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Heavy Ion Beams for Inertial Confinement Fusion

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Plasma Physics Seminar

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Heavy Ion Beams For Inertial Confinement Fusion

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Fusion Context & Parameters

Beam Currents & Accelerator

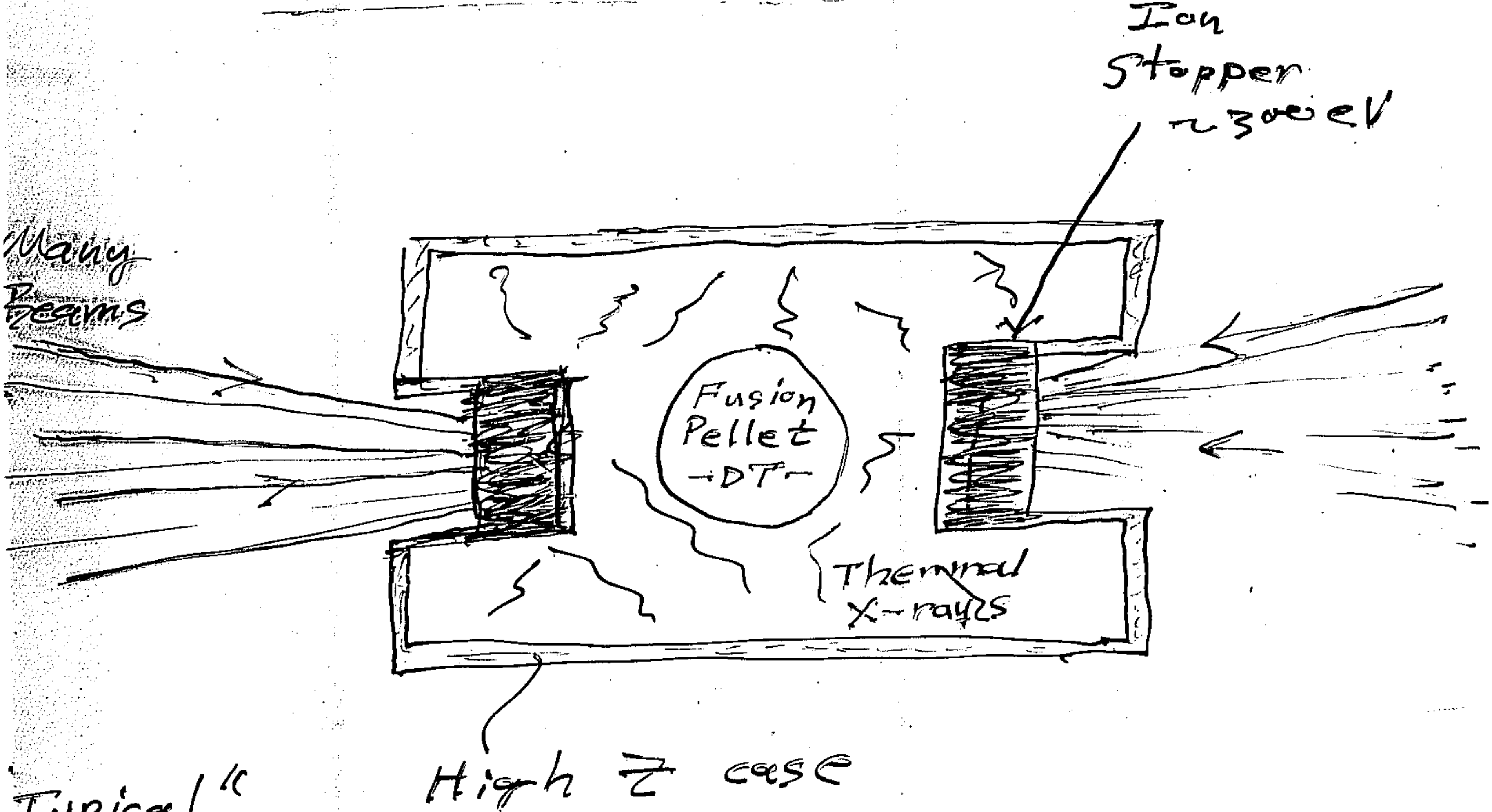
Beam Transport: Analogy with Photons

Quadrupole Magnet Transport

Role of Space Charge

Breathing Mode Instability

Indirect Drive Target



"Typical"

4 MJ of Ions × Gain of 75 = 300 MJ Yield

× 10 Hz Rep Rate = 3000 MW Fusion Energy

× $\frac{1}{3}$ Plant Efficiency = 1000 MW Electric Power Plant

Parameters

Heavy Ions Have Short Range
In the Stopper at High Energy

$T = 4.0$ GeV Cs^+
$\rightarrow$ Range $\approx$ .1 gm/cm^2

$$\text{Total Ion Charge} = \frac{4.0 \text{ MJ}}{4.0 \text{ GeV}} = 10^{-3} \text{ C}$$

Ions into Target in 10^{-8} s

$\rightarrow$ 4×10^{14} Watts beam power
100,000 Amperes beam current

Relativistic Factor $\quad \beta\gamma = \sqrt{\left(\frac{T}{mc^2}\right)^2 + 2\left(\frac{T}{mc^2}\right)}$

$= .256$

— Only Slightly Relativistic —

Currents

100 000 Amperes is divided among many beams to reduce space charge effects

Say $N_{beam} = 50$

$\rightarrow I_{beam} = \frac{100\,000}{50} = 2000$ Amperes

At Source $\begin{cases} T \approx 1.6 \text{ MeV} \\ I_{beam} \approx .5 \text{ Ampere} \end{cases}$

Accelerate to 4000 MeV

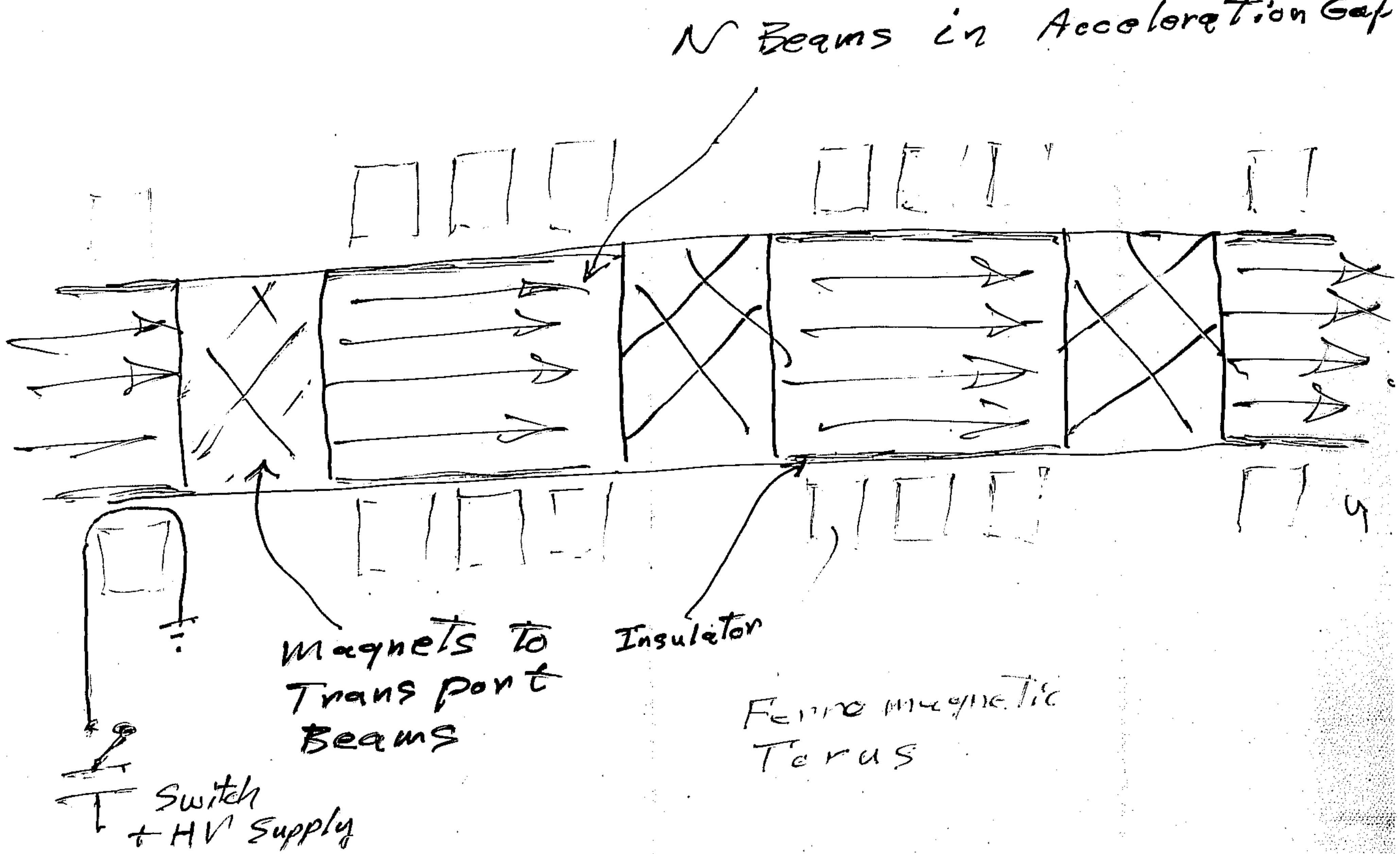
$\rightarrow$ I increases by $\times \sqrt{\frac{4000}{1.6}} = 50$

Compress During Acceleration by $\times 4$

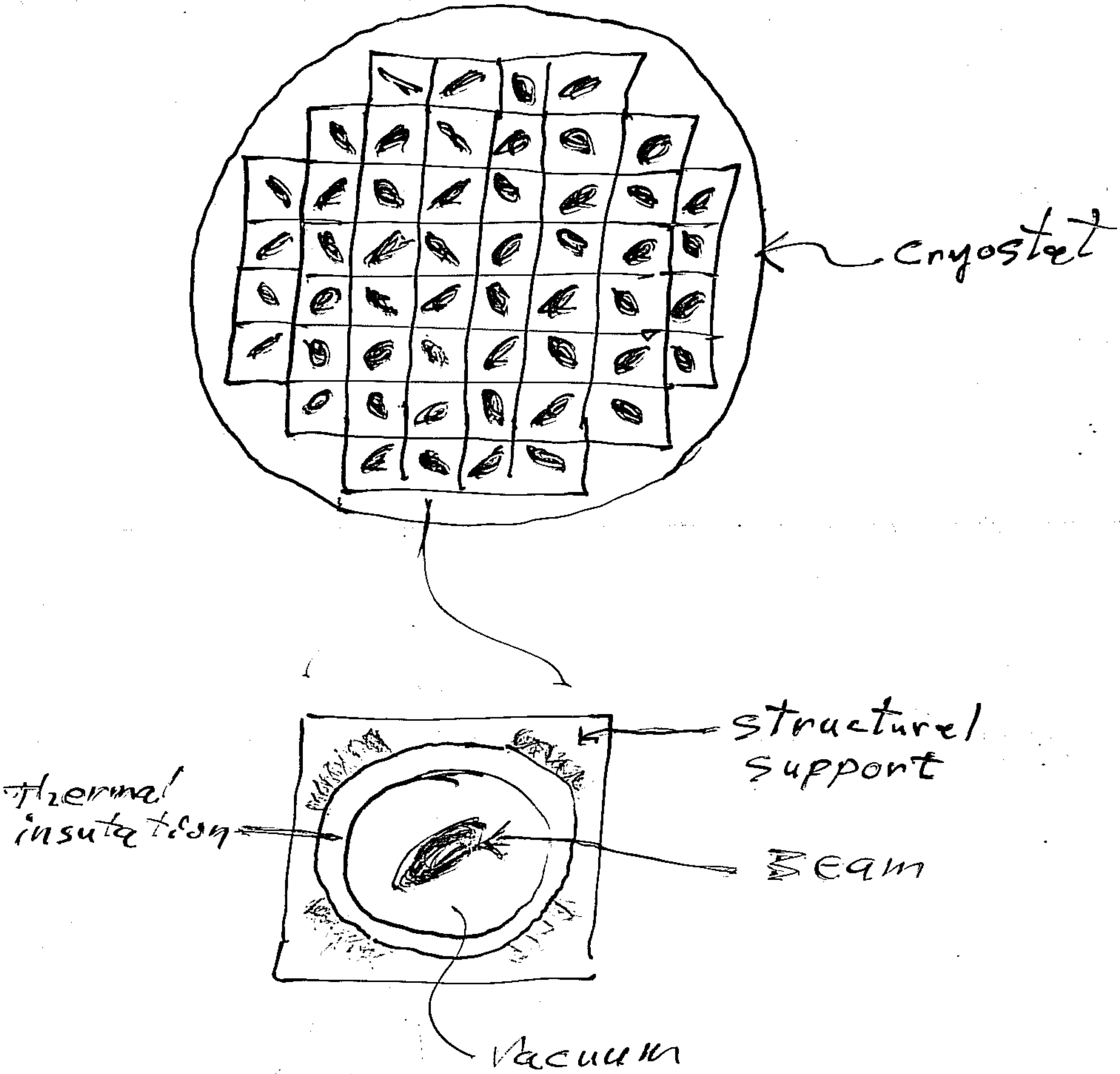
End of Accelerator $I_{beam} = 4 \times 50 \times .5 = 100$ Amp

Compress another $\times 20$ from Accelerator to Target $\rightarrow$ 2000 Amps

Induction Linear Accelerator

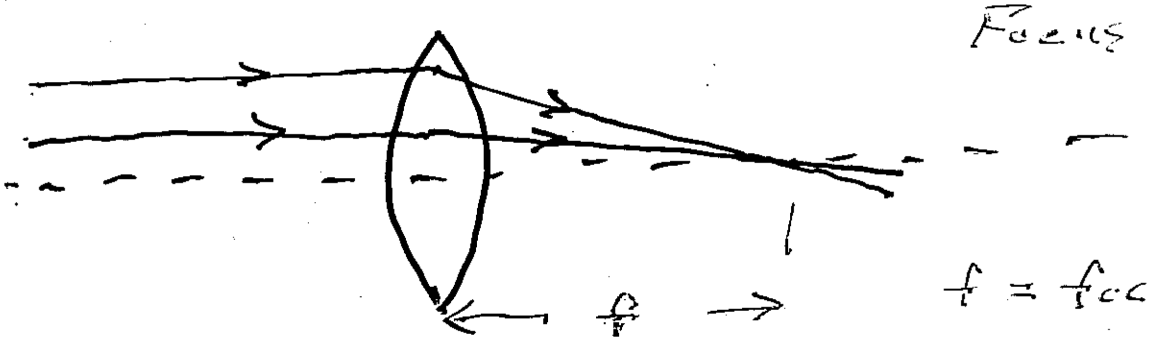


52 Beam (efficient) Bundle In Magnet



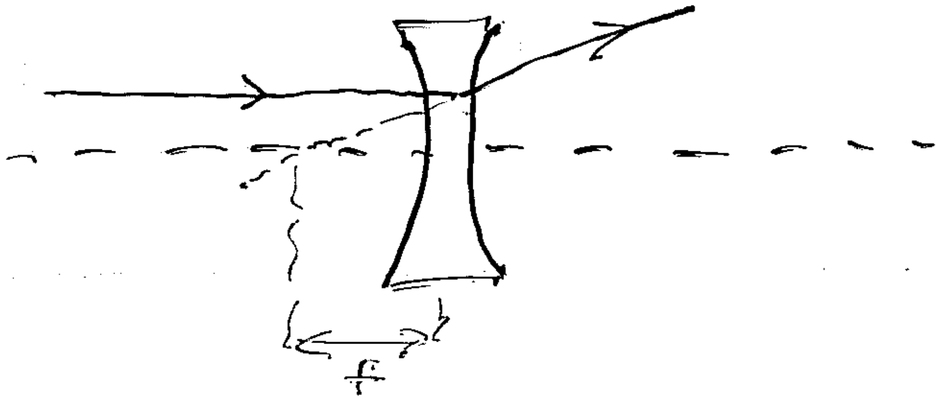
Ions Are Transported by a Periodic System of Magnetic Lenses

Analogy With Photons



Focus Lens

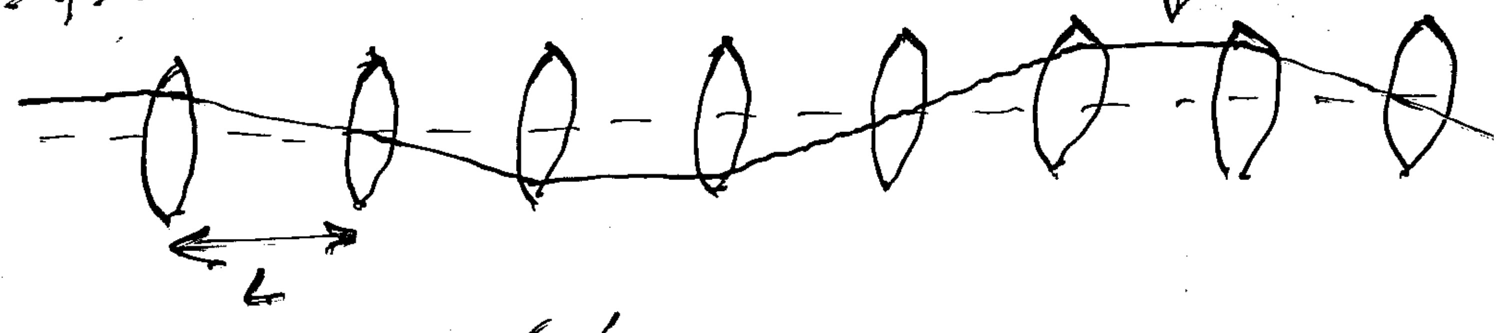
f = focal length



Defocus Lens

Periodic System case of $\frac{2}{f} = L$

photon displacement $x(z)$



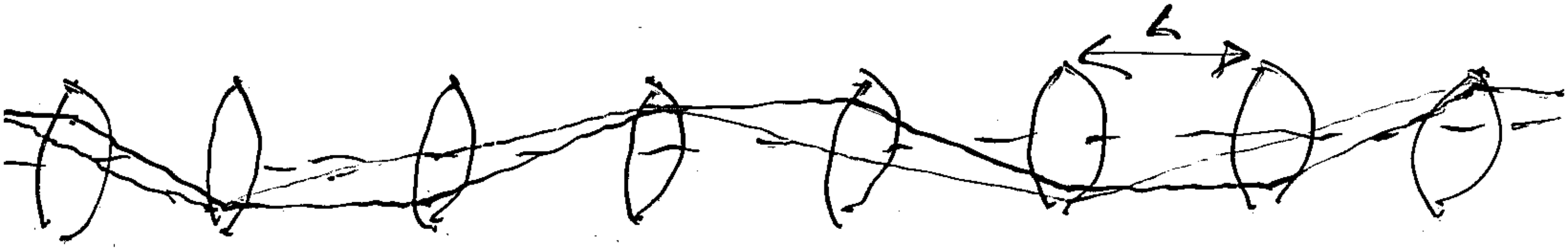
Period of Orbit $= 6L$

$$X(z) \sim \cos\left(2\pi \frac{z}{6L}\right)$$

Case of $f = L$ cont.

Define $\sigma_0 = \begin{matrix}\text{phase advance} \\ \text{per lattice period}\end{matrix} = \frac{2\pi}{6} = 60^\circ$

Try $f = L/2$:



Orbit period $= 4L \rightarrow \sigma_0 = 90^\circ$

Try $f = L/4$:

Orbit period $= 2L \rightarrow \sigma_0 = 180^\circ$

General Formula:

$$\cos(\sigma_0) = 1 - \frac{L}{2f}$$

L/f	σ_0
0	0
1	60°
2	90°
4	180°

Photon Analogy - Cont.

Problem: Show $\cos(\sigma_0) = 1 - \frac{L}{2f}$

Use $\begin{pmatrix} X(L) \\ X'(L) \end{pmatrix} = M \begin{pmatrix} X(0) \\ X'(0) \end{pmatrix}$

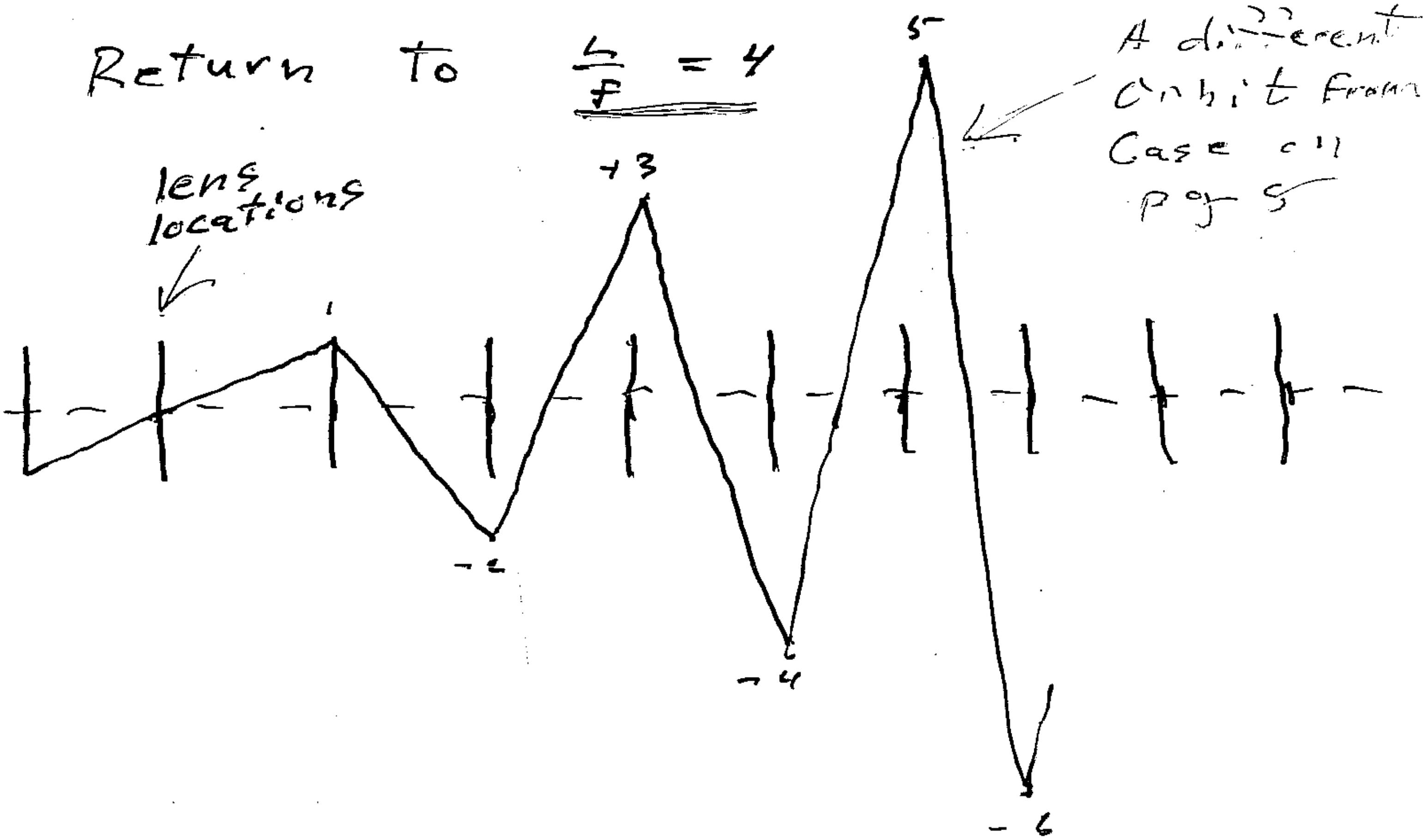
+ Find Eigenvalues of M, etc..

What about $\frac{L}{f} > 4 \rightarrow \cos\sigma_0 < -1$

? ? ? ?

– Unstable Orbit –

Return to $\underline{\underline{\frac{L}{f} = 4}}$



$\frac{L}{f} = 4$ Cont.

This Orbit Shows Liner Growth On the Average!

→ We Are At a Half Integer Resonance ←

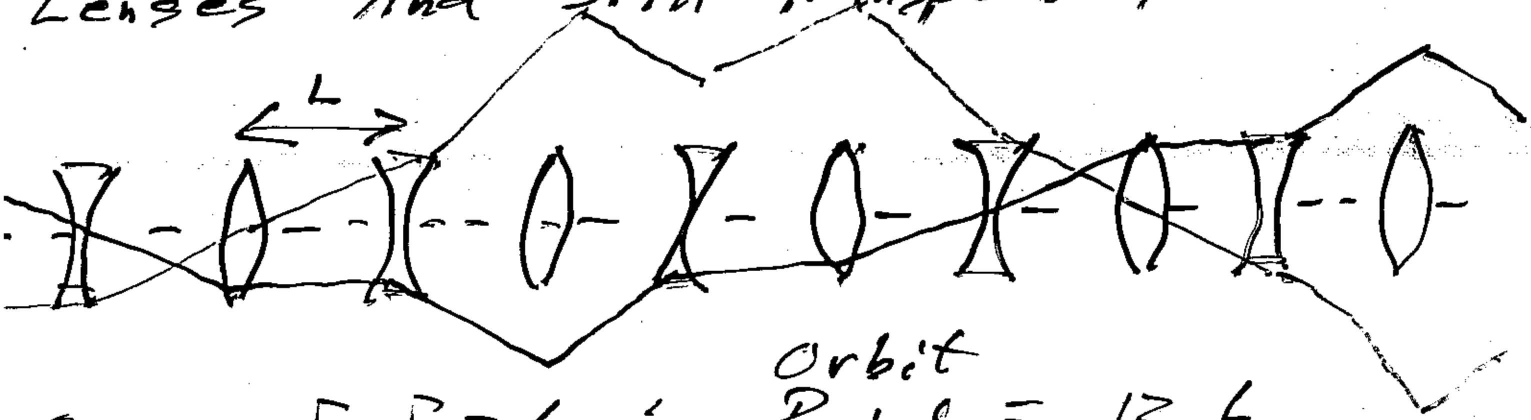
\# of oscillations of 1st orbit = $\frac{1}{2}$ \# of lattice periods

∴ We Are At Edge of Stop Band

For $\sigma_c < 180°$, Stable Oscillations But Large Local ~~Variations~~ Deviations From $\cos(\sigma_0 z/L)$ As $\sigma_0 \to 180°$

$$\frac{d^2 x}{dz^2} \simeq -\left(\frac{\sigma_0}{L}\right)^2 x$$

We Can Also Alternate Focus + Defocus Lenses And Still Transport Photons



Case of $f = L$; Orbit Period $= 12L$

Phase Advance $= 360^\circ$ in $12L$

But Lattice Period $= 2L$ Now

$$\therefore \quad \sigma_0 = \frac{360^\circ}{6} = 60^\circ$$

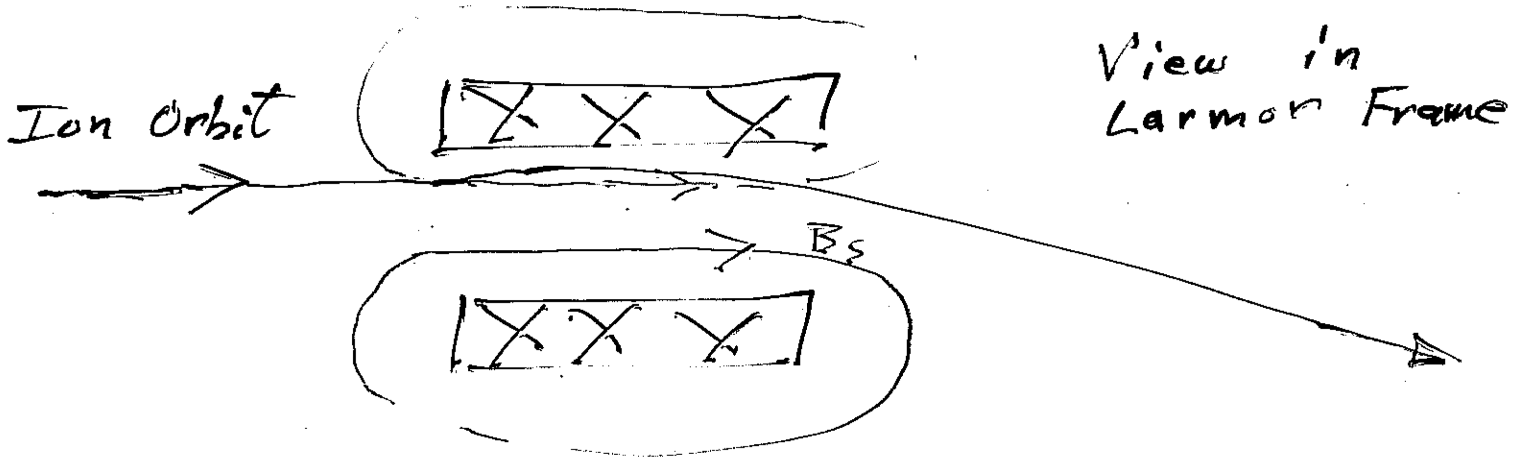
General Formula ;

$$\cos(\sigma_0) = 1 - \frac{L^2}{2f^2}$$

L/f	σ_0
0	0
1	60°
$\sqrt{2}$	90°
2	180°
> 2	unstable

For Charge Particles

In Principle Lenses Can Be Short Solenoids



$$\frac{1}{f} \cong l_s \left(\frac{B_s}{2[BR]} \right)^2$$

l_s = solenoid length

B_s = " field

$[BR] = \frac{\text{momentum}}{\text{charge}}$ = "Rigidity"

$= 3.107\ \beta\gamma\ A/Z$

A = mass in amu

Z = charge state

Consider 4.0 GeV Cs^+ $\begin{cases} \beta\gamma = .256 \\ A = 133 \\ Z = 1 \end{cases}$

$$[BR] = 3.107 \times .256 \times \frac{133}{1} = 106 \text{ T-m}$$

Say $B_\perp = 5.0$ T

$\rightarrow$ Radius of Curvature $R = \frac{106}{5} \simeq 21$ m

Generally $R \gg$ Magnet Lens Size

— Opposite of Magnetic Confinement of Plasmas ←

Say $\begin{cases} l_s = 1.0 \text{ m} \\ [BR] = 106 \text{ T-m} \\ B_s = 10 \text{ T} \end{cases}$

$$\rightarrow \frac{1}{f} = 1.0 \times \left(\frac{10}{2 \times 106}\right)^2 = \frac{1}{450 \text{ m}}$$

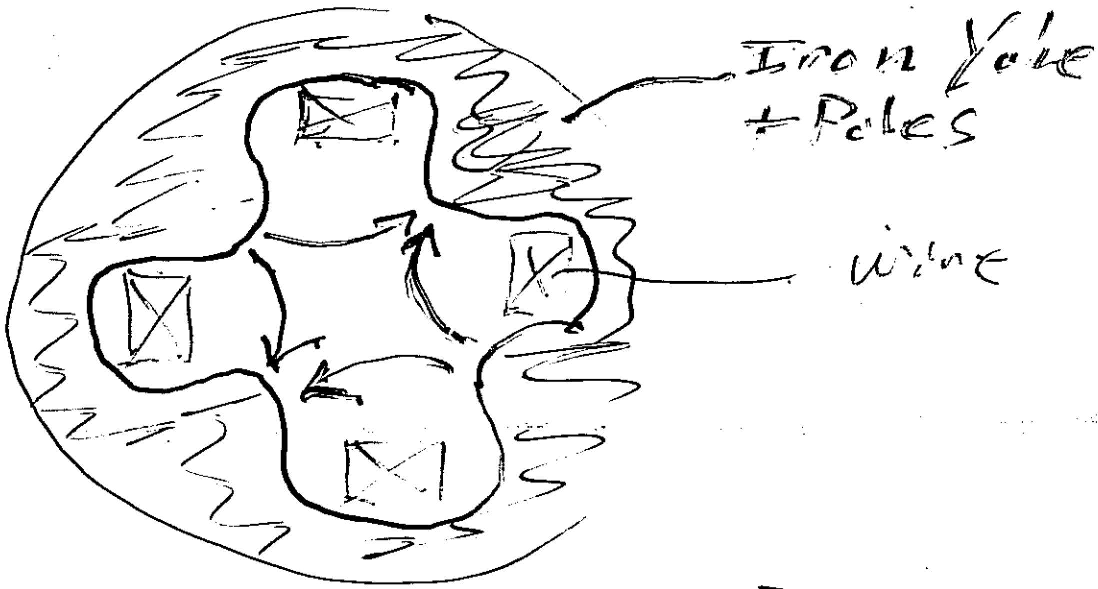
— Not Very Useful Here ←

Good For Electrons!

Big Idea: Use Magnetic Quadrupoles

The Solenoid is Weak Because $\underset{\sim}{B}$ is Nearly Parallel To Beam

For Quadrupole $\underset{\sim}{B}$ is Normal



$$B_x \simeq G y \qquad\qquad \nabla_\perp \cdot B_\perp \simeq 0$$
$$B_y \simeq G x \qquad\qquad \underset{\sim}{\nabla}_\perp \times \underset{\sim}{B}_\perp \simeq 0$$

G = Quadrupole Gradient

$$\frac{dP_x}{dt} = -Ze\,v_z\,B_y$$
$$\frac{dP_y}{dt} = +Ze\,v_z\,B_x$$

cont

Quadrupole – Cont

$$\frac{d^2x}{dz^2} = -\frac{G}{[BR]}\,x$$

$$\frac{d^2y}{dz^2} = +\frac{G}{[BR]}\,y$$

$$\rightarrow \quad \frac{1}{F} = \frac{G\,l_{quad}}{[BR]}$$

Focus in One Plane + Defocus In The Other → Alternate Polarity To Get Transport In Both Planes (Recall Photons)

Say $G = 50$ T/m
$l_{quad} = .5$ m
$[BR] = 106$ T-m

$$\rightarrow \quad \frac{1}{f} = \frac{50 \times .5}{106} = \frac{1}{4.25\text{ m}}$$

– Good –

Summary So Far

- Use Alternating Quad Polarities

- $$\frac{d^2x}{dz^2} \simeq -\left(\frac{\sigma_0}{2L}\right)^2 x$$ same for y

- $$\cos\sigma_0 \simeq 1 - \frac{2L^2}{f^2}$$ Thin lens

- Keep $\sigma_0 < 180^\circ$ * phase advance per lattice period 2L

- $$\frac{1}{f} = \frac{G\, l_{quad}}{[BR]}$$

- $l_{quad} \ll L$ = spacing of lattice (half period)

- $$G \lesssim \frac{3\ \text{Tesla}}{\text{Beam Radius}}$$ Quad Gradient

- $[BR] = 3.107\ \beta\gamma\ A/Z$ Rigidity

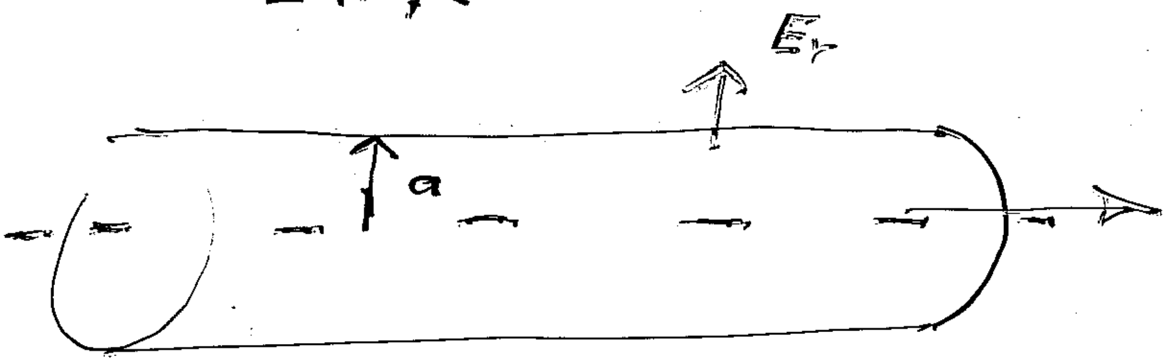
* σ_0 To be reduced later

Why Hold The Beam Together ?

(1) Transverse Thermal Pressure
- Small -

(2) Transverse Space Charge Force
- Large -

Slug of charge moving at $v_z = \beta c$



Roughly $\quad \frac{1}{r}\frac{\partial}{\partial r} r E_r = \frac{\rho}{\epsilon_0}$

Assume Uniform $\rho \quad 0 < r < a = \text{edge}$

$$E_r \approx \frac{\rho r}{2\epsilon_0} \qquad \rho \doteq \frac{\lambda}{\pi a^2}$$

$$\frac{dP_x}{dt} = Ze\left(E_x - v_z B_y\right)$$

$$\simeq Ze\, E_r \frac{x}{r} \frac{1}{\gamma^2} \leftarrow \text{From } v_z B_y$$

$$\cong Ze \frac{\rho}{2\epsilon_0} \frac{1}{\gamma^2}\, x$$

$$\cong \left(\frac{Ze\lambda}{2\pi\epsilon_0 a^2 \gamma^2}\right) x$$

Recall
$$I = \lambda \beta c \quad , \quad P_x = \gamma m \frac{dx}{dt}$$
$$v_z = \beta c$$
$$\frac{d}{dt} = v_z \frac{d}{dz}$$

$$\rightarrow \quad \frac{d^2 x}{dz^2} = \frac{G}{a^2} x$$

$$G = \frac{2 Z e I}{(\beta\gamma)^3 4\pi\epsilon_0 M c^3} = \text{"Dimensionless Perveance"}$$

$$= \frac{2 Z (I / 31.07\times 10^6)}{(\beta\gamma)^3 \; A}$$

Say
$$\begin{cases} I = 100 \text{ Amperes} \\ \beta\gamma = .256 \\ A = 133 \\ Z = 1 \end{cases}$$

$$\rightarrow \quad G = 2.88 \times 10^{-6}$$

Looks Small But It IS Important

With No Quadrupoles

$$Z_{\text{blow up}} \simeq \frac{a}{\sqrt{G}} \simeq 18 \text{m} \qquad \begin{cases} a = 3 \text{ cm} \\ G = 2.9\times 10^{-6} \end{cases}$$

Combine Equations of Motion

$$\frac{d^2x}{dz^2} = -\left(\frac{\sigma_o}{2L}\right)^2 x + \frac{G}{a^2} x$$

Quads Space Charge

For a = constant = a_o

$$x \sim \cos\left(\frac{\sigma}{2L} z\right)$$

where $\left(\frac{\sigma}{2L}\right)^2 = \left(\frac{\sigma_o}{2L}\right)^2 - \frac{G}{a_o^2}$

σ = "Depressed Tune"

Typical Fusion $\quad \sigma_o \approx 70^\circ$ $\quad \sigma \approx 7^\circ$

$\sigma \ll \sigma_o$ Because Thermal Pressure Is Low

But Thermal Pressure becomes Important when Beam is Focused onto the Fusion Target

Transport Limit

$\sigma \to 0$ (Cold Beam)

$\rightarrow Q \le \left(\frac{\sigma_0}{2L}\right)^2 a_0^2$

Recall $Q \sim \frac{Z\, I}{(\beta\gamma)^3 A}$

$\rightarrow I_{max} \sim \sigma_0^2 \left(\frac{a}{L}\right)^2 (\beta\gamma)^3 \frac{A}{Z}$

Application of this formula to Cost Optimization Is Not Obvious

$I_{max} \simeq (1465 \text{ Amps}) \left(\frac{\beta\gamma}{.256}\right)^3 \left(\frac{\sigma_0}{1.3 \text{ rad}}\right)^2 \left(\frac{a/L}{.01}\right)^2 \left(\frac{A/133}{Z/1}\right)$

At Low Energy (1.6 MeV) $\begin{cases} \beta\gamma \simeq .005 \\ a/L \simeq .07 \end{cases}$ Typical

$\rightarrow I_{max} \simeq .53$ Amps Cs^+

But I_{max} Rises Rapidly with $\beta\gamma$

Breathing Mode Instability

Recall $$\frac{d^2x}{dz^2} = -\left(\frac{\sigma_0}{2L}\right)^2 x + \frac{G}{a^2} x$$

For cold Beam ($\sigma \to 0$) An Ion At The Beam Edge Stays There

$$x \longrightarrow a$$

Let $a = a(z)$

$$\Rightarrow \frac{d^2a}{dz^2} = -\left(\frac{\sigma_0}{2L}\right)^2 a + \frac{G}{a}$$

Equilibrium $\quad a = a_0$

$$0 = -\left(\frac{\sigma_0}{2L}\right)^2 a_0 + \frac{G}{a_0} \quad \left\{ \begin{array}{l} \text{Previous} \\ \text{Result} \end{array} \right.$$

Perturb $\quad a \cong a_0 + \delta a(z)$

$$\frac{d^2 \delta a}{dz^2} = -\left(\frac{\sigma_0}{2L}\right)^2 \delta a - \frac{G}{a_0^2}\delta a$$

$$= -2\left(\frac{\sigma_0}{2L}\right)^2 \delta a$$

Breathing Mode - Cont

$$\delta q \sim e^{-i\Omega z}$$

$$\rightarrow \Omega^2 = 2\left(\frac{\sigma_0}{2L}\right)^2$$

Phase Advance of Breathing Per Lattice Period (2L) $\sigma_{breath} = \sqrt{2}\,\sigma_0$

- I have cheated -

The equilibrium and perturbed equations have modulations of period 2L from Quadrupoles

$\rightarrow$ Half Integer Resonance and Stop Band At

$$\sigma_{breath} \gtrsim 180^\circ$$

$$\rightarrow \text{Need} \quad \sigma_0 \leq \frac{180^\circ}{\sqrt{2}} = 127^\circ$$

Experiments And PIC Simulations Show Trouble For $\sigma_0 \gtrsim 85^\circ$

- Unresolved Mystery -