Conf-950293--1

LA-UR-95- 1489

Title: **THE ROTATIONALLY IMPROVED SKYRMION, OR "RISKY"**

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Submitted to: **PROCEEDINGS OF INTERNATIONAL WORKSHOP ON NUCLEAR & PARTICLE PHYSICS HELD IN SEOUL, SOUTH KOREA, FEBRUARY 6-10, 1995.**

The Rotationally Improved Skyrmion, or "RISKY"

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Abstract

The perceived inability of the Skyrme model to reproduce pseudovector pion-baryon coupling has come to be known as the "Yukawa problem." In this talk, we review the complete solution to this problem. The solution involves a new configuration known as the rotationally improved Skyrmion, or "RISKY," in which the hedgehog structure is modified by a small quadrupole distortion. We illustrate our ideas both in the Skyrme model and in a simpler model with a global $U(1)$ symmetry.

Introduction. The Skyrme model [1, 2] provides an approximate description of the baryon spectrum of QCD in the large-N_c limit. The semiclassical expansion about the Skyrmion corresponds directly to the $1/N_c$ expansion in the underlying gauge theory. In the previous talk [3], we established a direct connection between effective Lagrangian models, in which baryons are represented by pointlike Dirac spinor fields, and Skyrme-type models, in which baryons are instead pictured as solitons in the field of mesons. The connection is by means of the large-N_c Renormalization Group; we reviewed under what circumstances Skyrme-type models are, or are not, the ultraviolet fixed points of the effective field theories, under the action of this RG flow.

Here we discuss the equivalence in the *opposite* direction: starting with the Skyrme model, we explain how it bootstraps itself into an equivalent effective meson-baryon Lagrangian, with explicit (rather than solitonic) baryon fields. In other words, this talk supplies the "missing third leg" of Fig. 1 from the preceding lecture.

Obviously the very first effective vertex one needs to recreate is the pseudovector Yukawa coupling of the pion to the $I = J$ tower of large-N_c baryons. Such a vertex is needed to account, not only for the hadronic decay of the Δ, that is $\Delta \to \pi + N$, but also for the virtual processes $N \to \pi + N$ or $\Delta \to \pi + \Delta$. Yet reproducing this vertex directly from Skyrmion quantization has proved to be the most longstanding headache in the Skyrme-model literature, and has come to be known as the "Yukawa problem."[1] The origin of this problem lies in the fact that first variations off the static hedgehog Skyrmion vanish, as the Skyrmion is, by definition, a solution to the Euler-Lagrange equations. In this talk we review the complete solution to the Yukawa problem, following our recent work [6, 7].[2] At the same time, we present the solution to another longstanding problem, not just with the Skyrme model, but with the entire large-N_c program: the fact that, as $N_c \to \infty$, the spectrum of ground-state baryons contains not only the nucleon and the Δ as desired, but also unwanted, unseen states with $I = J = \frac{5}{2}, \frac{7}{2}, \frac{9}{2}, \cdots$.

While tied to the previous talk through Fig. 1, the physics issues involved here are actually quite different. We will be interested in the *analytic properties* of Skyrmions, and will find that while the usual hedgehog Skyrmion is inadequate, a close relative, which we call the *rotationally improved Skyrmion*, or RISKY, serves our purpose well. In a nutshell, the RISKY is obtained by including the (iso)rotational kinetic energy term in the minimization of the static Hamiltonian. It is characterized by an interesting small quadrupole distortion away from the hedgehog ansatz (as has been derived independently, using other physics entirely, by Schroers [8]).

The Yukawa problem. It has long been suspected that the effective S-matrix element describing $\Delta \to \pi + N$, $N \to \pi + N$, etc., in the Skyrme model may be expressed in a particularly compact form. In [2] it is argued from general considerations of symmetry and mass dimensions that the *effective* coupling of soft pions to the Skyrmion should be of the form

$$\mathcal{L} = \frac{3g_{\pi NN}}{4\pi f_\pi M_N} \partial_i \pi^a \mathrm{Tr}\, \tau_i A^\dagger \tau_a A \ , \tag{0.1}$$

where $A \in SU(2)$ is the (iso)rotational collective coordinate of the Skyrmion and the overall normalization of the coupling is determined by evaluating its matrix element between initial and final nucleon states. We stress the word *effective* because, obviously, this coupling is not present in the Skyrme Lagrangian itself. The interaction

[1] Surprisingly, the next-most-complicated effective vertex, in which two pions are attached to the baryon, is less sensitive to the Yukawa problem, and indeed constitutes one of the early phenomenological successes of the Skyrme model [4, 5].

[2] For a comprehensive list of references to other approaches to the Yukawa problem, see Ref. [7].

(0.1) yields predictions for the couplings of pions to the whole $I = J$ tower of baryons which arise when the isorotational motion of the Skyrmion is quantized. In particular, the effective vertex for Δ decay comes with a coupling constant $g_{\pi N\Delta} = \frac{3}{2} g_{\pi NN}$; this relation yields a value for the width of the Δ within a few MeV of its experimentally measured value. More generally, (0.1) embodies the "proportionality rule" which fixes the ratios of the pion couplings to all the baryons in the large-N_c $I = J$ tower, and which may be derived in many different ways (see preceding lecture for a discussion).

For many reasons, therefore, we should expect that the leading-order semi-classical S-matrix elements for pion-baryon interactions in the Skyrme model should coincide with the tree-level effective Yukawa couplings implied by (0.1). But as mentioned earlier, the problem of showing, from first principles of soliton quantization, that this is actually the case presents some difficulty. Before presenting our solution to this Yukawa problem it is convenient to review some of the basic issues in the context of Δ decay. The obvious starting point for calculating the S-matrix element for Δ decay is the one-point Green's function of the pion field evaluated between an incoming Δ-state and an outgoing nucleon, $G^a(\mathbf{x}, t) = \langle N|\pi^a(\mathbf{x}, t)|\Delta\rangle$. When quantizing the Skyrmion it is customary to split up the pion field as $\vec{\pi} = \vec{\pi}_{\text{cl}} + \delta\vec{\pi}$ where $\pi^a_{\text{cl}}(\mathbf{x}, t) = \sin F(r) D^{(1)}_{ab}(A(t))\hat{x}^b$ is the background Skyrmion field configuration and $\delta\vec{\pi}$ is the fluctuating part of the pion field. This division of the field defines two contributions to the one-point function which we now examine in turn.

Early approaches to this problem identified the physical pion field with the fluctuating part $\delta\vec{\pi}$ [9]. A contribution of this sort requires a term linear in $\delta\vec{\pi}$ in the expansion of the Skyrme Lagrangian. However, precisely because the static Skyrmion is a solution of the field equation, such a term is absent at leading order in $1/N_c$ and only appears when the rotation of the Skyrmion is included. The resulting coupling is proportional to $\dot{A}$ and (since the large-N_c Skyrmion (iso)rotates very slowly) is therefore down in the $1/N_c$ expansion relative to the desired result (0.1).

Because of these difficulties, several authors [10, 11] suggested that the effective Yukawa interaction comes instead from the classical contribution to the pion one-point function, $G^a_{\text{cl}}(\mathbf{x}, t) = \langle N|\pi^a_{\text{cl}}(\mathbf{x}, t)|\Delta\rangle$. However, as it stands, this idea cannot be correct either. To see why not, we must consider the corresponding momentum space one-point function. Because the Skyrmion profile function decays exponentially at large r, its Fourier transform will have the following pole contribution:

$$\tilde{G}_{\text{cl}}(\mathbf{k}, \omega) \sim \frac{\delta(\omega - M_\Delta + M_N)}{|\mathbf{k}|^2 + m_\pi^2} . \tag{0.2}$$

The position of the pole is always at an imaginary value of the pion momentum $|\mathbf{k}| = \pm i m_\pi$. However, so long as $m_\pi < M_\Delta - M_N$, Δ decay occurs at the *real* value of the momentum dictated by energy and momentum conservation. Thus the naive semiclassical Green's function (0.2) cannot contribute to the on-shell S-matrix element via the LSZ reduction formula. Put another way, LSZ requires that the denominator in (0.2) read $|\mathbf{k}|^2+m_\pi^2-\omega_\pi^2$ rather than just $|\mathbf{k}|^2+m_\pi^2$, where ω_π denotes the energy of the emitted pion, or equivalently, the difference between the initial and final Skyrmion energies.

In summary, neither of the two contributions to the pion Green's function identified above can, by itself, reproduce the effective Yukawa vertex (0.1). In the remainder of this talk, we will explain how, when taken together, the Yukawa problem is solved, in a rather surprising way.

Skyrmion quantization in 3+1 and 1+1 dimensions. In order to motivate our solution it will be convenient to consider a simpler model than the $SU(2)$ Skyrme model which has only an abelian global internal symmetry. In the following we will develop both models in parallel for maximum clarity. We will consider the case of a real two-component scalar field $\vec{\phi}=(\phi_1,\phi_2)$ in two space-time dimensions,

$$\mathcal{L}=\frac{1}{2}\partial_\mu\vec{\phi}\cdot\partial^\mu\vec{\phi}-\frac{m^2}{2}|\vec{\phi}|^2-W(\vec{\phi})\ . \tag{0.3}$$

In contrast, the Skyrme model is described by the following Lagrangian for an $SU(2)$ valued matrix field in four-dimensional spacetime:

$$\mathcal{L}=\frac{f_\pi^2}{16}\mathrm{Tr}\,\partial_\mu U^\dagger\partial^\mu U+\frac{1}{32e^2}\mathrm{Tr}\,[U^\dagger\partial_\mu U\,,\,U^\dagger\partial_\nu U]^2+\frac{m_\pi^2 f_\pi^2}{8}\mathrm{Tr}\,(U-1)\ . \tag{0.4}$$

We will choose to rewrite the model in terms of the pion field as $U=\exp(2i\vec{\pi}\cdot\vec{\tau}/f_\pi)$. The Lagrangian then takes the general form

$$\mathcal{L}=\frac{1}{2}\dot{\pi}^i g_{ij}(\vec{\pi})\dot{\pi}^j-V(\vec{\pi},\partial_i\vec{\pi}) \tag{0.5}$$

In fact, it will not be necessary to specify the target space metric $g_{ij}(\vec{\pi})$ or the potential $V(\vec{\pi},\partial_i\vec{\pi})$. Our analysis will apply to any chiral soliton model which can be written in the above form.

Provided the potential W has its minima at $\vec{\phi}=0$, the model (0.3) has an unbroken $U(1)$ symmetry, $\vec{\phi}\to\mathcal{M}(\theta)\cdot\vec{\phi}$, where

$$\mathcal{M}(\theta)=\begin{pmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{pmatrix} \tag{0.6}$$

Correspondingly, the Skyrme model has an unbroken $SU(2)$ symmetry $U\to AUA^\dagger$. In terms of the pion field of (0.5) this symmetry takes the form $\vec{\pi}\to D^{(1)}(A)\cdot\vec{\pi}$ where

$$D^{(1)}_{ij}(A)=\frac{1}{2}\mathrm{Tr}\,\tau_i A\tau_j A^\dagger\ . \tag{0.7}$$

In either theory, soliton solutions are found by minimizing the corresponding mass functionals:

$$\begin{aligned} \mu[\vec{\phi}] &= \int dx \, \frac{1}{2}|\vec{\phi}'|^2 + \frac{m^2}{2}|\vec{\phi}^2| + W(|\vec{\phi}|) \ , \\ M[\vec{\pi}] &= \int d^3\mathbf{x} \, V(\vec{\pi}, \partial_i \vec{\pi}) \ . \end{aligned} \tag{0.8}$$

In the $U(1)$ model we will assume that this yields a soliton solution of the form $\vec{\phi}^{\rm cl} = (\phi_S(x), 0)$ [3]. For reasons that will become clear below, we will sometimes write the profile function as $\phi_S(x; m)$ to emphasize its parametric dependence on the meson mass m. In the Skyrme model the Euler-Lagrange equation $\delta M[\vec{\pi}]/\delta \pi^a = 0$ yields a chiral soliton solution with the characteristic hedgehog form

$$\vec{\pi}_{\rm cl} = \frac{f_\pi}{2} \sin F(r) \, \hat{\mathbf{r}} \ . \tag{0.9}$$

In both models the ansatz chosen is the one of maximal symmetry; our assumption that the $U(1)$ soliton can be chosen to lie entirely in the first component of the field $\vec{\phi}$ is analogous to the hedgehog ansatz in this respect. The profile functions, $\phi_S(x)$ and $F(r)$ must be determined by solving a non-linear ODE. However, in both cases, the spatial asymptotics of these functions can be determined by solving the corresponding linearized equation. Thus $\phi_S(x; m) \to A \exp(-mx)$ as $|x| \to \infty$, while the large-r asymptotics of F are given by

$$F(r) \to B \cdot \left(\frac{m_\pi}{r} + \frac{1}{r^2} \right) \exp(-m_\pi r) \ . \tag{0.10}$$

The constants A and B must determined by solving the corresponding non-linear equation numerically.

In the $U(1)$ model the full set of static one-soliton solutions is obtained by acting on the solution $\vec{\phi}^{\rm cl}$ with a $U(1)$ rotation and a translation:

$$\vec{\phi}^{\rm cl}(x; X, \theta) = \mathcal{M}(\theta) \cdot \vec{\phi}^{\rm cl}(x - X) \tag{0.11}$$

while, for the Skyrme model, the full set is parametrized by the collective coordinates $A \in SU(2)$ and $\mathbf{X} \in \mathcal{R}^3$:

$$\vec{\pi}^{\rm cl}(\mathbf{x}; \mathbf{X}, A) = D^{(1)}(A) \cdot \vec{\pi}^{\rm cl}(\mathbf{x} - \mathbf{X}) \tag{0.12}$$

We are ignoring the Lorentz contraction of the soliton for notational simplicity; moreover the translational collective coordinates will not play an important role in what

[3] Actually, in order to get such a soliton it is necessary to include in the model an additional scalar field [12]. However, this field is neutral under the $U(1)$ symmetry and its presence does not affect our analysis in any way

follows and we will suppress them below. Instead we will concentrate on the dynamics of the internal collective coordinates θ and A.

It is convenient to define, in each model, a moment of inertia which is a functional of the field:

$$\begin{aligned} \lambda[\vec{\phi}] &= \int dx\, \vec{\phi}\cdot\vec{\phi} \\ \Lambda_{ij}[\vec{\pi}] &= \int d^3\mathbf{x}\, \epsilon_{abi}\pi^a g_{bd}(\vec{\pi})\epsilon_{jcd}\pi^c \end{aligned} \tag{0.13}$$

In both models the semi-classical baryon spectrum is obtained by allowing the rotational collective coordinate to be time-dependent. In the Skyrme case the effective Lagrangian for this degree of freedom is[4]

$$L = -M + \Lambda \mathrm{Tr}\, \dot{A}\dot{A}^\dagger \tag{0.14}$$

where both M and $\Lambda \sim N_c$. Quantizing this system gives the standard rotor spectrum first obtained by Adkins, Nappi and Witten: an infinite tower of states $|I = J, i_z, s_z\rangle$ with masses,

$$M(J) = M + \frac{J(J+1)}{2\Lambda} \tag{0.15}$$

In the $U(1)$ model it is trivial to carry out the same procedure; in this case the effective dynamics of the time-dependent angle $\theta(t)$ is just that of a free particle moving on a circle,

$$L = -\mu + \frac{1}{2}\lambda\dot{\theta}^2\ . \tag{0.16}$$

In order to exploit the analogy to the Skyrme model to the full, we will choose the N_c dependence of the coupling constants in (0.3) so that $\mu \sim N_c$ and $\lambda \sim N_c$ (recognizing that in this toy model N_c no longer corresponds to "quark number" of any sort, but merely parametrizes the semiclassical expansion). The resulting spectrum of states is labeled by a conserved $U(1)$ charge $q = 0, \pm 1, \pm 2\ldots$ analogous to the (iso)spin quantum numbers of the Skyrme model baryon states. The corresponding mass spectrum and wave-functions are given by,

$$\mu(q) = \mu + \frac{q^2}{2\lambda} \qquad\qquad \langle q|\theta\rangle = \frac{1}{\sqrt{2\pi}}\exp(iq\theta) \tag{0.17}$$

Rotationally improved Skyrmions. In the $U(1)$ model there is a process which is precisely analogous to Δ decay: the state $|q+1\rangle$ can decay on shell to the state $|q\rangle$ with the emission of a single physical meson, provided, of course, that the meson mass m is less than the splitting between these states. Before describing how

[4] When evaluated on a hedgehog, the moment of inertia tensor collapses to a scalar: $\Lambda_{ij} = \Lambda\,\delta_{ij}$. Deviations from hedgehog structure only affect this result at higher order in $1/N_c$.

the S-matrix element for Δ decay can be calculated in the Skyrme model, we will consider this simple process which contains many of the features of the Skyrme case. As described earlier, the relevant Green's function, from which the S-matrix element can be extracted, is the one point function of the meson field sandwiched between the initial and final soliton states.

$$g_a(x,t) = \langle q|\phi_a(x,t)|q+1\rangle \tag{0.18}$$

The key idea introduced in Ref [6] is that this Greens function is given to leading order in the semiclassical approximation by replacing $\vec{\phi}$ with a saddle-point field configuration $\vec{\phi}_{\rm sp}(x;\theta,q)$ which depends both on the collective coordinate θ of the soliton and the conserved $U(1)$ charge q. The relevant saddle-point equation is obtained by minimizing the the mass functional $\mu[\vec{\phi}]$ augmented by a correction term due to the rotational motion of the soliton:

$$\frac{\delta}{\delta\phi_a}\left(\mu[\vec{\phi}] + \frac{q^2}{2\lambda[\vec{\phi}]}\right) = 0 \tag{0.19}$$

This works out to,

$$(\Box + m^2)\phi_a + \frac{\delta W}{\delta\phi_a} - \frac{q^2\phi_a}{\lambda^2[\vec{\phi}]} = 0 \tag{0.20}$$

The only modification, therefore, of the static equation of motion is a shift in the meson mass term; $m^2 \to m^2 - q^2/\lambda^2[\vec{\phi}]$. It is straightforward to relate the solution of (0.20) to the solution $\phi_S(x;m)$ of the static field equation. Writing $\vec{\phi}_{\rm sp} = \mathcal{M}(\theta)\cdot(\varphi,0)$, the saddle-point profile function is given by $\varphi(x) = \phi_S(x, \sqrt{m^2 - q^2/\lambda^2[q]})$ where $\lambda[q]$ must be determined self-consistently:

$$\lambda[q] = \int dx\, \phi_S^2(x; \sqrt{m^2 - q^2/\lambda^2[q]}) \tag{0.21}$$

In [6] we showed that, with certain very mild assumptions, this equation always has a real solution for q sufficiently small. For the present purposes it is only necessary to observe that the self-consistency equation can be expanded in powers of $1/N_c$ giving $\lambda[q] = \lambda \cdot [1 + O(1/N_c)]$. The net result of this shift in the effective meson mass is that the profile of the rotationally improved soliton has a slower exponential fall off at large $|x|$ than its static counterpart,

$$\varphi(x) \to A\exp\left(-\sqrt{m^2 - q^2/\lambda^2}\,|x|\right) \tag{0.22}$$

This "swelling" of the soliton is just its response to the centrifugal forces produced by its internal rotation.

Just as in the simple $U(1)$ case described above, the leading-order Green's function which contributes to Δ decay is dominated by a single field configuration;

the Rotationally Improved Skyrmion or RISKY which satisfies a modified equation of motion,

$$\frac{\delta}{\delta\pi^a}\left(M[\vec{\pi}]+\frac{1}{2}J^m\Lambda^{-1}_{mn}[\vec{\pi}]J^n\right)=0 \tag{0.23}$$

where $\mathbf{J}$ is the classical angular momentum vector of the Skyrmion. Although we cannot solve this equation analytically it is straightforward to find the leading modification of the spatial asymptotics of the static Skyrmion. Writing $\vec{\pi}_{\rm sp}=D^{(1)}(A)\cdot\vec{\pi}(\mathbf{x};\mathbf{J})$, as $r\to\infty$ we have,

$$\begin{aligned}\vec{\pi}(\mathbf{x},\mathbf{J}) \;\to\; & \frac{B}{J^2}\cdot\left(\frac{m_\pi}{r}+\frac{1}{r^2}\right)\exp\left(-m_\pi r\right)(\mathbf{J}\cdot\hat{\mathbf{r}})\,\mathbf{J}\\ &+\frac{B}{J^2}\cdot\left(\frac{\sqrt{m_\pi^2-J^2/\Lambda^2}}{r}+\frac{1}{r^2}\right)\exp\left(-\sqrt{m_\pi^2-J^2/\Lambda^2}\cdot r\right)(\mathbf{J}\times\hat{\mathbf{r}}\times\mathbf{J})\end{aligned} \tag{0.24}$$

which properly collapses to the hedgehog in the limit $J^2\to 0$, as the reader may verify. Here the situation is somewhat more complicated than the $U(1)$ case; the RISKY field configuration is a superposition of two different tensor structures. The coefficient of the first tensor structure in (0.24), which is parallel to $\mathbf{J}$, has the same exponential fall-off as the static Skyrmion. In contrast the coefficient of the second tensor structure, which is perpendicular to $\mathbf{J}$ has a modified tail analogous to that of the rotating $U(1)$ soliton: the pion mass is shifted as $m_\pi^2\to m_\pi^2-J^2/\Lambda^2$. The physical interpretation is clear; the rotating Skyrmion "swells" in the directions perpendicular to its axis of rotation due to centrifugal forces.

We are still only half way to evaluating the required Green's function; we have calculated the saddle-point field configuration which gives the dominant semiclassical contribution for a Skyrmion with collective coordinate A and conjugate angular momentum $\mathbf{J}$, but it is still necessary to quantize these collective coordinate degrees of freedom. Again we begin by treating the simpler $U(1)$ case. The collective coordinate θ and its conjugate momentum q become quantum operators $\hat{\theta}$, $\hat{q}$ with $[\hat{\theta},\hat{q}]=i$. The problem of evaluating the leading semiclassical contribution to the Green's function (0.18) reduces to that of calculating a quantum mechanical expectation value of the saddle point field; $g^i(x,t)=\langle q|\phi^i_{\rm sp}(x,t;\hat{\theta},\hat{q})|q+1\rangle+O(1/N_c)$ which gives

$$g^i(x,t)=\langle q|\mathcal{M}_{i1}(\hat{\theta})\cdot\phi_S(x;\sqrt{m^2-\hat{q}^2/\lambda^2})|q+1\rangle \tag{0.25}$$

However, as it stands, this expression is ambiguous; we need to specify an ordering prescription for the non-commuting operators $\hat{\theta}$ and $\hat{q}$. This ordering problem is quite generic to soliton quantization where the introduction of collective coordinates always involves a nonlinear change of variables involving both the coordinates and their conjugate momenta [13]. The standard resolution of this problem is to choose the Weyl ordering prescription which is known to have certain desirable properties; in the case of translational motion of a soliton in one dimension this prescription is required to preserve Lorentz invariance [14]. Using standard identities, the net result of Weyl ordering the saddle-point field operator in (0.25) is that $\hat{q}$ can be replaced everywhere by its midpoint value $\bar{q} = (q + 1/2)$.

Taking a Fourier transform, the resulting semiclassical Green's function for the emission of a positively charged meson with energy ω and momentum k contains a pole contribution which is dictated by the spatial asymptotics of $g^i(x,t)$.

$$\tilde{g}(k,\omega) = \delta(\omega - \mu(q+1) + \mu(q)) \cdot \frac{2iAk}{k^2 + m^2 - \bar{q}^2/\lambda^2} + \text{Non-pole terms} \qquad (0.26)$$

In order for a non-vanishing contribution to the S-matrix it is necessary that the pole position coincides with the meson mass shell condition $k^2 + m^2 = \omega^2 = (\mu(q+1) - \mu(q))^2$. This follows immediately because $\mu(q+1) - \mu(q) = ((q+1)^2 - q^2)/2\lambda = \bar{q}/\lambda$. Hence we see that, when the operator ordering problem is correctly resolved, the rotational improvement of the soliton profile has the effect of shifting the meson pole to exactly the position required by the LSZ reduction formula. This means that there will be a leading order contribution to the S-matrix for the decay process $|q+1\rangle \to |q\rangle +$ one meson which coincides exactly with the expected Yukawa vertex.

In the Skyrme case, the one-pion Green's function is dominated by the RISKY, which solves equation (0.23). By analogy with our treatment of the $U(1)$ case above, we will now examine the pole contribution to the Fourier transform of this field configuration which is dictated by the asymptotic form (0.24),

$$\tilde{\vec{\pi}}(\mathbf{k};\hat{\mathbf{J}}) \sim \frac{4\pi i B}{|\mathbf{k}|}\left[\frac{1}{|\mathbf{k}|^2 + m_\pi^2}\frac{(\hat{\mathbf{J}}\cdot\mathbf{k})\hat{\mathbf{J}}}{\hat{J}^2} + \frac{1}{|\mathbf{k}|^2 + m_\pi^2 - \hat{J}^2/\Lambda^2}\frac{\hat{\mathbf{J}}\times\mathbf{k}\times\hat{\mathbf{J}}}{\hat{J}^2}\right] \qquad (0.27)$$

The resulting momentum-space configuration has two separate poles, corresponding to the two tensor structures which contribute to the RISKY. However, this is appropriate in the Skyrme case because there are two separate processes allowed by spin and isospin conservation. As well as the real process of Δ decay, $\Delta \to N + \pi$ there are also virtual processes $N \to N + \pi$ and $\Delta \to \Delta + \pi$, etc. As we showed in detail in [7] there exists an ordering prescription for the non-commuting operators $\hat{\mathbf{J}}$ and $\hat{A}$, analogous to the Weyl ordering chosen in the $U(1)$ case, which produces exactly the required analytic structure for the saddle-point Green's function evaluated between states of initial and final spin J and J',

$$\tilde{\mathbf{G}}_{\text{sp}}(\mathbf{k},\omega) = 4\pi i B\mathbf{k}\cdot\left[\frac{\delta_{J,J'}\delta(\omega)}{|\mathbf{k}|^2 + m_\pi^2} + \frac{\delta_{J,J'\pm1}\delta(\omega - M(J) + M(J'))}{|\mathbf{k}|^2 + m_\pi^2 - (M(J) - M(J'))^2}\right] \qquad (0.28)$$

Thus we see that the two tensor structures give poles corresponding exactly to the two allowed processes. The linear dependence on $\mathbf{k}$ means pseudovector coupling as desired—even beyond the soft-pion kinematic regime where this is required by Adler's rule. Once again, large-N_c reasoning allows an extrapolation to higher energies. The constant B is fixed by (0.1) to be $3g_{\pi NN}/8\pi M_N$.

Width calculations and the unwanted $I = J$ baryons. In sum, we have introduced a new configuration, the RISKY, whose analytic properties are precisely such that the Skyrme model maps onto an effective meson-baryon Lagrangian (at least at the level of the Yukawa vertex (0.1)). Furthermore, armed with this effective coupling, we can also confront an important phenomenological objection to large-N_c physics, namely the existence of the unwanted $I = J$ baryons with $I \geq \frac{5}{2}$. They are simply too broad to be seen! We can address this issue precisely because Eq. (0.1) may be sandwiched between any of the $I = J$ baryons, not just the nucleon or Δ. Furthermore, all these decay amplitudes will be proportional to $g_{\pi NN}$ which sits out in front (the "proportionality rule" described in the previous lecture). We refer the interested reader to Ref. [7] for the (non-illuminating) calculational details, and conclude by summarizing the principal findings:

1. With $g_{\pi NN}$ drawn from experiment, the width of the Δ works out to 114 MeV in the Skyrme model, within a few MeV of the actual value.

2. With this same value for $g_{\pi NN}$, the widths of the higher-spin baryons rises rapidly, thus $\Gamma_{5/2} \sim 800\,\text{MeV}$, $\Gamma_{7/2} \sim 2600\,\text{MeV}$, $\Gamma_{9/2} \sim 6400\,\text{MeV}$, etc. These are so broad that *there is no conflict whatever with phenomenology.*

References

[1] T. H. R. Skyrme, *Proc. Roy. Soc.* **A260** (1961) 127.

[2] G. Adkins, C. Nappi and E. Witten, *Nucl. Phys.* **B228** (1983) 552.

[3] N. Dorey and M. P. Mattis, *The large-N_c renormalization group*, to appear in the Proceedings of the 1995 International Workshop on Nuclear & Particle Physics: Chiral Dynamics in Hadrons & Nuclei.

[4] M. Mattis and M. Karliner *Phys. Rev.* **D31** (1985) 2833; M. Mattis and M. Peskin, *Phys. Rev.* **D32** (1985) 58.

[5] A. Hayashi, G. Eckart, G. Holzwarth and H. Walliser, *Phys. Lett.* **B147** (1984) 5.

[6] N. Dorey, J. Hughes and M. P. Mattis, *Phys. Rev.* **D49** (1994) 3598.

[7] N. Dorey, J. Hughes and M. P. Mattis, *Phys. Rev.* **D50** (1994) 5816.

[8] B. J. Schroers, *Zeit. Phys.* **C61** (1994) 479.

[9] H. Verschelde, *Phys. Lett.* **B209** (1988) 34; G. Holzwarth, A. Hayashi and B. Schwesinger, *Phys. Lett.* **B191** (1987) 27; S. Saito, *Prog. Theor. Phys.* **78** (1987) 746.

[10] D. I. Dyakonov, V. Yu. Petrov, and P. B. Pobylitsa, *Phys. Lett.* **B205** (1988) 372.

[11] A. Hayashi, S. Saito and M. Uehara, *Phys. Lett.* **B246** (1990) 15; *Phys. Rev.* **D43**, 1520 (1991); *ibid.*, **D46**, 4856 (1992); *Prog. Theor. Phys. Supp.* **109** (1992) 45.

[12] R. Rajaraman and E. Weinberg, *Phys. Rev.* **D11** (1975) 2950.

[13] J. Gervais and A. Jevicki *Nucl. Phys.* **B110** (1976) 93.

[14] E. Tomboulis, *Phys. Rev.* **D12** (1975) 1678.