

Modular Invariant Gaugino Condensation in the Presence of an Anomalous $U(1)^*$

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Abstract

Starting from the previously constructed effective supergravity theory below the scale of $U(1)$ breaking in orbifold compactifications of the weakly coupled heterotic string, we study the effective theory below the scale of supersymmetry breaking by gaugino and matter condensation in a hidden sector. Questions we address include vacuum stability and the masses of the various moduli fields, including those associated with flat directions at the $U(1)$ breaking scale, and of their fermionic superpartners. The issue of soft supersymmetry-breaking masses in the observable sector presents a particularly serious challenge for this class of models.

*This work was supported in part by the Director, Office of Science, Office of High Energy and Nuclear Physics, Division of High Energy Physics of the U.S. Department of Energy under Contract DE-AC03-76SF00098, in part by the National Science Foundation under grants PHY-0098840 and INT-9910077, and in part by National Science and Engineering Research Council of Canada.

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1 Introduction

In previous articles [?, ?] the effective supergravity theory obtained in the presence of an anomalous $U(1)$, which for the remainder of this article we will denote $U(1)_X$, was studied. In the cases investigated, scalar fields charged under $U(1)_X$ as well as other $U(1)$'s, acquired nonvanishing vacuum expectation values (*vev*'s). Associated chiral multiplets were “eaten” by the $U(1)$ vector multiplets, including the $U(1)_X$ multiplet, to form massive vector multiplets. Tree-level exchange of these massive vector multiplets were eliminated by redefinitions that removed linear couplings between the heavy and light fields. It was demonstrated that these redefinitions can be made at the superfield level, while maintaining manifest *modular invariance*¹, local supersymmetry and the (modified) linearity conditions,

$$(\bar{\mathcal{D}}^2 - 8R)L = -\sum_a (\mathcal{W}\mathcal{W})_a, \quad (\mathcal{D}^2 - 8\bar{R})L = -\sum_a (\overline{\mathcal{W}\mathcal{W}})_a, \quad (1.1)$$

for the linear superfield L , whose lowest component is the real scalar associated with the dilaton. A comparison with redefinitions at the component field level provided assurances that the superfield approach was reliable [?].

Our motivation for studying these theories stems from the prevalence of a $U(1)_X$ factor in the string scale gauge group of semi-realistic string compactifications; for example, in a recent study [?] of a certain class of standard-like heterotic Z_3 orbifold models, it was found that 168 of the 175 models in the class had an anomalous $U(1)_X$. Thus the additional ingredient of a $U(1)_X$ factor is an important modification of the string-inspired effective supergravity worked out by Binétruy, Gaillard and Wu (BGW) [?, ?]. Indeed, we expect low energy phenomenological aspects of these models—general features of the superpartner spectrum [?–?], cosmology of the models [?, ?], implications for accelerator searches [?—to be modified by the presence of a $U(1)_X$ at the high scale. In [?] complications were addressed that arise in the semi-realistic models that we seek to understand, since the scalars that get *vev*'s due to the $U(1)_X$ are typically charged under several $U(1)$ factors and multiple scalars must generally get *vev*'s in order for the *D-terms* of the several $U(1)$'s to (approximately) vanish. However in that article only the supersymmetric phase was examined; the nonperturbative dynamics in a hidden sector—which ultimately leads to supersymmetry breaking by gaugino condensation—was not addressed. The purpose of this paper is to examine the effective supergravity theory when these important effects are accounted for; *i.e.*, we intend to study the effective theory below the scale of gaugino condensation. As has been noted previously, the supersymmetric vacuum is approximately the stable vacuum in the case where dynam-

¹The modular transformations on the fields of the effective theory are defined in (??) below.

ical supersymmetry breaking *via* gaugino condensation occurs in an effective supergravity context [?]. Thus the tools developed already in [?, ?] will prove useful here.

In Section 2 we review those aspects of References [?] and [?] that are needed for the present discussion. The string scale theory is first defined; it is the same as in the BGW models [?, ?] except that now a $U(1)_X$ is present. As a consequence large *vev*'s are induced and some fields get masses of order 10^{-1} to 10^{-2} m_P , where $m_P = 1/\sqrt{8\pi G} \approx 2.44 \times 10^{18}$ GeV is the reduced Planck mass. (In the remainder of this article we work in units where $m_P = 1$.) These large fields are integrated out from the one-loop effective action by a sequence of field redefinitions that are chosen so as to obtain an effective theory below the $U(1)_a$ -breaking scale that is manifestly modular invariant and locally supersymmetric and preserves the modified linearity condition for the linear multiplet L . We conclude Section 2 by summarizing these redefinitions which were worked out in Refs. [?, ?].

In Section 3 we add effective terms to the Lagrangian that describe the leading contributions from gaugino condensation in a hidden sector. Here again the description is basically that of the BGW models, except that we must be careful about the $U(1)_X$ when anomaly matching is considered. This leads to important and interesting constraints. We construct the effective bosonic Lagrangian and conclude Section 3 with discussions of the potentials for the T-moduli and the dilaton and the masses of their superpartners.

In Section 4 we discuss scalar masses in the observable sector under different assumptions for the Kähler potential for matter. Our results confirm those of earlier analyses [?]-[?], that did not specify the mechanism for supersymmetry breaking, in that the D-term contribution to these masses is generically dominant, resulting in an unacceptably large scalar/gaugino mass hierarchy as well as the possibility of large charge and color breaking *vev*'s. In Section 5 we discuss parameterizations of string nonperturbative effects and their influence on the scales of coupling constant unification and condensation and on the gravitino and scalar masses. D-moduli masses are addressed in Section 6, and in Section 7 we summarize our results and discuss future lines of investigation. Detailed calculations are relegated to appendices.

Throughout this article we use the linear multiplet formulation [?, ?] for the dilaton and the $U(1)_K$ superspace formalism [?, ?, ?] of supergravity, except that, for reasons explained in [?], we do not use $U(1)_K$ superspace for the Abelian gauge groups that are broken at the string scale by the anomalous $U(1)_X$. (For a review of the $U(1)_K$ superspace formalism see [?]; for a review of the linear multiplet formulation see [?].)

2 Review

In this section we review elements of Refs. [?] and [?]. We unify our notation and present enough details to render the present article reasonably self-contained.

2.1 String scale effective theory

We start with the effective theory at the string scale defined as in [?]:

$$\mathcal{L} = \int d^4\theta \tilde{L} + \mathcal{L}_Q + \mathcal{L}_{th}, \quad (2.1)$$

where $\tilde{L}$ is the real superfield functional

$$\tilde{L} = E[-3 + 2Ls(L) + L(bG - \delta_X V_X)] = E[-3 + 2LS], \quad (2.2)$$

and the Kähler potential is given by

$$\begin{aligned} K &= k(L) + G + \sum_A K_{(A)}, & K_{(A)} &= e^{G^A + 2 \sum_a q_A^a V^a} |\Phi^A|^2, \\ G &= \sum_I g^I, & G^A &= \sum_I q_I^A g^I, \\ g^I &= -\ln(T^I + \bar{T}^I), & k(L) &= \ln L + g(L). \end{aligned} \quad (2.3)$$

In the dual chiral formulation $s(L) \rightarrow \text{Re}(s)$; the *vev* $\langle s(L) \rangle = g_s^{-2}$ determines the coupling at the string scale. Canonical normalization of the Einstein term requires:

$$k'(L) = -2Ls'(L). \quad (2.4)$$

Since the underlying theory is anomaly free, it is known that the apparent anomalies are canceled by a four-dimensional version [?, ?] of the Green-Schwarz (GS) mechanism [?]. This leads to a Fayet-Illiopoulos (FI) term in the effective supergravity Lagrangian. Ignoring nonperturbative corrections² to the dilaton Kähler potential, the D-component of the $U(1)_X$ vector supermultiplet is given by

$$D_X = \sum_A \frac{\partial K}{\partial \phi^A} q_A^X \phi^A + \xi, \quad \xi = \frac{g_s^2 \text{Tr } T_X}{192\pi^2}, \quad (2.5)$$

where K is the Kähler potential, q_A^X is the $U(1)_X$ charge of the scalar matter field ϕ^A , ξ is the FI term, T_X is the charge generator of $U(1)_X$, g_s is the unified (string scale) gauge coupling.

²The modification in the presence of nonperturbative corrections will be noted below.

Up to perturbative loop effects, the chiral dilaton formulation has $g_s^2 = 1/\text{Re}\langle s \rangle$, where $s = S|$ is the lowest component of the chiral dilaton superfield S . However, once higher order and nonperturbative corrections are taken into account the chiral dilaton formulation becomes inconvenient. The dual linear multiplet formulation—which relates a (modified) linear superfield L to $\{S, \bar{S}\}$ through a duality transformation—provides a more convenient arrangement of superfield degrees of freedom due to the neutrality of L with respect to target-space duality transformations (hereafter called *modular transformations*):

$$\begin{aligned} T^I &\rightarrow \frac{a^I T^I - i b^I}{i c^I T^I + d^I}, \\ \Phi^A &\rightarrow e^{-\sum_I q_I^A \mathcal{F}^I} \Phi^A, \\ a^I d^I - b^I c^I &= 1, \quad a^I, b^I, c^I, d^I \in \mathbf{Z} \quad \forall \quad I = 1, 2, 3, \\ \mathcal{F}^I &= \ln(i c^I T^I + d^I). \end{aligned} \tag{2.6}$$

The parameters a^I , *etc.*, may be taken as independent, or subject to additional constraints, depending on the details of the string construction. In the limit of vanishing nonperturbative corrections to the dilaton Kähler potential, $g_s^2 = 2\langle \ell \rangle$, where $\ell = L|$.

In the linear multiplet formulation, including nonperturbative corrections to the dilaton Kähler potential, the FI term becomes

$$\xi(\ell) = \frac{2\ell \text{Tr } T_X}{192\pi^2}. \tag{2.7}$$

Consequently, the background dependence of the FI term in (??) arises from $\langle \ell \rangle = \langle L|$. The FI term induces nonvanishing *vev*'s for some scalars ϕ^A as the scalar potential drives $\langle D_X \rangle \rightarrow 0$, if supersymmetry is unbroken. In general a total number m of $U(1)_a$'s are broken at the same time. The nonvanishing *vev*'s in the supersymmetric vacuum phase can be related to the FI term. Then $\langle L|$ serves as an order parameter for the vacuum and all nontrivial *vev*'s can be written as some function of $\langle L|$. However the vacuum value $\langle L|$ is not determined at the $U(1)_a$ -breaking scale. The conditions

$$\langle D_a \rangle = 0 \tag{2.8}$$

require only that $m - 1$ linear combinations of the modular invariant functions $\langle K_A|$ vanish and that one linear combination is equal to (2.7), that is, proportional to ℓ , which, like the T-moduli, remains a dynamical field of the effective supergravity theory below the $U(1)_a$ -breaking scale. To account for this fact, following [?, ?] we promote (??) to a superfield relation. Thus we impose the superfield identity

$$\left(\frac{\partial K}{\partial V_a} + 2L \frac{\partial S}{\partial V_a} \right)_{\Delta_A=0} = \left(\frac{\partial K}{\partial V_a} \right)_{\Delta_A=0} - L \delta_X \delta_{Xa} = 0, \tag{2.9}$$

where Δ_A are superfields, to be defined below, that vanish in the supersymmetric vacuum. This assures vanishing of the D-terms at the $U(1)_a$ symmetry breaking scale while maintaining manifest local supersymmetry below that scale. The latter point was demonstrated in detail, at both the superfield and the component field levels, for the toy model studied in [?].

$\mathcal{L}_Q$ is the quantum correction [?, ?, ?] that contains the field theory anomalies canceled by the GS terms:

$$\mathcal{L}_Q = - \int d^4\theta \frac{E}{8R} \sum_a \mathcal{W}_a^\alpha P_\chi B_a \mathcal{W}_\alpha^a + \text{h.c.}, \quad (2.10)$$

$$B_a(L, V_X, g^I) = \sum_I (b - b_a^I) g^I - \delta_X V_X + f_a(L), \quad (2.11)$$

where P_χ is the chiral projection operator [?]: $P_\chi \mathcal{W}^\alpha = \mathcal{W}^\alpha$, that reduces in the flat space limit to $(16\Box)^{-1} \bar{\mathcal{D}}^2 \mathcal{D}^2$, and the L -dependent piece $f_a(L)$ is the “2-loop” contribution [?]. The string-loop contribution is [?]

$$\mathcal{L}_{th} = - \int d^4\theta \frac{E}{8R} \sum_{a,I} b_a^I (\mathcal{W}\mathcal{W})_a \ln \eta^2(T^I) + \text{h.c.} \quad (2.12)$$

For each Φ^A , the $U(1)_X$ charge is denoted q_A^X while q_I^A are the modular weights. The conventions chosen here imply $U(1)_X$ gauge invariance under the transformation

$$V_X \rightarrow V'_X = V_X + \frac{1}{2} (\Theta + \bar{\Theta}), \quad \Phi^A \rightarrow \Phi'^A = e^{-q_A^X \Theta} \Phi^A. \quad (2.13)$$

The GS coefficients b and δ_X must be chosen to cancel the quantum field anomalies under modular and $U(1)_X$ transformations that would be present in the absence of the GS counterterms [?, ?]. It is not hard to check that the correct choices are given by:

$$\delta_X = -\frac{1}{2\pi^2} \sum_A C_{a \neq X}^A q_A^X = -\frac{1}{48\pi^2} \text{Tr } T_X, \quad (2.14)$$

$$8\pi^2 b = 8\pi^2 b_a^I + C_a - \sum_A (1 - 2q_I^A) C_a^A \quad \forall I = 1, 2, 3 \quad \text{and} \quad \forall a. \quad (2.15)$$

2.2 Field redefinitions

In this section we review the field redefinitions of [?, ?], phrased in the notation used in the present article. We state here the general case, which was treated in Section 3.2 of [?].

We introduce a vector superfield V_a ($a = 1, \dots, m$) for each of the $U(1)_a$ gauge groups that are broken by the presence of the FI term $-\delta_X L V_X$ in (??). One of these is assumed to be anomalous; we denote it by $U(1)_X$ and the corresponding vector superfield by V_X . In

addition, there are a number of chiral superfields Φ^A that carry nontrivial charge under the $U(1)$'s. Define the modular invariant vev 's³

$$\langle e^{G^A} |\Phi^A|^2 \rangle = |C_A|^2, \quad (2.16)$$

where C_A is a complex constant. For $C_A \neq 0$ ($A = 1, \dots, n$) we may define chiral superfields Θ^A through the identification

$$\Phi^A = C_A e^{\Theta^A}. \quad (2.17)$$

Then the (composite) superfield whose vev appears in (??) can be written

$$|\Phi^A|^2 e^{G^A} = |C_A|^2 \exp(G^A + \Theta^A + \bar{\Theta}^A). \quad (2.18)$$

This motivates the definition of the modular invariant real superfield

$$\Sigma^A = \Theta^A + \bar{\Theta}^A + G^A, \quad (2.19)$$

that satisfies $\langle \Sigma^A \rangle = 0$. By contrast, we generically have $\langle G^A \rangle \neq 0$ and $\langle \Theta^A \rangle \neq 0$. The basis (g^I, V^a, Σ^A) is equivalent to the basis $(g^I, V^a, |\Phi^A|)$, but the fields Σ^A that have replaced $|\Phi^A|$ are modular invariant superfields with vanishing vev 's; the usefulness of this feature is apparent when we expand about a given vacuum.

The corresponding contribution to the Kähler potential (??) now takes the form

$$K_{(A)} = |C_A|^2 \exp\left(\Sigma^A + 2 \sum_a q_A^a V^a\right). \quad (2.20)$$

Note that $\langle K_{(A)} \rangle = |C_A|^2$ if we take V^a in Wess-Zumino gauge. However for the condition (??) to hold in the effective theory that is operative between the $U(1)_a$ -breaking scale and the condensation scale where the vev of the dilaton ℓ is determined, (??) is not fixed at a constant value but rather as a functional of L . This is most easily achieved by absorbing a dependence on L in the vector fields V_a , as will be done below.

From (??) it is evident that each V^a will generically “eat” some combination of the Σ^A when we go to unitary gauge, since linear couplings between V^a and Σ^A are implied. It is possible to identify a set of vector superfields that do not couple linearly to the massless matter superfields. To this end we make the following field redefinitions:

$$V_a = U'_a - \Sigma_a, \quad \Sigma_a = \sum_A T_{aA} \Sigma^A, \quad \Sigma'^A = \Sigma^A - 2 \sum_a q_A^a \Sigma_a, \quad (2.21)$$

³By the vev of a superfield we mean that the component fields in the expansion should be evaluated at their vev 's.

where T_{aA} is a projection from the n -dimensional space of chiral superfields with nonvanishing vev 's onto the m -dimensional $U(1)^m$ space of linearly independent generators of the spontaneously broken $U(1)_a$'s. It is defined by:

$$T_{aA} = \frac{1}{2} B_A \sum_b M_{ab}^{-1} q_A^b, \quad M_{ab} = \sum_A q_A^a q_A^b B_A. \quad (2.22)$$

Eq. (??) then becomes:

$$K_{(A)} = |C_A|^2 \exp \left[2 \sum_a q_A^a (U'_a - \Sigma_a) + \Sigma^A \right] = |C_A|^2 \exp \left[2 \sum_a q_A^a U'_a + \Sigma'^A \right]. \quad (2.23)$$

The linear dependence of m of the uneaten matter fields is apparent in the Σ'^A basis, for it is easy to check that

$$\sum_A q_A^a B_A \Sigma'^A = 0 \quad \forall a. \quad (2.24)$$

Thus, only $n - m$ of the Σ'^A are linearly independent.⁴

While (??) is not a gauge transformation, it can be related to one. To arrive at this result, for the fields Σ'^A that appear in (??) we make the identification

$$\Sigma'^A \equiv G'^A + \Theta'^A + \bar{\Theta}'^A, \quad (2.25)$$

where Θ'^A is a chiral superfield and G'^A is a function of the g^I . From the transformation (??) we read off:

$$\Theta'^A = \Theta^A - 2 \sum_a q_A^a \Theta_a, \quad G'^A = G^A - 2 \sum_a q_A^a G_a. \quad (2.26)$$

$$\Theta_a = \sum_A T_{aA} \Theta^A, \quad G_a = \sum_A T_{aA} G^A. \quad (2.27)$$

This leads us to rewrite the vector superfield shift that appears in (??) in the following way:

$$V_a = (U'_a - G_a) - (\Theta_a + \bar{\Theta}_a) \equiv V'_a - (\Theta_a + \bar{\Theta}_a). \quad (2.28)$$

The shift $V_a \rightarrow V'_a$ is a gauge transformation, provided we simultaneously shift *all* the gauge-charged fields correspondingly:

$$\Phi'^A = \Phi^A \exp \left(-2 \sum_a q_A^a \Theta_a \right) \quad \forall A. \quad (2.29)$$

⁴For this reason the V', Σ' basis was referred to as “quasi-unitary” gauge in [?]; the conventional unitary gauge will be recovered below when certain conditions are imposed on the constant, real parameters B_A .

Indeed with the identification

$$\Phi'^A = C_A e^{\Theta'^A}, \quad A = 1, \dots, n, \quad (2.30)$$

the shift in (??) is precisely the change of variables (??), for the fields that get *vev*'s.

From (??) we have $U'_a = V'_a + G_a$. Thus U'_a “eats” the combination of Kähler moduli G_a ; it is this shift $V'_a \rightarrow U'_a$ that is not a gauge transformation. With this redefinition we obtain corrections to the effective action that do not cancel between the GS term and the one-loop quantum correction; we include these explicitly in our total effective Lagrangian. (As noted in [?]), once the nonperturbative dynamics of the hidden sector stabilizes the T^I , modular invariance is broken and we are free to instead take $U'_a = V'_a + \langle G_a \rangle$, which is just a gauge transformation if $\langle G_a \rangle$ is a homogeneous background field.)

To account for the required L -dependence of $K_{(A)}$, we shift to a new (unprimed) vector superfield basis

$$U'_a = U_a + h_a(L) + \sum_B b_{aB}(L) \Sigma'^B, \quad (2.31)$$

where $\langle U_a \rangle = 0$ by definition. We determine the functions $h_a(L)$ from the requirement that the D-term *vev*'s vanish:

$$\sum_A \left\langle q_A^b e^{G'^A} |\Phi'^A|^2 \exp \left(\sum_a 2q_A^a \left[U_a + h_a(L) + \sum_B b_{aB} \Sigma'^B \right] \right) \right\rangle_{(L,T)} = \frac{\delta_X}{2} L \delta_{bX}, \quad (2.32)$$

where the subscript (L, T) indicates that the dilaton and moduli superfields L, T^I are left as quantum variables; that is, the superfield “*vev*'s” $\langle |\Phi^A|^2 \rangle_{(L,T)} = |C_A|^2 e^{-G^A}$ and $\langle V_a \rangle_{(L,T)} = h_a(L)$ are defined as functionals of the superfields (L, T) . Since by assumption $\langle U_a \rangle_{(L,T)} = \langle \Sigma'^A \rangle_{(L,T)} = 0$, (??) gives a set of equations for the functionals h_a :

$$\sum_A q_A^b |C_A|^2 \exp \left(2 \sum_a q_A^a h_a(L) \right) \equiv \sum_A q_A^b x^A = \frac{\delta_X}{2} L \delta_{bX}. \quad (2.33)$$

Evaluated at the vacuum values $U = \Sigma = 0$, the shifts (??) in the U'_a modify the functions that appear in (??) and (??):

$$\begin{aligned} k(L) &\rightarrow \tilde{k}(L) = k(L) + \delta k(L), \\ 2Ls(L) &\rightarrow 2L\tilde{s}(L) = 2Ls(L) + 2L\delta s(L), \end{aligned} \quad (2.34)$$

with [?], using (??),

$$\delta k(L) = \sum_A x^A, \quad \frac{\partial}{\partial L} \delta k(L) = 2 \sum_{A,b} q_A^b h'_b x^A = h'_X \delta_X L, \quad (2.35)$$

and⁵

$$2L\delta s = -\delta_X L h_X, \quad 2L \frac{\partial}{\partial L} \delta s = -\delta_X L h'_X = -\frac{\partial}{\partial L} \delta k, \quad (2.36)$$

so the Einstein condition (??) is satisfied for $U = \Sigma = 0$. The Kähler potential for matter is

$$\begin{aligned} K(\Phi) &= \sum_A e^{G'^A + 2 \sum_a q_A^a [U_a + h(L)_a + \sum_B b_{aB} \Sigma'^B]} |\Phi'^A|^2 \\ &= \delta k(L) + L \delta_X U_X + \sum_A \Sigma'^A \left(x^A + b_{XA} \delta_X L \right) \\ &\quad + 2 \sum_{A,a} \Sigma'^A U_a \left(q_A^a x^A + 2 \sum_{B,b} b_{bA} x^B q_B^b q_B^a \right) + O(U^2, \Sigma'^2, |\Phi'^A|^{>n}). \end{aligned} \quad (2.37)$$

The linear coupling of Σ'^A to the vector multiplets U_a is given by:

$$K \ni 2 \sum_a U_a \left[\sum_A \Sigma'^A \left(q_A^a x^A + 2 \sum_{B,b} b_{bA} x^B q_B^b q_B^a \right) \right]. \quad (2.38)$$

We can exploit (??) to eliminate U, Σ' mixing. We choose the constants b_{aA} in (??) such that⁶

$$f_a(L) q_A^a B_A = q_A^a x^A + 2 \sum_{B,b} b_{bA} x^B q_B^b q_B^a. \quad (2.39)$$

Then the term in brackets in (??) vanishes identically for each $a = 1, \dots, m$. Since the $U(1)_a$ are assumed to be broken, the vectors $q^a = (q_1^a, \dots, q_n^a)$ are linearly independent, and the matrix

$$N_{ab} = \sum_B x^B q_B^a q_B^b \quad (2.40)$$

has an inverse. This allows us to uniquely determine the required constraints:

$$b_{aA}(L) = \frac{1}{2} \sum_b q_A^b N_{ab}^{-1} [f_b(L) B_A - x^A(L)]. \quad (2.41)$$

From (??) we have the sum rule

$$\sum_A b_{aA} \Sigma'^A = -\frac{1}{2} \sum_{b,A} N_{ab}^{-1} x^A q_A^b \Sigma'^A. \quad (2.42)$$

It is convenient to identify the part of b_{aA} that actually contributes to the right-hand side of (??):

$$\hat{b}_{aA} = -\frac{1}{2} \sum_b N_{ab}^{-1} x^A q_A^b, \quad \sum_A q_A^c \hat{b}_{aA} = -\frac{1}{2} \delta_a^c. \quad (2.43)$$

⁵A factor $\delta_X/2$ is missing from the right hand side of both equations in (3.9) of [?].

⁶The functional $f_a(L)$ introduced here is not to be confused with the one in (??).

Then the kinetic terms (??) can finally be written as

$$K_{(A)} = |C_A|^2 \exp \left(\Sigma'^A + \sum_a 2q_A^a \left[U_a + h_a(L) + \sum_B \hat{b}_{aB}(L) \Sigma'^B \right] \right). \quad (2.44)$$

The set of gauge transformations and field redefinitions leading to (??) defines a gauge that is closely related to the “true” unitary gauge, as can be seen [?] by writing (??) in the form

$$\begin{aligned} K_{(A)} &= x^A(L) \exp \left(\hat{\Sigma}^A + \sum_a 2q_A^a U_a \right), \\ \hat{\Sigma}^A &= \Sigma'^A + 2 \sum_{aB} q_A^a \hat{b}_{aB} \Sigma'^B = \Sigma^A - \sum_{abB} q_A^a x^B q_B^b N_{ab}^{-1} \Sigma^B, \end{aligned} \quad (2.45)$$

$$0 = \sum_A q_A^a x^A(L) \hat{\Sigma}^A(L). \quad (2.46)$$

Here the m massive vectors U_a and the $n - m$ linearly independent uneaten supermultiplets $\hat{\Sigma}^A$ appear as the physical states. This reduces to the conventional unitary gauge when L is replaced by the *vev* ℓ_0 of the dilaton $\ell = L$, which is determined only at the condensation scale.

However, there are two simple examples where the constraint (??) leads directly to “true” unitary gauge. First suppose that

$$q_A^a = q_{A_0}^a, \quad \forall A = 1, \dots, n; \quad (2.47)$$

that is, all fields acquiring *vev*’s have the same charges. Then the condition (??) reads

$$q_{A_0}^{a \neq X} = 0, \quad \sum_A |C_A|^2 = \frac{\delta_X}{2q_{A_0}^X} L \exp \left(-2q_{A_0}^X h_X(L) \right) = C, \quad x^A = L |C_A|^2 \frac{\delta_X}{2q_{A_0}^X C}, \quad (2.48)$$

where C is a constant, so that $\hat{\Sigma}$ is independent of L : $\hat{\Sigma}(L) = \hat{\Sigma}(\ell_0)$.

Next suppose that $m = n$; *i.e.*, the number of fields acquiring *vev*’s corresponds to the number of spontaneously broken $U(1)$ generators. Then there are no uneaten fields among the Σ^A . The charge matrix q_A^a is invertible, and we can uniquely define solutions to the equations

$$\sum_a q_A^a Q_a^B = (qQ)_A^B = \delta_A^B, \quad \sum_A q_A^a Q_a^B = (Qq)_b^a = \delta_b^a. \quad (2.49)$$

In this case we can invert (??) to obtain

$$x^A(L) = \frac{1}{2} \delta_X L Q_X^A, \quad \hat{b}_{aA} = -\frac{1}{2} Q_a^A, \quad \hat{\Sigma}^A = 0. \quad (2.50)$$

In general $m \leq n$; if $m < n$ it is not possible to invert the conditions (??), and $\hat{b}_{aA}$ depends on L if the $U(1)_a$ charges are not degenerate.

Note that the constant parameters B^A do not appear in (??). As noted in [?], these parameters are unphysical and must simply be chosen such that the constraint (??) is satisfied. The constants C_A are also not physical since they can be shifted by gauge transformations with constant parameters [?] that leave the physically relevant variables x^A unchanged; in particular one could choose $B_A = |C_A|^2$. On the other hand, with the choice⁷ [?]

$$b_{aA}(\ell_0) = 0, \quad f_a(\ell_0) = f, \quad B_A = x^A(\ell_0)/f, \quad (2.51)$$

the gauge defined by (??) with $L \rightarrow \ell_0$ is the same as the one defined by (??) up to the shift $h(\ell_0) = U' - U(\ell_0)$. For the special case of degenerate $U(1)_a$ charges, this is satisfied by $B_A = |C_A|^2$, $f_a(L) = L\delta_X/2a_{A_0}C$, while for the case of minimal *vev*'s this requires $B_A = cQ_X^A$, $f_a(L) = L\delta_X/2c$, $c = \text{constant}$, suggesting $|C_A|^2 = cQ_X^A$ as a convenient choice.

The above field redefinitions also modify the kinetic terms for all $U(1)_a$ -charged chiral fields Φ^B . We have

$$\begin{aligned} V_a &= U_a + h_a(L) + \hat{\Sigma}_a, \quad \hat{\Sigma}_a = \sum_A \hat{b}_{aA} \Sigma^A = \hat{\Theta}_a + \hat{\bar{\Theta}}_a + \hat{G}_a, \\ \hat{\Theta}_a &= \sum_A \hat{b}_{aA} \Theta^A, \quad \hat{G}_a = \sum_A \hat{b}_{aA} G^A. \end{aligned} \quad (2.52)$$

The kinetic term for Φ^B takes the form⁸

$$K_{(B)} = e^{G^B + 2 \sum_a q_B^a (h_a(L) + \hat{\Sigma}_a)} |\Phi^B|^2 + O(U_a). \quad (2.53)$$

First consider the case where $\hat{b}_{aA}$ is independent of L . Then the term proportional to $\hat{\Theta}_a + \hat{\bar{\Theta}}_a$ in the shift (??) in the vector field is just a gauge transformation and the corresponding term in the exponent in (??) is absorbed in the redefinition of Φ^B under the gauge transformation:

$$\Phi'^B = \Phi^B \exp \left(2 \sum_a q_B^a \hat{\Theta}_a \right). \quad (2.54)$$

Since the exponent in (??) is not modular invariant, the modular weights of Φ' are modified with respect to those of Φ , as reflected by the T -dependence of the exponent in (??):

$$\begin{aligned} K_{(B)} &= e^{G'^B + 2 \sum_a q_B^a h_a(L)} |\Phi'^B|^2 + O(U_a), \quad G'^B = \sum_I q_I'^B g^I, \\ q_I'^B &= q_I^B + 2 \sum_{aA} q_B^a \hat{b}_{aA} q_I^A = q_I^B + \delta q_I^B. \end{aligned} \quad (2.55)$$

⁷The first equality in (3.32) of [?] should read $\langle b_{aA} \rangle = 0$.

⁸The presence of terms linear in U_a in the exponent in (??) induces terms of $O(|\Phi^B \Phi^C|^2)$ in the effective low energy Kähler potential defined by integrating out the massive vector fields.

In the more general case where $\hat{b}_{aA}$ depends on L and unitary gauge is defined *a posteriori* after the *vev* $\langle L \rangle = \ell_0$ is fixed we have

$$\hat{b}_{aA}(L) = \hat{b}_{aA}(\ell_0) + O(\Delta L), \quad \Delta L = L - \ell_0. \quad (2.56)$$

Then the gauge transformation (??) and the redefinition (??) of modular weights are defined by the replacement $\hat{b}_{aA} \rightarrow \hat{b}_{aA}(\ell_0)$. The additional terms of order ΔL generate higher dimension couplings of the Φ'^B to the dilaton and to the eaten superfields $\hat{\Sigma}_a$, whose components in unitary gauge are identified with the longitudinally polarized components of the massive vector supermultiplet U_a . When the U_a are integrated out they generate operators of dimension eight (*e.g.*, $|\Phi|^6 \hat{\ell}^2$) and higher in the low energy theory, and we may neglect them. As noted above, the m “Goldstone modes” $\hat{\Sigma}_a$ disappear from the Lagrangian due to overall gauge invariance. The $n - m$ uneaten physical states⁹ $\hat{\Sigma}^A$; $\langle \hat{\Sigma} \rangle = 0$, introduced in (??) may be expressed [?] in terms of chiral and anti-chiral fields $D^A, \bar{D}^{\bar{A}}$:

$$\begin{aligned} \hat{\Sigma}^A &= \hat{\Theta}^A + \hat{\bar{\Theta}}^{\bar{A}} + \hat{G}^A = D^A + \bar{D}^{\bar{A}} + O\left(\left[\hat{T}^I + \hat{\bar{T}}^{\bar{I}}\right]^2 / \langle t^I + \bar{t}^{\bar{I}} \rangle^2\right), \\ D^A &= \hat{\Theta}^A + \langle \hat{G}^A \rangle + \left\langle \frac{\partial \hat{G}^A}{\partial t^I} \right\rangle \hat{T}^I, \quad \langle D^A \rangle = 0, \\ 0 &= \sum_A q_A^a x^A(\ell_0) \hat{\Sigma}^A = \sum_A q_A^a x^A(\ell_0) \hat{\Theta}^A = \sum_A q_A^a x^A(\ell_0) \hat{G}^A. \end{aligned} \quad (2.57)$$

where $T^I = \langle t^I \rangle + \hat{T}^I$.

2.3 Weyl transformation

Of chief concern in [?, ?] was the maintenance of the canonical normalization for the Einstein term—concurrent to field redefinitions. Here we recall the general prescription given in those papers for determining the necessary Einstein condition from $\mathcal{L}$ rewritten in a new field basis. The relevant part of the Lagrangian is (??). We define M to stand collectively for the fields that are to be regarded as independent of L in a given basis. We then define the functional S by the identification

$$\tilde{L} \equiv E[-3 + 2LS(L, M)]. \quad (2.58)$$

⁹There is a term in K that is linear in $\hat{\Sigma}$ whose only effect is to slightly modify the Kähler metric for the T -moduli: see (2.44)-(2.47) and (3.33) of [?]. D^A is a singlet of the surviving gauge group, and terms in the Kähler potential that are linear in an uncharged chiral superfield do not contribute to the Lagrangian. Shifting this term from K to S *via* the Weyl transformation has no effect because linear terms in the effective Kähler potential $K - 2LS$ are Weyl invariant.

The Einstein condition holds provided

$$\left(\frac{\partial K}{\partial L}\right)_M + 2L \left(\frac{\partial S}{\partial L}\right)_M = 0. \quad (2.59)$$

Here the subscripts on parentheses instruct us to hold constant under differentiation the fields denoted collectively by M .

As explained in [?], since the redefinition (??) involves L , some care is required if we are to retain a canonical Einstein term. In (??) we saw that the condition (??) is automatically satisfied to zeroth order in the fields $\Delta = U, \Sigma$ with vanishing vev 's after the field redefinitions of the previous subsection, provided (??) is satisfied. To ensure (??) holds at nontrivial orders in Δ requires a Weyl transformation that redefines the linear multiplet $L \rightarrow \hat{L}(L, \Delta)$ such that the linearity condition (??) holds for $\hat{L}$ in the new Weyl basis; this transformation eliminates the lowest dimension terms linear in Δ , so that tree exchange of these fields may be neglected when they are integrated out. These results apply to the supersymmetric phase. When we introduce supersymmetry breaking through gaugino condensation, we do not expect (??) to remain strictly true. The required Weyl transformation to assure (??) in the case of small nonvanishing D-terms is worked out in Appendix ??.

3 Gaugino condensation

After we make the gauge transformations and field redefinitions of the previous section, as summarized in (??), as well as the requisite Weyl transformation, the density $\tilde{L}$, Eq. (??), is modified to read¹⁰

$$\begin{aligned} \tilde{L} &= E \left[-3 + 2L\tilde{s}(L) + L \left(bG - \delta_X \hat{G}_X - \delta_X U_X \right) \right] = \hat{E} \left[-3 + 2\hat{L}\tilde{s}(\hat{L}) + \tilde{L}_{GS} \right] + O(U^2), \\ \tilde{L}_{GS} &= \hat{L} \left(bG - \delta_X \hat{G}_X \right), \quad \tilde{s}(L) = s(L) - \frac{\delta_X}{2} h_X(L), \end{aligned} \quad (3.1)$$

where here U refers collectively to all the heavy modes with vanishing vev 's that we have integrated out, and $\hat{G}_X$ is defined in (??). The quantum Lagrangian (??) is also modified. For the term quadratic in the field strength of the unbroken gauge group factor $\mathcal{G}_u$, it is of

¹⁰In this section we study the vacuum in the parameter space of the dilaton, T -moduli and static condensates, and set to zero all fields with vanishing vev 's. Therefore the term linear in $\hat{\Sigma}^A$ noted in the previous footnote is irrelevant here. However when we shift the vev 's $\langle \Phi^A \rangle$ by small amounts Δ^A to allow $D_a \sim |u|^2 \neq 0$, there are terms linear in Δ^A ; these are treated exactly in Section ?? and Appendix ??.

the form (??) with

$$B_u = \sum_I (b - b_u^I) g^I - \delta_X \hat{G}_X - \delta_X h_X(\hat{L}) + f_u(\hat{L}) + O(U). \quad (3.2)$$

The shift $h_X(L)$ restores the gauge field kinetic energy term to its original form; *i.e.* just multiplied by the original function $s(L)$. The anomalous modular transformation of the $\delta_X \hat{G}_X$ term in B_u is canceled by the corresponding shift in $\tilde{L}_{GS}$:

$$bG - \delta_X \hat{G}_X = \sum_I g^I (b + \delta b_I) \equiv \sum_I g^I b_I. \quad (3.3)$$

The shift in the moduli dependence of B_u corresponds to the shifted modular weights given in (??) of the $U(1)_a$ -charged fields in the loops that contribute to the β -function for $\mathcal{G}_u$. That is, referring to (??) and (??), the shift in B_u is given by

$$\begin{aligned} 4\pi^2 \delta B_u &= \sum_B C_u^B \sum_I \delta q_B^I g^I = 2 \sum_{B,a} C_u^B q_B^a \sum_{A,I} \hat{b}_{aA} q_I^A g^I = 2 \sum_{B,a,A} C_u^B q_B^a \hat{b}_{aA} G^A \\ &= 2 \sum_{B,a} C_u^B q_B^a \hat{G}_a = -4\pi^2 \delta_X \hat{G}_X = 4\pi^2 \sum_I \delta b_I g^I, \end{aligned} \quad (3.4)$$

where C_u^B is the quadratic Casimir for the representation B of $\mathcal{G}_u$, and we used the fact that

$$\sum_B C_u^B q_B^{a \neq X} = 0, \quad (3.5)$$

since by assumption only $U(1)_X$ is anomalous. The terms in (??) that are quadratic in the field strengths of the broken $U(1)_a$'s generate terms in the low energy theory that are of very high dimension in derivative of fields and in auxiliary fields, as discussed in [?], and we neglect them. We next construct the effective Lagrangian for condensation at the scale where one gauge group $\mathcal{G}_c$ becomes strongly coupled, extending the approach of [?]. A number of new features arise, which we describe below.

3.1 Construction of the effective theory at the condensation scale

Here we follow the construction of [?]. As shown there the physics of condensation is dominated by the group $\mathcal{G}_c$ with the largest β -function coefficient b_c unless there are two groups with nearly equal β -functions. Therefore, for simplicity, we consider here the case with just one condensing simple group $\mathcal{G}_c$ in the hidden sector. Here we set to zero fields with vanishing vev 's, in which case the new Weyl basis discussed in the previous section is equivalent to the original one. The term in (??) that is quadratic in the strongly coupled gauge field

strength $\mathcal{W}_c^\alpha$ is replaced by an effective VYT [?] action, generalized [?, ?] to the case of local supersymmetry, that is manifestly invariant under the nonanomalous symmetries of the underlying quantum field theory:

$$\mathcal{L}_{VYT} = \frac{1}{8} \int d^4\theta \frac{E}{R} U_c \left[b'_c \ln(e^{-K/2} U_c) + \sum_\alpha b_c^\alpha \ln \Pi^\alpha \right] + \text{h.c.}, \quad (3.6)$$

where U_c and Π^α are nonpropagating¹¹ gauge and matter condensate chiral superfields, with Kähler chiral weights 2 and 0, respectively:

$$U_c \simeq \mathcal{W}_c^\alpha \mathcal{W}_\alpha^c, \quad \Pi^\alpha \simeq \prod_B (\Phi_c^B)^{n_\alpha^B}. \quad (3.7)$$

The chiral superfields Φ_c^B that condense are charged under the strongly coupled gauge group $\mathcal{G}_c$. The effective theory is modular invariant; the modular anomaly matching condition between the effective Lagrangian (??) and the underlying quantum Lagrangian (??) reads [?]

$$b'_c + \sum_{\alpha, B} b_c^\alpha n_\alpha^B q_I^B = \frac{1}{8\pi^2} \left[C_c - \sum_B C_c^B (1 - 2q_I^B) \right] \quad \forall I. \quad (3.8)$$

The strongly coupled Yang-Mills sector also possesses a residual global $U(1)_a$ invariance that is broken only by superpotential couplings, such as (??) below, that involve those chiral superfields that get *vev*'s at the $U(1)_a$ -breaking scale. They enter the $\mathcal{G}_c$ gauge coupling RGE only through chiral field wave function renormalization, which is a two-loop effect that is encoded in the expression (??) for the gaugino condensate through the appearance¹² of the superpotential coefficients W_α . We therefore impose the $U(1)_a$ anomaly matching conditions

$$\sum_{\alpha, B} b_c^\alpha n_\alpha^B q_B^a = \delta_{aX} \sum_B \frac{C_c^B}{4\pi^2} q_B^X, \quad (3.9)$$

where again the sum over B includes only the chiral superfields Φ_c^B that are charged under $\mathcal{G}_c$. Finally, assigning canonical dimensions $(\frac{3}{2}, 1)$ to $\mathcal{W}_c^\alpha, \Phi_c^B$, the standard trace anomaly requires [?]

$$3b'_c + \sum_{\alpha, B} b_c^\alpha n_\alpha^B = 3b'_c + \sum_\alpha b_c^\alpha d_\alpha = \frac{1}{8\pi^2} \left(3C_c - \sum_B C_c^B \right) + O(\Lambda_c/m_P), \quad (3.10)$$

¹¹The dynamical condensate case was studied in ref. [?] with just an E_8 gauge condensate. After correctly integrating out the heavy bound state degrees of freedom, that have masses larger than the condensation scale Λ_c , one recovers the theory with a static E_8 condensate [?]. We expect this result to be generic [?].

¹²See (2.29)–(2.33) of [?] and (14)–(16) of [?] and the related discussions.

where d_α is the dimension of Π_α , and Λ_c is the condensation scale. The matter condensates are invariant under all the unbroken, nonanomalous symmetries, and therefore the same monomials can appear in the superpotential¹³

$$W(\Pi) = \sum_\alpha W_\alpha \Pi^\alpha, \quad (3.11)$$

where W_α is a function of the $\mathcal{G}_c$ -neutral unconfined chiral multiplets. Solving for the condensates gives [?]

$$\begin{aligned} |u|^2 &= \bar{U}_c U_c = e^{-2b'_c/b_c} e^K e^{-2s(\ell)/b_c - b \sum_I g^I/b_c} \prod_I |\eta(t^I)|^{4(b-b_c)/b_c} \prod_\alpha |b_c^\alpha/4W_\alpha|^{-2b_c^\alpha/b_c}, \\ \pi^\alpha &= \Pi^\alpha = -\frac{e^{-K/2} b_c^\alpha}{4W_\alpha} u, \quad b_c \equiv b'_c + \sum_\alpha b_c^\alpha. \end{aligned} \quad (3.12)$$

The expression for $|u|^2 \simeq \Lambda_c^6$ is consistent [?, ?] with instanton calculations in supersymmetric Yang-Mills theories provided b_c is the coefficient of the β -function:

$$\begin{aligned} b_c &= -\frac{2}{3g_c^3(\mu)} \mu \frac{\partial g_c(\mu)}{\partial \mu} = \frac{1}{8\pi^2} \left(C_c - \frac{1}{3} \sum_B C_c^B \right) \\ &= b'_c + \sum_\alpha b_c^\alpha = \frac{1}{8\pi^2} \left(C_c - \frac{1}{3} \sum_B C_c^B \right) + \sum_\alpha b_c^\alpha \left(1 - \frac{d_\alpha}{3} \right) + O(\mu/m_P), \end{aligned} \quad (3.13)$$

which is satisfied if only matter condensates of dimension three have $b_c^\alpha \neq 0$. This is consistent with the fact that operators of dimension $3 + \delta_\alpha$ are suppressed in the superpotential (??) by a factor¹⁴ $W_\alpha \sim m_P^{-\delta_\alpha}$: the second equation in (??) shows that condensation can occur only if $b_c^\alpha \rightarrow 0$ when $W_\alpha \rightarrow 0$. Operators Π_2^α of dimension two may be generated by the *vev*'s of fields Φ^A that break the $U(1)_a$'s through superpotential couplings of the form

$$W_M = c_R(T^I) \left\langle \prod_A \Phi^A \right\rangle \Phi_c^R \Phi_c^{\bar{R}} = M_R \Phi_c^R \Phi_c^{\bar{R}} \simeq M_R \Pi_2^R. \quad (3.14)$$

where $R, \bar{R}$ denote a representation of $\mathcal{G}_c$ and its conjugate. In this case, for finite b_c^R, u , (??) requires

$$\pi_2^R = -e^{-K/2} b_c^R u / M_R \rightarrow 0, \quad M_R \gg \Lambda_c = |u\bar{u}|^{\frac{1}{6}}, \quad (3.15)$$

¹³The anomaly matching conditions are satisfied if one generalizes the Π^α in (??) to a linear combination of monomials: $\Pi^\alpha = \sum_i c_\alpha^i \Pi_i^\alpha$ with the same dimension, $U(1)_X$ charge and modular weights, for fixed α , and superpotential $W(\Pi) = \sum_{\alpha,i} W_\alpha^i \Pi_i^\alpha$. The only change is that the second equation in (??) is replaced by $\pi^\alpha = -b_c^\alpha c_\alpha^i / 4W_\alpha^i u$, $\forall \alpha, i$, requiring c_α^i / W_α^i independent of i for fixed α if condensation is to occur.

¹⁴If $U(1)_a$ -breaking generates masses M such that $\Lambda_c^2 \ll M^2 < m_P^2$ the lowest of these would be expected to replace m_P everywhere.

in conformity with conventional wisdom.

The most straightforward solution to (??) is¹⁵

$$b'_c = \frac{1}{8\pi^2} \left(C_c - \sum_B C_c^B \right), \quad \sum_{\alpha \in \Pi_d} b_c^\alpha = \sum_{B \in \Pi_d} \frac{C_c^B}{4d\pi^2}. \quad (3.16)$$

It is interesting to note that in this case, if $b_c^R \neq 0$, the contribution of $\Phi^{R,\bar{R}}$ to b'_c exactly cancels the contribution of b_c^R in the expression (??) for b_c , in conformity with the decoupling theorem: heavy states with $M_R > \Lambda_c$ do not contribute to the running of the gauge couplings. However they also do not contribute to the anomaly coefficients (??), (??) and (??) in the effective theory below Λ_c . Therefore we should set $b_c^R = 0$. The contribution of $\Phi_c^R, \Phi_c^{\bar{R}}$ to the anomaly at the string scale is still present in the form of a field-dependent ultra-violet cut-off [?] M_{sR} in the standard loop integral, but the infra-red cut-off should be the mass M_R , giving a net contribution to the effective action

$$\begin{aligned} \mathcal{L}_R &= -\frac{C_c^R}{16\pi^2} \int d^4\theta \frac{E}{R} U_c \ln(M_{sR}/M_R) + \text{h.c.}, \\ M_R &= M_{sR} W_{R\bar{R}} = c_R(T) \prod_A C_A e^{\hat{\Theta}^A} M_{sR}, \end{aligned} \quad (3.17)$$

where M_R is modular covariant (but not in general $U(1)_a$ covariant when C_A is held fixed) and M_{sR} contains the contribution to the anomalies; up to a constant factor of order one in Planck units

$$M_{sR}^2 = e^{K-G^R-G^{\bar{R}}-2\sum_a(q_R^a+q_{\bar{R}}^a)V_a} \rightarrow e^{\hat{K}-\hat{G}^R-\hat{G}^{\bar{R}}-2\sum_a(q_R^a+q_{\bar{R}}^a)h_a(L)}, \quad (3.18)$$

where the second expression is obtained after integrating out the massive vectors with the field redefinitions of Section ??. The $\hat{\Theta}^A$ are the ‘‘D-moduli’’ that remain massless at the string scale in the case when there are more scalar vev ’s than broken $U(1)_a$ ’s.

In the next section we construct the effective potential under the assumption that there are no mass terms of the form (??) in the strongly coupled sector and take $b^\alpha \neq 0$ only if $d_\alpha = 3$. Note that the exponents n_α^B need not be integers, because the effective superpotential (??) contains terms that, like the superpotential (??) for U_c , are generated by nonperturbative effects. For example we will sometimes use as a concrete example a model with a strongly

¹⁵This expression for b'_c also produces the correct anomaly under the phase transformation $\theta \rightarrow e^{i\alpha}\theta$ on the fermionic superspace coordinates. Simultaneous solutions to (??) and (??) may require constraints on the $\mathcal{G}_c$ -charged matter; they are trivially solved if each Π^α is composed of Φ^B ’s with the same Casimir C_B . This is the case for $d = 2$ and also for the $d = 3$ condensates in the FIQS $SO(10)$ model considered below, as well as the toy E_6 and $SU(3)$ models considered in [?].

coupled $SO(10)$ and matter in three fundamental spinorial representations ξ^B ; the lowest dimension $SO(10)$ -invariant matter-composite operators O_α have four factors of ξ^B . No masses are generated for these fields by $U(1)_a$ -breaking, and we require $\Pi^\alpha \sim O_\alpha^{\frac{3}{4}}$.

In Appendix ?? we give the corrections to this construction when terms like (??) are present. The sum rules (??) and (??) given below are modified by the removal of the $\Phi_c^R, \Phi_c^{\bar{R}}$ contributions. This is compensated for by new contributions from (??) in such a way that, aside from the usual renormalization group factor $\Lambda_c^2 \sim e^{-2/3bcg_s^2}$ that depends on the β -function factor for the massless spectrum of the strongly coupled sector below the $U(1)_a$ -breaking scale, the effective potential is determined by parameters defined in terms of the modular weights and gauge charges of the full spectrum of the effective theory at the string scale.

The superpotential (??) is made modular invariant by incorporating an appropriate T^I dependence in W_α as in [?]. It is not $U(1)_a$ invariant; we follow the standard approach for an effective theory with a broken symmetry: since the symmetry breaking arises only from vev 's of $\mathcal{G}_c$ singlets, we first construct a $U(1)_a$ -invariant superpotential by including appropriate powers of these fields, and then replace them by their (T-moduli and dilaton dependent) vev 's. Then when we solve for the condensate vev 's to get (??), the effects of $U(1)_a$ -breaking appear in $|u|^2$ at two-loop level through the superpotential couplings, as they should. The vev 's $\langle \Phi^A \rangle = V_0^A(L, T^I)$ obtained in Section ?? assured vanishing D-terms at the $U(1)_a$ -breaking scale. Once supersymmetry is broken at the condensation scale, one also expects D-terms to be generated. Therefore we replace those vev 's by $V^A = V_0^A(L, T^I) + \delta V^A$, with δV^A , like U_c, Π^α taken to be nonpropagating superfields to be determined by solving the overall equations of motion.

3.2 Solving for the vacuum at the supersymmetry-breaking scale

Once supersymmetry is broken we cannot demand *a priori* that $\langle D_a \rangle = 0$. We may also generate F-terms associated with the $U(1)_a$ -charged chiral fields that get vev 's. To include these, we slightly modify the field redefinitions (??) and (??) as follows:

$$\Phi'^A = C_A e^{\Theta'^A + \Delta^A}, \quad U'_a = U_a + h_a(L) + \sum_B b_{aB} \Sigma'^B + \Delta_a, \quad (3.19)$$

where Δ_a and Δ^A are vector and chiral superfields, respectively with only constant scalar components:

$$\delta_a = \Delta_a|, \quad F_a = -\frac{1}{4} D^2 \Delta_a|, \quad D_a = \frac{1}{8} \mathcal{D}^\alpha (\bar{\mathcal{D}}^2 - 8R) \mathcal{D}_\alpha \Delta_a|,$$

$$\delta^A = \Delta^A|, \quad F^A = -\frac{1}{4} D^2 \Delta^A|, \quad (3.20)$$

and we take

$$\langle U_a \rangle = \langle \Sigma'^A \rangle = 0 \quad (3.21)$$

as before. It is clear that we can make $U(1)_a$ gauge transformations with constant chiral superfields Λ_a to eliminate the scalar and F components from Δ_a :

$$\delta_a = F_a = 0, \quad (3.22)$$

which just redefines δ^A, F^A . The field redefinitions in Section ?? were chosen to eliminate terms linear in the heavy modes U , neglecting terms in $\mathcal{L}_Q$ that are proportional to the squared YM field strengths. When $\mathcal{G}_c$ -charged fields condense, the term quadratic in $\mathcal{W}_c^\alpha$ becomes a contribution to the potential proportional to $|u|^2$ with couplings to the U 's, including, in particular, a coupling linear in the $U(1)_X$ gauge potential U_X . Thus if $S_0(\tilde{U})$, with $\tilde{U} = U + \Delta$, represents the effective action at the $U(1)_a$ -breaking scale, we require $\delta S_0/\delta U|_{U=\Delta=0} = 0$. At the condensate scale the effective action becomes $S_c(\tilde{U}) = S_0(\tilde{U}) + \Delta S(\tilde{U})$, where ΔS is the condensate contribution. The condition that there be no terms linear in the heavy fields U now reads $\delta S_c/\delta U|_{U=0, \tilde{U}=\Delta} = 0$; it is automatically satisfied if we minimize the effective theory with respect to Δ keeping $U = 0$. However the shifts needed to go to unitary gauge are slightly modified. While we impose the constraints (??) on the zeroth order vacuum values, here denoted by k^A :

$$k^A = |C_A|^2 e^2 \sum_a q_A^a h_a, \quad \sum_A q_A^a k^A = \frac{\delta_{aX} \delta_X L}{2}, \quad (3.23)$$

we use the true vacuum values x^A :

$$x^A = k^A e^{\Delta^A + \bar{\Delta}^A + 2 \sum_a q_A^a \Delta_a} = k^A [1 + O(\Delta)], \quad (3.24)$$

in the conditions (??) for going to unitary gauge. We have

$$\sum_A q_A^a x^A = \frac{\delta_{aX} \delta_X L}{2} + \sum_A q_A^a k^A (\Delta^A + \bar{\Delta}^A) + 2 \sum_b N^{ab} \Delta_b + O(\Delta^2), \quad (3.25)$$

using the notation introduced in (??). Now we have a theory defined by

$$\begin{aligned} K &= k + \sum_A x^A + G, \\ \tilde{L} &= \hat{E} [2Ls - \delta_X L (\hat{G}_X + h_X + \Delta_X) + bLG - 3] \equiv E \left[2LS(L, \Delta) + L \sum_I b_I g^I - 3 \right], \\ K' &= k' + 2 \sum_{A,a} h'_a q_A^a x^A = -2LS' + \delta_X \Delta_X / 2 + 2 \sum_{a,A} q_A^a h'_a k^A (\Delta^A + \bar{\Delta}^A) + O(\Delta^2), \end{aligned} \quad (3.26)$$

where we used [?] the relations,¹⁶ with N_{ab} defined in (??),

$$2 \sum_a h'_a N_{ab} = \delta_{bX} \frac{\delta_X}{2}, \quad 2h'_a = N_{aX}^{-1} \frac{\delta_X}{2}, \quad \frac{\delta_X}{2} \sum_a N_{aX}^{-1} x^A q_A^a = 2x^A \sum_a q_A^a h'_a, \quad (3.27)$$

that follow from the L -derivative of (??), and prime denotes differentiation with respect to L . Thus we have to perform a further Weyl transformation to eliminate noncanonical Einstein terms of order Δ . When we eliminate the auxiliary fields in the standard way we will obtain expressions for $D_a(\hat{\ell}, x^A) \sim \Delta$, and the full scalar potential found below takes the form

$$V = \frac{s}{2} \sum_a D_a^2(\hat{\ell}, x^A) + \frac{|u(\hat{\ell}, t^I, x^A)|^2}{16} [w(x^A) + v(\hat{\ell}) + O(\delta)]. \quad (3.28)$$

Since, as we shall see below, $\partial u / \partial \delta^A \sim u$, $\partial D_a / \partial \delta^A$, $\partial w / \partial \delta^A \sim 1$, the minimization equations for δ^A imply $D_a \sim \Delta \sim |u|^2 \ll 1$ in reduced Planck mass units, and we need only keep terms up to order Δ^2 in the Weyl transformation. The details of this transformation are given in Appendix ??, following the procedure described in Appendix B of [?]. Working in the new Weyl basis, we obtain for the nonderivative part of the bosonic Lagrangian

$$\begin{aligned} e^{-1} \mathcal{L}_B &= \sum_I \frac{1 + b_I \hat{\ell}}{(t^I + \bar{t}^I)^2} \bar{F}^I F^I - \frac{1}{16\hat{\ell}} \left[\partial_{\hat{\ell}} \hat{K} \bar{u} u - 4e^{\hat{K}/2} \partial_{\hat{\ell}} \hat{K} \hat{\ell} (W \bar{u} + u \bar{W}) \right] + \sum_{AB} \tilde{K}_{A\bar{B}} \bar{F}^{\bar{B}} F^A \\ &\quad + \frac{1}{9} (\hat{\ell} \partial_{\hat{\ell}} \hat{K} - 3) \left[\bar{M} M - \frac{3}{4} \left\{ \bar{M} (b'_c u - 4W e^{\hat{K}/2}) + \text{h.c.} \right\} \right] - \frac{b'_c}{8} \partial_{\hat{\ell}} \hat{K} \bar{u} u \\ &\quad + \left\{ \frac{u}{4} \left[\sum_{\alpha} b_c^{\alpha} \frac{F^{\alpha}}{\pi^{\alpha}} + \sum_I \left[\frac{b'_c - b_I}{2 \text{Ret}^I} - 2b_c^I \zeta(t^I) \right] F^I - b'_c \sum_A \hat{K}_A F^A \right] + \text{h.c.} \right\} \\ &\quad + \frac{1}{8} L_c (F_c - u \bar{M} + \text{h.c.}) + \frac{1}{2} \hat{S}_A (\bar{u} \bar{F}^{\bar{A}} + u F^A) + \sum_a \left(\frac{s}{2} D_a^2 + \tilde{K}_a D^a \right) \\ &\quad + e^{\hat{K}/2} \left[\sum_I F^I (W_I + \hat{K}_I W) + \sum_{\alpha} F^{\alpha} W_{\alpha} + \sum_A F^A (W_A + \hat{K}_A W) + \text{h.c.} \right], \\ L_c &= 2\hat{S} + b'_c \ln(e^{2-\hat{K}} \bar{u} u) + \sum_{\alpha} b_c^{\alpha} \ln(\pi^{\alpha} \bar{\pi}^{\alpha}) + \sum_I [b_I g^I - 2b_c^I \ln |\eta(t^I)|^2], \end{aligned} \quad (3.29)$$

where hatted variables refer to the new Weyl basis in which the Einstein term is canonically normalized, $\zeta(t) = \partial \ln \eta(t) / \partial t$ and

$$\begin{aligned} \hat{K}(\hat{\ell}, t, \delta^A) &= \hat{K}|, \quad \hat{S}(\hat{\ell}, \delta^A) = \hat{S}|, \quad \partial_{\hat{\ell}} \hat{K} = \partial \hat{K} / \partial \hat{\ell}, \quad \hat{S}^A = \partial \hat{S} / \partial \delta^A, \quad \tilde{K} = \hat{K} + 2L\hat{S}, \\ \tilde{K}_a &= \frac{1}{2} \frac{\partial \tilde{K}}{\partial \Delta_a} \Big|, \quad \tilde{K}_A = \frac{\partial \tilde{K}}{\partial \delta^A}, \quad \tilde{K}_{A\bar{B}} = \frac{\partial \tilde{K}}{\partial \delta^A \partial \bar{\delta}^{\bar{B}}}, \quad W_A = \frac{\partial W}{\partial \delta^A}, \quad \text{etc.} \end{aligned} \quad (3.30)$$

¹⁶A factor $x^A q_A^b$ is missing from the next to last expression in Eq.(3.29) of [?].

The equations of motion for the auxiliary fields give

$$\begin{aligned}
F^\alpha : \quad & \pi^\alpha e^{\hat{K}/2} W_\alpha + \frac{u}{4} b_c^\alpha = 0 = e^{\hat{K}/2} W + \frac{u}{4} (b_c - b'_c), \\
F^c : \quad & \bar{u} u = e^{-2b'_c/b_c} e^{\hat{K}-2\hat{S}/b_c} \prod_\alpha |b_c^\alpha/4W_\alpha|^{-2b_c^\alpha/b_c} \prod_I (2\text{Ret}^I)^{b_I/b_c} [|\eta(t^I)|^4]^{b_c^I/b_c}, \\
M : \quad & M = \frac{3}{4} u b_c = 3m_{\tilde{G}}, \\
F^I : \quad & F^I = -\frac{2\text{Ret}^I}{1+b_I\hat{\ell}} \frac{\bar{u}}{4} \left[b_c - b_I + 2\text{Ret}^I \left(\sum_\alpha b_c^\alpha \frac{\partial}{\partial \bar{t}^I} \ln \bar{W}_\alpha - 2\bar{\zeta}(\bar{t}^I) b_c^I \right) \right], \\
F^A : \quad & F^A = -\frac{\bar{u}}{4} \sum_B \tilde{K}^{AB} \left[2\hat{S}_{\bar{B}} - \sum_\alpha b_c^\alpha \frac{\partial}{\partial \bar{\delta}^{\bar{B}}} \ln \bar{W}_\alpha - \hat{K}_{\bar{B}} b_c \right], \\
D^a : \quad & D_a = -\frac{1}{s} \tilde{K}_a, \tag{3.31}
\end{aligned}$$

where $m_{\tilde{G}}$ is the gravitino mass. Invariance under modular and $U(1)_a$ transformations requires that the part of W_α that is nonvanishing in the vacuum takes the form¹⁷

$$W_\alpha = c_\alpha \prod_I [\eta(t^I)]^{2(q_I^\alpha + p_I^\alpha - 1)} \prod_A (\phi^A)^{q_\alpha^A}, \quad \sum_A q_A^a q_\alpha^A = -q_\alpha^a, \quad p_I^\alpha = \sum_A q_I^A q_\alpha^A. \tag{3.32}$$

Note that once the $U(1)$'s are broken, the Lagrangian is still gauge invariant (before one picks a gauge), but the symmetry is nonlinearly realized. In unitary-gauge the anomaly matching conditions read:

$$\begin{aligned}
\sum_\alpha b_c^\alpha q_I^\alpha &= \sum_B b_c^\alpha n_\alpha^B q_I^B = \sum_B \frac{C_c^B}{4\pi^2} q_I^B = b_I - b'_c - b_c^I, \quad \sum_\alpha b_c^\alpha = b_c - b'_c, \\
\sum_\alpha b_c^\alpha q_\alpha^a &= \sum_B b_c^\alpha n_\alpha^B q_B^a = \sum_B \frac{C_c^B}{4\pi^2} q_B^a = -\frac{1}{2} \delta_X \delta_{aX}, \quad b_I = b + \delta b_I. \tag{3.33}
\end{aligned}$$

Writing

$$|\phi^A| = \sqrt{x^A} e^{-G^A/2 - \sum_a q_a^A h_a}, \tag{3.34}$$

we have

$$\prod_A |\phi^A|^{-2 \sum_\alpha b_c^\alpha q_\alpha^A} = \exp \left(- \sum_A p_A \ln x^A + \sum_I p_I g^I - \delta_X h_X \right), \tag{3.35}$$

where we define

$$p_I = \sum_\alpha b_c^\alpha p_I^\alpha, \quad p_A = \sum_\alpha b_c^\alpha q_\alpha^A, \tag{3.36}$$

¹⁷There may be additional factors of modular invariant holomorphic functions $f(T^I)$ that we are ignoring here.

and we used the second anomaly matching condition in (??) which implies

$$2 \sum_A q_A^a p_A = 2 \sum_{\alpha, A} b_c^\alpha q_\alpha^A q_A^a = -2 \sum_\alpha b_c^\alpha q_\alpha^a = \delta_X \delta_{aX}. \quad (3.37)$$

Then we obtain

$$\begin{aligned} \bar{u}u &= e^{-2b'_c/b_c} e^{\kappa-2(\hat{S}-\delta s)/b_c} \prod_\alpha |b^\alpha/4c_\alpha|^{-2b^\alpha/b_c} \prod_I \left[2\text{Ret}^I |\eta(t^I)|^4 \right]^{(b_I-b_c+p_I)/b_c} e^{-\sum_A p_A \ln x^A/b_c}, \\ F^I &= -\frac{2\text{Ret}^I}{1+b_I\hat{\ell}} \frac{\bar{u}}{4} (b_c - b_I - p_I) \left[1 + 4\text{Ret}^I \zeta(\bar{t}^I) \right], \\ F^A &= -\frac{\bar{u}}{4} \sum_B \tilde{K}^{AB} \left[2\hat{S}_B - p_B - \hat{K}_B b_c \right], \quad \kappa = \hat{K} - G. \end{aligned} \quad (3.38)$$

Note that the factor $\langle e^{-2(\hat{S}-\delta s)/b_c} \rangle = \langle e^{-2s(\hat{\ell})/b_c} \rangle + O(\delta) = e^{-2/g_s^2 b_c} + O(\delta)$ has the standard dependence on the β -function at the condensate scale $\Lambda_c^6 = \langle u\bar{u} \rangle$. The auxiliary fields F^A are evaluated in Appendix ??; the full potential takes the form

$$\begin{aligned} V &= \frac{1}{2s} \sum_a \tilde{K}_a^2 + \frac{|u|^2}{16} v(\hat{\ell}, \delta) + \sum_I \frac{1+b_I\hat{\ell}}{(t^I + \bar{t}^I)^2} \bar{F}^I F^I, \\ v(\hat{\ell}, \delta) &= \frac{\partial_{\hat{\ell}} \hat{K}}{16\hat{\ell}} (1+b_c\hat{\ell})^2 - 3b_c^2. \end{aligned} \quad (3.39)$$

In the remainder of this section we consider aspects of this potential as well as the modular fermion masses.

3.3 The moduli potential

The potential is modular invariant, with a similar t -dependence as in [?], so the moduli are still stabilized at self-dual points $t_1 = 1$, $t_2 = e^{i\pi/6}$, with $\langle F^I \rangle = 0$. The T-moduli masses are determined by the coefficients of F^I in (??). Setting¹⁸

$$t^I = \frac{1}{\sqrt{2}} (\tau^I + ia^I), \quad (3.40)$$

we obtain

$$\begin{aligned} m_{\tau,a}^I &= \frac{|(b_I + p_I - b_c)u|\mu_{\tau,a}^I(t^I)|}{4(1+b_I\hat{\ell})} = \frac{|(b_I + p_I - b_c)|\mu_{\tau,a}^I(t^I)|}{b_c(1+b_I\hat{\ell})} m_{\tilde{G}}, \\ \mu_\tau^I(t^I) &= -8\text{Ret}^I \left[\zeta(t^I) + \text{Ret}^I \zeta'(t^I) \right], \quad \mu_a^I(t^I) = -8 \left(\text{Ret}^I \right)^2 \zeta'(t^I), \\ \mu_\tau^I(t_1) &\approx 2.56, \quad \mu_a^I(t_1) \approx .56, \quad \mu_\tau^I(t_2) \approx 3.02, \quad \mu_a^I(t_2) \approx 1.02. \end{aligned} \quad (3.41)$$

¹⁸The value of m_t quoted in [?] is the average mass for $t = 1$ in the approximation $\eta(1) \approx e^{-\pi/12}$.

where $\tilde{G}$ is the gravitino, and here we neglect $O(\delta)$ corrections. In the absence of an anomalous $U(1)$, $p_I = 0$, $b_I = b$. This was the case studied in [?] using $b = b_{E_8} \approx 10b_c$, yielding a welcome mass hierarchy, $m_\tau/m_{\tilde{G}} \approx 30$, $m_a/m_{\tilde{G}} \approx 6$ –10, thereby evading cosmological difficulties that arise if there are many moduli degenerate with the gravitino. However a large ratio b/b_c is not generic to models with gauge symmetry breaking by Wilson lines. For example in the FIQS model, [?] considered below, $b = b_c$, which would give massless T-moduli without the additional contributions in (??). Using (??), the contribution,

$$\delta b_I = -\delta_X \hat{G}_X = -\delta_X \sum_A \hat{b}_{XA} q_I^A = \frac{\delta_X}{2} \sum_{A,a} N_{aX}^{-1} q_A^a x^A q_I^A, \quad (3.42)$$

reflects the shifts (??) in the modular weights, as in (??). We also have, using (??)

$$p_I = \sum_{\alpha,A} b_c^\alpha q_I^A q_A^\alpha = \sum_A q_I^A p^A = \frac{\delta_X}{2} \sum_A q_I^A p^A / \sum_B q_X^B p^B, \quad (3.43)$$

From the definition (??), one generally expects $b_I/p_I > 0$, and we can get a hierarchy between the moduli masses and the gravitino mass without requiring $b \gg b_c$ if $p_I \sim b_I \gg b_c$. For example, in minimal models with $n = m$ discussed at the end of Section ??

$$\delta b_I = p_I, \quad b_I + p_I = b + \delta_X q_I^X + p_I = b + 2p_I. \quad (3.44)$$

Specifically in the FIQS model introduced in the next subsection, with $b = b_c$, $p_I \approx 2b$, the T-moduli masses (??) are given by

$$\frac{m_\tau^I}{m_{\tilde{G}}} \approx \left\{ \begin{matrix} 10 \\ 12 \end{matrix} \right., \quad \frac{m_a^I}{m_{\tilde{G}}} \approx \left\{ \begin{matrix} 2 \\ 4 \end{matrix} \right., \quad \text{for } \langle t^I \rangle = \left\{ \begin{matrix} t_1 \\ t_2 \end{matrix} \right., \quad (3.45)$$

if $b_I \ell \ll 1$.

3.4 The dilaton potential

From the results of Appendix ??, the potential for ℓ at the condensation scale is, evaluated at the moduli self-dual points,

$$\begin{aligned} V &= \frac{1}{2s} \sum_a \tilde{K}_a^2 + \frac{|u|^2}{16} v(\ell) + O(\delta|u|^2) & v(\ell) &= w + \frac{k'}{\ell} (1 + b_c \ell)^2 - 3b_c^2, & \delta &\sim |u|^2, \\ w &= \sum_A w(k^A), & w(k^A) &= b_c^2 k^A + 2p_A b_c + \frac{p_A^2}{k^A} = \frac{(p_A + b_c k^A)^2}{k^A}. \end{aligned} \quad (3.46)$$

For this potential the positivity condition on v found in [?] is sufficient, but not necessary, since the other terms in V are positive-semi-definite. In fact we have to let v go negative to

cancel the cosmological constant if $F, D \neq 0$. Since $D \ll F$, we require $w + v \approx 0$. Since $w(k^A)$ is minimized at $k^A = p_A/b_c$,

$$w \geq 4b_cp, \quad p = \sum_A p_A = \sum_{A,\alpha} b_c^\alpha q_\alpha^A = \frac{1}{12\pi^2} \sum_{A,\alpha} C_c^A q_\alpha^A, \quad (3.47)$$

for the solution (??) of the anomaly constraints, giving the constraint

$$k'(1 + b_c\ell) \leq \ell(3b_c^2 - 4b_cp), \quad (3.48)$$

at the vacuum. The right hand side of (??) need not be positive. A negative vacuum value for k' would be unviable if k were actually the dilaton Kähler potential, because the dilaton metric would be proportional to k' , requiring $\langle k' \rangle > 0$. However since the actual Kähler potential is $\tilde{k}$, we have the weaker condition

$$\tilde{k}' > 0, \quad k' > -\delta k' = -\sum_A k'^A = -\delta_X \ell h'_X. \quad (3.49)$$

Together (??) and (??) require

$$\delta_X \ell h'_X + (3b_c^2 - 4b_cp) / (1 + b_c\ell) > 0. \quad (3.50)$$

If we neglect nonperturbative contributions, $k' = \ell^{-1}$, and (??) requires $\ell b_c > 1$, which is strong coupling since $\ell = g_s^2/2$ in this case, so we still need to include nonperturbative effects if we wish to stabilize the dilaton at weak coupling. However, unlike the models studied in [?, ?, ?], these contributions are not needed to stabilize the potential at very large ℓ , because the D-term grows with ℓ unless some $k^A \sim \ell$ also get large; but in this case some F -terms become large, so $V \sim \ell$ in the large coupling limit. This allows more freedom in the parameterization of the nonperturbative effects. The vacuum value $\langle \tilde{k}' \rangle$ is an important parameter for phenomenology. A small value increases the ratio $m_\ell/m_{\tilde{G}}$, where m_ℓ is the universal dilaton mass, and suppresses the universal axion coupling; both are welcome features for a viable modular cosmology. However small $\langle \tilde{k}' \rangle$ also suppresses gaugino masses, which at tree level are the same as those found in [?] with the substitution $k' \rightarrow \tilde{k}'$: $m_{\tilde{g}}/m_{\tilde{G}} \propto \langle \tilde{k}' \rangle$. This can become problematic since the squark and slepton masses are generally of order $m_{\tilde{G}}$ or larger.

If the only flat direction at the string scale corresponds to a minimal set $n = m$ of n chiral multiplet vev 's that break m $U(1)_a$'s, the conditions (??) and (??) have a unique solution:

$$\begin{aligned} k^A &= \ell p^A = \frac{\ell \delta_X}{2} Q_X^A, & \delta k' &= p = \sum_A p_A, \\ w &= \ell^{-1} p (1 + b_c\ell)^2 = \ell^{-1} \delta k' (1 + b_c\ell)^2. \end{aligned} \quad (3.51)$$

Then the vacuum condition (??) determines the vacuum value of the dilaton metric as

$$\ell^{-1}\tilde{k}' = \frac{3b_c^2}{(1+b_c\ell)^2}, \quad (3.52)$$

which is precisely the value found in [?], resulting in suppressed gaugino masses and axion coupling, and an enhanced dilaton mass. The latter two features are welcome, but there would be more phenomenologically viable parameter space available if the gauginos masses were, say, just a factor two larger. (Increasing the *vev* of $\tilde{k}'/\ell$ affects the dilaton and axion parameters only in proportion to its square root). However this would require adding a *negative* contribution to the potential. It is much easier to find additional *positive* contributions. This is simply a consequence of the positivity of the vacuum energy in global supersymmetry: negative contributions are necessarily connected to higher dimension operators of local supersymmetry. For example an extra superpotential $W^\phi = f(T)\prod_A\Phi^A$ gives a negative contribution: $V \ni -3e^K|W^\phi|^2$, but since we need $\langle W^\phi \rangle \leq 10^{-15}$, at least one Φ must have a very small *vev*, so there will be at least one larger F-term: $V \ni F^A F_A \gg e^K|W^\phi|^2$. As we note in the concluding section, loop corrections from nonrenormalizable couplings with large coefficients are not expected to significantly change this analysis.

If the vacuum is degenerate at the string scale ($n > m$), we may write

$$k^A = \ell p_A + y^A, \quad \sum_A q_A^a y^A = 0, \quad (3.53)$$

where the last equality assures that if (??) is satisfied, so is (??). Then at the condensation scale, the (approximate) vacuum values will be those that minimize $w(k^A)$ with respect to the y^A subject to the condition in (??). If $y^A = 0 \forall A$, the dilaton potential is identical to the minimal case, (??).

The conditions (??) are most easily implemented by separating out a minimal subset of the Φ^A with nonvanishing p_A :

$$k^A, \quad A = 1, \dots, n, \quad \rightarrow \quad (k^A, k^M), \quad A = 1, \dots, m, \quad M = 1, \dots, n - m. \quad (3.54)$$

Then defining Q_a^A as in (??), the constraint in (??) reads

$$y^A = -\sum_M \zeta_M^A y^M, \quad \zeta_M^A = \sum_a Q_a^A q_a^M, \quad (3.55)$$

and we may take the y^M as independent variables. We have (neglecting order δ terms)

$$\frac{\partial V}{\partial y^M} = \frac{\partial w}{\partial k^M} - \sum_A \zeta_M^A \frac{\partial w}{\partial k^A} = -(p_M/k^M)^2 + b_c^2 + \sum_A \zeta_M^A [(p_A/k^A)^2 - b_c^2]. \quad (3.56)$$

If $p_M = 0$, we require $k^M = y^m \geq 0$, and

$$\left. \frac{\partial V}{\partial y^M} \right|_{y=0} = b_c^2 + \sum_A \zeta_M^A (\ell^{-2} - b_c^2). \quad (3.57)$$

If $\ell b_c^2 < 1$ and $\sum_A \zeta_M^A \geq 0$, the minimum indeed corresponds to $y = 0$. However if $\sum_A \zeta_M^A < 0$, the minimum corresponds to a smaller w with $y \neq 0$. For example if $y^M = y'^A$, $q'^A = -q^A$, $\zeta_M^A = -\delta_M^A$, the minimum occurs for $y'^A = y^A = p_A (\ell + 1/\sqrt{2}b_c)$. The matrix N_{ab} is scaled by a factor $(k^A + y^A)/2\ell p_A = 3 + \sqrt{2}/(b_c\ell)$ with respect to the minimal case. In the approximation $b_c\ell \ll 1$, this gives

$$\begin{aligned} w &\approx \sqrt{2}b_cp(2 + \sqrt{2}), & \delta k' &= \delta_X \ell h'_X = \frac{1}{4}\delta_X^2 N_{XX}^{-1} \approx b_cp/\sqrt{2}, \\ \tilde{k}' &\approx \delta k - \ell w < 0. \end{aligned} \quad (3.58)$$

If $p_M \neq 0$ we have instead of (??)

$$\left. \frac{\partial V}{\partial y^M} \right|_{y=0} = \left(\sum_A \zeta_M^A - 1 \right) (\ell^{-2} - b_c^2). \quad (3.59)$$

In this case $y = 0$ is the minimum if and only if $\sum_A \zeta_M^A = 1$, for example $y^M = y_i^A$, $i = 2, \dots, N$, $n = Nm$, with $\vec{q}_i^A = \vec{q}^A$, $\zeta_M^A = \delta_M^A$. If $\sum_A \zeta_M^A > 0$, the minimum will in general shift slightly from $y = 0$. For example with a single $U(1)_X$ and two chiral superfields with $p_1 = p_2$, The minimum occurs for $y_2 = -q_1 y_1 / q_2 = q_1 / q_2 - 1$. The dilaton potential for these cases is not substantially different from the minimal case. On the other hand if $\sum_A \zeta_M^A < 0$, the situation is similar to the case with $p_M = 0$: the minimum occurs for larger k^A , with lower values for both the potential w and $\delta k'$ such that it becomes difficult to maintain $\tilde{k}' > 0$, *i.e.*, positivity of the dilaton metric.

To get an idea what values the various parameters might take, consider the FIQS model described in section 4.2 of [?], with the p_A along the $(S_1, S_2, S_3, S_6, S_8, Y_1)$ sector. In this sector $Q_3^F = 0$, $6Y = \frac{1}{3}Q_1^F - \frac{1}{2}Q_2^F + Q_4^F = 0$, where, with the $U(1)_a$ -charge sign conventions used here, the properly normalized charges are

$$q'_a = -\frac{1}{12\sqrt{3}}Q_a^F \times \begin{cases} \sqrt{2} & \text{for } a = 1, X \\ \sqrt{3} & \text{for } a = 2, 6, 7 \\ \sqrt{6} & \text{for } a = 3, 4, 5 \end{cases}. \quad (3.60)$$

Then

$$-Y = \sqrt{\frac{2}{3}}q'_1 - q'_2 + \sqrt{2}q'_4. \quad (3.61)$$

We define the independent $\vec{q}$'s that are nonzero in this sector and orthogonal to $\vec{Y}$ as $(\vec{q}^1, \vec{q}^2, \vec{q}^5, \vec{q}^6, \vec{q}^7, \vec{q}^X)$, with

$$\begin{aligned} q^1 &= \frac{1}{2\sqrt{11}} (\sqrt{3}q'_1 + 4\sqrt{2}q'_2 + 3q'_4) = -\frac{1}{12\sqrt{22}} (Q_1^F + 4Q_2^F + 3Q_4^F), \\ q^2 &= \frac{1}{2} (\sqrt{3}q'_1 - q'_4) = -\frac{1}{12\sqrt{2}} (Q_1^F - Q_4^F), \\ q^a &= q'_a, \quad a = 5, 6, 7, X. \end{aligned} \quad (3.62)$$

Then the matrices defined in (??) are

$$\begin{aligned} \mathbf{q}^T &= \frac{1}{18\sqrt{2}} \begin{pmatrix} 0 & 3\sqrt{11} & 3\sqrt{11} & -3\sqrt{11} & -3\sqrt{11} & 0 \\ 12 & -3 & -3 & -9 & -9 & 12 \\ 0 & 6 & -6 & 6 & -6 & 0 \\ -6\sqrt{2} & 0 & 6\sqrt{2} & 0 & -3\sqrt{2} & 3\sqrt{2} \\ 0 & -6\sqrt{2} & 6\sqrt{2} & 3\sqrt{2} & -3\sqrt{2} & 0 \\ 8\sqrt{3} & 8\sqrt{3} & 8\sqrt{3} & 8\sqrt{3} & 8\sqrt{3} & -4\sqrt{3} \end{pmatrix}, \\ Q^T &= \frac{\sqrt{2}}{6} \begin{pmatrix} 3/\sqrt{11} & 3 & 0 & -4\sqrt{2} & 2\sqrt{2} & \sqrt{3} \\ 6/\sqrt{11} & 0 & 3 & 2\sqrt{2} & -4\sqrt{2} & \sqrt{3} \\ 6/\sqrt{11} & 0 & -3 & 2\sqrt{2} & 2\sqrt{2} & \sqrt{3} \\ -12/\sqrt{11} & 0 & 6 & 2\sqrt{2} & 2\sqrt{2} & \sqrt{3} \\ -12/\sqrt{11} & 0 & -6 & 2\sqrt{2} & -4\sqrt{2} & \sqrt{3} \\ -18/\sqrt{11} & 6 & 0 & 8\sqrt{2} & -4\sqrt{2} & \sqrt{3} \end{pmatrix}. \end{aligned} \quad (3.63)$$

Then we get $p = \delta_X \sqrt{3/2} = 6p_A$. In this model the above states come in degenerate groups¹⁹ of 3, that we label by k_α^A , $\alpha = 1, 2, 3$ and set $k_\alpha^A = \ell p_A^\alpha$. If N of the p_A^α are nonvanishing we have $p = \sqrt{\frac{3}{2}}\delta_X = 6 \sum_{\alpha=1}^N p_A^\alpha$. We can also calculate the parameters relevant for T-moduli masses:

$$p_I = \sum_A q_I^A p_A = \frac{2}{3}p + (p_6^1, p_6^2, p_6^3), \quad \sum_\alpha p_6^\alpha = \frac{p}{6}, \quad (3.64)$$

with

$$\delta_X = \frac{3\sqrt{6}}{4\pi^2} = \sqrt{6}b_c = \sqrt{6}b \quad p = 3b. \quad (3.65)$$

In this model there are additional F-flat and D-flat directions associated with “invariant blocks” $\mathcal{B}$ of fields such that

$$\sum_{A \in \mathcal{B}} q_A^a + \sum_{M \in \mathcal{B}} q_a^M = 0. \quad (3.66)$$

¹⁹There is an additional three-fold degeneracy for $Y_{1,2,3}$.

It is clear that if we choose Φ^M that form invariant blocks with the Φ^A , at least some $\zeta_M^A < 0$. The conditions in (??) are met for the choice

$$\Phi^A = (S_1, S_2, S_3, S_6, S_8, Y_1), \quad \Phi^M = (S_4, S_7, Y_2, Y_3, S_5, S_9). \quad (3.67)$$

The numerical solution to the minimization equations with just one set of k^A, p^A nonzero gives $\langle \tilde{k}' \rangle < 0$. We expect this problem to be generic. We therefore restrict here²⁰ the class of viable models to be the minimal models,²¹ defined by models in which the potential $w(k^A)$ is minimized at $k^A = \ell p_A$. For example in the FIQS model if we set $p_A^\alpha \neq 0 \forall \alpha$, condensation can occur only if $k_\alpha^A \neq 0 \forall \alpha$, and it is likely that the directions $k^M \neq 0$ are no longer F-flat [?] so that the minimal scenario described above is viable. For these models the vacuum conditions $\langle V \rangle = \langle V' \rangle = 0$ reduce to

$$\tilde{k}'' = \frac{\tilde{k}'(1 - b_c \ell)}{\ell(1 + b_c \ell)} = \frac{3b_c^2(1 - b_c \ell)}{(1 + b_c \ell)^3}, \quad (3.68)$$

and condition for a local minimum $\langle V' \rangle > 0$ reads

$$\tilde{k}''' > \frac{2\tilde{k}'(2 - 2b_c \ell + b_c^2 \ell^2)}{\ell(1 + b_c \ell)^2} = \frac{6b_c^2(2 - 2b_c \ell + b_c^2 \ell^2)}{(1 + b_c \ell)^4} \equiv \mu_0^2. \quad (3.69)$$

The dilaton mass

$$m_\ell = \sqrt{\frac{2}{\tilde{k}'}} \mu = \sqrt{\frac{2}{3}} \frac{\mu}{b_c^2} (1 + b_c \ell)^2 m_{\tilde{G}}, \quad \mu^2 = \tilde{k}''' - \mu_0^2, \quad (3.70)$$

can be considerably larger [?] than the gravitino mass if $\mu \sim 1$.

If the Kähler potential has terms of order higher than quadratic in fields with large v_{ev} 's, the minimal form assumed in (??) may not be valid. As shown in Appendix ??, if $K = k + G + f(x^A)$, the dilaton potential takes the form (??) with, neglecting $O(\delta)$ corrections,

$$\begin{aligned} w &= \sum_{AB} K^{AB} (p_A + b_c K_A) (p_B + b_c K_B), \\ \delta_{aX} \ell \frac{\delta_X}{2} &= \ell \sum_A q_A^a p_A = \sum_A q_A^a K_A = \ell \sum_A q_A^a K'_A = 2\ell \sum_{ABb} q_A^a q_B^b h'_b K_{AB}, \\ \delta k' &= 2 \sum_{Aa} q_A^a h'_a K_A = h'_X \delta_X = -2\delta s'. \end{aligned} \quad (3.71)$$

²⁰If the dilaton metric goes through zero, one should rewrite the theory in terms of the canonically normalized field, in terms of which the zero of the metric becomes a singularity in the potential. It is not clear that there might not be some viable region of parameter space in this case.

²¹We have also assumed a minimal Kähler potential for matter fields, by which we mean that the matter field Kähler potential is the minimal one consistent with modular invariance, as given in (??); more general forms will be considered below.

For a minimal set we can use (??) to obtain from the constraints in (??)

$$\begin{aligned} Q_X^A \frac{\delta_X}{2} &= p_A = 2 \sum_{Bb} K_{AB} q_B^b h'_b, & \sum_B K^{AB} p_B &= 2 \sum_b q_A^b h'_b, \\ \sum_{AB} K^{AB} p_A p_B &= \delta_X \sum_{Ab} Q_X^A q_A^b h'_b = \delta_X h'_X. \end{aligned} \quad (3.72)$$

Then since $K_A = \ell p_A$ for minimal models we get $w = \delta k'$ and the dilaton potential is identical to that for the case of a minimal Kähler potential. In the general case the F-term is

$$\sum_{AB} F^A K_{A\bar{B}} \bar{F}^{\bar{B}} = w - \delta k' + O(\delta) \geq 0, \quad (3.73)$$

so a deviation from the minimal case can only give an additional positive contribution to the vacuum energy, making it difficult to maintain a positive dilaton metric with vanishing vacuum energy, as discussed above for the minimal case.

3.4.1 Modular fermion masses

The mass matrix for the fermion superpartners of the dilaton and moduli is given in (??). In the FIQS model, with $b_c^I = 0$ and $p_I \approx 2b_c$ and $b_I \approx 3b_c$ nearly independent of I , this reduces to the simpler form given in (??) and (??) in the case that all the moduli are stabilized at the same self-dual point: $t_1 = 1$ or $t_2 = e^{i\pi/6}$. In this approximation there are two linear combinations χ_b of the “T-modulini” that do not mix with the dilatino and that have approximately the same mass:

$$|m_{\chi_b}(t_1)| \approx \frac{8+6z}{1+3z} m_{\tilde{G}}, \quad |m_{\chi_b}(t_2)| \approx \frac{10+6z}{1+3z} m_{\tilde{G}}, \quad (3.74)$$

where $z = b_c \ell = .08\ell$ in the FIQS model. The third modulino χ_0 , which is approximately an equal admixture of the χ^I , mixes with the dilatino with via the mass matrix given in (??) with, in this model,

$$\begin{aligned} m_{\chi_0} &= m_{\chi_b} + m' \approx m_{\chi_b} + \frac{m_{\tilde{G}}}{1+3z}, & m_{\chi_\ell \chi^I} &= \sqrt{6} \frac{3+7z+z^2-z^3}{(1+z)^2(1+2z)\sqrt{1+3z}}, \\ m_{\chi_\ell} &\approx -\frac{1-13z+24z^2+7z^3+35z^4+54z^5+6z^6}{3z(1+z)^3(1+2z)} m_{\tilde{G}}. \end{aligned} \quad (3.75)$$

If $z = .11$, the mass eigenvalues are

$$|m_1| = 14.2 m_{\tilde{G}}, \quad |m_2| = 8.7 m_{\tilde{G}}, \quad (3.76)$$

for $\langle t^I \rangle = t_1$ or t_2 ; these numbers decrease monotonically with z for $z < 1$ dropping to 6.1, .6 at $z = .6$. Since the choice of the self-dual point at which the moduli are stabilized has a

minor effect on the masses, we expect the same results to hold if they are not all stabilized at the same point. All the masses in the dilaton/T-moduli sector decrease somewhat as z increases.

4 Observable sector scalar masses

Here we consider only the minimal models defined above. Chiral fields Φ^M with vanishing vev 's have Kähler potential

$$K(\Phi^M, \bar{\Phi}^{\bar{M}}) = K_{(M)} = x^M(\ell) = e^{G'^M + 2 \sum_a q_M^a h_a(\ell)} |\Phi^M|^2, \quad \ell = \ell(\hat{\ell}, \delta^A, x^N). \quad (4.1)$$

Terms bilinear in these fields appear in the potential only through the functional $\hat{S}$ and the effective Kähler potential $\tilde{K}$ in the canonically normalized Weyl basis; at lowest order in x^M, δ^A :

$$\tilde{K}_{(M)} = \hat{K}_{(M)} + 2L\hat{S}_{(M)} = K_{(M)} + 2LS_{(M)} = x^M(\ell), \quad S_M = 0, \quad 2\hat{S}_M = \partial_\ell x^M(\ell). \quad (4.2)$$

Since $\partial V / \partial \phi \propto \bar{\phi}$ vanishes in the vacuum, the mass matrix is diagonal:

$$m_M^2 = \left(\frac{\partial \tilde{K}}{\partial \phi^M \partial \bar{\phi}^{\bar{M}}} \right)^{-1} \frac{\partial V}{\partial \phi^M \partial \bar{\phi}^{\bar{M}}} = \left(\frac{\partial x^M}{\partial \phi^M \partial \bar{\phi}^{\bar{M}}} \right)^{-1} \frac{\partial V}{\partial \phi^M \partial \bar{\phi}^{\bar{M}}} = \frac{\partial V}{\partial x^M}. \quad (4.3)$$

The complex scalar masses are evaluated in Appendix ??:

$$m_M^2 = m_{\tilde{G}}^2 \left(1 + \zeta_M \frac{1 - z^2}{z^2} \right), \quad (4.4)$$

where

$$\zeta_M = \sum_a q_M^a Q_a = \sum_{a,A} q_M^a Q_a^A, \quad z = b_c \ell, \quad (4.5)$$

$m_{\tilde{G}} = b_c |u|/4$ is the gravitino mass, and the matrix Q_a^A is defined in (??). Unless $|\zeta_M| < z^2$, the D-term contribution to the scalar masses strongly dominates [?]-[?] over the supergravity contribution ($\mu_M = 1$ if $\zeta_M = 0$) for weak coupling ($z \ll 1$), as has been extensively discussed in the literature. Moreover positivity of Standard Model scalar squared masses with weak coupling would require $\zeta^M > 0 \forall M$ [?]; this is not a generic feature of orbifold compactifications (see, *e.g.* the FIQS example below). Therefore models that are viable in the weak coupling regime *a priori* require vanishing or very small values of $|\zeta_M|$ for the standard model particles.

As a concrete example, in the FIQS model discussed above we have

$$Q_a = \sum_A Q_a^A = \left(-\frac{9}{\sqrt{22}}, \frac{3}{\sqrt{2}}, 0, 4, -2, \sqrt{6} \right). \quad (4.6)$$

In the same model the left-handed $SU(2)$ doublets $Q = Q_L$ have

$$q_Q^a = \left(\frac{5}{2\sqrt{22}}, \frac{1}{2\sqrt{2}}, 0, 0, 0, 0 \right), \quad \zeta_Q = -\frac{3}{11}. \quad (4.7)$$

There are two candidates u_i for the left-handed anti-up quarks u^c , two candidates d_i for the left-handed anti-down quarks d^c , five candidates ℓ_i for the left-handed leptons ℓ^+ , four candidates $\tilde{G}_i$ for the Higgs doublet H_d , and five candidates G_i from which to choose the lepton doublets L and the Higgs doublet H_u . Each of these comes in a triplet of states with identical gauge quantum numbers. To avoid flavor-changing neutral currents we assume states with the same Standard Models gauge quantum numbers also have the same $U(1)_a$ charges. We obtain the smallest values of $|\zeta_M|$ with the identification

$$(Q, u^c, d^c, \ell^+, L, H_u, H_d)_L = (Q_L, u_2, d_2, \ell_i, G_j, G_k, \tilde{G}_{3 \text{ or } 4}), \quad i, j, k \neq 5. \quad (4.8)$$

With this choice we have for the left-handed anti-quarks:

$$\begin{aligned} q_u^a &= \left(\frac{1}{\sqrt{22}}, -\frac{1}{3\sqrt{2}}, 0, \frac{1}{6}, 0, -\frac{2}{3\sqrt{6}} \right), \quad \zeta_u = -\frac{10}{11}, \\ q_d^a &= \left(-\frac{1}{2\sqrt{22}}, \frac{1}{6\sqrt{2}}, \frac{1}{3\sqrt{2}}, 0, -\frac{1}{3}, -\frac{2}{3\sqrt{6}} \right), \quad \zeta_d = \frac{5}{11}. \end{aligned} \quad (4.9)$$

The fields G_i, ℓ_i , $i = 1, \dots, 4$, and $\tilde{G}_{3,4}$ have different $U(1)_a$ charges, but degenerate values of ζ_M :

$$\zeta_{\ell^+} = -\frac{7}{11}, \quad \zeta_{H_d} = \frac{2}{11}, \quad \zeta_{H_u} = \zeta_L = -\frac{13}{11}. \quad (4.10)$$

In the FIQS model we have $b_c \approx .08$, so an acceptable mass spectrum would require $\ell \approx 12$, much larger than the classical value $\ell = g_s/2 \approx 1/4$. Such a large value of ℓ suggests strong coupling in the hidden sector, a conclusion that will be revisited in Section 5. Note that the “best fit” [?, ?] to the model of [?] requires a smaller value $b_c \approx .03$, which would require still larger ℓ and/or smaller values of $|\zeta_M|$. However those analyses may be modified in the class of models considered here.

These conclusions are not significantly modified if we take a nonminimal form for the Kähler potential. Taking inspiration from the known form of the Kähler potential for the untwisted sector:

$$K_{untw}(\Phi, \bar{\Phi}) = -\sum_I \ln \left(1 - \sum_A x_I^A \right), \quad (4.11)$$

we assume a Kähler potential of the form

$$K = k + G + \sum_{\alpha} K^{\alpha}, \quad K^{\alpha} = -C_{\alpha} \ln \left[1 - C_{\alpha}^{-1} \left(\sum_A x_{\alpha}^A + \sum_M x_{\alpha}^M \right) \right]. \quad (4.12)$$

The scalar masses in this case are given in (??). The corrections due to the modification of the Kähler potential are subleading in the weak coupling limit. The general expression is rather complicated and we will just consider some illustrative examples for minimal sets of *vev*'s. First assume that the fields with large *vev*'s belong to the same set α , $C_{\alpha} = C$. In the FIQS model, the quarks Q_L are in the untwisted sector. Then from (??) not more than one Q_L can belong to the set α ; then none of them can if we require that they all have the same mass so as to avoid flavor changing neutral currents. If we assume that all of the Standard Model chiral multiplets have $\beta \neq \alpha$, we obtain ($\lambda = 1$ in the FIQS model)

$$\begin{aligned} \mu_{M\beta}^2 &= \frac{m_{M\beta}^2}{m_{\tilde{G}}^2} = 1 + \\ &\zeta_M^{\beta} \left(\frac{1}{z^2} - 1 + \frac{3\lambda(1+z)[2C - 3\lambda z]^2(1-z)}{(1+2z)(1+3\lambda z)} \right) \left(1 - \frac{3\lambda^2 z^2 (1+z)^2}{(C + 3\lambda z)^2} \right). \end{aligned} \quad (4.13)$$

If instead we have three copies of a minimal set with $p_{\alpha} = p/3$, we can assign each generation of standard model particles to one of these sets. Then consistency with (??) for the untwisted sector particles (*e.g.*, Q_L in the FIQS model) requires $C_{\alpha} = 1$, and we obtain, *e.g.*, for $\lambda = 1$,

$$\mu_{M\beta}^2 = \zeta_M^{\beta} \left(\frac{1}{z^2} - \frac{(1+z)}{(1+2z)} \right) - \frac{z(1+z)}{(1+2z)}. \quad (4.14)$$

These corrections to the scalar masses relative to the “minimal” Kähler potential case (??) are negligible except for $|\zeta_M/z| \sim 1$.

There are many other possible parameterizations, since the Kähler potential involving twisted sector fields is not known beyond the quadratic terms. It might be that a set α contains only fields with the same modular weights, as in (??). Alternatively the Kähler potential could be invariant under Heisenberg transformations on the untwisted sector fields:

$$\begin{aligned} K &= k + G_{un} + f(X^A), \quad X^A = |\Phi_{tw}^A|^2 e^{G_{un}^A}, \quad G_{un} = G + K_{untw} = \sum_I G_{un}^I, \\ G_{un}^A &= \sum_I q_I^A G_{un}^I, \quad G_{un}^I = g^I + f^I, \end{aligned} \quad (4.15)$$

where the Φ_{tw}^A are twisted sector fields, and $\exp G_{un}^I$ is a radius of compactification in string units. It has been argued [?, ?] that the effective supergravity theory from the weakly coupled heterotic string may allow a viable inflationary scenario if the Kähler potential is

of this form. In this case, the contribution to untwisted sector squared masses $m_{(MI)}^2$ from the terms evaluated in Appendix ?? is somewhat more complicated, but the corrections are still subleading with respect to (??). However under the ansatz (??) there is an additional contribution to the untwisted sector masses from the shift (??) and the expression (??) with now g^I replaced by G^I :

$$\delta\mu_{(MI)}^2 = \left[(\delta b_I + p_I)^2 / b_c^2 \right] [1 + O(z)], \quad (4.16)$$

with, *e.g.*, $(\delta b_I + p_I)/b_c \approx 6$ in the FIQS model; this would give a large positive contribution to μ_Q . (In the FIQS model discussed in the previous section, the only untwisted sector fields other than the $\mathcal{G}_c$ -charged fields are the quark doublets Q and u_2 , $\tilde{G}_1$, which are candidates for u_L^c , H_d , respectively; they have larger values of μ_M than the fields $u_2, \tilde{G}_{3 \text{ or } 4}$ that were identified with these states in the previous section.) If the GS term is also Heisenberg invariant, there is a further contribution

$$\delta\mu_{(MI)}'^2 = \left[(b - b_c)^2 / b_c^2 \right] [1 + O(z)]. \quad (4.17)$$

If instead the full moduli + matter Kähler potential couples to the GS term: $G \rightarrow K - k$ in (??), all squared masses for the x^M are shifted by (??); this expressions vanishes identically in the FIQS model but it is possible that effects such as these in generic models could make all the masses positive. Even so, the large scalar/gaugino mass hierarchy remains problematic although one-loop corrections [?] can significantly increase the gaugino masses in the presence of scalar couplings in the GS term.

5 Parameterizations of nonperturbative effects

It has been argued [?] on general grounds that the effective supergravity Lagrangian from compactified string theory receives corrections $\propto e^{-\beta/g_s}$, arising from string nonperturbative effects. This was indeed shown [?] to occur in an explicit compactification of the heterotic string. Even in the absence of these effects, one expects corrections $\propto e^{-\beta/g_s^2}$ from both string and field theory [?] nonperturbative effects.

To parameterize these effects, a simple form was assumed in [?, ?] for the functional²²

$$f(L) = 2Ls(L) - 1 = \sum_n a_n x^n e^{-x}, \quad x = \beta/\sqrt{L}, \quad (5.1)$$

²²This is not the most general ansatz since there can be different parameters in the exponents.

from which the functional $g'(L) = k'(L) - L^{-1}$ was inferred using the condition (??). Retaining just one or two terms in the sum (??), a fit to the vacuum conditions $v = v' = 0$, $g_s^2 \approx .5$, where found with order one values of $\beta, a_{0,1}$. However it might be more justified to start with a simple form for the functional $g(L)$:

$$g(x) = \sum_n A_n x^n e^{-x}, \quad (5.2)$$

since this parameterizes the corrections to the actual Kähler potential. The conditions that the functions $g, f \rightarrow 0$ in the weak coupling limit $L \rightarrow 0$ are met if one assumes both (??) and (??). Then the constraint (??) relates the coefficients by

$$nA_n - A_{n-1} + (2+n)a_n - a_{n-1} = 0. \quad (5.3)$$

If we also require that they are finite in the strong coupling limit $x \rightarrow 0$, we set $A_{n<0} = a_{n<0} = 0$. Then (??) is satisfied by

$$a_n = -\frac{n}{n+2}A_n + 2 \sum_{p=0}^{n-1} \frac{(p+1)!}{(n+1)!} A_p. \quad (5.4)$$

This requires $a_0 = 0$, which could not be imposed in the model of [?, ?] because in those papers the nonperturbative terms were needed to make the potential positive. This is not the case for the models considered here.

On the other hand, in the presence of nonperturbative corrections the true string coupling constant is $g_s = 1/\sqrt{s(\ell)} \neq \sqrt{2\ell}$, suggesting we should set $x = \beta\sqrt{s(L)}$ in (??) and (??). Moreover, if it is the various terms $\mathcal{L}_i$ in the *Lagrangian* $\mathcal{L} = \sum_i \mathcal{L}_i$ that receive corrections of the form $L_i \rightarrow (1 + h_i)\mathcal{L}_i$, with $h_i(x)$ a function of the form (??), one can show that this corresponds to a correction to the Kähler potential of the form

$$k = -\ln(2s) - \sum_i \alpha_i \ln(1 + h_i). \quad (5.5)$$

All parameterizations of the nonperturbative corrections are equivalent if these corrections are small, but in fact the vacuum condition (??) requires either large coupling or a significant correction to the classical Kähler potential $k = \ln(\ell)$. To see this note that the condition

$$k' + 2\ell s' = \tilde{k}' + 2\ell \tilde{s}' = 0 \quad (5.6)$$

implies

$$\frac{\partial k}{\partial s} = \frac{\partial \tilde{k}}{\partial \tilde{s}} = -2\ell, \quad \frac{\partial k}{\partial \ell} = \frac{\partial s}{\partial \ell} \frac{\partial k}{\partial s} = 4\ell \left(\frac{\partial^2 k}{\partial s^2} \right)^{-1}, \quad \kappa \equiv \frac{1}{\ell} \frac{\partial k}{\partial \ell} = 4 \left(\frac{\partial^2 k}{\partial s^2} \right)^{-1}, \quad (5.7)$$

with all relations holding for $k, s \leftrightarrow \tilde{k}, \tilde{s}$. In the classical limit we just have $k = -\ln(2s)$, $\kappa = \ell^{-2}$, so the vacuum condition $\kappa = 3rb_c^2/(1 + \ell b_c)^2$, $r \approx 1-4$, requires $\ell \sim b_c^{-1}/\sqrt{3r}$. The parameterization of nonperturbative effects in [?, ?] allowed a fit with $f \sim 1$, $\kappa \sim b_c^{-2} \ll \ell \sim 1$. In that parameterization $g_s = \sqrt{2\ell/(1+f)}$, $f > 0$, so $g_s \ll \ell$ requires $f \gg 1$, *i.e.*, nonperturbative effects must dominate the dilaton potential at the vacuum, which is rather implausible in the context of weak coupling. A different parameterization might allow $1/2s \ll \ell \sim b_c/\sqrt{3r}$ for moderate nonperturbative contributions, and thus some suppression of the scalar masses found in Section ??.

Suppose that for the observed value $s \approx 2$ of the coupling constant, the sum in (??) is dominated near the vacuum by a single term:

$$k = -\ln(2s) - \alpha \ln(1+h). \quad (5.8)$$

This gives, for $\alpha = 1$,

$$\ell = \frac{1}{2s} + \frac{h'}{2(1+h)}, \quad \kappa = 4 \left(\frac{1}{s^2} - \frac{h''}{(1+h)} + \frac{h'^2}{(1+h)^2} \right)^{-1}, \quad (5.9)$$

where here prime means differentiation with respect to s . If $h = \epsilon - 1 > -1$, $h' > 0$, one can get an enhancement of ℓ and a suppression of κ for small ϵ . For example if we take a monomial $h = -Ae^{-\sqrt{s}} < 0$, we have $h' > 2(1+h)$ if $3.5 < A < 4.1$. This gives $1.5 \leq \ell \leq 6.8$, $.51 \geq \kappa \geq .02$ for $3.6 \leq A \leq 4$. Alternatively if we take $h = -A\sqrt{s}e^{-\sqrt{s}} < 0$, we have $h' > 2(1+h)$ if $2.4 < A < 2.9$, and $1.6 \leq \ell \leq 5.9$, $.60 \geq \kappa \geq .03$ for $2.5 \leq A \leq 2.8$.

If canonical normalization of the Einstein term is imposed in the the dual formulation for the dilaton in terms of a chiral superfield S , the condition (??) arises from the solution to the equations of motion for L in the duality transformation:

$$2\text{Re}S = \int \frac{dL}{2L} k'(L). \quad (5.10)$$

When the GS term is included, this is modified to read

$$2\text{Re}S + V_{GS} = \int \frac{dL}{2L} k'(L), \quad V_{GS} = bG - \delta_X V_X. \quad (5.11)$$

When we shift V_X from its (L -dependent) *vev*, $S = s(L) \rightarrow \tilde{s}(L)$, and $k(L) \rightarrow \tilde{k}(L)$, up to terms of order δ and terms quadratic in the heavy fields that we integrate out. This suggests that the parameterization (??) should apply to $\tilde{k}, \tilde{s}$ rather than to k, s . The fact that the gauge coupling is really s rather than $\tilde{s}$ comes from a compensating term from field theory quantum corrections that appear to be unrelated to the duality transformation $L \leftrightarrow S + \bar{S}$.

The vacuum condition in the minimal models now reads

$$\tilde{\kappa} = \kappa + 3\ell^{-1}\lambda b_c = 3rb_c^2, \quad (5.12)$$

which requires $\kappa < 0$ unless $\lambda < rb_c\ell$, which for example in the FIQS model would require $\ell > 13/r$; for the favored value $b_c \approx .03$ in the analyses of [?, ?], this constraint would be more stringent. On the other hand the parameterization (??) of k requires positive κ in order to enhance ℓ and suppress $|\kappa|$. Therefore this parameterization seems to be viable only if it applies to $\tilde{k}$.

Another important parameter of the effective low energy theory is the scale μ_s of unification of the Standard Model gauge couplings. The boundary condition found in [?], that gives essentially the standard result [?] $\mu_s^2 \approx g_s^2/2e$ in the $\overline{MS}$ scheme, is changed due to modifications of the Kähler potential from both string nonperturbative and $U(1)_a$ -breaking effects. The string nonperturbative correction to μ_s^2 was found [?] to be less than 1% in the model of [?]. In the present case we have new corrections as well as a wider range of possible parameterizations for the string nonperturbative effects.

Making the appropriate modifications to Eq. (5.9) of [?], comparison with the renormalization group invariant [?] gives, following the analysis of [?], the two-loop renormalization group equations²³

$$\begin{aligned} g_a^{-2}(\mu) &= s - \frac{1}{16\pi^2}(C_a - C_a^\chi)\tilde{k}(\ell) - \frac{2}{16\pi^2} \sum_M C_a^M \ln[1 - P_M(\ell)] \\ &\quad + \frac{1}{16\pi^2}(3C_a - C_a^\chi) \ln(e\mu^2) - \frac{2C_a}{16\pi^2} \ln g_a^2(\mu) - \frac{2}{16\pi^2} \sum_M C_a^M \ln Z_M(\mu), \\ &= g_s^{-2} + \frac{C_a}{8\pi^2} \ln(e^{\tilde{k}} g_a^{-2}(\mu_s)) - \frac{1}{8\pi^2}(3C_a - C_a^\chi) \ln \frac{\mu_s}{\mu} + \frac{C_a}{8\pi^2} \ln \frac{g_a^2(\mu_s)}{g_a^2(\mu)} \\ &\quad + \frac{1}{8\pi^2} \sum_M C_a^M \ln \frac{Z_M(\mu_s)}{Z_M(\mu)}, \quad C_a^\chi = \sum_M C_a^M, \end{aligned} \quad (5.13)$$

where C_a^χ is the chiral matter quadratic Casimir and the Z_M are the renormalization factors for the matter fields Φ^M . In writing the last equality we made the identifications

$$\mu_s^2 = e^{\tilde{k}-1}, \quad s = g_s^{-2}, \quad Z_M^{-1}(\mu_s) = 1 - P_M(\ell), \quad (5.14)$$

²³Here we neglect moduli-dependent string threshold corrections to the gauge kinetic terms. The universal terms [?] drop out of the unification constraints. The nonuniversal terms (??) are absent in many quasi-realistic orbifold models such as the FIQS model, and in any case are small when the moduli are stabilized at their self-dual points as is the case here. The factor e relates [?] the scale λ of the external momentum ($-p^2 = \lambda^2$) to the scale μ of the $\overline{MS}$ scheme: $\lambda^2 = e\mu^2$.

where the functions $P_M(\ell)$ arise from the dilaton-dependent term in the reparameterization connection derived from the effective Kähler metric $\tilde{K} = K + 2\hat{L}S$; they depend on the $U(1)_a$ charges q_A^a and on the couplings of Φ^A in the GS terms. If we neglect two-loop running in the vicinity of coupling unification, this contribution drops out, and we may set $g_s^{-2}(\mu_s^2) = s$ in the second term on the RHS of (??). Then defining μ_{ab} by

$$g_a(\mu_{ab}) = g_b(\mu_{ab}), \quad (5.15)$$

we find in this approximation

$$\begin{aligned} \mu_{ab} &\approx e^{(\tilde{k}-1)/2} (e^{\tilde{k}} s)^{-\epsilon_{ab}} = \frac{g\lambda_{ab}}{2^{\epsilon_{ab}} \sqrt{e}}, \\ \lambda_{ab} &= (e^{\tilde{k}} 2s)^{\epsilon_{ab}}, \quad \epsilon_{ab} = \frac{1}{2} - \frac{C_a - C_b}{3C_a - C_a^\chi - 3C_b + C_b^\chi}. \end{aligned} \quad (5.16)$$

The parameters λ_{ab} measure the deviations from the classical string theory prediction without string nonperturbative and $U(1)_a$ -breaking effects: $\tilde{k} \rightarrow k \rightarrow \ln(2s)$. Using the MSSM values for the gauge and matter Casimirs:

$$(C_a, C_a^\chi) = (0, 6.6), (2, 7), (3, 6), \quad a = 1, 2, 3, \quad (5.17)$$

gives

$$\epsilon_{12} = \frac{1.5}{10.1}, \quad \epsilon_{23} = .25, \quad \epsilon_{31} = \frac{1.3}{6.6}. \quad (5.18)$$

Thus although some parameterizations of nonperturbative effects considered above have μ_s considerably larger than its classical value $g/\sqrt{2e}$, the effect on coupling unification is much less significant. Taking the most extreme of those cases (assuming, as argued above, that the parameterization (??) applies to $\tilde{k}, \tilde{s}$)

$$\begin{aligned} \tilde{k} &= -\ln(2\tilde{s}) - \ln(1 - 4e^{-\sqrt{\tilde{s}}}), \quad \tilde{s} = s - \frac{\delta_X}{4} \ln \ell \approx s \approx 2, \\ 2se^{\tilde{k}} &\approx (1 - 4e^{-\sqrt{\tilde{s}}})^{-1} \approx 36, \end{aligned} \quad (5.19)$$

we obtain

$$\lambda_{12} = 1.66, \quad \lambda_{23} = 2.45, \quad \lambda_{31} = 2.03. \quad (5.20)$$

As is well known, the classical prediction $\lambda = 1$ is in contradiction with experiment since the measured unification scale is lower than predicted by about an order of magnitude if the running is due only to MSSM particles. Orbifold models predict additional particle content with masses somewhere between the electroweak scale and the string scale, and many analyses [?, ?] have shown that these masses can be adjusted to reconcile the theory

with experiment. We expect that similar fits can be made in the present case, even with slightly higher “unification” scales, as in (??).

The scale of condensation $\Lambda_c = |u\bar{u}|^{\frac{1}{6}}$ and the gravitino mass $m_{\tilde{G}} = b_c u/4$ are also governed by the factor $e^\kappa = e^{\tilde{\kappa}} + O(\delta)$ in $u\bar{u}$ given in (??); in the “extreme” parameterization (??), $e^{\tilde{\kappa}} \approx 9$. However there is a much stronger ℓ -dependence that is unrelated to nonperturbative effects; this is the factor $\exp[-(\sum_A p_A \ln x^A)/b_c]$ in (??). In the minimal models studied here that are subject to the constraints (??), we have $x^A = \ell p^A$, so this contribution grows as ℓ decreases, suggesting that these scenarios are more viable if ℓ is considerably larger than its classical value of $g_s^2/2$. For example, taking the FIQS model with $g_s^2 = .5$, $t^I = 1$, $c_\alpha = 1$, gives a gravitino mass of 3000 TeV if $\ell = 6$, and this grows dramatically as ℓ decreases. Therefore a viable model in this class requires a smaller β -function, as was also found in studies [?, ?] of models without an anomalous $U(1)$, that were unable to accomodate the $SO(10)$ condensing gauge group of the FIQS model.

6 D-moduli masses

It has been pointed out [?] that there is generally a large degeneracy of the vacuum associated with $U(1)_a$ breaking, resulting in many massless chiral multiplets, that we call D-moduli, between the $U(1)_a$ -breaking and supersymmetry-breaking scales. Moreover, in the absence of superpotential couplings a number of these remain massless even after supersymmetry breaking. Here we show that couplings of the D-moduli to the matter condensates *via* the superpotential defined by (??) and (??) is sufficient to lift the degeneracy and give masses to the *real* parts of the D-moduli scalars as well as the fermions, while the imaginary parts of the scalars (“D-axions”) remain massless in the absence of other superpotential couplings. This remaining degeneracy may be at least partially lifted by D-moduli couplings to other unconfined, $\mathcal{G}_c$ -neutral chiral supermultiplets.

Since our purpose in this section is only to establish that D-moduli masses are generated, we will restrict our analysis to the simple case in which there are N minimal sets of chiral fields with the same $U(1)_a$ charges so that $k^A = \ell p^A$, and we assume the minimal Kähler potential (??). Among the $n = Nm$ chiral fields with $\langle \phi^A \rangle \neq 0$, there are $n - m = (N - 1)m$ chiral superfields

$$D^i = (\sigma^i, a^i, \chi^i), \quad (6.1)$$

that are the physical states orthogonal to the m eaten Goldstone bosons. The D^i are defined in (??) in terms of the chiral superfields $\hat{\Theta}^A$ introduced in (??). The relevant Lagrangian

for these fields is given in (??) and (??) of the appendix. Their masses

$$m_a = 0, \quad m_\sigma = \frac{\sqrt{1+z^2}}{1-4z} m_\chi = \frac{\sqrt{1+z^2}}{z} m_{\tilde{G}} \quad (6.2)$$

are generated by the F-terms associated with the superpotential (??), (??) and therefore satisfy the sum rule

$$m_a^2 + m_\sigma^2 = 2m_\chi^2 [1 + O(z)], \quad (6.3)$$

where the $O(z)$ corrections vanish in the limit of vanishing gravitino mass: $m_{\tilde{G}} \approx z m_\chi \rightarrow 0$ if $z \rightarrow 0$.

For example in the FIQS model discussed in Section ??, there are three identical sets Φ_α^A , $\alpha = 1, 2, 3$, defined as Φ^A in (??). If one of these sets, say Φ_1^A , has no couplings to the condensates, $p_1^A = 0$, then as shown in Section ?? the minimum of the potential will have $\langle \phi_1^A \rangle = 0$. Then the 6 corresponding fermions χ_1^A will remain massless, and the complex scalars will acquire masses given by (??) with $\zeta_{A_1} = 1$:

$$m_{A_1}^2 = \frac{m_{\tilde{G}}^2}{z^2}, \quad (6.4)$$

while those Φ_α^A with nonvanishing p_α^A and nonvanishing vev 's will have masses as in (??). It was argued that Section ?? that it may be necessary for all the Φ^A to have nonvanishing p^A in order to stabilize the vacuum against otherwise dangerous flat directions in the space of the superfields Φ^M in (??). In this case the ϕ^M as well the scalar components of other superfields that form invariant blocks [?] with the Φ^A all acquire squared-mass terms of the form (??) with

$$\zeta_{S_4} = \zeta_{S_7} = \zeta_{Y_2} = \zeta_{Y_3} = -2, \quad \zeta_{S_5} = \zeta_{S_9} = \zeta_{S_{10}} = \zeta_{S_{11}} = 1. \quad (6.5)$$

There are three copies each of the superfields S_i and nine copies of the Y_i . Since S_4 and Y_2 have the same $U(1)$ charges, the D-term potential for (S_4, S_7, Y_2, Y_3) at the condensation scale has an approximate²⁴ $SO(24) \times SO(18) \times SO(6)$ invariance, resulting in 33 (approximately) massless (pseudo) Goldstone bosons when linear combinations of these fields acquire vev 's, if there is no other source for their masses. The positive squared masses are safely large for weak coupling, $z \ll 1$. However if we try to make the observable sector viable in this model by substantially increasing z , we would have an additional 27 scalars with masses

²⁴The symmetry is reduced to $[SO(6)]^8$ when the differences in modular weights is taken into account, and could be further reduced by the choice of Kähler potential; these effects involve higher dimensional couplings and should generate very small masses.

uncomfortably close to the gravitino mass. In any case there would be 30 massless complex Weyl fermions. However there is a possible source of much larger masses for at least some of these superfields. Although terms trilinear in the three fields that form each invariant block are forbidden [?], supersymmetric masses of order $\langle \phi^A \rangle^4 / m_P^3$ for S_4, S_7, Y_2, Y_3 would be generated if the superpotential includes quadratic terms in the invariants

$$(S_4 S_3 Y_1), \quad (S_7 S_2 Y_1), \quad (Y_2 S_3 Y_1), \quad (Y_3 S_6 Y_1), \quad (6.6)$$

thus possibly eliminating 3/5 of the troublesome light states in this scenario. Cosmological issues associated with massless and TeV-scale fermions were discussed in [?].

7 Conclusions and future directions

We have studied a class of models based on the weakly coupled heterotic string with an anomalous $U(1)$ and supersymmetry breaking by condensation in a strongly coupled hidden gauge sector. In contrast to the models [?, ?] previously considered without an anomalous $U(1)$, dilaton stabilization is assured by the presence of D-terms, but string nonperturbative corrections to the dilaton Kähler potential are still needed to stabilize the dilaton at weak coupling. Several promising features of the earlier studies persist: enhancement of the dilaton and T-moduli masses relative to the gravitino mass, masslessness of the universal axion and a suppression of its coupling constant, dilaton mediated supersymmetry breaking that avoids potential problems with flavor changing neutral currents.

However some new difficulties arise unless the observable sector is uncharged under the broken $U(1)_a$'s [or its charges are orthogonal to the inverse charges of the fields with large vev 's: $\zeta_M = 0$ in (??)]. As noted in earlier studies [?]-[?], [?], there is considerable tension in maintaining *i*) a vanishing cosmological constant, *ii*) a positive dilaton metric and *iii*) positive and acceptably small scalar masses in the observable sector on the one hand, while requiring *iv*) weak coupling and *v*) acceptably large D-moduli/fermion masses on the other hand. Some of this tension might be attenuated by relaxing the requirement of a vanishing cosmological constant. However if one invokes an unknown mechanism to cancel the cosmological constant there is no reason to assume that it will not also contribute to scalar masses, making any predictions meaningless. Moreover, as discussed in the text, one needs a *negative* contribution to the vacuum energy, which is very hard to achieve, except by simply adding a constant to the superpotential. Such a term could arise from the vev of the three-form of ten-dimensional supergravity: [?] $\langle \int d\sigma^{lmn} H_{lmn} \rangle \neq 0$. Such a contribution has been considered in the past but

was abandoned in the weak coupling context after the realization that this *vev* is quantized in Planck scale units [?]. On the other hand it has been pointed out [?] that quadratically divergent quantum corrections may induce a significant contribution to the cosmological constant. These were calculated in [?] for an arbitrary supergravity theory (with at most two space-time derivatives at tree level), giving for the leading terms in the number N, N_G of chiral and gauge supermultiplets, respectively:²⁵

$$\langle V_{\text{quad}} \rangle \ni \left[N_G \left(\langle V_{\text{tree}} \rangle - m_{\tilde{g}}^2 \right) + N \left(m_{\tilde{G}}^2 - \frac{g_s^2}{2} \sum_a \langle D_a^2 \rangle \right) \right] \frac{\Lambda^2}{16\pi^2}. \quad (7.1)$$

For the class of standard-like Z_3 orbifold models studied in [?], we have

$$N \geq 3N_G + 223, \quad (7.2)$$

suggesting, since $|D_a| \ll |m_{\tilde{g}}| < |m_{\tilde{G}}|$, that this contribution, dominated by the term proportional to $Nm_{\tilde{G}}^2$, is always positive and of the same order as tree level terms if $\Lambda \sim 1$ in reduced Planck units. However, when the theory is regulated in a way that preserves local supersymmetry, the cut-off Λ^2 is replaced by

$$\Lambda_{eff}^2 = 2 \sum_i \eta_i m_i^2 \ln m_i^2, \quad (7.3)$$

where η_i is the signature and m_i is the mass of a Pauli-Villars (PV) regulator superfield (and additional terms quadratic in the PV masses are generated). It was shown in Appendix C of [?] that Λ_{eff}^2 has an indeterminate sign if there are four or more terms in the sum over the PV fields that regulate any one contribution to the quadratically divergent part of the one-loop effective action. Cancellation of all UV divergences in realistic string-derived supergravity models requires [?] at least 5 PV chiral supermultiplets for each chiral supermultiplet of the low energy theory and 51 PV chiral superfields for each light gauge superfield, as well as gauge singlet PV chiral superfields and Abelian PV vector superfields. This proliferation of regulator fields is not surprising, since the PV contributions parametrize those from infinite towers of string and Kaluza-Klein modes of the underlying string theory. It might therefore seem reasonable to include [?] an arbitrary constant of order $m_{\tilde{G}}^2$, which need not be positive, in the effective potential at the condensation scale. However the terms in (??), together with their supersymmetric completion, respect supersymmetry only to one-loop order; they are the $O(\epsilon = \Lambda^2/16\pi^2)$ corrections to the potential due to a shift [?] $K \rightarrow K + \epsilon \Delta K$ in

²⁵The result quoted in [?] does not take into account the quadratically divergent renormalization of the Einstein term. The Weyl transformation necessary to put the Einstein term in canonical form gives additional corrections to the potential.

the Kähler potential. If the coefficient of the correction is large enough to be important, it must be retained to all orders in evaluating the effective Lagrangian. Then it can be shown that the effect on the vacuum energy is negligible [?].

The generically large scalar-to-gaugino mass ratio might be reduced by introducing more general forms of the Kähler potential for chiral superfields, and/or including couplings of matter superfields in the GS term. Our current predictive power in this respect is unfortunately limited by uncertainties in our knowledge of the string-scale couplings. In addition, the scalar masses can be reduced relative to the gravitino mass by increasing the *vev* of the dilaton ℓ . Larger ℓ for fixed g_s is also favored by requiring that the gravitino mass and condensation scale be sufficiently low. While this suggests strong coupling, we showed that with the increased flexibility in the parameterization of string nonperturbative effects in the presence of D-terms, ℓ can be considerably larger than its classical value $g_s^2/2$ while maintaining weak coupling: $g_s^2 \approx 1/2$. However either mechanism for reducing the scalar-to-gaugino mass ratio also reduces the D-moduli masses for fixed gaugino masses.

The weakly coupled heterotic string theory that we are using can be obtained as the limit of zero separation between the hidden and observable ten-dimensional boundaries of (suitably compactified generalizations of) the Hořava-Witten (HW) scenario [?]. One might consider relaxing this strict weak coupling limit by allowing a small separation between the ten-dimensional boundaries of the hidden and observable effective supergravity theories. Besides generating different corrections to the coupling constants of these two sectors, it is conceivable that some tuning of 11d moduli might allow for a suppression of the overlap factor ζ_M that governs observable sector masses. However we do not expect either of these effects to be significant unless we approach the very strongly coupled HW limit, where we cannot use perturbative results and instead have to appeal to 11-d supergravity to extract data. On the other hand, 11-d supergravity-based calculations [?]-[?] show that one gets an effective 4d supergravity theory very similar to those we are studying. As a consequence, our results can easily be extended to effective 4d descriptions of the strongly coupled heterotic string, since the features of our effective field theory are general enough to accommodate scenarios that occur in that context, to some level of approximation. A drawback to this approach is the greatly weakened predictive power with respect to the case of the weakly coupled heterotic string, since one does not have available the genus zero conformal field theory calculations of the massless spectrum and couplings.

Several other avenues for future investigation suggest themselves. In general left-handed fermions and anti-fermions have different $U(1)_a$ -charges resulting in different masses for their scalar superpartners; possible constraints on these mass differences from precision elec-

troweak data need to be investigated. Analyses [?, ?] of the model of [?] regarding a viable vacuum at the electroweak symmetry breaking scale, neutralinos as dark matter, a viable inflationary scenario and the Affleck-Dine mechanism for baryogenesis need to be revisited with the FI D-term included. Indeed the inflationary scenario of [?] evoked such a term for dilaton stabilization during inflation. Including $U(1)_a$ breaking provides new possibilities, such as D-axions as possible candidates for quintessence and a see-saw mechanism for neutrino masses if the right handed neutrinos acquire masses through Yukawa couplings at the $U(1)_a$ -breaking scale. There are also possible new mechanisms for proton decay [?]. We have used the FIQS model [?] as a benchmark to illustrate what might be representative numerical results in a realistic model. However this model cannot reproduce the observed Standard Model Yukawa textures [?], and in the present context gives implausibly large values for $m_{\tilde{G}}$, and Λ_c , as well as an unacceptable pattern of soft supersymmetry breaking scalar masses in the observable sector. The most extensively studied models [?, ?] have $SU(3) \otimes SU(2) \otimes U(1)^5$ as the gauge group in the observable sector. One might consider models in which the Standard Model is embedded in a larger gauge group that is broken to the Standard Model, at a scale considerably larger than the gravitino mass, by the v_{ev} of some field whose squared mass is driven negative at the condensation scale. This could give viable observable sector scalar masses provided $\zeta_M \ll 1$ for squarks and sleptons, or if these particles are quasi-Goldstone bosons of an approximate global symmetry that is broken at the same scale.

Acknowledgments

We wish to thank Pierre Binétruy, Brent Nelson and Erich Poppitz for helpful discussions. This work was supported in part by the Director, Office of Science, Office of High Energy and Nuclear Physics, Division of High Energy Physics of the U.S. Department of Energy under Contract DE-AC03-76SF00098, in part by the National Science Foundation under grants PHY-0098840 and INT-9910077, and in part by the National Science and Engineering Research Council of Canada.

Appendices

A Weyl transformations

In this appendix we work out the Weyl transformations necessary for obtaining the potential given in (??).

A.1 General formalism

We start with the Lagrangian defined by²⁶

$$\begin{aligned} K &= k + G + \sum_A |C_A|^2 e^{(\Delta^A + \bar{\Delta}^{\bar{A}})} \prod_a e^{2q_A^a(h_a + \Delta_a)} = k + G + \sum_A x^A = \tilde{k} + G, \\ S &= s + \frac{1}{2} [\tilde{G} - \delta_X(h_X + \Delta_X)] = \tilde{s} + \frac{1}{2} (\tilde{G} - \delta_X \Delta_X), \quad \tilde{G} = bG - \delta_X \hat{G}_X, \\ \tilde{L} &= E(2LS - 3). \end{aligned} \tag{A.1}$$

The Einstein term is canonical in zeroth order in Δ ; to put the remaining terms in canonical form we make a Weyl transformation

$$\begin{aligned} K &= \hat{K} + \Delta k, \quad E = e^{-\Delta k/3} \hat{E}, \quad L = e^{\Delta k/3} \hat{L}, \\ 2\hat{L}\hat{S}(\hat{L}, g^I, \Delta) &= 2\hat{L} S(L, g^I, \Delta) \Big|_{L=e^{\Delta k/3}\hat{L}} + 3(1 - e^{-\Delta k/3}) \\ &= 2\hat{L} S(L, g^I, \Delta) - 2\hat{L} \Delta s. \end{aligned} \tag{A.2}$$

The condition for a canonical Einstein term in the new Weyl basis is:

$$\begin{aligned} 0 &= \left(\frac{\partial \hat{K}}{\partial \hat{L}} \right)_\Delta + 2\hat{L} \left(\frac{\partial \hat{S}}{\partial \hat{L}} \right)_\Delta \\ &= \left(\frac{\partial L}{\partial \hat{L}} \right)_\Delta [K'(L) + 2\hat{L} S'(L)] - \left(\frac{\partial \Delta k}{\partial \hat{L}} \right)_\Delta - 2\hat{L} \left(\frac{\partial \Delta s}{\partial \hat{L}} \right)_\Delta \\ &= e^{\Delta k/3} \left[1 + \hat{L} \left(\frac{\partial \Delta k}{3 \partial \hat{L}} \right)_\Delta \right] [K'(L) + 2\hat{L} S'(L) + 2\Delta s e^{-\Delta k/3}], \end{aligned} \tag{A.3}$$

where $F'(L) = (\partial F / \partial L)_\Delta$, and the subscript on derivatives indicates the variable(s) held fixed, aside from the moduli that are held fixed throughout. This gives

$$\begin{aligned} 0 &= K'(L) + 2\hat{L} S'(L) + 2\Delta s e^{-\Delta k/3} \\ &= K'(L) + 2\hat{L} S'(L) - \frac{3}{\hat{L}} (1 - e^{-\Delta k/3}) e^{-\Delta k/3} \end{aligned}$$

²⁶Here we are assuming nonvanishing vev 's only for fields with $O(\delta_X^{\frac{1}{2}})$ vev 's at the string scale. We will consider additional vev 's of order Δ in Appendix ?? below.

$$\begin{aligned}
&= K'(L) + 2\hat{L}S'(L) - \frac{3}{L} \left(1 - e^{-\Delta k/3}\right) \\
&= K'(L) + 2\hat{L}S'(L) - \frac{3}{L} \left(1 - \frac{\hat{L}}{L}\right).
\end{aligned} \tag{A.4}$$

Defining $\partial_\alpha F = \partial F / \partial \Delta^\alpha$, $\alpha = a, A, \bar{A}$, the Δ derivatives of the effective Kähler potential $\tilde{K} = \hat{K} + 2\hat{L}\hat{S}$ satisfy

$$\begin{aligned}
\tilde{K}_\alpha &= \left(\partial_\alpha \tilde{K}\right)_{\hat{L}} = (\partial_\alpha K)_L + 2\hat{L}(\partial_\alpha S)_L + (\partial_\alpha L)_{\hat{L}} \left[K'(L) + 2\hat{L}S'(L)\right] - \left(\partial_\alpha [\Delta k + 2\hat{L}\Delta s]\right)_{\hat{L}} \\
&= (\partial_\alpha K)_L + 2\hat{L}(\partial_\alpha S)_L + (\partial_\alpha \Delta k/3)_{\hat{L}} \left\{L \left[K'(L) + 2\hat{L}S'(L)\right] - 3 + 3e^{-\Delta k/3}\right\} \\
&= (\partial_\alpha K)_L + 2\hat{L}(\partial_\alpha S)_L.
\end{aligned} \tag{A.5}$$

The D-terms in the scalar potential are determined in (??) by the scalar component of $\tilde{K}_a$:

$$\begin{aligned}
\tilde{K}_a \Big| &= 2 \sum_A q_A^a x^A(\ell, \delta_A, \bar{\delta}_{\bar{A}}) - \delta_X \hat{\ell} \delta_{aX} \\
&= \delta_X \delta_{aX} (\ell - \hat{\ell}) + 2 \sum_A q_A^a k^A (\delta^A + \bar{\delta}^A) + O(\delta^2) \sim \delta,
\end{aligned} \tag{A.6}$$

where δ_{aX} is the Kronecker delta-function. Since $(\partial_A S)_L = 0$, for $\alpha = A$ we have simply

$$\tilde{K}_A \Big| = x^A(\ell, \delta^A, \bar{\delta}^{\bar{A}}). \tag{A.7}$$

To evaluate the F-terms, we need the Kähler metric $\tilde{K}_{A\bar{B}} \Big|$. In the following we define

$$\begin{aligned}
x^A &\equiv x^A(\ell, \delta_A, \bar{\delta}_{\bar{A}}) = k^A(\ell) e^{\delta^A + \bar{\delta}^{\bar{A}}}, \quad x'^A = (\partial x^A / \partial \ell)_\delta, \quad etc., \\
\ell &= e^{\Delta k/3} \hat{\ell}, \quad \ell_A = (\partial_A \ell)_{\hat{\ell}}, \quad \ell_{\hat{\ell}} = (\partial_{\hat{\ell}} \ell)_\delta.
\end{aligned} \tag{A.8}$$

Differentiation of the $\theta = 0$ component of (??) gives²⁷

$$\begin{aligned}
0 &= \ell_{\hat{\ell}} \left[K''(\ell) + 2\hat{\ell} \tilde{s}''(\ell) + 3/\ell^2 - 6\hat{\ell}/\ell^3 \right] + 2\tilde{s}'(\ell) + 3/\ell^2 \\
&= \ell_A \left[K''(\ell) + 2\hat{\ell} \tilde{s}''(\ell) + 3/\ell^2 - 6\hat{\ell}/\ell^3 \right] + x'^A, \\
\ell_A &= x'^A \frac{\ell_{\hat{\ell}} \ell^2}{3 - \ell \tilde{k}'(\ell)} = \ell_{\hat{\ell}} c(\ell) x'^A,
\end{aligned} \tag{A.9}$$

since $\tilde{k}'(\ell) = -2\ell \tilde{s}'(\ell)$. Then we obtain

$$\tilde{K}_{A\bar{B}} \Big| = \left(\partial_{\bar{B}} \partial_A \tilde{K} \right)_{\hat{\ell}} = \left[\partial_{\bar{B}} x^A(\ell, \delta^A, \bar{\delta}^{\bar{A}}) \right]_{\hat{\ell}} = \delta_{AB} x^A + c \ell_{\hat{\ell}} x'^A x'^B, \tag{A.10}$$

²⁷In [?, ?] we parametrized the Weyl transformation by the function $\alpha(\ell) = 3\delta_X c(\ell)/\ell$.

and the inverse metric $\tilde{K}^{A\bar{B}}$ is

$$\tilde{K}^{A\bar{B}} = \frac{\delta^{AB}}{x^A} - \frac{x'^A x'^B}{x^A x^B} \left[(\ell_{\hat{c}})^{-1} + \sum_C \frac{(x'^C)^2}{x^C} \right]^{-1}. \quad (\text{A.11})$$

Above the condensate scale, quadratic terms in the Δ_α appear only through the combination of functionals $\tilde{K} = \hat{K} + 2\hat{L}\hat{S}$. When a gaugino condensate potential is added, quadratic terms in Δ_A appear through a different linear combination of $\hat{K}, \hat{S}$. Thus we need to evaluate these separately. Since $(\partial_A S)_L = 0$, we have

$$\begin{aligned} \hat{K}_A &= (\partial_A \hat{K})_{\hat{L}} = \tilde{K}_A - 2\hat{L}\hat{S}_A, \\ \hat{S}_A &= (\partial_A \hat{S})_{\hat{L}} = (\partial_A L)_{\hat{L}} S'(L) - (\partial_A \Delta s)_{\hat{L}}, \quad \Delta s = 3(L^{-1} - \hat{L}^{-1})/2, \\ \hat{S}_A| &= \ell_A [\tilde{s}'(\ell) + 3\ell^{-2}/2] = \ell_A [3 - \ell\tilde{k}'(\ell)]/2\ell^2 = \frac{\ell_{\hat{c}}}{2} x'^A. \end{aligned} \quad (\text{A.12})$$

To study the dilaton potential we can drop terms of order δ and set $x^A = k^A, ; \hat{\ell} = \ell$. In this approximation the auxiliary field F^A in (??) is just:

$$F_A = \sum_B \tilde{K}_{A\bar{B}} \bar{F}^{\bar{B}} = -\frac{\bar{u}}{4} \left[(k'^A (1 + b_c \hat{\ell}) - p_A - b_c k^A) + O(\delta) \right]. \quad (\text{A.13})$$

It follows from the definitions (??) and (??) and the constraints (??) and (??) that

$$\sum_A p_A k'^A / k^A = \ell^{-1} \sum_A k'^A = \sum_A (k'^A)^2 / k^A = \delta_X h'_X(\ell), \quad (\text{A.14})$$

and therefore that

$$\sum_A F_A x'^A / x^A = O(\delta \bar{u}) \quad (\text{A.15})$$

is negligible in this approximation, giving

$$\begin{aligned} \sum_A F_A F^A &= \frac{|u|^2}{16} \left[w(k) - (1 + b_c \hat{\ell})^2 \delta_X h'_X(\ell) + O(\delta) \right], \\ w(x) &= \sum_A \frac{(p_A + x^A b_c)^2}{x^A}. \end{aligned} \quad (\text{A.16})$$

Setting the moduli at their self-dual points $F^I = 0$, the full potential for the dilaton is

$$V = \frac{|u|^2}{16} v(\ell) + O(\delta^2, \delta|u|), \quad v(\ell) = w[k(\ell)] + \ell^{-1} k'(\ell) (1 + b_c \ell)^2 - 3b_c^2. \quad (\text{A.17})$$

To evaluate the vev 's of δ and the D-terms $\tilde{K}_a$, we need only retain terms linear in the coefficient of $|u|^2$, since $\tilde{K}_a \sim \delta \sim |u|^2$. It follows from (??) that

$$\sum_A F_A F^A = \sum_A |F_A|^2 / x^A + O(\delta^2), \quad (\text{A.18})$$

Taking the ℓ derivative of (??) gives

$$2 \sum_A q_A^a x'_A = \tilde{K}'_a + \ell_{\hat{\ell}}^{-1} \delta_X \delta_{aX} = \tilde{K}'_a + \delta_X \delta_{aX} \ell_{\hat{\ell}}^{-1}. \quad (\text{A.19})$$

Then again using (??) and (??) we have

$$\begin{aligned} \sum_A |F_A|^2 / x^A &= w(x) + 2(1 + b_c \hat{\ell}) \sum_A \left[(1 + b_c \hat{\ell}) x'^A h'_a q_A^a - 2p_A h'_a q_A^a - b_c x'^A \right] \\ &= w(x) - (1 + b_c \hat{\ell})^2 h'_X \delta_X \ell_{\hat{\ell}} \\ &\quad + (1 + b_c \hat{\ell}) \ell_{\hat{\ell}} \sum_a h'_a \left[\ell_{\hat{\ell}} (1 + b_c \hat{\ell}) \tilde{K}'_a - 2b_c \tilde{K}_a \right]. \end{aligned} \quad (\text{A.20})$$

To obtain the remaining contribution to $v(\hat{\ell}, \delta)$ we use the lowest component of (??) which reads

$$\hat{\ell} - \ell = -\tilde{c} \sum_a h'_a(\ell) \tilde{K}_a, \quad \tilde{c}(\ell) = \frac{\ell^2}{3 - \ell k'(\ell)} = \frac{1}{c^{-1} + \delta_X h'_X(\ell)}, \quad (\text{A.21})$$

$$\hat{K}'(\ell) = k'(\ell) + \sum_a h'_a \tilde{K}_a + \delta_X h'_X \hat{\ell} + 3\partial_{\ell} \ln \left(1 - \ell^{-1} \tilde{c} \sum_a h'_a \tilde{K}_a \right). \quad (\text{A.22})$$

In addition, from (??) or (??),

$$\ell_{\hat{\ell}} = \left[1 - \partial_{\ell} \left(\tilde{c} \sum_a h'_a \tilde{K}_a \right) \right]^{-1} = 1 + \partial_{\ell} \left(\tilde{c} \sum_a h'_a \tilde{K}_a \right) + O(\delta^2), \quad (\text{A.23})$$

so

$$\begin{aligned} \ell_{\hat{\ell}} \hat{K}'(\ell) &= k'(\ell) + (k'(\ell) - 3\ell^{-1}) \partial_{\ell} \left(\tilde{c} \sum_a h'_a \tilde{K}_a \right) + \sum_a h'_a \tilde{K}_a (1 + 3\ell^{-2} \tilde{c}) + \ell_{\hat{\ell}} \delta_X h'_X \hat{\ell} + O(\delta^2) \\ &= k'(\ell) - \frac{\ell}{\tilde{c}} \partial_{\ell} \left(\tilde{c} \sum_a h'_a \tilde{K}_a \right) + \sum_a h'_a \tilde{K}_a (2 + \tilde{c} \ell^{-1} k') + \ell_{\hat{\ell}} \delta_X h'_X \hat{\ell} + O(\delta^2), \end{aligned} \quad (\text{A.24})$$

where in the last equality we used the definition of $\tilde{c}$ in (??). Terms in $\tilde{K}'_a$ and terms proportional to $\delta_X h'_X$ cancel between (??) and (??) in the contribution $v(\hat{\ell}, \delta)$ to the potential (??):

$$\begin{aligned} v(\hat{\ell}, \delta) &= \hat{\ell}^{-1} (1 + b_c \hat{\ell}^2)^2 \ell_{\hat{\ell}} \hat{K}'(\ell) + \sum_A |F_A|^2 / x^A - 3b_c^2 + O(\delta^2) \\ &= \hat{\ell}^{-1} (1 + b_c \hat{\ell}) \sum_a \tilde{K}_a \left\{ h'_a \left[(1 + b_c \hat{\ell}) \left(1 + \frac{\tilde{c} k'}{\ell} - \frac{\ell \tilde{c}'}{\tilde{c}} \right) - 2b_c \hat{\ell} \right] - \ell h''_a \right\} \\ &\quad + w(x) + (1 + b_c \hat{\ell}^2)^2 \hat{\ell}^{-1} k'(\ell) - 3b_c^2 + O(\delta^2). \end{aligned} \quad (\text{A.25})$$

We may further expand (??) using (??) to write

$$k'(\ell) = k'(\hat{\ell}) + \tilde{c} \sum_a h'_a \tilde{K}_a k'' + O(\delta^2), \quad w[x(\ell)] = w[x(\hat{\ell})] + \tilde{c} \sum_a h'_a \tilde{K}_a w' + O(\delta^2). \quad (\text{A.26})$$

Then using the minimization equation for $\hat{\ell}$ and vanishing of the vacuum energy

$$\begin{aligned} v'(\langle \hat{\ell} \rangle) &\equiv v'(\langle \hat{\ell} \rangle, 0) = \left[w' + (1 + b_c \hat{\ell})^2 \hat{\ell}^{-1} k''(\hat{\ell}) - \hat{\ell}^{-2} (1 - b_c^2 \hat{\ell}^2) k'(\hat{\ell}) \right] \Big|_{\hat{\ell}=\langle \hat{\ell} \rangle} = O(\delta), \\ v(\langle \hat{\ell} \rangle) &\equiv v(\langle \hat{\ell} \rangle, 0) = \left[w + (1 + b_c \hat{\ell})^2 \hat{\ell}^{-1} k'(\hat{\ell}) - 3b_c^2 \right] \Big|_{\hat{\ell}=\langle \hat{\ell} \rangle} = O(\delta), \end{aligned} \quad (\text{A.27})$$

gives

$$v(\langle \hat{\ell} \rangle, \delta) = w[x(\hat{\ell})] - w[k(\hat{\ell})] + \sum_a \tilde{K}_a A_a(\langle \hat{\ell} \rangle) + O(\delta^2), \quad (\text{A.28})$$

where

$$A_a(\ell) = \ell^{-1} (1 + b_c \ell) \left\{ h'_a \left[1 - b_c \ell + 2 \frac{\tilde{c} k'}{\ell} - (1 + b_c \ell) \frac{\ell \tilde{c}'}{\tilde{c}} \right] - \ell h''_a \right\}. \quad (\text{A.29})$$

Then, with the dilaton at its vacuum value, the potential for δ is

$$\begin{aligned} V(\langle \hat{\ell} \rangle, \delta) &= \frac{1}{2s(\ell)} \sum_a \tilde{K}_a^2 + \frac{|u|^2}{16} v(\langle \hat{\ell} \rangle, \delta), \quad v(\langle \hat{\ell} \rangle, \delta) = \sum_A (\delta^A + \bar{\delta}^A) v_A(\langle \hat{\ell} \rangle) + O(\delta^2), \\ v_A(\ell) &= w_A(\ell) + \sum_a A_a(\ell) \tilde{K}_{aA}, \quad w_A(\ell) = b_c^2 k^A(\ell) - \frac{p_A^2}{k^A(\ell)}, \end{aligned} \quad (\text{A.30})$$

with, using (??) and (??),

$$\begin{aligned} \tilde{K}_{aA} &= (\partial_A \tilde{K}_a)_{\hat{\ell}} = 2q_A^a k^A + \delta_X \delta_{aX} \tilde{c} \sum_b h'_b \tilde{K}_{bA} + O(\delta) \\ &= 2q_A^a k^A + \delta_{aX} \frac{\tilde{c} \delta_X k'^A}{1 - \tilde{c} \delta_X h'_X} = 2q_A^a k^A + \delta_{aX} c \delta_X k'^A. \end{aligned} \quad (\text{A.31})$$

The vacuum conditions, in addition to (??), are [using (??)]

$$\begin{aligned} \frac{|u|^2}{16} v_A(\hat{\ell}) &= -\frac{2}{s(\hat{\ell})} k^A(\hat{\ell}) \sum_a q_A^a [\tilde{K}_a + c h'_a \delta_X \tilde{K}_X] + O(\delta^2), \quad \tilde{K}_a \sim \delta \sim |u|^2, \\ \sum_A v_A(\hat{\ell}) &= -\frac{16c \delta_X \hat{\ell}}{|u|^2 \tilde{c} s(\hat{\ell})} \tilde{K}_X + O(\delta^2), \\ \sum_A q_A^a v_A(\hat{\ell}) &= -\frac{32}{|u|^2 s(\hat{\ell})} \left(\sum_b N^{ab} \tilde{K}_b + \frac{c}{4} \delta_X^a \delta_X^2 \tilde{K}_X \right) + O(\delta^2), \end{aligned} \quad (\text{A.32})$$

which may be inverted to evaluate the D-terms:

$$\tilde{K}_X = -\frac{|u|^2 \tilde{c}s}{16c\delta_X \hat{\ell}} \sum_A v_A, \quad \tilde{K}_a = -\frac{|u|^2 s}{32} \sum_{A,b} N_{ab}^{-1} q_A^b v_A - c\delta_X h'_a \tilde{K}_X, \quad (\text{A.33})$$

where we used (??). Consistency of these equations for $a = X$ requires

$$\sum_A v_A = 2\ell \sum_{A,a} q_A^a h'_a v_A, \quad (\text{A.34})$$

which is automatically satisfied for the minimal models with $n = m$ discussed at the end of Section ??, since [see (??) and (??) below] $2\ell \sum_a q_A^a h'_a = \sum_{a,B} Q_a^B q_A^a = 1$ in these models. In general, we have from (??)

$$\sum_A v_A = \sum_A w_A + \delta_X \ell \frac{c}{\tilde{c}} A_X(\ell). \quad (\text{A.35})$$

Using the definition of N_{ab} in (??) gives

$$2 \sum_A q_A^b \tilde{K}_{aA} = 4 \left(N_{ab} + \delta_{aX} c \delta_X \sum_c h'_c N_{bc} \right). \quad (\text{A.36})$$

Then using the second equality in (??) gives

$$\begin{aligned} 2\ell \sum_{Ab} q_A^b h'_b \tilde{K}_{aA} &= \frac{c}{\tilde{c}} \ell \delta_X \delta_{aX}, \\ 2\ell \sum_{Ab} q_A^b h'_b v_A &= 2\ell \sum_{Ab} q_A^b h'_b w_A + \delta_X \ell \frac{c}{\tilde{c}} A_X(\ell), \end{aligned} \quad (\text{A.37})$$

and the consistency condition (??) becomes

$$\sum_A w_A = 2\ell \sum_{A,a} q_A^a h'_a w_A. \quad (\text{A.38})$$

Making the same decomposition as in (??), the minimization conditions (??) give

$$\begin{aligned} w_A &= k^M \sum_A \zeta_M^A w_A / k^A, \\ \sum_A w_A + \sum_M w_M &= \sum_A w_A \left(1 + \sum_M \zeta_M^A k^M / k^A \right) = \frac{\delta_X \ell}{2} \sum_A w_A Q_X^A / k^A, \end{aligned} \quad (\text{A.39})$$

where the last equality follows from the conditions (??). In addition

$$\begin{aligned} 2\ell \sum_a h'_a \left(\sum_A q_A^a w_A + \sum_M q_M^a w_M \right) &= 2\ell \sum_a h'_{aA} w_A \left(q_A^a + \sum_M q_M^a \zeta_M^A k^M / k^A \right) \\ &= 2\ell \sum_a h'_{aA} w_A / k^A \sum_b Q_b^A N_{ba} = \frac{\ell \delta_X}{2} \sum_A w_A Q_X^A / k^A, \end{aligned} \quad (\text{A.40})$$

where we again used the definition (??) of N_{ab} and the second equality in (??).

A.2 Minimal models

For these models we have, at leading order in δ ,

$$\begin{aligned}
k^A(\ell) &= \ell p_A = \frac{\delta_X}{2} Q_X^A \ell, \quad h'_a = \frac{1}{2\ell} Q_a = -\ell h''_a, \quad Q_a = \sum_A Q_a^A \\
\delta k' &= \sum_A k'^A = \delta_X h'_X \ell = p = \sum_A p_A, \quad \delta k = \sum_A k^A = \delta_X h'_X \ell^2, \\
w(k) &= \ell^{-1} p (1 + b_c \ell)^2 = \delta_X h'_X (1 + b_c \ell)^2, \\
A_a &= A h'_a, \quad \frac{\ell A(\ell)}{1 + b_c \ell} = -3b_c \ell + \frac{\tilde{c} k'}{\ell} (1 - b_c \ell) - \tilde{c} k''.
\end{aligned} \tag{A.41}$$

To simplify notation we also define

$$z = b_c \langle \ell \rangle, \quad \langle \ell^2 h'_X \rangle \delta_X = Q_X \delta_X \langle \ell \rangle / 2 = 3\lambda z. \tag{A.42}$$

For example, $\lambda = 1$ in the FIQS model considered in the text. Then at the vacuum, at leading order in δ , using (??) and (??) we obtain from (??)

$$\begin{aligned}
\tilde{K}_a &= \frac{|u|^2 s}{32\ell^2} Q_a \left[\frac{\tilde{c}}{c} (1 - z^2) + a(z) \right], \quad a(z) = -\langle \ell A(\ell) \rangle = (1 + z) (\langle \ell \tilde{c} w' \rangle - 3z). \\
\langle \ell \tilde{c} w' \rangle &= -\frac{\lambda z (1 - z^2) (1 + z)^2}{(1 + z)^2 (1 + \lambda z) - z^2}, \quad \frac{\tilde{c}}{c} = \frac{1 + 2z}{(1 + z)^2 (1 + \lambda z) - z^2}.
\end{aligned} \tag{A.43}$$

These results are unchanged if we replace a minimal set $k^A = \ell p_A$ with N minimal sets $k^i = \ell p_A^i$ with the same $U(1)$ charges such that $\sum_{i=1}^N p_A^i = p_A$.

A.3 Additional $O(\delta)$ vev 's

The parameterization of the Kähler potential in (??) is valid only if $|C_A|^2 \sim \delta^0$. Once we allow the D-terms to be nonvanishing, $|\tilde{K}_a| \sim \delta$, we might expect additional order δ terms to occur from vev 's of fields Φ^M that are $U(1)$ charged, but lie in F-flat directions, and have $p_M = 0$. Since these fields acquire vev 's at the condensation scale where t^I and ℓ are also determined, modular invariance, the linearity condition and local supersymmetry are broken, and the formalism of [?, ?] does not apply. The exact treatment of the minimization equations involves mixing among all the “light” fields (ϕ^M, t^I, ℓ) . However, for the purpose of determining the parameters that define the vacuum, we can set to zero any field that vanishes in the vacuum. This allows us to parameterize the contributions in a fashion analogous to the Φ^A with large vev 's such that the above results still hold. For the fields Φ^M , define

$$\begin{aligned}
x^M &= \epsilon^M e^{(\Delta^M + \bar{\Delta}^M)} \prod_a e^{2q_M^a (h_a + \Delta_a)} = \langle e^{G^M} |\Phi^M|^2 \rangle \prod_a e^{2q_M^a (h_a + \Delta_a)}, \\
\epsilon^M &\sim \delta, \quad |\Delta^M| = \delta^M \sim 1, \quad k^M = 0.
\end{aligned} \tag{A.44}$$

Then the results given above as expansions in δ are unchanged.

A.4 $O(\delta^2)$ terms

In order to determine if the extrema found above are true minimal, we need to evaluate the terms quadratic in δ . The same terms are needed to evaluate masses of D-moduli in nonminimal cases. Writing

$$V = \frac{1}{2s(\ell)} \sum_a \tilde{K}_a^2 + \frac{|u|^2}{16} \left[v(\hat{\ell}) + \sum_A \delta^A v_A(\hat{\ell}) + \frac{1}{2} \sum_{AB} \delta^A \delta^B v_{AB}(\hat{\ell}) + O(\delta^3) \right], \quad (\text{A.45})$$

and recalling that the vacuum conditions require $v(\hat{\ell})$, $\partial_{\hat{\ell}} v(\hat{\ell})$, $\tilde{K}_a \sim \delta \sim |u|^2$, we have

$$\begin{aligned} V_{\hat{\ell}\hat{\ell}} &= \mu_{\hat{\ell}\hat{\ell}}^2 = \frac{|u|^2}{16} v''(\ell) + O(\delta^2), \\ V_{\hat{\ell}A} &= \mu_{\hat{\ell}A}^2 = \frac{1}{s} \sum_a \left[\tilde{K}_a \partial_{\hat{\ell}} \tilde{K}_{aA} + \tilde{K}_{aA} \left(\tilde{K}'_a - \frac{s'}{s} \tilde{K}_a \right) \right] + \frac{|u|^2}{16} \left(\partial_{\hat{\ell}} v_A + v_A \partial_{\hat{\ell}} \ln |u|^2 \right) + O(\delta^2), \\ V_{AB} &= M_{AB}^2 + \mu_{AB}^2, \quad M_{AB}^2 = \frac{1}{s} \sum_a \tilde{K}_{aB} \tilde{K}_{aA}, \\ \mu_{AB}^2 &= \frac{1}{s} \sum_a \tilde{K}_a \tilde{K}_{aAB} + \frac{|u|^2}{16} \left(v_{AB} + v_A \partial_B \ln |u|^2 + v_B \partial_A \ln |u|^2 \right) + O(\delta^2). \end{aligned} \quad (\text{A.46})$$

The matrix V_{AB} is the mass matrix for the real fields Σ^A introduced in (??). In the minimal case, its properly normalized eigenvalues are the squared masses of the $U(1)_a$ vector bosons which are positive. In this case the only requirement for the vacuum to be a local minimum is $v''(\ell) > 0$; since $M^2 \sim \delta^0$ and $\mu^2 \sim \delta$, $\text{Det} V = v'' \text{Det} M^2 + O(\delta^2)$, and similarly for any submatrices on the diagonal.

For the general case, we can write the mass terms for the eaten Goldstone bosons $\hat{\Sigma}_a$ and the D-moduli $\hat{\Sigma}^A$ defined in (??)–(??):

$$\Sigma^A = \hat{\Sigma}^A - 2 \sum_a q_A^a \hat{\Sigma}_a, \quad \sum_A q_a^A x^A \hat{\Sigma}^A = 0. \quad (\text{A.47})$$

We have

$$\tilde{K}_{aA} = 2x^A \left(q_A^a + c\delta_{aX} \delta_X \sum_b q_A^b h'_b \right) = 2x^A q_A^a + c\delta_{aX} \delta_X x'^A, \quad \sum_A \tilde{K}_{aA} \hat{\Sigma}^A = 0, \quad (\text{A.48})$$

so the relevant squared-mass matrix for the light $\hat{\ell}, \hat{\Sigma}^A$ sector is μ^2 , and mixing of these states with $\hat{\Sigma}_a$ is negligible. From the last equality in (??), which also implies

$$\sum_A x'^A \hat{\Sigma}^A = 0, \quad x^A = k^A + O(\delta), \quad (\text{A.49})$$

we may drop terms proportional to $q_A^a k^A, q_B^a k^B, k'^{A,B}$ in $\mu_{\hat{\ell}A}$ and μ_{AB} . Thus we may drop terms proportional to $\tilde{K}_{aA}$. This means in particular that in the expansion of $v(\hat{\ell}, \delta)$ to order δ^2 we may drop all terms of containing $\tilde{K}_a \tilde{K}_b$ or $\tilde{K}_a \tilde{K}'_b$. Then there are no new relevant $O(\delta^2)$ terms in the expression (??) for $\hat{K}'$. Since $\ell_{\hat{\ell}} \delta_X h'_X$ cancels between (??) and (??), the only relevant additional terms in (??) are, using (??),

$$\left[\ell_{\hat{\ell}} \hat{K}'(\ell) \right]_{\delta^2} = (k' - 3\ell^{-1}) \left(\tilde{c} \sum_a h'_a \tilde{K}'_a \right)^2 = -\ell \tilde{c} \left(\sum_a h'_a \tilde{K}'_a \right)^2. \quad (\text{A.50})$$

From (??) we get a contribution

$$\left[\sum_A |F_A|^2 / x^A \right]_{\delta^2} = \frac{|u|^2}{16} \left[w_{\delta^2} + 2\tilde{c} (1 + b_c \ell)^2 \left(\sum_a h'_a \tilde{K}'_a \right)^2 \right]. \quad (\text{A.51})$$

Finally, there is a contribution from the second term in (??):

$$F_A F^A \ni -\tilde{c} \left(\sum_A F_A x'^A / x^A \right)^2 + O(\delta^3 |u|^2) = -\frac{|u|^2}{16} \tilde{c} (1 + b_c \ell)^2 \left(\sum_a h'_a \tilde{K}'_a \right)^2 + O(\delta^3 |u|^2). \quad (\text{A.52})$$

Further expansion of (??) in $\ell - \hat{\ell}$ gives only terms that are higher order in $\tilde{K}_a$. Projection onto the D-moduli sector also gives

$$\begin{aligned} v_A &\rightarrow w_A & \tilde{K}'_{aA} &\rightarrow 2k'^A \left(q_A^a + c \delta_X \delta_{aX} \sum_b q_A^b h'_b \right), \\ \tilde{K}_{aAB} &= \delta_{AB} \tilde{K}_{aA} + \ell_B \partial_{\hat{\ell}} \tilde{K}_{aA} \rightarrow 0, \end{aligned} \quad (\text{A.53})$$

since [see (??)] $\ell_B \propto x'^B$. Then collecting (??), (??) and (??), terms quadratic in $\tilde{K}'_a$ also cancel, and v_{AB} reduces to

$$v_{AB} = w_{AB}, \quad (\text{A.54})$$

while the contributions to $\partial_{\hat{\ell}} v_A$ are just the ℓ -derivatives of v_A in (??) with

$$\partial_{\hat{\ell}} \left[A_a(\hat{\ell}) \tilde{K}_{aA} \right] \rightarrow A_a(\hat{\ell}) \tilde{K}'_{aA}. \quad (\text{A.55})$$

From (??), (??), the first condition in (??) and the expression (??) for $\delta k'$, we have

$$\begin{aligned} \partial_{\hat{\ell}} \ln |u|^2 &= \tilde{k}' - 2s'/b_c - \sum_A p_A k'^A / b_c k^A = \delta k' + k'(b_c \ell - 1)/b_c \ell - \delta_X h'_X / b_c \\ &= \tilde{k}'(b_c \ell - 1)/b_c \ell, \\ \partial_A \ln |u|^2 &= -p_A / b_c + \hat{K}_A - 2\hat{S}_A / b_c + \ell_A \partial_{\hat{\ell}} \ln |u|^2 \rightarrow -p_A / b_c + k^A. \end{aligned} \quad (\text{A.56})$$

Then using the vacuum conditions (??) and the results (??) we obtain the matrix elements of μ .

These expressions simplify further for the case of N copies of a minimal set with vacuum values that satisfy $k^A = \ell p^A$, $q_A^a k'^A = \ell^{-1} q_A^a k^A$. In this case we may also drop $\tilde{K}'_a$ when evaluating the squared mass matrix for $\hat{\ell}, \hat{\Sigma}^A$. Then the above expressions reduce to

$$\begin{aligned} v_A &\rightarrow -p_A^2/k^A + b_c^2 k_A = -\ell^{-2} k^A (1 - z^2), \\ \partial_{\hat{\ell}} v_A &\rightarrow k'_A [(p_A/k^A)^2 + b_c^2] = \ell^{-2} k'^A (1 + z^2) \rightarrow 0, \\ v_{AB} &\rightarrow \delta_{AB} \ell^{-2} k^A (1 + z^2), \end{aligned} \quad (\text{A.57})$$

and, using the vacuum value (??) for $\tilde{k}'$, the relevant elements of μ in (??) reduce to

$$\begin{aligned} \mu_{\hat{\ell}\hat{\ell}}^2 &= \frac{|u|^2}{16} v''(\ell), \quad \mu_{\hat{\ell}A}^2 = \frac{|u|^2}{16} \ell^{-2} k^A \frac{(1 - z)^2}{1 + z}, \\ \mu_{AB}^2 &= \frac{|u|^2}{16} \left[\delta_{AB} \ell^{-2} k^A (1 + z^2) + \frac{2}{z} \ell^{-2} k^A k^B (1 - z)^2 (1 + z) \right]. \end{aligned} \quad (\text{A.58})$$

In the case under consideration, the condition (??) becomes

$$\begin{aligned} 0 &= \sum_{A,\alpha} q_A^a k_\alpha^A \hat{\Sigma}_\alpha^A = \sum_{A,\alpha,a} Q_a^B q_A^a k_\alpha^A \hat{\Sigma}_\alpha^A \\ &= \sum_\alpha k_\alpha^B \hat{\Sigma}_\alpha^B \quad \forall B, \end{aligned} \quad (\text{A.59})$$

and there is no mixing of the dilaton with the D-moduli. Defining chiral fields D^A as in (??) and setting to zero the m eaten Goldstone modes $\hat{\Sigma}_a$, the mass term for the D-moduli²⁸ $\sigma'^A = \Sigma'^A$ or $d^A = D^A$ is given by

$$\mathcal{L}_{m_D} = -\frac{1}{2} \sigma'^A \mu^2 \sigma'^B = -\frac{1}{2} m_G^2 \frac{1 + z^2}{z^2} \sum_{A=1}^n k^A (d^A + \bar{d}^{\bar{A}})^2. \quad (\text{A.60})$$

The Kähler potential for the D-moduli is²⁹

$$K(D, \bar{D}) = \frac{1}{2} \sum_{A=1}^n k^A (D^A + \bar{D}^{\bar{A}})^2. \quad (\text{A.61})$$

When we reexpress the d^A in terms of an orthonormal set D^i subject to the constraint (??):

$$D^A = \sum_{i=1}^{n-m} c_i^A D^i, \quad \sum_A k^A c_i^A = 0, \quad (\text{A.62})$$

²⁸The prime on Σ refers to the field redefinitions make in Section ?? and does not denote differentiation with respect to ℓ .

²⁹The second term in Eq. (3.33) of [?] is missing a factor 1/2

the Kähler metric and the squared mass matrix

$$K_{i\bar{j}} = \sum_A c_i^A k^A c_j^A, \quad \mu_{ij}^2 = m_G^2 \frac{1+z^2}{z^2} \sum_A c_i^A k^A c_j^A, \quad (\text{A.63})$$

are diagonalized by the same unitary transformation:

$$D^i \rightarrow D^i = U_j^i D^j, \quad d^i = D^i \Big| = N_d(\sigma^i + i a^i), \quad (\text{A.64})$$

where the normalization constant N_d is chosen to make the kinetic energy term canonically normalized. Then the Lagrangian quadratic in the scalar D-moduli reads

$$\mathcal{L}_D = \frac{1}{2} \sum_i \left[\partial_\mu \sigma^i \partial^\mu \sigma^i + \partial_\mu a^i \partial^\mu a^i - m_G^2 \frac{1+z^2}{z^2} (\sigma^i)^2 \right]. \quad (\text{A.65})$$

A.5 Scalar masses

For fields Φ^M with vanishing vev 's and Kähler potential

$$K = \sum_M x^M, \quad x^M = e^{G'^M + 2 \sum_a q_M^a h_a} |\Phi^M|^2, \quad (\text{A.66})$$

referring to (??), the complex scalars have masses

$$m_M^2 = \frac{\partial V}{\partial x^M} = V_M = \frac{|u|^2}{16} v_M + \frac{1}{s} \sum_a \tilde{K}_a \tilde{K}_{aM}, \quad (\text{A.67})$$

where everywhere the subscript M denotes differentiation with respect to x^M . We have

$$\tilde{K}_M = \hat{K}_M + 2L\hat{S}_M = K_M + 2LS_M = K_M = 1. \quad (\text{A.68})$$

Since x^M and δ appear in the functionals $\tilde{K}$ and $\hat{S}$ in the same way, the terms linear in x^M can be directly extracted from the formulate for those linear in δ_A . The x^M derivatives are obtained from the δ^A derivatives by the replacements

$$K_A = k^A \rightarrow K_M = 1, \quad K'_A = k'^A \rightarrow K'_M = \frac{x'^M}{x^M} = 2 \sum_a h'_a q_M^a \equiv 2h'_M, \quad p_A \rightarrow 0. \quad (\text{A.69})$$

Thus we obtain from (??), to zeroth order in δ :

$$v_M = b_c^2 + \sum_a A_a(\ell) \tilde{K}_{aM}, \quad \tilde{K}_{aM} = 2q_M^a + 2c\delta_{aX} \delta_X h'_M. \quad (\text{A.70})$$

Using (??) and (??) to solve for the D-terms in the vacuum gives

$$\tilde{K}_a = -\frac{|u|^2 s}{16} \left(\sum_A w_A B_{Aa} + A_a \right), \quad B_{Aa}(\ell) = \frac{1}{2} \sum_b N_{ab}^{-1} q_A^b - \frac{\tilde{c} h'_a}{\ell}. \quad (\text{A.71})$$

The functions $A_a(\ell)$ drop out of the scalar masses:

$$m_M^2 = \frac{|u|^2}{16} \left[b_c^2 - \sum_{Aa} w_A B_{Aa} \tilde{K}_{aM} \right]. \quad (\text{A.72})$$

For the minimal models (??) reduces to (??), and

$$\begin{aligned} v_M &= b_c^2 - \ell^{-1} a(z) \sum_a h'_a \tilde{K}_{aM} = b_c^2 - 2h'_M \frac{ca(z)}{\tilde{c}}, \\ \sum_a \tilde{K}_a \tilde{K}_{aM} &= \frac{|u|^2 s}{8\ell} h'_M \left[1 - z^2 + \frac{ca(z)}{\tilde{c}} \right], \end{aligned} \quad (\text{A.73})$$

giving

$$\begin{aligned} m_M^2 &= \frac{|u|^2 s}{16\ell} [\ell b_c^2 + h'_M(1 - z^2)] = \left(\zeta_M \frac{1 - z^2}{z^2} + 1 \right) m_G^2, \\ \zeta_M &= \sum_{a,A} q_M^a Q_a^A. \end{aligned} \quad (\text{A.74})$$

A.6 Nonminimal Kähler potential for matter

First consider the toy model with just one charged superfield Φ . Following [?] set

$$\delta K \equiv K(\Phi, \bar{\Phi}) = f(x), \quad x = e^{G_q + 2qV} |\Phi|^2 = e^{G_q + 2qV'} = e^{2qU'}. \quad (\text{A.75})$$

We require $\langle D_X \rangle = 0$ at the Planck scale, where

$$D_X = qK_\Phi \Phi - \frac{\delta_X L}{2} = qx f'(x) - \frac{\delta_X L}{2}, \quad \langle D_X \rangle_L = \left\langle qe^{2qh} f'(qe^{2qh}) - \frac{\delta_X L}{2} \right\rangle_L \quad (\text{A.76})$$

where

$$U' = h(L) + U, \quad \langle U \rangle = 0. \quad (\text{A.77})$$

Then we have

$$\begin{aligned} \delta k &= f(e^{2qh}), \quad \delta s = -\frac{\delta_X h}{2}, \\ \delta k' &= 2qh' e^{2qh} f'(e^{2qh}) = h' \delta_X L, \quad \delta s' = -\frac{\delta_X h'}{2}, \quad \delta k' + 2L\delta s' = 0. \end{aligned} \quad (\text{A.78})$$

So the Einstein condition is again satisfied for $U = 0$. Next consider the terms linear in U :

$$\delta K = f(x) = \delta k + U \left. \frac{\partial f}{\partial U} \right|_{U=0}, \quad \left. \frac{\partial f}{\partial U} \right|_{U=0} = 2qe^{2qh} f'(e^{2qh}) = \delta_X L, \quad \delta S = \delta s - \frac{\delta_X}{2} U. \quad (\text{A.79})$$

The linear terms are the same as in [?], and are removed by the same Weyl transformation.

Next consider the general case. Allowing for $O(|u|^2)$ D-terms, we have

$$\begin{aligned}
\delta K &= f(x^A), \quad D_a = \sum_A q_A^a x^A f_A(x) - \frac{\delta_X L}{2} \delta_{Xa}, \\
f_A &= \frac{\partial f}{\partial x^A}, \quad x^A = |\Phi^A|^2 e^{G^A+2\sum_a q_A^a V_a}, \\
\langle x^A \rangle_{(L,T)} &= |C_A|^2 e^{d_A+\bar{d}_A} \prod_a e^{2q_A^a(h_a(L)+\Delta_a)} = k^A e^{\tilde{\Delta}_A}, \quad \tilde{\Delta}_A = d_A + \bar{d}_A + 2\sum_a q_A^a \Delta_a = O(|u|^2), \\
0 &= \sum_A q_A^a k^A f_A(k) - \frac{\delta_X L}{2} \delta_{Xa}, \quad \delta k = f(k^A) + O(|u|^2), \\
\delta k'(k) &= 2\sum_{A,b} q_A^b h'_b k^A f_A(k) = \delta_X L h'_X = -2L\delta s'. \tag{A.80}
\end{aligned}$$

To evaluate the condensate-induced potential, we set $x^A = k^A e^{\tilde{\Delta}_A}$ and expand in $\tilde{\Delta}_A$ as before:

$$\begin{aligned}
\tilde{K}_A &= K_A = \frac{\partial f}{\partial d_A} \Big| = x^A f_A(x), \quad \tilde{K}_a = 2\sum_A q_A^a K_A - \hat{\ell} \delta_X \delta_{aX}, \\
\hat{K}_A &= \tilde{K}_A - 2\hat{L}\hat{S}_A, \quad \hat{S}_A = \frac{\ell_{\hat{\ell}}}{2} K'_A, \quad \tilde{K}_{A\bar{B}} = K_{A\bar{B}} + c\ell_{\hat{\ell}} K'_A K'_B, \\
K_{A\bar{B}} &= K_{AB} = K_A \delta_{AB} + x^A x^B f_{AB}, \quad f_{AB} = \frac{\partial^2 f}{\partial x^A \partial x^B}, \\
F^A &= -\frac{\bar{u}}{4} \tilde{K}^{A\bar{B}} \left[(1 + b_c \hat{\ell}) \ell_{\hat{\ell}} K'_B - p_B - K_B b_c \right]. \tag{A.81}
\end{aligned}$$

To obtain the potential we need the inverse $\tilde{K}^{A\bar{B}}$ of $\tilde{K}_{A\bar{B}}$; defining $K^{A\bar{B}}$ to be the inverse of $K_{A\bar{B}}$, we obtain

$$\tilde{K}^{A\bar{B}} = K^{AB} - \sum_{C,D} K^{AC} K'_C K'^{BD} K'_D \left[(c\ell_{\hat{\ell}})^{-1} + \sum_{E,F} K^{EF} K'_E K'_F \right]^{-1}. \tag{A.82}$$

The $K_A(k)$ satisfy the same constraints as k^A in the case of a minimal Kähler potential, although now $K_A(x) \neq x^A$ in general. The potential is the same as before except for the replacements $x^A \rightarrow K_A$, $\delta^{AB}/x^A \rightarrow K^{AB}$. In particular, the relations

$$\begin{aligned}
2\sum_A q_A^a K_A &= \delta_X \delta_{aX} \hat{\ell} + \tilde{K}_a, \quad K'_A = 2\sum_{B,b} h'_b q_B^b K_{AB}, \\
2\sum_A q_A^a K'_A &= 4\sum_{ABb} q_A^a q_B^b h'_b K_{AB} = \ell_{\hat{\ell}}^{-1} \delta_X \delta_{aX} + \tilde{K}'_a, \\
\delta k' &= 2\sum_{A,a} h'_a q_A^a K_A = h'_X \delta_X \hat{\ell} + \sum_a h'_a \tilde{K}_a, \tag{A.83}
\end{aligned}$$

give

$$\begin{aligned}
\sum_{AB} K^{A\bar{B}} K'_A K'_B &= 2 \sum_{A,a} h'_a q_A^a K'_A = h'_X \delta_X + \sum_a h'_a \tilde{K}'_a, \\
\sum_{AB} K^{A\bar{B}} K'_A K_B &= 2 \sum_{A,a} h'_a q_A^a K_A = h'_X \delta_X \hat{\ell} + \sum_a h'_a \tilde{K}_a, \\
\sum_{AB} K^{A\bar{B}} K'_A p_B &= 2 \sum_{A,a} h'_a q_A^a p_A = h'_X \delta_X.
\end{aligned} \tag{A.84}$$

Thus

$$\sum_A F_A K^{A\bar{B}} K'_B = O(\bar{u}\delta), \tag{A.85}$$

and we obtain the result in (??) with

$$w = K^{A\bar{B}} (p_A + K_A b_c) (p_B + K_B b_c), \tag{A.86}$$

which in minimal models reduces to

$$w = K^{A\bar{B}} (1 + b_c \ell)^2 p_A p_B. \tag{A.87}$$

In this case we can invert the equations in (??) to obtain, dropping order δ terms,

$$2K_A = 2\ell p_A = 2\ell K'_A = \delta_X Q_X^A \ell, \quad \sum_B K^{A\bar{B}} p_B = 2 \sum_a h'_a q_A^a, \quad \sum_{AB} K^{A\bar{B}} p_A p_B = \delta_X h'_X. \tag{A.88}$$

Then in these models

$$w = (1 + z)^2 \delta_X h'_A + O(\delta), \quad \sum_{AB} F_A \tilde{K}^{A\bar{B}} \bar{F}_B = \sum_{AB} F_A K^{A\bar{B}} \bar{F}_B + O(\delta^2) = O(\delta), \tag{A.89}$$

as before, and the dilaton potential is unchanged with respect to the case of a minimal Kähler potential for matter. Using (??) for the general case, the expression for $\sum_{AB} F_A K^{A\bar{B}} \bar{F}_B$ is the same as the right hand side of (??), with $w(x)$ now given by (??), and we obtain (??)–(??) with the same substitution. The results (??)–(??) and (??) are modified as follows:

$$\begin{aligned}
w_A &= 2b_c (p_A + b_c K_A) - K^{D\bar{C}} K_{\bar{C}AE} K^{E\bar{B}} (p_D + b_c K_D) (p_B + b_c K_B), \\
\tilde{K}_{aA} &= 2 \sum_B q_B^a K_{AB} + c \delta_{aX} \delta_X K'_A + O(\delta), \\
\frac{|u|^2}{16} v_A(\hat{\ell}) &= -\frac{2}{s(\hat{\ell})} \sum_{a,B} K_{AB} q_B^a [\tilde{K}_a + c h'_a \delta_X \tilde{K}_X] + O(\delta^2), \\
\sum_{AB} K_B K^{BA} q_B^a v_A(\hat{\ell}) &= -\frac{32}{|u|^2 s(\hat{\ell})} \left(\sum_b N^{ab} \tilde{K}_b + \frac{c}{4} \delta_X^a \delta_X^2 \tilde{K}_X \right) + O(\delta^2), \\
\sum_{AB} K_B K^{BA} v_A(\hat{\ell}) &= -\frac{16c \delta_X \hat{\ell}}{|u|^2 \tilde{c} s(\hat{\ell})} \tilde{K}_X + O(\delta^2), \quad N^{ab} = \sum_A q_A^a q_A^b K_A(k),
\end{aligned}$$

$$\begin{aligned}
\tilde{K}_a &= -\frac{|u|^2 s}{32} \sum_{ABb} K_B K^{BA} N_{ab}^{-1} q_B^b v_A - \frac{c}{4} \delta_X^2 N_{aX}^{-1} \tilde{K}_X = -\frac{|u|^2 s}{16} \left(\sum_A B_{Aa} w_A + A_a \right), \\
\tilde{K}_X &= -\frac{|u|^2 \tilde{c} s}{16 c \delta_X \ell} \sum_{AB} K_B K^{BA} v_A, \quad B_{Aa} = \sum_B K_B K^{BA} \left(\sum_b N_{ab}^{-1} q_B^b - \frac{\tilde{c}}{2\ell} \delta_X N_{aX}^{-1} \right), \quad (\text{A.90})
\end{aligned}$$

where we used, instead of (??),

$$\sum_{AB} K_B K^{BA} q_B^a \tilde{K}_{Aa} = 2 \sum_b N_{ab} (1 + c \delta_X \delta_{aX} h'_b), \quad \sum_{AB} K_B K^{BA} \tilde{K}_{Aa} = \frac{c}{\tilde{c}} \delta_X \delta_{aX}. \quad (\text{A.91})$$

With a Kähler potential of the form (??)

$$\begin{aligned}
K_A^\alpha &= x_\alpha^A e^{K^\alpha/C_\alpha}, \quad K_{AB}^\alpha = K_A^\alpha \delta_{AB} + C_\alpha^{-1} K_A^\alpha K_B^\alpha, \\
K_\alpha^{AB} &= \frac{1}{K_A^\alpha} \delta^{AB} - \frac{1}{C_\alpha + \sum_C K_C^\alpha}, \\
w_A^\alpha &= b_c^2 K_A^\alpha - (p_A^\alpha)^2 / K_A^\alpha - C_\alpha^{-1} (p_A^\alpha + b_c K_A^\alpha) w_\alpha, \\
w &= \sum_\alpha w_\alpha, \quad w_\alpha = \sum_A \frac{1}{K_A^\alpha} (p_A^\alpha + b_c K_A^\alpha)^2 - \frac{[\sum_A (p_A^\alpha + b_c K_A^\alpha)]^2}{C_\alpha + \sum_B K_B^\alpha}. \quad (\text{A.92})
\end{aligned}$$

For a minimal set (??) we have $N_{ab}^{-1} = \sum_A Q_a^A Q_b^A / K_A$, $N_{aX}^{-1} = 2Q_a / \delta_X \ell$, and we obtain

$$\begin{aligned}
w_\alpha &= \frac{C_\alpha p_\alpha (1+z)^2}{\ell (C_\alpha + \ell p_\alpha)}, \quad p_\alpha = \sum_A p_A^\alpha = Q_X^\alpha \frac{\delta_X}{2} \equiv \frac{3z\lambda_\alpha}{\ell}, \quad Q_a^\alpha = \sum_A Q_a^{A\alpha}, \\
w_A^\alpha &= \sum_B K_{AB}^\alpha \frac{\partial w^\alpha}{\partial K_B^\alpha} = \ell^{-2} \sum_B K_{AB}^\alpha g_\alpha, \quad \sum_A B_{Aa} w_A = \frac{1}{\ell^2} \sum_\alpha g_\alpha \left(Q_a^\alpha - \frac{\tilde{c}}{\ell} Q_a p_\alpha \right) \\
g_\alpha &= z^2 - 1 + 3z^2(1+z)\lambda_\alpha \frac{3\lambda_\alpha(1-z) - 2C_\alpha}{(C_\alpha + 3z\lambda_\alpha)^2}. \quad (\text{A.93})
\end{aligned}$$

Finally we can solve for h'_a using

$$\begin{aligned}
K_A^\alpha &= \frac{k_\alpha^A}{1 - C_\alpha^{-1} \sum_B k_\alpha^B} = \ell p_A^\alpha, \quad k_\alpha^A = \frac{\ell p_A^\alpha}{1 + C_\alpha^{-1} \ell p_A^\alpha}, \\
h'_a &= \frac{1}{2} \sum_{A\alpha} Q_a^{A\alpha} \frac{k_\alpha'^A}{k_\alpha^A} = \frac{1}{2\ell} \sum_\alpha \frac{Q_a^\alpha}{1 + C_\alpha^{-1} \ell p_\alpha}, \quad h'_X = \frac{1}{\ell \delta_X} \sum_\alpha \frac{C_\alpha p_\alpha}{C_\alpha + \ell p_\alpha}. \quad (\text{A.94})
\end{aligned}$$

Then at the vacuum

$$c = \frac{\ell^2}{3 - \ell k' - \delta_X \ell^2 h'_X} = \frac{\ell^2}{3 + (\ell^2 w - 3z)/(1+z)^2 - \delta_X \ell^2 h'_X} = \frac{\ell^2 (1+z)^2}{3(1+2z)}. \quad (\text{A.95})$$

is unchanged with respect to the case of a minimal Kähler potential. When scalars Φ_β^M with vanishing vev 's are included in the Kähler potential (??), we have (in order δ^0)

$$\tilde{K}_M^\beta = \hat{K}_M^\beta + 2L\hat{S}_M^\beta = K_M^\beta = \left(\partial_M^\beta K^\beta \right)_\ell = \frac{1}{1 - C_\beta^{-1} \sum_A k_\beta^A} = 1 + C_\beta^{-1} \ell p_\beta, \quad (\text{A.96})$$

The scalar masses are now given by

$$\begin{aligned} m_{M\beta}^2 &= (K_M^\beta)^{-1} \frac{\partial V}{\partial x_\beta^M} = (K_M^\beta)^{-1} V_M^\beta, \\ V_M^\beta &= \frac{|u|^2}{16} v_M^\beta + \frac{1}{s} \sum_a \tilde{K}_a \tilde{K}_{a,M\beta}, \quad v_M^\beta = w_M^\beta + \sum_a A_a \tilde{K}_{a,M\beta}. \end{aligned} \quad (\text{A.97})$$

As before the terms the terms proportional to $\sum_a \tilde{K}_a \tilde{K}_{aM}$ cancel out, and we obtain

$$m_{M\beta}^2 = (K_M^\beta)^{-1} \frac{|u|^2}{16} \left(w_M^\beta - \sum_{A\beta a} w_A^\beta B_{A\beta a} \tilde{K}_{aM} \right). \quad (\text{A.98})$$

We now have

$$\begin{aligned} w_M^\beta &= 2b_c^2 K_M^\beta - K_\beta^{D\bar{C}} K_{\bar{C}ME}^\beta K_\beta^{E\bar{B}} (p_D^\beta + b_c K_D^\beta) (p_B^\beta + b_c K_B^\beta) \\ &= \sum_B K_{MB}^\alpha \frac{\partial w^\alpha}{\partial K_B^\alpha} = \ell^{-2} K_M^\beta [1 + g_\beta (1 + C_\beta^{-1} \ell p_\beta)] + O(x_M), \\ \tilde{K}_{a,M\beta} &= 2 \sum_B q_{B\beta}^a K_{MB}^\beta + c \delta_{aX} \delta_X K_M'^\beta + O(\delta, x_M) \\ &= 2 K_M^\beta \sum_b \left(q_{M\beta}^b + C_\beta^{-1} \sum_A q_{A\beta}^b K_A^\beta \right) (\delta_{ab} + c \delta_{aX} \delta_X h'_b), \end{aligned} \quad (\text{A.99})$$

where in the first equalities the sums are over all chiral multiplets, and in the second equalities we specialized to the Kähler potential (??). For the minimal case we obtain

$$\begin{aligned} \frac{m_{M\beta}^2}{m_{\tilde{G}}^2} &= \frac{1}{z^2} [1 + g_\beta (1 + C_\beta^{-1} \ell p_\beta)] \\ &\quad - \frac{1}{z^2} \sum_\alpha \left(\zeta_{M\beta}^\alpha + C_\beta^{-1} \ell \sum_A p_A^\beta \zeta_{A\beta}^\alpha \right) \sum_\gamma \left[\delta_{\alpha\gamma} + \frac{3cz\lambda_\gamma C_\alpha}{\ell^2 (C_\alpha + 3z\lambda_\alpha)} \right] \left(g_\gamma - \frac{3z\tilde{c}}{\ell^2} \sum_\delta \lambda_\delta g_\delta \right), \\ \zeta_{M\beta}^\alpha &= \sum_a q_{M\beta}^a Q_a^\alpha, \quad \zeta_M^\beta = \sum_\alpha \zeta_{M\beta}^\alpha, \quad \zeta_{A\beta}^\alpha = \sum_a q_{A\beta}^a Q_a^\alpha. \end{aligned} \quad (\text{A.100})$$

It is straightforward to check that (??) reduces to (??) for $C_\alpha \rightarrow \infty$, $3z\lambda_\alpha \rightarrow \ell^2 \delta_X h'_X$. In the examples considered in the text, each sector α with some $k_A^\alpha \neq 0$ includes a complete minimal set with charges $q_{A\alpha}^a = q_A^a$, so $\zeta_{A\beta}^\alpha = 1$.

B Fermion masses

The general expression for the Yukawa couplings of the fermionic superpartners of the D-moduli

$$\chi^A = \frac{1}{\sqrt{2}} \mathcal{D}D^A| \quad (\text{B.1})$$

is somewhat complicated and involves the reparameterization connection derived from the effective Kähler potential $\tilde{K}$:

$$\Gamma_{BC}^A = \tilde{K}^{AD} \tilde{K}_{DBC}. \quad (\text{B.2})$$

However, due to the condition (??), in the simple model considered here, the Yukawa term simplifies considerably since we can drop terms proportional to $\sum_A k^A \chi^A$, and $k^A = \ell p^A = \ell k'^A$, $k''^A = 0$. Therefore the connection (??) drops out, as do terms proportional to $\sum_A W_A \chi^A$ and $\sum_A \hat{K}_A \chi^A$. Defining

$$\chi^\alpha = \frac{1}{\sqrt{2}} \mathcal{D}\Pi^\alpha|, \quad \chi_c = \frac{1}{\sqrt{2}} \mathcal{D}U|, \quad \chi_\ell = \sqrt{2} \mathcal{D}L|, \quad \chi^I = \frac{1}{\sqrt{2}} \mathcal{D}T^I|, \quad (\text{B.3})$$

the Yukawa couplings take the form³⁰ [?] (in the gauge $\sigma_m \psi^m = 0$ for the gravitino)

$$\begin{aligned} \mathcal{L}_Y = & -\frac{1}{2} \sum_{AB} (\chi^A \chi^B) \left\{ e^{\hat{K}/2} (W_{AB} + \hat{K}_{AB} W) + \frac{u}{4} [2\hat{S}_{AB} (1 - 2b_c \ell) - b'_c \hat{K}_{AB}] \right\} \\ & + \sum_\alpha \left\{ (\chi^\alpha \chi^\alpha) \frac{u}{8} \frac{b_c^\alpha}{(\pi^\alpha)^2} - e^{\hat{K}/2} \left[\sum_I (\chi^\alpha \chi^I) (W_{\alpha I} + \hat{K}_I W_\alpha) + \sum_A (\chi^\alpha \chi^A) W_{\alpha A} \right] \right\} \\ & - e^{\hat{K}/2} \left\{ \frac{1}{2} \sum_{IJ} (\chi^I \chi^J) [W_{IJ} + (2W_I \hat{K}_J + \hat{K}_I \hat{K}_J W) (1 - \delta_{IJ})] + \sum_{IA} (\chi^A \chi^I) W_{AI} \right\} \\ & + \frac{1}{4} \sum_I (\chi^I \chi_c) [(b'_c - b_I) \hat{K}_I + 2b_c^I \zeta(t^I)] + \frac{1}{16\ell} \left[2(1 + b'_c \ell) \tilde{k}' + \tilde{k}'' \ell + \frac{(\tilde{k}')^2 \ell}{\tilde{k}' \ell - 3} \right] (\chi_c \chi_\ell) \\ & + \frac{u}{8} \sum_I (\chi^I \chi^I) (\partial_I - 2\hat{K}_I) \{ [b'_c - b_I (1 - 2b_c \ell)] \hat{K}_I + 2b_c^I \zeta(t^I) \} \\ & - \frac{1}{8} (\chi_\ell \chi_\ell) \left\{ \frac{u}{4\ell^2 (3 - \ell \tilde{k}')} [3\tilde{k}' - \ell (\tilde{k}')^2 (1 - b_c \ell) - \ell \tilde{k}'' (3 - 2\ell \tilde{k}')] (1 + b_c \ell)] \right\} \\ & \quad + (\tilde{k}'' + \tilde{k}'^2) e^{\hat{K}/2} W - \frac{b'_c u}{4} \tilde{k}'' \left\} - \sum_\alpha \frac{b_c^\alpha}{4\pi^\alpha} (\chi^\alpha \chi_c) - \frac{b'_c}{8u} (\chi_c \chi_c) \right. \\ & \left. - \frac{1}{2} e^{\hat{K}/2} \tilde{k}' \left[\sum_\alpha W_\alpha (\chi_\ell \chi^\alpha) + \sum_I (W_I + \hat{K}_I W) (\chi_\ell \chi^I) \right] + \text{h.c.}, \right. \quad (\text{B.4}) \end{aligned}$$

where we used

$$\Gamma_{II}^I = 2K_I = 2\hat{K}_I = -(\text{Ret}^I)^{-1}, \quad \hat{K}_{IJ} = \delta_{IJ} \hat{K}_I^2, \quad (\text{B.5})$$

³⁰A factor $u_{(a)}$ is missing from the second and third term on the right hand side of (D.2) in [?].

and we dropped terms proportional to F^I which vanishes in the vacuum. Eliminating the static fields χ^α, χ_c by their equations of motion gives

$$\begin{aligned}
\chi^\alpha &= \frac{\pi^\alpha}{u} \chi_c + \frac{4(\pi^\alpha)^2}{ub_c^\alpha} e^{\hat{K}/2} \left[\sum_I \chi^I (W_{\alpha I} + \hat{K}_I W_\alpha) + \sum_A \chi^A W_{\alpha A} + \frac{\tilde{k}'}{2} \chi_\ell W_\alpha \right], \\
\chi_c &= \frac{u}{4b_c' \ell} \left[2(1 + b_c' \ell) \tilde{k}' + \tilde{k}'' \ell + \frac{(\tilde{k}')^2 \ell}{\tilde{k}' \ell - 3} \right] \chi_\ell \\
&\quad + \frac{u}{b_c'} \sum_I \left[\hat{K}_I (b_c' - b_I) + 2b_c^I \zeta(t^I) \right] \chi^I - \sum_\alpha \frac{ub_c^\alpha}{b_c' \pi^\alpha} \chi^\alpha \\
&= \left(1 - \frac{b_c}{b_c'} \right) \chi_c + \frac{u}{4b_c' \ell} \left(2\tilde{k}' (b_c \ell + 1) + \tilde{k}'' \ell + \frac{(\tilde{k}')^2 \ell}{\tilde{k}' \ell - 3} \right) \chi_\ell \\
&\quad + \frac{u}{b_c'} \sum_I \chi^I \left[(b_c - b_I) \hat{K}_I + 2(b_I + p_I - b_c) \zeta(t^I) \right], \tag{B.6}
\end{aligned}$$

where we used the equation of motion for F^α in (??), the constraints (??) and the definitions (??); in particular

$$\sum_{A,\alpha} b_c^\alpha q_\alpha^A \chi^A = \sum_A p_A \chi^A = 0. \tag{B.7}$$

Evaluating (??) at the moduli vacuum values

$$F^I = 0 = 1 + 4\text{Ret}^I \zeta(t^I), \quad \hat{K}_I = 2\zeta(t^I), \tag{B.8}$$

gives

$$\begin{aligned}
X^\alpha &\equiv e^{\hat{K}/2} W_\alpha \chi^\alpha = \frac{b_c^\alpha u}{4} \left[2 \sum_K \chi^K \zeta(t^K) (q_I^\alpha + p_I^\alpha) + \sum_A \chi^A q_\alpha^A + \frac{\tilde{k}'}{2} \chi_\ell - \frac{\chi_c}{u} \right] \\
&= \frac{b_c^\alpha u}{4} \left[2 \sum_I \chi^I \zeta(t^I) \left(q_I^\alpha + p_I^\alpha - \frac{p_I}{b_c} \right) + \sum_A \chi^A q_\alpha^A - \frac{1}{4b_c \ell} \left(2\tilde{k}' + \tilde{k}'' \ell + \frac{(\tilde{k}')^2 \ell}{\tilde{k}' \ell - 3} \right) \chi_\ell \right], \\
\chi_c &= \frac{u}{4b_c \ell} \left[2\tilde{k}' (b_c \ell + 1) + \tilde{k}'' \ell + \frac{(\tilde{k}')^2 \ell}{\tilde{k}' \ell - 3} \right] \chi_\ell + \frac{2u}{b_c} \sum_I \chi^I p_I \zeta(t^I). \tag{B.9}
\end{aligned}$$

Using (??) and (??), we have

$$\begin{aligned}
e^{\hat{K}/2} W_{AB} &= -\frac{u}{4} \sum_\alpha b_c^\alpha q_A^\alpha q_B^\alpha, \quad e^{\hat{K}/2} W_{AI} = -\frac{u}{2} \sum_\alpha b_c^\alpha q_A^\alpha (q_I^\alpha + p_I^\alpha), \\
e^{\hat{K}/2} W_{IJ} &= -u \zeta(t^I) \zeta(t^J) \left[\sum_\alpha b_c^\alpha (q_I^\alpha + p_I^\alpha) (q_J^\alpha + p_J^\alpha) - 2b_I - 2p_I + 2b_c^I + b_c + b_c' \right] \\
&\quad - \frac{u}{2} \zeta'(t^I) (b_I + p_I - b_c - b_c^I) \delta_{IJ}. \tag{B.10}
\end{aligned}$$

Then defining

$$\begin{aligned}
\mathcal{L}_\alpha &\equiv -\frac{1}{2} \sum_\alpha X^\alpha \left\{ 2 \sum_I \chi^I \zeta(t^I) (q_I^\alpha + p_I^\alpha) + \sum_A \chi^A q_A^\alpha + \chi_\ell \frac{\tilde{k}'}{2} \right\} \\
&= e^{\hat{K}/2} \left[\frac{1}{2} \sum_{IJ} (\chi^I \chi^J) W_{IJ} + \frac{1}{2} \sum_{AB} (\chi^A \chi^B) W_{AB} + \sum_{IA} (\chi^I \chi^A) W_{IA} \right] \\
&\quad - \frac{u}{2b_c} \sum_{IJ} (\chi^I \chi^J) \zeta(t^I) \zeta(t^J) \left[(b_I + p_I - b_c^I) (2b_c - p_J) - b_c^2 - b_c' (b_c - p_J) \right] \\
&\quad + \frac{u}{4} \sum_I (\chi^I \chi^I) \zeta'(t^I) (b_I + p_I - b_c - b_c^I) \\
&\quad + \frac{u}{16b_c \ell} \sum_I \zeta(t^I) \left\{ (b_c - b_c') \left(2\tilde{k}' + \tilde{k}'' \ell + \frac{(\tilde{k}')^2 \ell}{\tilde{k}' \ell - 3} \right) \right. \\
&\quad \quad \left. - 2\ell \tilde{k}' [b_c (b_I - b_c' - b_c^I) + p_I b_c'] \right\} (\chi^I \chi_\ell) \\
&\quad + \frac{u \tilde{k}'}{64b_c \ell} (b_c - b_c') \left(2\tilde{k}' + \tilde{k}'' \ell + \frac{(\tilde{k}')^2 \ell}{\tilde{k}' \ell - 3} \right) (\chi_\ell \chi_\ell) \\
\mathcal{L}_c &\equiv \frac{1}{8} \left\{ \sum_I (\chi^I \chi_c) [(b_c' - b_I) \hat{K}_I + 2b_c^I \zeta(t^I)] + \frac{b_c' \tilde{k}'}{2} (\chi_c \chi_\ell) \right\} \\
&= \frac{u p_I}{2b_c} \sum_{IJ} (\chi^I \chi^J) (b_c^I - b_I + b_c') \zeta(t^I) \zeta(t^J) \\
&\quad + \frac{u}{16b_c \ell} \sum_I (\chi^I \chi_\ell) \left\{ \left[2(1 + b_c \ell) \tilde{k}' + \tilde{k}'' \ell + \frac{(\tilde{k}')^2 \ell}{\tilde{k}' \ell - 3} \right] (b_c^I - b_I + b_c') + 2\ell b_c' p_I \right\} \zeta(t^I), \\
&\quad + \frac{u b_c' \tilde{k}'}{64b_c \ell} \left[2(1 + b_c \ell) \tilde{k}' + \tilde{k}'' \ell + \frac{(\tilde{k}')^2 \ell}{\tilde{k}' \ell - 3} \right] (\chi_\ell \chi_\ell) \tag{B.11}
\end{aligned}$$

we see that the second derivatives of the superpotential W drop out of the Yukawa coupling:

$$\mathcal{L}_Y = \mathcal{L}(\chi^A, \chi^I, \chi_\ell) + \mathcal{L}_\alpha + \mathcal{L}_c + \text{h.c.} = \mathcal{L}(\chi^A) + \mathcal{L}(\chi^I, \chi_\ell) + \text{h.c.} \tag{B.12}$$

Referring to (??) and (??), the D-fermion masses are determined by

$$\mathcal{L}(\chi^A) = -\frac{u}{8} \sum_A (\chi^A \chi^A) [k'^A (1 - 3b_c \ell) - b_c k^A] = -\frac{m_{\tilde{G}}}{2z} \sum_A (\chi^A k^A \chi^A) (1 - 4z), \tag{B.13}$$

Here we evaluated (at leading order in δ) $S_{AB}|$ from (??) using (??) to obtain

$$\ell_{\hat{\ell}A} = 2c \sum_a h'_a q_A^a k'^A + c \sum_a \tilde{K}_{aA} [\tilde{c}^{-1} \tilde{c}' h'_a + h''_a] = c k''^A + c' k'^A = (\partial_{\hat{\ell}} \ell_A)_A, \tag{B.14}$$

where the last equality follows directly from (??). When we make the fermion field redefinitions analogous to (??)–(??), the Lagrangian quadratic in D-fermions reads

$$\mathcal{L}_{fD} = \frac{1}{2} \sum_A \bar{\chi}^i \left(i \not{\partial} - m_{\tilde{G}} \frac{1 - 4z}{z} \right) \chi^i, \tag{B.15}$$

where $z = b_c \ell$ and here $\chi^i = C(\bar{\chi}^i)^T = C(\bar{\chi}^i)^T$ is a four-component Weyl fermion. For the dilatino and T-modulini, we have, using (??) in (??),

$$\begin{aligned}
\mathcal{L}(\chi^I, \chi_\ell) = & -\frac{u}{64\ell^2} \left[2\tilde{k}' - \ell\tilde{k}'^2 (1 + 2b_c\ell) - \ell^2\tilde{k}'' (2b_c\ell + 3\tilde{k}' + 2b_c\ell\tilde{k}') \right. \\
& \left. + \frac{\ell}{3 - \tilde{k}'\ell} \left\{ \ell\tilde{k}' (2b_c\tilde{k}' + 4(\tilde{k}')^2 + \tilde{k}''(1 + b_c\ell)) \right\} \right] (\chi_\ell \chi_\ell) \\
& + \frac{u}{2b_c} \sum_{IJ} (\chi^I \chi^J) p_I p_J \zeta(t^I) \zeta(t^J) \\
& + \frac{u}{4} \sum_I (\chi^I \chi^I) \left\{ (b_I + p_I - b_c) \zeta'(t^I) - 2\zeta^2(t^I) [b_I (1 + 2b_c\ell) + p_I + b_c] \right\} \\
& + \frac{u}{16b_c\ell} \sum_I \zeta(t^I) (b_c + b_c^I - b_I) \left(2\tilde{k}' + \tilde{k}''\ell + \frac{(\tilde{k}')^2\ell}{\tilde{k}'\ell - 3} \right) (\chi^I \chi_\ell). \tag{B.16}
\end{aligned}$$

To determine the fermion masses we evaluate this expression at the vacuum values $v'(\ell) = v(\ell) = 0$. Using (??), (??), and the vacuum conditions (??), (??), we obtain

$$\begin{aligned}
\mathcal{L}(\chi^I, \chi_\ell) = & \frac{u}{4} \sum_I (\chi^I \chi^I) \left[(b_I + p_I - b_c) \zeta'(t^I) - \frac{p_I + b_c + b_I(1 + 2b_c\ell)}{8(\text{Ret}^I)^2} \right] \\
& - \frac{u}{32} \sum_{IJ} (\chi^I \chi^J) \frac{p_I p_J}{(\text{Ret}^I)^2} - \frac{3zu(3 + 7z + z^2 - z^3)}{64\ell(1 + z)^3(1 + 2z)} \sum_I (\chi^I \chi_\ell) \frac{b_c + b_c^I - b_I}{\text{Ret}^I} \\
& - \frac{z^2(1 - 13z + 24z^2 + 7z^3 + 35z^4 + 54z^5 + 6z^6)u}{64\ell^3(1 + z)^5(1 + 2z)} (\chi_\ell \chi_\ell). \tag{B.17}
\end{aligned}$$

The corresponding kinetic energy terms [?] are given in Dirac notation in terms of 4-component Majorana spinors by

$$\begin{aligned}
\mathcal{L}_{KE}(\chi^I, \chi_\ell) = & \frac{i\tilde{k}'}{16\ell} \bar{\chi}_\ell \not{\partial} \chi_\ell + \frac{i}{2} \sum_I (1 + b_I\ell) \hat{K}_I^2 \bar{\chi}^I \not{\partial} \chi^I \\
= & \frac{3iz^2}{16(1 + z)^2\ell^2} \bar{\chi}_\ell \not{\partial} \chi_\ell + \sum_I \frac{i(1 + b_I\ell)}{8(\text{Ret}^I)^2} \bar{\chi}^I \not{\partial} \chi^I, \tag{B.18}
\end{aligned}$$

and the mass matrix in terms of the canonically normalized fields χ_N^I is

$$\begin{aligned}
m_{\chi^I \chi^J} = & \frac{u}{4(1 + b_I\ell)} \left\{ \delta_{IJ} \left[8(\text{Ret}^I)^2 (b_I + p_I - b_c) \zeta'(t^I) - p_I - b_c - b_I(1 + 2b_c\ell) \right] + p_I p_J \right\}, \\
m_{\chi_\ell} = & -b_c u \frac{(1 - 13z + 24z^2 + 7z^3 + 35z^4 + 54z^5 + 6z^6)}{12z(1 + z)^3(1 + 2z)}, \\
m_{\chi_\ell \chi^I} = & -\sqrt{\frac{3}{2}} \frac{u(3 + 7z + z^2 - z^3)(b_c + b_c^I - b_I)}{4(1 + z)^2(1 + 2z)\sqrt{1 + b_I\ell}}. \tag{B.19}
\end{aligned}$$

In the FIQS model used in the text as an illustrative example, $b_c^I = 0$, and $b_I = b_c + p_I$. Since b_I, p_I are nearly independent of I , the mixing simplifies considerably in the case that all three moduli are stabilized at the same self-dual point. In this case the eigenstates of $m_{\chi^I \chi^J}$ are

$$\chi_0 = \sum_I \chi_N^I / \sqrt{3}, \quad (\text{B.20})$$

and two orthogonal combinations χ_b that have the same mass:

$$m_{\chi_b} = u \frac{8(\text{Re} t^I)^2 p_I \zeta'(t^I) - (p_I + b_c)(1 + b_c \ell)}{2(1 + b_I \ell)}. \quad (\text{B.21})$$

Only χ_0 mixes with the dilaton, with mass matrix

$$m = \begin{pmatrix} m_{\chi_\ell} & \sqrt{3} m_{\chi_\ell \chi^I} \\ \sqrt{3} m_{\chi_\ell \chi^I} & m_{\chi_b} + m' \end{pmatrix}, \quad m' = \frac{3up_I^2}{4(1 + b_I \ell)}. \quad (\text{B.22})$$

C Massive $\mathcal{G}_c$ -charged chiral multiplets

If some $\mathcal{G}_c$ -charged chiral multiplets acquire masses at the $U(1)_a$ -breaking scale as in (??), they do not contribute to the anomalies of the effective theory below the condensation scale, and the anomaly matching conditions (??), (??) and (??) are modified to read

$$\begin{aligned} \sum_\alpha b_c^\alpha q_I^\alpha &= b_I - b'_c - b_c^I + \sum_R \frac{C_c^R}{4\pi^2} (1 - q_I^R - q_I^{\bar{R}}), \\ \sum_\alpha b_c^\alpha q_\alpha^a &= -\frac{1}{2} \delta_X \delta_{aX} - \sum_R \frac{C_c^R}{4\pi^2} (q_R^a + q_{\bar{R}}^a), \\ 2 \sum_A q_A^a p_A &= \delta_X \delta_{aX} + \sum_R \frac{C_c^R}{2\pi^2} (q_R^a + q_{\bar{R}}^a), \end{aligned} \quad (\text{C.1})$$

where b_c, b'_c are defined as in Section ?? in terms of the Casimirs of the massless spectrum of the strongly coupled gauge group. Then (??) becomes

$$\prod_A |\phi^A|^{-2} \sum_\alpha b_c^\alpha q_\alpha^A = \exp \left[- \sum_A p_A \ln x^A + \sum_I p_I g^I - \delta_X h_X - \sum_{R,a} \frac{C_c^R}{2\pi^2} (q_R^a + q_{\bar{R}}^a) h_a \right], \quad (\text{C.2})$$

The contribution of (??)

$$\mathcal{L}_R = - \sum_R \frac{C_c^R}{16\pi^2} \int d^4\theta \frac{E}{R} U_c \ln \left\{ c_R \prod_I [\eta(T^I)]^{2(q_I^R + q_I^{\bar{R}} + p_I^R - 1)} \prod_A (\phi^A)^{q_R^A} \right\} + \text{h.c.}, \quad (\text{C.3})$$

gives an additional contribution to the bosonic Lagrangian (??):

$$\begin{aligned}
e^{-1}\mathcal{L}_B^R &= -u \sum_R \frac{C_c^R}{8\pi^2} \left[2 \sum_I (q_I^R + q_I^{\bar{R}} + p_I^R - 1) \zeta(t^I) F^I + q_R^A F^A \right] + \frac{1}{8} L_c^R (F_c - u \bar{M}) + \text{h.c.}, \\
L_c^R &= \sum_R \frac{C_c^R}{2\pi^2} \left[\ln |c_R|^2 + 2 \sum_I (q_I^R + q_I^{\bar{R}} + p_I^R - 1) \ln |\eta(t^I)|^2 \right. \\
&\quad \left. + \sum_A q_R^A \left(\ln x^A - G^A - \sum_a q_A^a h_a \right) \right], \tag{C.4}
\end{aligned}$$

where we used (??). Gauge invariance and modular covariance of the superpotential (??) imply

$$\sum_A q_R^A q_A^a = -q_R^a - q_{\bar{R}}^a, \quad p_I^R = \sum_A q_R^A q_I^A. \tag{C.5}$$

Then defining

$$p_I^r \equiv \sum_R \frac{C_c^R}{4\pi^2} p_I^R = \sum_A p_A^r q_I^A, \quad p_A^r \equiv \sum_R \frac{C_c^R}{8\pi^2} q_R^A q_A^a, \tag{C.6}$$

the equations of motion (??) for the auxiliary fields F_c, F^I and F^A are modified to read

$$\begin{aligned}
\bar{u}u &= e^{-2b'_c/b_c} e^{\kappa-2(\hat{S}-\delta s)/b_c} \prod_\alpha |b^\alpha/4c_\alpha|^{-2b^\alpha/b_c} \prod_I \left[2\text{Ret}^I |\eta(t^I)|^4 \right]^{(b_I - b_c + \hat{p}_I)/b_c} e^{-\sum_A \hat{p}_A \ln x^A/b_c}, \\
F^I &= -\frac{2\text{Ret}^I}{1+b_I \hat{\ell}} \frac{\bar{u}}{4} (b_c - b_I - \hat{p}_I) \left[1 + 4\text{Ret}^I \zeta(\bar{t}^I) \right], \\
F^A &= -\frac{\bar{u}}{4} \tilde{K}^{A\bar{B}} \left[2\hat{S}_{\bar{B}} - \hat{p}_B - \hat{K}_{\bar{B}} b_c \right], \tag{C.7}
\end{aligned}$$

where

$$\hat{p}_I = p_I - p_I^r = \sum_A \hat{p}_A q_I^A, \quad \hat{p}_A = p_A - p_A^r, \quad \sum_A \hat{p}_A q_A^a = \delta_X \delta_{aX}. \tag{C.8}$$

Therefore the effective potential is determined by parameters defined in terms of the modular weights and gauge charges of the full spectrum of the effective theory at the string scale, except for the renormalization group factor $\Lambda_c^2 \sim e^{-2/3b_c g_s^2}$ that depends on the β -function factor for the massless spectrum of the strongly coupled sector below the $U(1)_a$ -breaking scale.

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