2$ expansion. There are, however, non-vanishing corrections arising at the next-to-leading order, which depend on unknown threshold effects at the scale $M_{\rm GUT}$. The existence of higher order corrections also raises the question if similar corrections to the other scalars, specifically to the top squarks, give too large contributions to $\delta m_{H_u}^2$ through renormalization group evolution because there is no reason that the large logarithm, $\ln(M_{\rm GUT}/m_{\tilde{t}})$, disappears for such contributions. In order to discuss fine-tuning based on Eq. (33), all these corrections to $m_{H_u}^2$, including the one from higher loop renormalization group evolution, must be smaller than the larger of the two quantities in the dominator of Eq. (33). We find, however, that if the higher order corrections to the soft masses are smaller than about v^2, which is quite natural given that they are $O(M_0^2/8\pi^2)$ effects, all the unknown contributions to $m_{H_u}^2$ can be made sufficiently small. This implies that we can simply treat $m_{H_u}^2$ and $m_{H_d}^2$ as free parameters at the scale $M_{\rm mess}$ and analyze fine-tuning using Eq. (33). The sizes of these parameters are bounded roughly as $m_{H_u}^2, m_{H_d}^2 \lesssim (M_{\rm Higgs}^2/2)/20\% \approx (200~{\rm GeV})^2$ if we require $\Delta^{-1} \geq 20\%$. The unknown corrections also induce uncertainties for the predictions of the squark and slepton masses, but they are at most of order v^2/M_0 and thus small.[4]

The natural sizes of μ and B parameters in supergravity are $O(m_{3/2})$, which is unacceptably large. This requires some mechanism of suppressing these parameters such as the ones proposed in Refs. [20, 30]. In fact, the sizes of μ and B are severely constrained in our framework. The μ parameter is subject to the bound $|\mu| \lesssim 190$ GeV, given by the naturalness requirement of $M_{\rm Higgs}^2/2|\mu|^2 \geq 20\%$. The value of B must also be small. Since both $|\mu|^2$ and $m_{H_d}^2$ are bounded by $\approx (200~{\rm GeV})^2$, the value of B must be $\approx (350/\tan\beta)$ GeV or smaller. Since $\tan\beta$ must be larger than about 5, as we will see later, this requires rather small values for the B parameter.

Is it possible to obtain such a small B without fine-tuning? Suppose there is a singlet field Σ that is subject to an approximate shift symmetry and couples to the Higgs doublets as

$$\delta S = \int d^4x \sqrt{-g} \int d^4\theta\, C^\dagger C \Big(\kappa(\Sigma + \Sigma^\dagger) H_u H_d + {\rm h.c.}\Big), \tag{34}$$

where κ is a constant and we have normalized Σ as a dimensionless field. Suppose also that the Σ field has a vanishing VEV in the lowest component, but has a non-vanishing VEV of order

[4] We assume that these higher order corrections are flavor universal, at least approximately, so that the supersymmetric flavor problem is not reintroduced.

M_0 in the F component that is real in the basis where F_C is real: $\langle\Sigma\rangle = \theta^2 F_\Sigma$. In this case the interaction of Eq. (34) gives

$$\mu = \kappa F_\Sigma, \tag{35}$$

$$B = (\gamma_{H_u} + \gamma_{H_d}) m_{3/2}, \tag{36}$$

at the scale $\mu_R \simeq M_{\rm GUT}$, so that μ and B of order $F_\Sigma \sim M_0 \ll m_{3/2}$ are naturally obtained [20].[5] Note that the contribution from the moduli F-term, F_T, is absent in Eq. (36) with our choice of $r_{H_u} = r_{H_d} = 0$. This size of B, however, is still too large, since we need $B \approx (10 \sim 70)$ GeV while M_0 is of order 500 GeV $\sim$ 1 TeV, as we will see later.

A further suppression of B, however, arises through renormalization group evolution. Given the B parameter of Eq. (36) at $\mu_R \simeq M_{\rm GUT}$, one-loop renormalization group evolution of B gives

$$B(\mu_R) = -2M_0 \Big\{ \gamma_{H_u}(\mu_R) + \gamma_{H_d}(\mu_R) \Big\} \ln\left(\frac{M_{\rm mess}}{\mu_R} \right), \tag{37}$$

at lower energies, where we have used Eqs. (22 – 24) for the other soft masses. We find that the B parameter vanishes at the scale $M_{\rm mess}$, which is close to TeV. Since there is no large correction to the μ parameter, we can naturally obtain the desired values of $\mu \approx (100 \sim 200)$ GeV and $B \approx (10 \sim 70)$ GeV with an $O(1)$ value for κ. In fact, this property is quite general. Any mechanism that eliminates the classical contribution to B of order $m_{3/2}$ and leaves the "anomaly mediated" contribution of Eq. (36) leads to $B \approx 0$ at $\mu_R \simeq M_{\rm mess}$. This is important to eliminate fine-tuning entirely from electroweak symmetry breaking. Having seen Eqs. (35 – 37), we treat μ and B as free parameters in our analysis below.[6]

Now we analyze if we have a region of M_0 satisfying the experimental bound $M_{\rm Higgs} \gtrsim 114.4$ GeV and $\Delta^{-1} \geq 20\%$ simultaneously. Out of four free parameters $m_{H_u}^2$, $m_{H_d}^2$, μ and B, one combination is fixed by the vacuum expectation value v, leaving three independent parameters, which we take to be μ, m_A and $\tan\beta$, where m_A is the mass of the pseudo-scalar Higgs boson. The summary of the analysis is presented in Fig. 1. In the left figure, we show the Higgs boson mass $M_{\rm Higgs}$, calculated using *FeynHiggs 2.2* [31], as a function of M_0 for various values of $\tan\beta$. The other parameters are fixed as $\mu = 170$ GeV, $m_A = 250$ GeV and $M_{\rm mess} = 1$ TeV. The sensitivity of $M_{\rm mess}$ to these parameters are rather mild. For the top quark mass, we have used the latest central value of $m_t = 172.7$ GeV [5]. From the figure, we obtain the lower bound on

[5]The Σ field should not have a coupling to the MSSM fields other than that in Eq. (34), at the lowest order. This can be easily achieved in higher dimensional theories.

[6]The explicit mechanism discussed here requires F_Σ to be (almost) real in the basis where F_C is real. Then, if the weak scale value of B is given by Eq. (37) without an additional contribution, there is no supersymmetric CP problem. It is interesting that the value of B in Eq. (37) is, in fact, consistent with large values of $\tan\beta$, e.g. $\tan\beta \gtrsim 10$, for $M_{\rm mess}$ about a TeV.

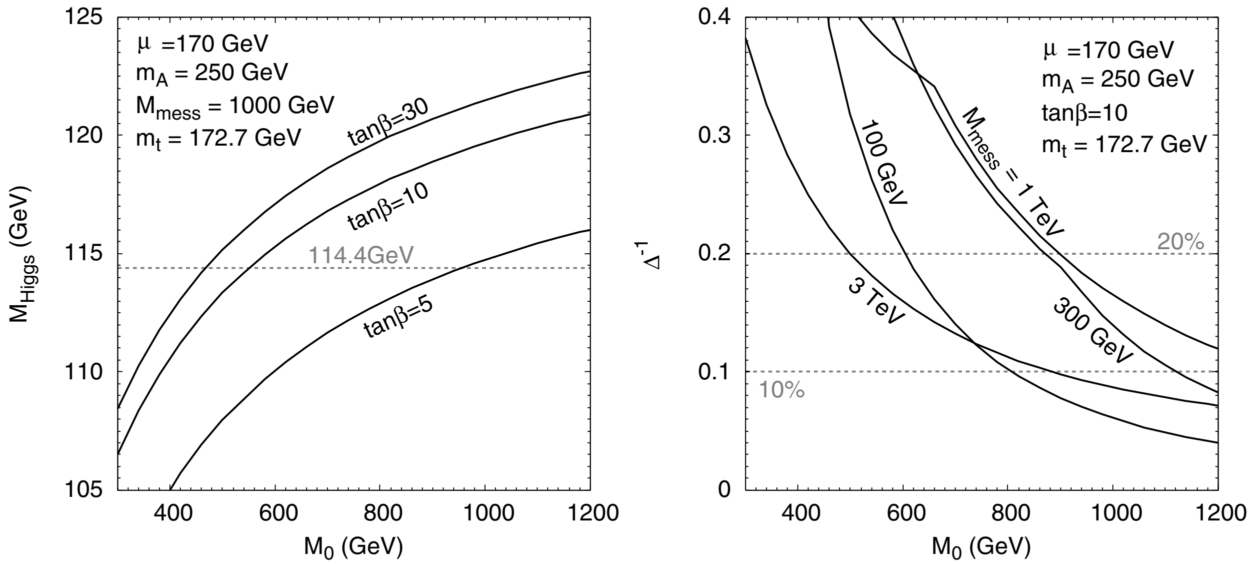

Figure 1: The Higgs boson mass, $M_{\rm Higgs}$, and the fine-tuning parameter, Δ^{-1}, as a function of M_0. The lower bound on M_0 is obtained from the left figure using the experimental bound of $M_{\rm Higgs} \gtrsim 114.4$ GeV, while the upper bound can be read off from the right figure with the requirement of $\Delta^{-1} \geq 20\%$.

M_0, depending on the value of $\tan\beta$. For $\tan\beta = 10$, for example, the bound is $M_0 \gtrsim 550$ GeV. The bound becomes weaker for larger $\tan\beta$; it becomes $M_0 \gtrsim 450$ GeV for $\tan\beta = 30$. The bound stays practically the same for $\tan\beta \gtrsim 30$.

It is remarkable that M_0 as low as $(450 \sim 550)$ GeV is allowed, which corresponds to $m_{\tilde{t}} \simeq M_0/\sqrt{2} \approx (320 \sim 390)$ GeV. Such low values of $m_{\tilde{t}}$ are consistent with the bound on the Higgs boson mass because of a large A_t, which necessarily arise as a consequence of lowering $M_{\rm mess}$. In fact, the masses of the two top squarks have appreciable splittings from $M_0/\sqrt{2}$ because of the large A_t: $m_{\tilde{t}_{1,2}} \simeq (M_0 \mp m_t)/\sqrt{2}$. The lighter top squark can thus be as light as $(200 \sim 270)$ GeV. This may be relevant for Run II at the Tevatron.

The M_0 dependence of the fine-tuning parameter Δ^{-1} is shown in the right figure. Here we have fixed $\tan\beta$ and plotted Δ^{-1} for various values of $M_{\rm mess}$. The sensitivities to the fixed parameters are again small. From the figure, we can obtain the upper bound on M_0. For a fixed value of Δ^{-1}, the maximum value of M_0 is obtained for $M_{\rm mess} \approx (300 \sim 1000)$ GeV; smaller values of M_0 are required both for larger and smaller $M_{\rm mess}$ because of the logarithmic enhancement of $m_{H_u}^2$ (the factor of $\ln(M_{\rm mess}/m_{\tilde{t}})$ in the denominator of Eq. (33)). Requiring the absence of fine-tuning, i.e. $\Delta^{-1} \geq 20\%$, the upper bound of $M_0 \lesssim 900$ GeV is obtained. Compared with the lower bound previously derived, we find that there is a region of M_0 in which the bound on the Higgs boson mass and the requirement from naturalness are both satisfied. For $\tan\beta = 30$ (10), it is 450 GeV $\lesssim M_0 \lesssim$ 900 GeV (550 GeV $\lesssim M_0 \lesssim$ 900 GeV). This shows that we have in fact solved the supersymmetric fine-tuning problem without extending the MSSM. In order to have an allowed range for M_0, the effective messenger scale $M_{\rm mess}$ must be in the range 50 GeV $\lesssim M_{\rm mess} \lesssim$ 3 TeV, and $\tan\beta \gtrsim 5$ is necessary. By requiring $M_0 \lesssim 900$ GeV, we can also read off the upper bound on $M_{\rm Higgs}$ from the left figure, giving $M_{\rm Higgs} \lesssim 120$ GeV.

If we relax the naturalness criterion to $\Delta^{-1} \geq 10\%$, M_0 can be as large as 1300 GeV. A larger range of $M_{\rm mess}$ is also allowed: 50 GeV $\lesssim M_{\rm mess} \lesssim$ 100 TeV. Here, the lower bound on $M_{\rm mess}$ comes from the fact that some of the superparticles become too light if $M_{\rm mess}$ is lowered beyond this value. The bound on $\tan\beta$ is relaxed to $\tan\beta \gtrsim 4$.

The dependence of our results on the value of the top quark mass is not strong. This is because the two loop effects on the Higgs boson mass, which give a sizable negative correction to $M_{\rm Higgs}$ for large superparticle masses, are not important here due to small values for M_0. If we use $m_t = 178.0$ GeV instead of 172.7 GeV, for example, the lower bound on M_0 decreases from ≈ 450 GeV to ≈ 400 GeV and the upper bound on $M_{\rm Higgs}$ increases from $\simeq 120$ GeV to $\simeq 124$ GeV, but there are no appreciable changes to the other numbers.

The naturalness bound of $\mu \lesssim 190$ GeV together with Eqs. (29 – 31) implies that the lightest supersymmetric particle is the Higgsino of mass about $|\mu|$. The mass splitting between the charged and neutral Higgsinos comes from the mixings with the gaugino states: $m_{\tilde{h}^\pm} - m_{\tilde{h}^0} = (m_Z^2/2M_0)(1+\epsilon)$, where $|\epsilon| \lesssim 0.2$ in the relevant parameter region, so that the neutral component is always lighter. A similar bound of $m_{H_d}^2 \lesssim (200~{\rm GeV})^2$ implies that the pseudo-scalar and charged Higgs bosons are relatively light: $m_A, m_{H^\pm} \lesssim 300$ GeV. This may have some implications on the rate of the rare $b \rightarrow s\gamma$ process, but the current theoretical status does not seem to allow us to make any definite statement [32]. The positive sign of μ, however, seems to be preferred over the other one.

The range of the gravitino mass is determined as

$$m_{3/2} \simeq 2M_0 \ln\left(\frac{M_{\rm Pl}}{m_{3/2}}\right) \simeq (30\sim 60)~{\rm TeV}. \tag{38}$$

For $\Delta^{-1} \geq 10\%$, the upper bound can be relaxed to 80 TeV. These values for $m_{3/2}$ are large enough to evade the cosmological gravitino problem. The mass of the moduli field is a factor of $\ln(M_{\rm Pl}/m_{3/2})$ larger than the gravitino mass, $m_T \approx 1000$ TeV, so that the cosmological moduli problem is also absent [22, 23]. The moduli field decays mainly into the gauge bosons, reheating the universe to about 100 MeV. A small fraction of moduli, however, also goes into the gravitino, which in turn produces the neutral Higgsino through its decay. This can provide dark matter of the universe, $\Omega_{\tilde{h}^0} \simeq 0.2$, for $m_{3/2}$ and m_T in the range considered here [33]. (The amount of the

Higgsino produced by the direct moduli decay is negligible.) This offers significant potential for the discovery at on-going dark matter detection experiments such as CDMS II.

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