UCRL-JC-132493
PREPRINT

Extension of a Thin-Wire Algorithm for Wires Moved Laterally Within a Mesh

Gerald J. Burke
David J. Steich

This paper was prepared for submittal to the
15th Annual Review of Progress in Applied Computational Electromagnetics
Monterey, CA
March 15-20, 1999

November 1998

Lawrence Livermore National Laboratory

DISCLAIMER

Extension of a Thin-Wire Algorithm for Wires Moved Laterally Within a Mesh

Gerald J. Burke and David J. Steich
Lawrence Livermore National Laboratory
Livermore, CA 94550

Introduction

While FDTD methods, such as that of Yee [1], are very versatile and have found widespread use in electromagnetics, modeling thin wires in such algorithms can present special problems. The wires are usually much thinner than the desired cell size, and reducing the cell size would require also reducing the time increment. Several methods have been used to treat thin wires in FDTD. Merewether and Fisher [2] and Umashankar *et al.* [3] used contour integration incorporating the $1/\rho$ field behavior. Parks *et al.* [4] and Holland [5] coupled the transmission line equations for the wire current to the FDTD mesh equations. Riley [6] and Mittra [7] solved integral equations for the wire current coupled to the mesh fields. These methods usually are best suited to wires that lie along edges for the electric or magnetic field in the mesh. When the wire can be moved, it often results in reduced accuracy. Requiring wires to lie on cell edges restricts the wire geometry when cells are orthogonal, and even with more general meshes it requires generation of a new mesh when a wire must be moved. This limitation may be a major reason that FDTD methods still are not nearly as popular as frequency domain integral equation and finite element techniques for modeling wire structures.

It would be very valuable to be able to locate wires independently of the mesh. This paper presents some efforts in that direction, considering the lateral movement of wires parallel to cell edges and also adjustment of the wire length with respect to the cell boundaries. The transmission line method used for the wire is similar to that described by Holland [5], but the averaging to couple the transmission line fields to the mesh fields is done in a way that can be generalized to wires moved with respect to cell edges. The treatment of wires passing through electric field edges is considered first, and then is generalized to wires moved from the edges. The accuracy when the wire is moved is demonstrated by comparisons with NEC [8].

Treatment of Wires on Cell Edges

In the method considered here the transmission line equations for current and voltage along the wire are coupled with the FDTD equations of the standard Yee algorithm [1]. The transmission line equations for the wire current are derived from the ϕ component of Maxwell's $\nabla \times \mathbf{E}$ equation and the ρ component of the $\nabla \times \mathbf{H}$ equation

$$\mu \frac{\partial H_\phi}{\partial t} = \frac{\partial E_z}{\partial \rho} - \frac{\partial E_\rho}{\partial z} \tag{1}$$

$$\epsilon \frac{\partial E_\rho}{\partial t} = \frac{1}{\rho}\frac{\partial H_z}{\partial \phi} - \frac{\partial H_\phi}{\partial z}. \tag{2}$$

For current I and charge Q on a wire on the z axis, the E_ρ and H_ϕ components for small ρ

can be approximated as

$$E_\rho = \frac{Q}{2\pi\epsilon\rho} \quad \text{and} \quad H_\phi = \frac{I}{2\pi\rho}.$$

Substituting for E_ρ and H_ϕ in equation (1) and integrating over ρ from the wire radius a to the transmission line radius r_t, with boundary condition $E_z(a) = 0$, yields

$$\frac{\mu \ln(r_t/a)}{2\pi}\frac{\partial I}{\partial t} = E_z(r_t) - \frac{\ln(r_t/a)}{2\pi\epsilon}\frac{\partial Q}{\partial z}$$

or, defining inductance $L_w = (\mu/2\pi)\ln(r_t/a)$,

$$\frac{\partial I}{\partial t} = \frac{1}{L_w}E_z(r_t) - \frac{1}{\mu\epsilon}\frac{\partial Q}{\partial z}. \tag{3}$$

At this point the transmission line radius r_t is an arbitrary distance to the outer shell of the transmission line that couples to the mesh fields. It will be found to drop out in the treatments that follow. Substituting for E_ρ and H_ϕ in equation (2), and averaging over ϕ to eliminate the $\partial H_z/\partial\phi$ term yields the continuity equation

$$\frac{\partial Q}{\partial t} = -\frac{\partial I}{\partial z}. \tag{4}$$

To reduce equations (3) and (4) to discrete form, the current and charge will be sampled at points spaced by Δz along the wire with time step Δt. Then I_k^n and Q_k^n represent the current and charge at sample point k and time step n, with charge points displaced from current by $\Delta x/2$. With a central difference approximation of the derivatives, and introducing an applied voltage V^{inc}, equations (3) and (4) become

$$I_k^{n+1/2} = I_k^{n-1/2} - \frac{\Delta t}{\Delta z\mu\epsilon}\left(Q_k^n - Q_{k-1}^n\right) + \frac{\Delta t}{\Delta z L_w}\left(E_{zk}^n\Delta z - V_k^{\text{inc}}\right) \tag{5}$$

$$Q_k^{n+1} = Q_k^n - \frac{\Delta t}{\Delta z}\left(I_{k+1}^{n+1/2} - I_k^{n+1/2}\right). \tag{6}$$

These can be written in terms of a voltage by defining capacitance $C_w = \mu\epsilon/L_w$ as

$$I_k^{n+1/2} = I_k^{n-1/2} - \frac{\Delta t}{\Delta z L_w}\left(V_k^n - V_{k-1}^n - E_{zk}^n\Delta z + V_k^{\text{inc}}\right) \tag{7}$$

$$V_k^{n+1} = V_k^n - \frac{\Delta t}{\Delta z C_w}\left(I_{k+1}^{n+1/2} - I_k^{n+1/2}\right). \tag{8}$$

The simplest situation for solving the transmission line equations coupled to the FDTD mesh is when the wire lies along the E_z edges. The transmission line current drives the mesh through the E_z update equation, and E_z from the mesh drives the transmission line through the E_{zk}^n term in equation (5). E_{zk}^n in equation (5) is interpreted as the mesh field averaged around the outer shell of the transmission line at radius r_t. However, the E_z value available from the mesh equations is the field averaged over a cell face. Holland [5] deals with this difference by averaging the equations (5) and (6), so that E_{zk}^n in equation (5) becomes the average field over the face, which is available, and L_w is averaged for ρ from a to $r_t = \sqrt{\Delta x\Delta y/\pi}$. The average inductance can be approximated as

$$\bar{L}_w = \frac{\mu}{2\pi}\frac{\int_a^{r_t}\ln(r/a)r\,dr}{\int_0^{r_t} r\,dr} = \frac{\mu}{2\pi}\left(\ln(r_t/a) - \frac{1}{2} + \frac{a^2}{2r_t^2}\right). \tag{9}$$

Holland also averages over a rectangular cell face and applies an empirical correction factor. This approach gives accurate results, but it is not clear how to generalize this averaging for a wire moved off of the E_z edge.

Another way that was found to work well for coupling the transmission line equations to the mesh fields with the wire on E_z edges is to use the exact inductance for $r_t = \Delta x/2$, and drive equation (5) with an average of the E_z mesh fields

$$E^n_{zk} = \tfrac{1}{2}\Big(E^n_{i_w,j_w,k} + \tfrac{1}{4}(E^n_{i_w-1,j_w,k} + E^n_{i_w+1,j_w,k} + E^n_{i_w,j_w-1,k} + E^n_{i_w,j_w+1,k})\Big) \qquad (10)$$

for a wire at mesh coordinates (i_w, j_w, k). In comparisons with NEC, equation (10) seemed to give slightly better accuracy than Holland's average inductance with correction factor, and remained stable to larger wire $a/\Delta x$. Another method used by Riley [6] is to locate the wire on the H_z edges and let the transmission line radius pass through the surrounding E_z points. Actually Riley had an unstructured mesh and r_t was the average distance to E_z samples. For cubical cells, $r_t = \Delta x/\sqrt{2}$, and E^n_{zk} in equation (5) is the average of the E_z mesh fields

$$E^n_{zk} = \tfrac{1}{4}(E^n_{i_w,j_w,k} + E^n_{i_w+1,j_w,k} + E^n_{i_w,j_w+1,k} + E^n_{i_w+1,j_w+1,k}) \qquad (11)$$

for the wire at $(i_w+1/2, j_w+1/2, k)$. It is not obvious how to generalize any of these averages for a wire at an arbitrary location in the x-y plane while maintaining the same accuracy. We also tried bi-quadratic interpolation on the 3×3 edges surrounding the wire on an E_z edge, and bi-cubic interpolation on the 4×4 E_z edges surrounding a wire on a H_z edge, but the results were not satisfactory.

Wires Displaced from Cell Edges

To generalize the wire location, we looked at the behavior of the fields in the mesh due to the wire excitation. The field component E_z at a distance 0, Δx and $2\Delta x$ from a wire located on the E_z edges is shown in Figure 1 for varying wire radius, with the computed fields connected by straight lines. The wire length was 1.4 m with a cell size of $\Delta x = 0.035$ m, and the center three segments of the wire were excited with 1/3 volts each, so that results could be compared for a cell size three times larger without changing the source gap size. The fields in Figure 1 were obtained using the average field of equation (10) to drive the transmission line. Similar results were obtained using Holland's average for a square cell with correction term [5, eq. 53] but that solution became unstable for radii of 0.01 and 0.014 m. The fields in Figure 1 represent the the peak values adjacent to the center of the wire due to a Gaussian pulse excitation with full-width-half-max (t_{fwhm}) of 2 ns. For all solutions in Figure 1 the wire current was in good agreement with the NEC model.

Examination of Figure 1 shows that E_z varies with x approximately as the average of $\ln(x/a)$ with the function going negative for $x < a$. This differs from the average in equation (9), where the field is assumed zero inside the wire. The transmission line equations, in effect, put a boundary condition $E_z(a) = 0$ on the fields that the mesh is averaging, but the field goes negative inside the wire as if it were produced by a filament of current on the wire axis. A simple model for comparison with the mesh fields is obtained by averaging $\log(\rho/a)$ over an annular region extending from $\rho = \max(0, r_0 - \Delta/2)$ to $r_0 + \Delta/2$. This yields

$$F_a(r_0, \Delta, a) = \begin{cases} -\frac{1}{2} + \frac{r_1^2 \ln(r_1/a) - r_2^2 \ln(r_2/a)}{r_1^2 - r_2^2} & \text{if } r_0 > \Delta/2 \\ -\frac{1}{2} + \ln(r_2/a) & \text{if } 0 \le r_0 \le \Delta/2 \end{cases} \qquad (12)$$

Table. 1. Relations of average E_z from the mesh and corresponding average of $\log(r/a)$ for a distance of 0 and Δx from a wire of radius a (E_0, E_1 and A_0, A_1, respectively).

Wire rad. (m)	E_0/E_1	A_0/A_1	$(E_0+E_1)/(2E_0)$	$\log(\frac{\Delta}{2a})/A_0$
0.014	−0.557	−0.551	−0.411	−0.425
0.01	−0.169	−0.143	−2.48	−2.963
0.003	0.380	0.411	1.806	1.737
0.001	0.486	0.592	1.380	1.354
10^{-5}	0.826	0.821	1.112	1.111
10^{-7}	0.885	0.886	1.065	1.066

where $r_1 = r_0 - \Delta/2$ and $r_2 = r_0 + \Delta/2$. The average could be computed over cell faces, but in either case it was found that a fudge factor was needed on the region size for best agreement with the observed fields and best agreement of the currents with NEC. For a mesh of cubical cells with size Δx, it was found that the mesh fields near the wire were matched best by $F_a(\rho, f_c\Delta x, a)$ with f_c approximately 0.78. This seems to work reasonably well over a wide range of wire radii and cell sizes. When averaging over a square cell face, rather than an annular region, the factor was approximately $0.70\Delta x$ to shrink the side length. Holland arrived at a similar correction by subtracting a constant from his average inductance to make it cross zero at an $a/\Delta x$ that matched a moment method solution for the wire. The factor of 0.78 for the annular average was chosen for best agreement of the dipole input impedance with NEC, although 0.80 would work about as well. Any significance of this factor is not understood at this point. However, it was noted that $f_c = 0.802188$ is the solution of the equation

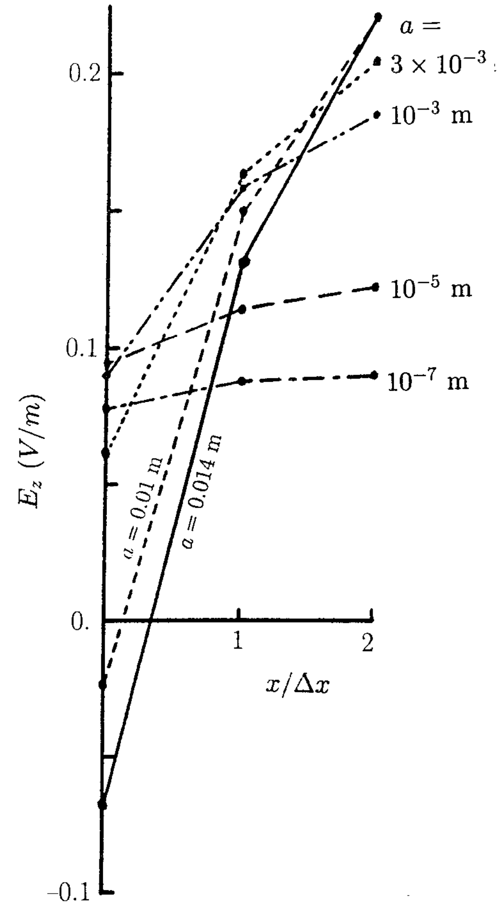

Fig. 1. E_z in the mesh at distances 0, Δx and $2\Delta x$ from a wire with radius a for $\Delta x =$ 0.035 m.

$$[F_a(0, f_c\Delta x, a) + F_a(\Delta x, f_c\Delta x, a)]/2 = \log(\Delta x/2a) \tag{13}$$

independent of Δx and a. That is, the average of the two averages is equal to $\log(\rho/a)$ at $\rho = \Delta x/2$.

The ratios of E_0/E_1 and A_0/A_1 are compared in Table 1, where E_0 and E_1 are the mesh fields E_z at the wire and one Δx away, and A_0 and A_1 represent $F_a(\rho, f_c\Delta x, a)$ at $\rho = 0$ and Δx. The correspondence shows that $F_a(\rho, f_c\Delta x, a)$ tracks the field well over the range of wire radii. For the case of Figure 1, where $\Delta x = 0.035$ m and $t_{\text{fwhm}} = 17\Delta x/c$ the function $F_a(\rho, f_c\Delta x, a)$ tracks the mesh fields reasonably well out to several Δx from the wire, while for the same wire with $\Delta x = 0.1$ m and $t_{\text{fwhm}} = 6\Delta x/c$ the average tracks the mesh fields from 0 to one Δx. In the last two columns of Table 1, the average of the mesh fields $(E_0 + E_1)/2$ and $\log(\Delta x/2a)$ are compared, both normalized by their average about the wire. The correspondence is expected, since the average of equation (10) works well in driving the transmission line equations.

Using $F_a(\rho, f_c\Delta x, a)$ as a model for the averaged mesh fields, the local E_z at a distance

ρ from the wire can be related to the mesh field at the wire location as

$$E_{zk}^{n}(\rho) = E_z^n(i_w, j_w, k)\ln(\rho/a)/F_a(0, f_c\Delta x, a). \tag{14}$$

Equation (14) can be used with $\rho = r_t$ to obtain E_{zk}^n driving the transmission line in equation (5). However, equation (14) applies only to the E_z field due to a current on the wire. When the wire is excited with a voltage source, changing the boundary condition on $E_z(a)$, a field component due to the derivative of charge is introduced that is more slowly varying than the form of equation (14). Hence the applied voltage should be removed from the E_z mesh field before applying the correction, and the source term in equation (5) becomes

$$\begin{aligned}(E_{zk}^n\Delta x - V_k^{\rm inc}) \rightarrow\ & \left[(E_{zk}^n\Delta x - V_k^{\rm inc})\ln(r_t/a)/F_a(0, f_c\Delta x, a) + V_k^{\rm inc}\right] - V_k^{\rm inc}\\ =&(E_{zk}^n\Delta x - V_k^{\rm inc})\ln(r_t/a)/F_a(0, f_c\Delta x, a).\end{aligned}$$

Equation (5) for the transmission line current then becomes

$$\begin{aligned}I_k^{n+1/2} =& I_k^{n-1/2} - \frac{\Delta t}{\Delta x\mu\epsilon}\left[Q_k^n - Q_{k-1}^n\right]\\ &+ \frac{\Delta t}{\Delta x L_w}\left[\Delta x E_z^n(i_w, j_w, k) - V_k^{\rm inc}\right]\ln(r_t/a)/F_a(0, f_c\Delta x, a)\Big].\end{aligned}$$

Since $L_w = \frac{\mu}{2\pi}\ln(r_t/a)$, the $\ln(r_t/a)$ factors cancel to yield

$$I_k^{n+1/2} = I_k^{n-1/2} - \frac{\Delta t}{\Delta x\mu\epsilon}\left[Q_k^n - Q_{k-1}^n\right] + \frac{2\pi\Delta t}{\Delta x\mu F_a(0, f_c\Delta x, a)}\left[\Delta x E_z^n(i_w, j_w, k) - V_k^{\rm inc}\right] \tag{15}$$

so that r_t is eliminated. Equation (15) is essentially the same as that obtained by Holland by averaging the equation, although the correction of the average is done in a different way.

When the wire is moved away from the E_z edge, Holland [5] suggested splitting the current by linear proportionality between the surrounding E_z edges. Thus for wire current I_k^n

$$J_z^n(i, j, k) = I_k^n(i + 1 - i_w)(j + 1 - j_w)/\Delta x\Delta y \tag{16a}$$
$$J_z^n(i + 1, j, k) = I_k^n(i_w - i)(j + 1 - j_w)/\Delta x\Delta y \tag{16b}$$
$$J_z^n(i, j + 1, k) = I_k^n(i + 1 - i_w)(j_w - j)/\Delta x\Delta y \tag{16c}$$
$$J_z^n(i + 1, j + 1, k) = I_k^n(i_w - i)(j_w - j)/\Delta x\Delta y \tag{16d}$$

are the current densities in the E_z update equations for the wire located at $i \le i_w < i + 1$ and $j \le j_w < j + 1$. Holland suggests using the same factors to interpolate the E_z fields from the mesh to drive equation (5). This approach gave reasonable agreement in the peaks of admittance, but poor agreement in peaks of impedance as the wire was moved within a cell.

The field average in equation (11) gave good agreement with NEC for a wire located on an H_z edge, where the current is split equally on to the four surrounding E_z edges. Examining the mesh fields from this solution showed that they do not behave as an average of a $\ln(\rho/a)$ function from the wire location, but rather behave as a sum of averaged $\ln(\rho/a)$ functions from the driven E_z edges. Generalizing to unequal edge excitations, the field averaged about a point $\mathbf{r}$ due to excited edges at $\mathbf{r}_i$ would be expected to have the form

$$\bar{E}_z(\mathbf{r}) = \sum_{i=1}^{4} c_i F_a(|\mathbf{r} - \mathbf{r}_i|, f_c\Delta x, a) \tag{17}$$

where the c_i represent excitation strengths. Assuming that the edge fields are excited in proportion to the currents driving them, the c_i will be in the ratio of the split currents. Then defining the factors

$$\begin{aligned}
x_1 &= x_{1,1} = (i+1-i_w)(j+1-j_w) \\
x_2 &= x_{2,1} = (i_w - i)(j+1-j_w) \\
x_3 &= x_{1,2} = (i+1-i_w)(j_w - j) \\
x_4 &= x_{2,2} = (i_w - i)(j_w - j)
\end{aligned}$$

equation (17) becomes

$$\bar{E}_z(\mathbf{r}) = C\sum_{i=1}^{4} x_i F_a(|\mathbf{r}-\mathbf{r}_i|, f_c\Delta x, a).$$

This leaves one unknown constant C that can be determined from the known average field at a mesh edge. Since we are assuming four excited edges, C can be evaluated as a weighted average as

$$C = \sum_{j=1}^{4} \alpha_j \bar{E}_z(\mathbf{r}_j) \qquad \text{where} \qquad \alpha_j = x_j \Big/ \sum_{i=1}^{4} x_i F_a(|\mathbf{r}_j - \mathbf{r}_i|, f_c\Delta x, a)$$

and $\sum x_j = 1$. The local field at a point $\mathbf{r}$ can then be evaluated as

$$E_z(\mathbf{r}) = C\sum_{i=1}^{4} x_i \ln(|\mathbf{r}-\mathbf{r}_i|/a).$$

The field driving the transmission line is this local field averaged over the outer shell of the transmission line. Assuming a transmission line with radius r_t located a distance r_d from an excited edge, the average of the log function is evaluated as

$$\begin{aligned}
\bar{E}_t &= \frac{1}{2\pi}\int_0^{2\pi} \ln\left[(r_t^2 + r_d^2 - 2r_t r_d\cos(\phi))/a\right) \, d\phi \\
&= \ln(r_{\max}/a)
\end{aligned}$$

where $r_{\max} = \max(r_t, r_d)$. Hence, if r_t is made large enough to enclose all driven edges, the average field for use in equation (5) in the k plane becomes

$$\begin{aligned}
E_{zk}^n =& C\ln(r_t/a) = \ln(r_t/a)\sum_{j=1}^{4} \alpha_j E_{zk}(\mathbf{r_j}) \\
=& \ln(r_t/a)\Big[\alpha_1 E_z^n(i_w, j_w, k) + \alpha_2 E_z^n(i_w+1, j_w, k) \\
& + \alpha_3 E_z^n(i_w, j_w+1, k) + \alpha_4 E_z^n(i_w+1, j_w+1, k)\Big].
\end{aligned} \tag{18}$$

Using (18) in equation (5), and subtracting the applied field from E_z results in the update equation

$$\begin{aligned}
I_k^{n+1/2} =& I_k^{n-1/2} - \frac{\Delta t}{\Delta x \mu\epsilon}\left(Q_k^n - Q_{k-1}^n\right) + \frac{2\pi\Delta t}{\Delta x\mu}\Big[\Delta x\Big(\alpha_1 E_z^n(i_w, j_w, k) + \alpha_2 E_z^n(i_w+1, j_w, k) \\
& + \alpha_3 E_z^n(i_w, j_w+1, k) + \alpha_4 E_z^n(i_w+1, j_w+1, k)\Big) - \alpha_{\text{sum}} V_k^{\text{inc}}\Big]
\end{aligned} \tag{19}$$

where $\alpha_{\text{sum}} = \sum_{i=1}^{4} \alpha_i$. The transmission line radius r_t is eliminated in the averaging. However, it would still occur in the equations involving voltage, since the transmission line voltage depends on the characteristic impedance of the line.

The input impedance of a a 1.4 m dipole modeled with cell size of 0.035 m is shown in Figure 2 as the wire is moved within a cell. The excitation was a voltage source spread over the center three segments (current nodes). The results in Figure 2a, using equation 19, show good agreement with NEC and independence of wire position. Similar results were obtained for the imaginary part of impedance and with other wire radii. Results in Figure 2b were obtained by splitting the wire current among the cell edges according to equation 16, as done for Figure 1a, but the electric field driving the transmission line equations was obtained as a weighted average of the surrounding mesh fields, using the average inductance of [5, eq. 53]. Much more variation is seen in the peaks of impedance as the wire is moved. However, when the results of Figure 1b are plotted as admittance, there is only a few percent variation in the admittance peaks as the wire is moved, similar to Holland's results.

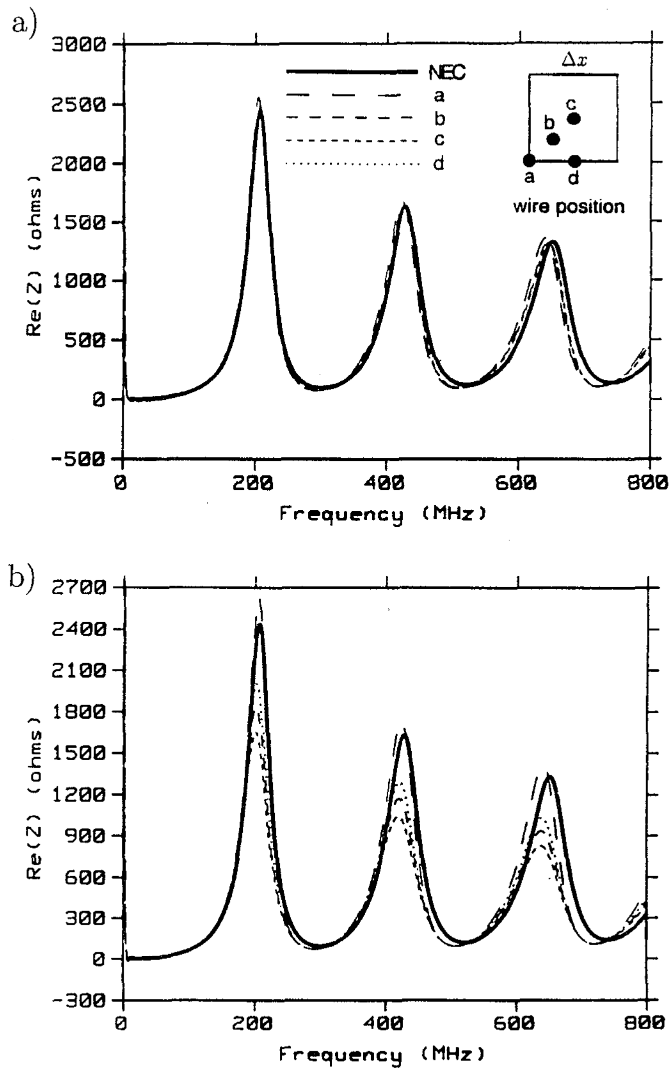

Fig. 2. Real part of input impedance as a 1.4 m dipole is moved within the mesh; a) result of equation (19), b) result using average cell inductance and linearly weighted average of mesh fields.

Adjustment of wire length

When equations (5) or (19) are solved with N charge (or voltage) nodes and fixed node spacing of Δz, the effective length of the wire is $N\Delta z$. The solution will include $N+1$ current nodes, including I_1 and I_{N+1} which have zero current and are located at $\Delta z/2$ before Q_1 and after Q_N, respectively. The length of the wire can be adjusted if the central difference for the derivate of current is replaced by a second-order approximation with arbitrary location of the zero current nodes. If the points where the current goes to zero are extended by δ from their normal positions on either ends of the wire, the derivatives are

$$\left.\frac{dI(z)}{dz}\right|_{z \text{ at } Q_1} = I_2 \frac{\Delta z - \delta}{\Delta z(\delta + \Delta z)} + I_3 \frac{\delta}{\Delta z(\delta + 2\Delta z)}$$
$$\left.\frac{dI(z)}{dz}\right|_{z \text{ at } Q_N} = -I_N \frac{\Delta z - \delta}{\Delta z(\delta + \Delta z)} - I_{N-1} \frac{\delta}{\Delta z(\delta + 2\Delta z)}$$

Results of adjusting the end locations by $\pm\Delta z/2$ from their normal positions on either end are shown in Figure 3, and are in good agreement with NEC. These results used $\Delta z = 0.1$

m, so the agreement with NEC is not as good as in Figure 2.

Conclusion

It was shown that the accuracy of results for wires moved laterally from mesh edges can be greatly improved by taking account of the behavior of the field in the vicinity of the wire. Also, the distance to the end of the wire can be adjusted within a cell by using a general second-order difference form for the derivative. Making the wire location completely independent of the mesh would require the additional ability to tilt the wire with respect to the edges. This seems to be a considerably more difficult problem than moving the wires laterally, since the component of the mesh field parallel to the wire gets mixed with the larger radial electric field due to charge on the wire. Simply averaging the mesh fields did not seem to work well, except in the case where the wire was tilted in one coordinate plane, and the mesh fields above and below the plane of the wire, on edges orthogonal to the wire normal, could be averaged. Further study is needed to develop a more general capability to tilt a wire with respect to the mesh.

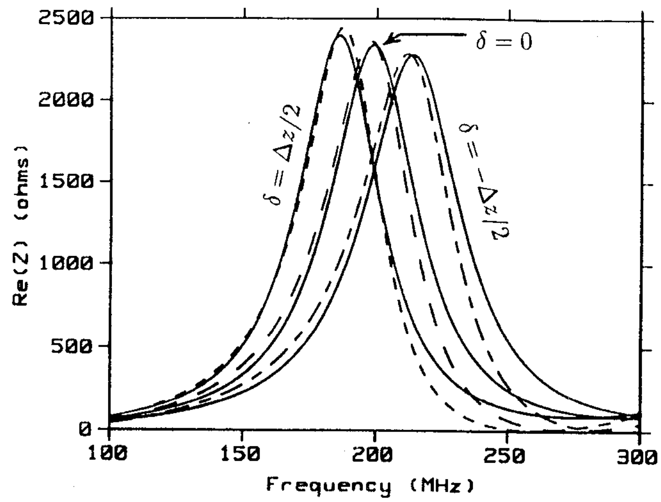

Fig. 3. Real impedance of a dipole antenna with radius 0.001 m, modeled with 14 charge nodes and Δz = 0.1 m, comparing NEC (solid) and FDTD (dashed). FDTD end points are extended by δ for effective lengths of 1.3, 1.4 and 1.5 m.

Work performed under the auspices of the U. S. Department of Energy by the Lawrence Livermore National Laboratory under Contract W-7405-Eng-48.

References

[1] K. S. Yee, "Numerical solution of initial boundary value problems involving Maxwell's equations in isotropic media," *IEEE Trans. Antennas Propagat.*, v. AP-14, pp. 302-307, May 1966.

[2] D. E. Merewether and R. Fisher, "Finite Difference Solutions of Maxwell's Equation for EMP Applications," Final Report on Contract DNA001-78-C-0231, April 22, 1980.

[3] K. R. Umashankar, R. A. Taflove and B. Beker, "Calculation and Experimental Validation of Induced Currents on Coupled Wires in an Arbitrary Shaped Cavity," *IEEE Trans. Antennas and Propagation*, v. 35, pp. 1248-1257, 1987.

[4] D. E. Parks, J. L. Gilbert and I. Katz, "A Technique for Treating Small Current-Carrying Struts in Electromagnetic Particle Codes," *Syst. Sci. Software*, SSS-R-76-2847, Jan. 1976.

[5] R. Holland, "Finite-Difference Analysis of EMP Coupling to Thin Struts and Wires," *IEEE Trans. Electromagnetic Compatibility*, v. EMC-23, no. 2, May 1981.

[6] D. J. Riley and C. D. Turner, "The *VOLMAX* Transient Electromagnetic Modeling System, Including Sub-Cell Slots and Wires on Random Non-Orthogonal Cells," *Proceedings of the 14th Annual Review of Progress in Applied Computational Electromagnetics*, Monterey, CA, March 1998.

[7] A. R. Bretones, R. Mittra and R. G. Martin, "A Hybrid Technique Combining the Method of Moments in the Time-Domain and FDTD," *IEEE Microwave and Guided Wave Letters*, v. 8, no. 8, pp. 281-283, Aug. 1998.

[8] G. J. Burke and A. J. Poggio, Numerical Electromagnetics Code (NEC) – Method of Moments, UCID-18834, Lawrence Livermore National Laboratory, CA, January 1981.