

Technicolorful Supersymmetry

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Technicolor achieves electroweak symmetry breaking (EWSB) in an elegant and natural way, while it suffers from severe model building difficulties. I propose to abandon its secondary goal to eliminate scalar bosons in exchange of solving numerous problems using supersymmetry. It helps to understand walking dynamics much better with certain exact results. In the particular model presented here, there is no light elementary Higgs boson and the EWSB is fully dynamical, hence explaining the hierarchy; There is no alignment problem and no light pseudo-Nambu-Goldstone bosons exist; The fermion masses are generated by a ultraviolet-complete renormalizable extended technicolor sector with techni-GIM mechanism and hence the sector is safe from flavor-changing-neutral-current constraints; The “ e^+e^- ” production of techni-states in the superconformal window is calculable; The electroweak precision observables are (un)fortunately not calculable.

INTRODUCTION

From the beautiful measurements of the triple gauge boson vertices at LEP-II [1] that agreed with the expectation in the non-abelian gauge theory, there is little doubt that the W and Z bosons are indeed gauge bosons of the electroweak $SU(2)_L \times U(1)_Y$ symmetry. Their finite mass implies that our Universe is filled with a Bose-Einstein condensate, analogous to Cooper pair condensates in superconductors. The analog of the finite penetration length in superconductors is the finite Compton wavelengths of W and Z bosons, making the interaction short-ranged. This is the physics of electroweak symmetry breaking (EWSB). The energy scale of the condensate is 250 GeV and understanding the nature of the condensate is the target of intense experimental activity in this and the next decade at Tevatron, LHC, and future electron-positron linear collider.

In the Standard Model of particle physics, the EWSB is accomplished by the condensation of elementary spinless particle called the Higgs boson. The model breaks down below TeV scale because the Higgs boson mass-squared receives a quadratic divergence rendering the theory unstable. It is possible to stabilize it using supersymmetry; in this conventional use of supersymmetry, the Higgs boson is still an elementary scalar while its mass-squared receives only a logarithmic divergence to all orders in perturbation theory. Recently, it has become a concern that the supersymmetry is getting already somewhat fine-tuned after the unsuccessful search for superparticles at LEP-II (see, *e.g.*, [2]), sometimes referred to as the “little hierarchy” problem.

Technicolor theories, on the other hand, attempt to describe the EWSB much more analogously to the superconductivity. The condensate is not elementary, but rather a pair of fermions. However, the lack of large Fermi surface in the Dirac sea in the relativistic theory of fermions forces us to introduce a new *strong* interaction. This point immediately poses difficulty in model

building, as the models rely on non-perturbative strong dynamics of gauge theories. In addition to this technical difficulty, there had been numerous other phenomenological difficulties. I refer readers to an excellent review for model-building attempts and an up-to-date list of references [3].

The main difficulties in the original QCD-like technicolor models are as follows:

- Technicolor itself does not provide a mechanism of generating masses of quarks and leptons. It has to be augmented by an extended technicolor (ETC) sector. The large top quark mass implies the ETC scale is as low as a few TeV, while the flavor-changing neutral current (FCNC) constraints require the ETC scale to be above 10–100 TeV. Some ETC models even suffer from proton instability.
- Most models have the alignment problem, namely that there are degenerate ground states in technicolor theory some of which do not successfully achieve the EWSB. Even if the correct ground state is chosen, there are light pseudo-Nambu-Goldstone bosons (PNGB) which are experimentally ruled out.
- Certain precisely measured electroweak observables measured are predicted to be out of the allowed ranges.

There had been efforts to solve these problems using non-QCD-like dynamics, especially using the “walking” theories [4, 5, 6] (see also [3] and references therein). In these theories, it is assumed that there are large anomalous dimensions that enhances the ETC interactions and solve the phenomenological problems listed above. Other possible directions include topcolor-assisted technicolor, extra dimensions, topseesaw, etc [7].

Apart from the EWSB itself, there has been another motivation for technicolor models: to eliminate spinless bosons from the theory. The argument is that we have

not yet seen any spinless elementary bosons in Nature, and their masses suffer from the quadratic divergences which render the theory unstable and fine-tuned. I consider this a secondary goal less important than the primary goal of the elegant EWSB. If the secondary goal is abandoned, it is attractive to consider the possibility that the technicolor model is supersymmetric. It necessarily introduces spinless bosons into the theory, while it will not cause dangerous quadratic divergences. On the other hand, non-perturbative dynamics of supersymmetric gauge theories are much better understood than the non-supersymmetric counter parts which provides better tools in model building (see [8] for a review). In fact, I will make use of the superconformal dynamics which allows exact predictions on the anomalous dimensions. Moreover, the presence of spinless bosons will provide simple solutions to the alignment problem and the successful construction of the ETC sector as I present below.

Here are the relevant energy scales in the particular model presented below:

- $m_{SUSY} \sim 0.2$ TeV.
- $\Lambda_{TC} \sim 2$ TeV.
- $\Lambda_{ETC} \sim 2$ TeV.

The EWSB is solely due to the strong dynamics of technicolor gauge interaction, and supersymmetry breaking is considered a small perturbation. The main advantage of using supersymmetry is to have clear predictions on the dynamics. Certain quantities are exactly calculable despite strong dynamics of the theory. Despite a relatively low ETC scale, supersymmetry allows a renormalizable implementation of the techni-GIM mechanism [16] to avoid the FCNC problems.

Earlier attempts to make use of supersymmetry in the context of technicolor have assumed $m_{SUSY} > \Lambda_{TC}$, and moreover were made before the dynamics of supersymmetric gauge theories were understood [7, 9]. The effect of soft supersymmetry breaking on dynamics had been worked out only relatively recently [10]. There is one important exception that used $m_{SUSY} < \Lambda_{TC}$ and made use of supersymmetric dynamics by Luty, Terning, and Grant [11] which, however, needed a light Higgs boson. The present work makes use of these developments.

TECHNICOLOR

The simplest choice of the technicolor group is $SU(2)_{TC}$ with the following particle content of chiral superfields (techniquarks)

$$T(\mathbf{2}, \mathbf{2}, 0), \quad t_+(\mathbf{2}, \mathbf{1}, +\frac{1}{2}), \quad t_-(\mathbf{2}, \mathbf{1}, -\frac{1}{2}) \quad (1)$$

where the quantum numbers are shown under the gauge group $SU(2)_{TC} \times SU(2)_L \times U(1)_Y$. This is the same

quantum number assignment as the minimal technicolor model in the non-supersymmetric case. The dynamics of technicolor treats two components of T , t_+ , and t_- all equal, and hence there is an $SU(4)$ global symmetry. The model described here was studied by Luty, Terning, and Grant in [11].

The low-energy effective theory of this supersymmetric gauge theory is known, and is described by the meson composites made of techniquarks [8]. It is known to lead to the so-called quantum modified constraint:

$$M_+ M_- - M_S M_s = \Lambda^4, \quad (2)$$

where the mesons are defined by

$$M_\pm = (T t_\pm), \quad M_S = (T T), \quad M_s = (t_+ t_-). \quad (3)$$

The contraction of technicolor indices is understood in each parentheses. At this point the model suffers from the alignment problem just like in the non-supersymmetric model. It is not clear how the desired ground state with expectation values in electroweak doublet composites M_+ and M_- is chosen over that with singlets M_S and M_s that do not break the electroweak gauge group at all. It is often assumed that the higher the remaining symmetry the lower the energy is; if so, it is more likely that the ground state does not achieve the EWSB. (Here, Λ is the holomorphic dynamical scale, which is different from the strong scale of technicolor theory. We will come back to this point later.)

In this model, it is very simple to solve the alignment problem. I introduce two singlet fields S and s , and a superpotential

$$W = S(TT) + s(t_+ t_-). \quad (4)$$

This superpotential is renormalizable and hence is ultraviolet complete. Once the technicolor interaction becomes strong at the scale Λ , the superpotential turns into mass terms of meson composites together with the introduced singlets of the order of Λ . The superpotential does not allow the $M_S = (T T)$ and $M_s = (t_+ t_-)$ mesons to acquire expectation values. This way, the desired ground state with the EWSB

$$\langle M_+ \rangle = \langle M_- \rangle = \Lambda^2 \quad (5)$$

is uniquely chosen together with the D -term potential, solving the alignment problem. By further making the soft masses for t_+ and t_- different in the ultraviolet, one can achieve different VEVs for M_\pm as well (*i.e.*, $\tan \beta \neq 1$).

Note that the dynamics has the custodial $SU(2)$ symmetry and hence leads to $\rho = 1$ naturally. Out of the $SU(4) \simeq SO(6)$ global symmetry, the superpotential breaks it to $SO(4) \simeq SU(2)_L \times SU(2)_R$. $U(1)_Y$ is embedded into $SU(2)_R$. Therefore, the custodial $SU(2)$ symmetry is broken only by the $U(1)_Y$ interaction and fermion

masses (see the next section), in complete analogy to the minimal Standard Model.

The effect of soft supersymmetry breaking in this model had been worked out by Luty and Rattazzi [10]. Out of five chiral superfields that survive the constraint, the five imaginary components acquire positive mass squared of $O(m_{SUSY}^2)$ while the five real components remain massless. The massless ones are the Nambu-Goldstone bosons of the spontaneously broken symmetry $SU(4)/Sp(2) = SO(6)/SO(5)$. Together with the terms from the superpotential, two full chiral multiplets become massive, leaving only three real and imaginary components left. The three massless imaginary components are eaten by the W and Z bosons as a consequence of the EWSB, while the real parts remain with supersymmetry-breaking scale. They are no Higgs bosons, however, as the Higgs boson is completely eliminated by the constraint. They are rather the analog of H^0 and H^\pm in two-doublet Higgs models while they lack scalar-scalar-vector vertices.

Therefore the dynamical EWSB works successfully in this model.

ETC

In the original QCD-like technicolor, the fermion masses are obtained through dimension-six four-fermion interactions giving

$$m_f \sim \frac{1}{(4\pi)^2} \frac{\Lambda_{TC}^3}{\Lambda_{ETC}^2}. \quad (6)$$

The trouble is that one also expects flavor-changing neutral currents from the ETC boson exchange suppressed by the same power, $1/\Lambda_{ETC}^2$. This causes the well-known dilemma; a large enough fermion mass, especially that for the top quark, is incompatible with the apparent lack of flavor-changing neutral current processes.

The fermion masses are obtained through dimension-five superpotential terms

$$W_{ETC} = h_u^{ij} \frac{1}{\Lambda_{ETC}} (Tt_+) Q_i u_j \quad (7)$$

$$+ h_d^{ij} \frac{1}{\Lambda_{ETC}} (Tt_-) Q_i d_j + h_l^{ij} \frac{1}{\Lambda_{ETC}} (Tt_-) L_i e_j. \quad (8)$$

In the supersymmetric case based on the model in the previous section, the ETC scale cannot be raised high [12]. The naive dimensional analysis is used to relate various scales in the problem [13, 14], as the holomorphy in supersymmetry is not powerful enough to constrain the Kähler potential. It suggests that the Lagrangian for the meson composites is given by

$$L = \frac{1}{(4\pi)^2} \left[\int d^4\theta \hat{M}_\pm^\dagger \hat{M}_\pm + \int d^2\theta \hat{X} (\hat{M}_+ \hat{M}_- - \Lambda_{TC}^2) + \Lambda_{TC} \hat{M}_- \frac{QU}{\Lambda_{ETC}} \right]. \quad (9)$$

The meson composite fields \hat{M}_\pm have different normalization from M_\pm as will be determined shortly below. It gives $m_W^2 \simeq g^2 v^2$ with $v \sim \Lambda_{TC}/4\pi$. This result is analogous to the result in non-supersymmetric QCD $f_\pi \sim \Lambda/(4\pi)$. \hat{X} is a Lagrange multiplier field to enforce the quantum modified constraint. The last term is the desired ETC interaction responsible for the top quark mass. It leads to the mass term

$$\frac{1}{(4\pi)^2} \frac{\Lambda_{TC}^2}{\Lambda_{ETC}} QU \quad (10)$$

which implies

$$m_f \sim \frac{1}{(4\pi)^2} \frac{\Lambda_{TC}^2}{\Lambda_{ETC}} \simeq \frac{v^2}{\Lambda_{ETC}}. \quad (11)$$

In order to reproduce the top quark mass, I need $\Lambda_{ETC} \sim v$. Hence the presumed high ETC scale is brought down to the electroweak scale and it does not fulfill the goal. It is also important to note that the strong scale $\Lambda_{TC} \simeq 4\pi v$ is different from Seiberg's holomorphic scale $\Lambda \sim v$ and the meson composites are related by $\hat{M}_\pm \sim 4\pi M_\pm/\Lambda$.

Another quantity that cannot be calculated rigorously is the contribution to the oblique electroweak parameters such as Peskin-Takeuchi S and T [15].

WALKING

As in the non-supersymmetric case, walking dynamics can be employed to enhance the ETC interaction to raise the ETC scale and suppress the possible FCNC effects. The main benefit of the supersymmetry is to allow me to calculate the anomalous dimension factors exactly.

In supersymmetric $SU(N_c)$ QCD for $\frac{3}{2}N_c < N_f < 3N_c$, the theory is in superconformal phase [8]. It has a dual magnetic description in terms of the dual gauge group $SU(N_f - N_c)$. The important point is that the wave function renormalization factor for quark fields is given exactly in the infrared as

$$Z = \left(\frac{\mu}{\Lambda} \right)^{(3N_c - N_f)/N_f}, \quad (12)$$

where μ (Λ) is the infrared (ultraviolet) scale. In particular, the case $N_c = 2$ and $N_f = 4$ will be used in the next section, giving

$$Z = \left(\frac{\mu}{\Lambda} \right)^{1/2}. \quad (13)$$

Note that the suppressed wave function renormalization factor Z at lower energies *enhances* the couplings of techniquarks.

AN OVERKILL ETC MODEL

The model introduces four additional chiral multiplets with the same quantum numbers as the Higgs doublets

H_u and H_d in the Minimal Supersymmetric Standard Model (MSSM). They are singlets under the technicolor gauge group.

$$\Phi_u, \Phi'_u(\mathbf{1}, \mathbf{2}, +\frac{1}{2}), \quad \Phi_d, \Phi'_d(\mathbf{1}, \mathbf{2}, -\frac{1}{2}). \quad (14)$$

They have the superpotential

$$W_{ETC} = \Lambda_{ETC}(\Phi_u \Phi'_d + \Phi'_u \Phi_d) + \Phi'_u(Tt_-) + \Phi'_d(Tt_+) + h_u^{ij} Q_i u_j \Phi_u + h_d^{ij} Q_i d_j \Phi_d + h_l^{ij} L_i e_j \Phi_d. \quad (15)$$

The only flavor-violating couplings are the Yukawa couplings h_u^{ij} , h_d^{ij} , h_l^{ij} , and hence the model realizes the requirement of the techni-GIM mechanism. By integrating out the massive Φ_u and Φ_d , I find

$$W_{ETC}^{eff} = \frac{1}{\Lambda_{ETC}} [(h_d^{ij} Q_i d_j + h_l^{ij} L_i e_j)(Tt_-) + h_u^{ij} Q_i u_j (Tt_+)]. \quad (16)$$

It has all the operators needed to generate fermion masses. This model does not lead to any phenomenologically problematic flavor-changing effects from the ETC operators.

As it stands now, combined with the $SU(2)$ technicolor model with two flavors in the earlier section, the ETC scale cannot be raised high, $\Lambda_{ETC} \sim v$. Therefore this overkill model actually predicts $\Phi_{u,d}$ are light Higgs doublets and Λ_{ETC} is nothing but the μ parameter [11]. The rest of the discussion is how the superconformal theory can be used to raise Λ_{ETC} and hence eliminate light Higgs from the spectrum.

Let me take the same $SU(2)$ technicolor model as before, but introduce two more flavors (four more doublets) to the model. The theory becomes strong at a scale Λ_4 (the subscript stands for four flavors) which is taken to be much higher than the ETC scale. Below Λ_4 , the $O(1)$ Yukawa interactions between the Φ doublets and the techniquarks $\Phi'_u(Tt_-) + \Phi'_d(Tt_+)$ are enhanced down to the mass Λ_{ETC} of Φ doublets as

$$\left(\frac{\Lambda_4}{\Lambda_{ETC}} \right)^{1/2}. \quad (17)$$

However, a too-large Yukawa interaction is likely to upset the delicate conformal dynamics. It is probably wise to restrict the growth of the Yukawa coupling to values less than 4π , which I take as the maximum allowed value of the Yukawa coupling at the ETC scale. After integrating out the ETC doublets $\Phi_{u,d}$, the effective interaction is further enhanced by an additional factor

$$\left(\frac{\Lambda_{ETC}}{\Lambda_{TC}} \right)^{1/2}. \quad (18)$$

In the end, an overall enhancement factor of

$$4\pi \left(\frac{\Lambda_{ETC}}{\Lambda_{TC}} \right)^{1/2} \quad (19)$$

can be obtained relative to the previous case. The technicolor model reduces to that of the two doublets discussed earlier by adding a mass term to the extra two doublets $m_{3,4}$ which I take to be a common mass m . Because the dynamics is already strong, Λ_{TC} is expected to be at the physical mass of the extra doublets $\Lambda_{TC} \sim m$, leading to the quantum modified constraint and hence the EWSB. The fermion mass is then enhanced to

$$m_f \sim \frac{v^2}{\Lambda_{ETC}} 4\pi \left(\frac{\Lambda_{ETC}}{\Lambda_{TC}} \right)^{1/2}, \quad (20)$$

and in order to obtain $m_f \sim v$ (top quark), I need

$$\Lambda_{ETC} \sim \Lambda_{TC} \sim \frac{\Lambda_4}{(4\pi)^2}. \quad (21)$$

Therefore, this model eliminates Higgs doublets at scale v and additional degrees of freedom such as $\Phi_{u,d}$ are pushed up to Λ_{TC} .

PHENOMENOLOGY

Phenomenology of the model presented is quite rich. Below TeV, the model looks very much like any supersymmetric models except that there is no light Higgs boson. There are analogs of heavy Higgses H^\pm and H^0 but not A^0 , even though they are not really Higgs bosons and there is no vector-vector-scalar vertex. On the other hand, the absence of light Higgs implies that the WW scattering amplitude grows and is unitarized only above TeV. It may even lead to some techni-resonances. If the ETC sector is not an overkill, there may be small deviations from the Standard Model in K - and B -physics, and some lepton-flavor violating signals.

The most striking prediction of this model is the presence of superconformal dynamics above Λ_{TC} . In particular, the “ e^+e^- ” cross section for producing techni-states can be predicted exactly [17] even though the S -matrix elements are ill-defined in conformal theories.

It is important to note that the usual flavor-changing and CP problems in supersymmetry exist also in this framework. The flavor-changing problems need to be solved by flavor-blind supersymmetry breaking mechanisms such as gauge mediation [18], anomaly mediation [19] (supplemented by $U(1)$ D -terms to make it viable [20]), or gaugino mediation [21]. The hierarchy is stable because of the technicolor rather than supersymmetry, and it may be possible to push the supersymmetry breaking scale to the technicolor scale or even beyond, while maintaining the natural hierarchy of the electroweak scale. If so, the flavor-changing problems of supersymmetry may be suppressed partly by decoupling, and also the little hierarchy problem can be ameliorated.

The overkill ETC model above should be considered only a toy model as it does not explain the origin of

flavor. It is conceivable that this framework can be combined with Froggatt–Nielsen mechanism [22] at the ETC scale. The effect of $O(1)$ Yukawa couplings on supersymmetry breaking parameters can still be made immune from FCNC effects using the anomaly mediation.

OPEN QUESTIONS

The result in this letter is a good start, but poses many open questions, some theoretical, some phenomenological, and some aesthetical. Here is an incomplete list:

- Is there a way to reliably calculate the electroweak precision observables such as S and T parameters?
- Is there a way to reliably relate the techni-pion decay constant to Λ_{TC} ?
- How far can the supersymmetry breaking scale be pushed up? Because the hierarchy is explained by technicolor rather than supersymmetry, it may be pushed up to Λ_{TC} , ameliorating the “little hierarchy” problem. If it is pushed even beyond Λ_{TC} , at some point the theoretical control thanks to supersymmetry is lost, but maybe reasonable extrapolation on dynamics is possible.
- Can one push up the ETC scale further?
- How is the coincidence understood that m_{SUSY} , Λ_{ETC} and Λ_{TC} (determined by the masses of extra doublets) are not very different? This is the analog of the μ -problem in the MSSM.
- The true ETC sector is supposed to explain the origin of flavor. Can a realistic model of flavor be implemented within this framework?
- Is there a grand-unifiable model?

CONCLUSION

I showed that the technicolor models with supersymmetry retain the beauty of the original technicolor model in explaining the EWSB dynamically with a natural origin for the hierarchy. It abandons one of the conventional motivation for technicolor to eliminate spinless bosons from the theory. On the other hand it solves many of the problems that have plagued technicolor models without supersymmetry by walking dynamics under good theoretical control. There is no alignment problem and no light pseudo-Nambu-Goldstone bosons exist. It is easy to generate fermion masses while suppressing the flavor-changing effects. In an overkill model presented in this letter, the fermion masses are generated by a ultraviolet-complete renormalizable extended technicolor sector with

techni-GIM mechanism and hence the sector is safe from flavor-changing neutral currents effects. The electroweak precision observables are not calculable and I cannot assess the phenomenological constraints at the moment. On the other hand the “ e^+e^- ” production of techni-sector can be calculable despite its walking dynamics. Given the model building is relatively simple, there may well be models consistent with grand unification.

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