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UCRL-TR-215700

Spherical Focusing Mirror for the VUV-FEL

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September 27, 2005

Disclaimer

This work was performed under the auspices of the U.S. Department of Energy by University of California, Lawrence Livermore National Laboratory under Contract W-7405-Eng-48.

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September 4, 2005

Based on analysis and ray-tracing that he did, Jacek Krzywinski has suggested that it should be possible to focus the 32 nm VUV-FEL beam down below 0.2 μm spot size with a normal-incidence multilayer-coated spherical mirror. There are advantages to a spherical mirror over an ellipsoid (or near-paraboloid) which are ease of manufacture and alignment. Off-axis aberrations are generally small, since for a beam that underfills the sphere's aperture, the beam itself defines the axis (rather than the optic). The dominant aberration for a sphere is spherical aberration, which decreases with increasing sphere radius of curvature. However, as the radius of curvature increases, so too does the focal length and f-number, and the diffraction-limited spot increases. Hence, as Jacek has pointed out, there is an optimum radius of curvature, to achieve the smallest possible spot, given a beam diameter. This optimum is determined by balancing the spread of the beam due to spherical aberration and the spread due to diffraction.

The wavefront aberration corresponding to spherical aberration, for a focal length f, and radial ray position a on the sphere is given by

$$w = -\frac{a^4}{32f^3}. \tag{1}$$

The focal length is $f = R/2$, where R is the mirror radius of curvature. For a uniform intensity beam filling an aperture of diameter a, the spot size (radius) in the circle of least confusion is

$$x_c = \frac{a^4}{32f^3}\frac{1}{\mathrm{NA}} = \frac{a^3}{32f^2}. \tag{2}$$

For a perfect imaging system with no aberrations, the diffraction-limited spot size for a uniform intensity beam is

$$x_d = 0.6\frac{\lambda}{\mathrm{NA}} = 0.6\frac{\lambda f}{a}. \tag{3}$$

The spot size will be a function that depends on both x_c and x_d, and will be a minimum close to $x_c = x_d$, whereby

$$f = f_{\min} = \left(\frac{a^4}{19.2\lambda}\right)^{1/3}. \tag{4}$$

For a beam radius of $a = 5$ mm, $\lambda = 32$ nm, we have $f_{\min} = 100$ mm, or a sphere with a radius of curvature of 200 mm.

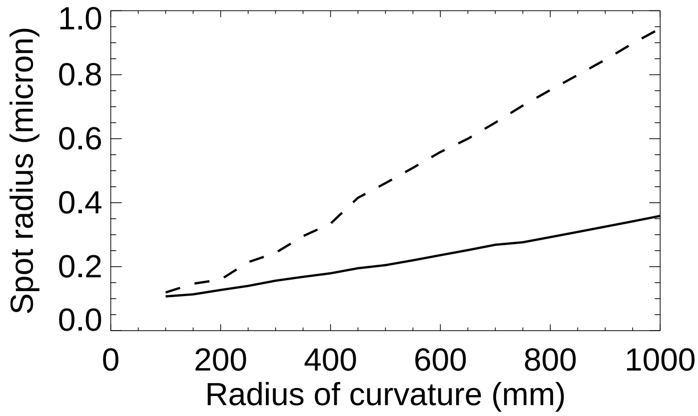

Figure 1: Radius of the focused spot at the $1/e^2$ intensity for a perfect spherical mirror (solid line) and a mirror with an RMS surface height error of 32 nm (dashed line).

We must consider that the VUV beam is Gaussian which in general makes optical systems more tolerant of aberrations. This can be crudely characterized using a smaller beam radius, and hence for example with $a = 3$ mm we have $f_{\min} = 50$ mm and a radius of curvature of 100 mm. A more accurate treatment can be made by ray-tracing, using the parameters of the VUV-FEL beam. In particular we consider single mode source with a $1/e$ waist radius of $w_s = 0.14$ mm for the intensity profile (FWHM = 0.19 mm), $\lambda = 32$ nm, and a distance to the source of $z_s = 70$ m. The profile of the intensity of the beam at this distance is given by

$$I(r) = \exp(-r^2/r_I^2)). \tag{5}$$

where $r_I = \lambda z_s/(\pi w_s) = 5.1$ mm. Ray-tracing is carried out by simulating the source as a point a distance z_s from the mirror, and computing differences in path-lengths to the image relative to a perfect reference wave. This aberration map is combined with $\sqrt{I(r)}$ and Fourier transformed to generate the amplitude in the image plane. We find the optimum image plane by varying the image distance and maximizing the spot intensity (minimizing the intensity-weighted wavefront error did not give the mimimum spot size). The spot size was quantified by fitting the spot intensity (squared amplitude) with a Gaussian. This procedure was repeated for several mirror radii of curvature, and the results are given in Fig. 1. For radii of curvature less than 100 mm, there are more than 100 waves of spherical aberration. This is hard to simulate by the diffraction calculation, due to the rapidly varying phase, and instead at these radii we omit the diffraction calculation and rely on tracing rays to the image plane.

As is seen in Fig. 1, the spot radius is about 0.1 μm for a mirror curvature of 100 mm. At smaller curvatures spherical aberration was severe but the fitted spot size continued to be about 0.1 μm. The image consisted of this small spot, with many rings which contained most of the energy. For most VUV-FEL experiments, it is not so important how small the spot is, but how high is the power density. We characterize this by computing the fraction of the reflected energy that is focused into an aperture of radius 0.1 μm, shown as the solid blue curve in Fig. 2. Although the spot size is smaller at a radius of curvature of 100 mm, the highest fluence can be achieved with a radius of curvature of 300 mm, where the spot radius is 0.16 μm. At curvatures

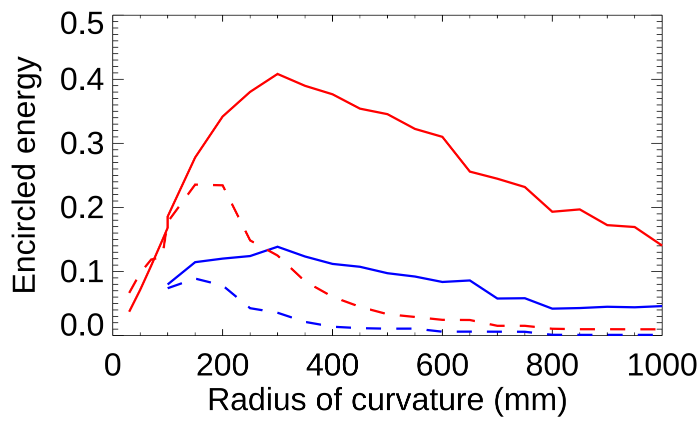

Figure 2: Fraction of total reflected energy focused into a 0.1 μm radius pinhole (blue) and 0.2 μm radius pinhole (red) for the perfect spherical mirror (solid lines) and the spherical mirror with 32 nm RMS surface height error (dashed lines).

below this, energy is spread out due to spherical aberration, and at curvatures greater than this the spot becomes larger due to diffraction.

It is well known that the smaller the focal length of a reflective system, the more tolerant it is to aberrations. For a radius of curvature of 100 mm, the spherical aberration is about 200 waves, so one may think that low order aberrations of this magnitude should not effect the performance too much. However, most of the error is at the periphery of the pupil, where the beam intensity is low. Spherical mirrors can be purchased (e.g. from REO Optics) at around $\lambda/20$ surface heigh error for $\lambda = 632$ nm (that is, about 1 wave of aberration at $\lambda = 32$ nm). We show the effect of a surface deviation with a root mean square (RMS) error of 32 nm, randomly distributed in 36 Zernike polynomials, by dashed lines in Figs. 1 and 2. For large radii of curvature the spot size almost trebles and the fraction of energy in a 0.2 μm diameter spot diminishes to below a few percent. As expected, the effect at small radii of curvature is less severe, and the optimum radius of curvature is reduced from 300 mm to about 150 mm (depending on the nature of the aberration). With a good-quality mirror it should be possible to focus up to 25% of the reflected beam into a 0.2 μm radius spot. Assuming a multilayer reflectivity of 40% and a pulse energy of 50 μJ, this would give an energy density of 4000 J/cm^2, or a power density of 10^{17} W/cm^2 for an 40 fs pulse.

We see from the simulations that the optimum mirror radius of curvature is between 150 and 200 mm (focusing the beam to a distance 75 to 100 mm from the mirror). Larger radii mirrors are generally cheaper to buy, but are less tolerant to manufacturing errors. The maximum angle of incidence for the 150 mm radius mirror is 3.9° for a beam width of 5.1 mm. The multilayer rocking curve for a Si – B_4C – Mo at normal incidence for 32 nm light is large enough that the multilayer coating can have a uniform d spacing across its surface.