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SINGULAR AND SMOOTH TIME-DEPENDENT ORBIFOLDS

Michal Fabinger
Stanford University and SLAC
Stanford, CA 94305/94309, USA
fabinger@stanford.edu

Abstract

We consider string theory in a smooth time-dependent orbifold of Minkowski space, which is known as the 'null-brane', and whose limit reproduces a spacetime with a null singularity – the parabolic orbifold. We show that adding particles of small enough energy to the geometry does not cause a gravitational collapse. The null-brane is a well-behaved background of string theory where it is possible to compute string scattering amplitudes using perturbation theory. We also mention another way of making the parabolic orbifold smooth – besides considering the null-brane.

1. INTRODUCTION

Formulating string theory in general time-dependent backgrounds is definitely a very difficult problem. For this reason it seems natural to study time-dependent backgrounds which might be relatively easy to understand, such as orbifolds of flat Minkowski space by a discrete subgroup of the Poincaré group. Many such orbifolds contain closed timelike curves, which raise unpleasant issues. Better in this regard is the model studied by Liu, Moore and Seiberg [1] which is an orbifold by $\mathbb{Z}$ generated by a parabolic element of $SO(1,2)$ and belongs to the class of models described by Horowitz and Steif [2]. The orbifold has a light-like singularity and contains closed light-like curves. It has a null Killing vector, which allows one to use light-cone quantization.

In this essay, I will briefly review part of a work done with John McGreevy [3]. (For partially overlapping work see [5, 6].) The main focus will be on a very closely related orbifold – the null-brane [4]. The

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generator of the orbifold group is a parabolic element of $SO(1,2)$ combined with a constant shift in a fourth direction. Its main virtue is that the orbifold group has no fixed points, and therefore the quotient space contains no singularities at all. This smooth orbifold provides a time-dependent string background which has a free world-sheet description, and in which the backreaction is under control. In the limit where the shift goes to zero we recover precisely the orbifold of [1].

2. CLASSICAL GEOMETRY

The null-brane geometry we will study is a $\mathbb{Z}$ orbifold of flat Minkowski space $\mathbb{R}^{1,3}$ (times $\mathbb{R}^6$, if we want to consider superstring theory). In terms of coordinates $x^{\pm} = (x^0 \pm x^1)/\sqrt{2}$, $x = x^2$, and $\chi = x^3$, the metric is

$$ds^2 = -2x^+x^- + dx^2 + d\chi^2 \tag{1.1}$$

We will write the generator of the orbifold group Γ_L as

$$g_L = \exp{(ivJ)}\ \exp{(iLp^\chi)}\,, \quad J \equiv \frac{1}{\sqrt{2}} J^{x^0x} + \frac{1}{\sqrt{2}} J^{x^1x} \tag{1.2}$$

This corresponds to a composition of a null Lorentz transformation of the (x^+, x, x^-) subspace and a translation by L in the χ-direction. In terms of the spacetime coordinates, g_L acts as

$$\begin{pmatrix} x^+ \\ x \\ x^- \\ \chi \end{pmatrix} \quad \rightarrow \quad \begin{pmatrix} x^+ \\ x + vx^+ \\ x^- + vx + \frac{1}{2}v^2x^+ \\ \chi + L \end{pmatrix} \tag{1.3}$$

For $L = 0$ the orbifold becomes the parabolic orbifold studied by Liu, Moore and Seiberg [1], which is singular at $x^+ = 0$. For non-zero L the orbifold is completely smooth and does not have any closed time-like or light-like curves.

The $\mathbb{R}^{1,3}/\Gamma_L$ orbifold in general preserves the subgroup of the four-dimensional Poincaré symmetry group generated by p^χ, J, and $p^+ = -p_-$. For non-zero L, the topology of the spacetime is simply $\mathbb{R}^3 \times S^1$.

3. BACKREACTION ON THE GEOMETRY

One of the most obvious questions that arise when one considers time-dependent orbifolds is whether the presence of a single particle does not cause the spacetime to gravitationally collapse. Placing one such particle of rest mass m (say $m \neq 0$) in the orbifold corresponds to adding to the universal covering space an infinite number of particles (the original one plus its images) which are boosted with respect to each other. Since the boost of distant particles goes to infinity, one might worry that the mass of a finite number of them might be larger than the corresponding Schwarzschild radius (which would be a clear sign of a large backreaction).

We will see that if $L \neq 0$ this does not happen here, provided m is not too large. Suppose we work in an inertial frame in which the 'original particle' is at rest. At any time x^0 the distance to its n-th image will be no smaller than nL, i.e. it grows at least linearly with n. The velocity of the n-th image is

$$v_n = \frac{nv}{4+n^2v^2}\sqrt{8+n^2v^2}, \tag{1.4}$$

and corresponds to energy

$$E_n = m\gamma_n = \frac{m}{\sqrt{1-v_n^2}} = m\left(1+\frac{1}{4}n^2v^2\right) \sim \frac{1}{4}m\,n^2v^2. \tag{1.5}$$

As a result, the total energy of the first $2n$ images grows like n^3. This energy is not the center-of-mass frame energy of the first $2n$ images, but even if it was, the corresponding Schwarzschild radius would not grow faster than $(n^3)^{1/10-3} = n^{3/7}$ since we work in 10d.[1] This is still a slower growth than the one of the smallest size nL of the region containing the first $2n$ images, provided $L \neq 0$. We see that adding a particle of small enough energy to the null-brane does not cause a gravitational collapse, unlike in the case of the parabolic orbifold. Moreover, a similar kind of reasoning leads to the conclusion that the null-brane admits scattering processes which can be studied perturbatively.

4. STRING THEORY IN THE NULL-BRANE

Thanks to a free worldsheet theory, strings in the null-brane geometry can be easily quantized [5, 3]. Here we just point out a few facts, leaving the details to [5, 3].

- Both the bosonic and fermionic parts of the torus partition function are finite.
- It is possible to construct a basis of states for which the tree lever scattering amplitudes are finite (up to slightly enhanced infrared divergencies).
- In the singular limit $L \to 0$ certain tree level amplitudes diverge.

[1]Actually, the center-of-mass frame energy grows like n^2, leading to gravitational radius of order $n^{2/7}$, as discussed in detail in [6].

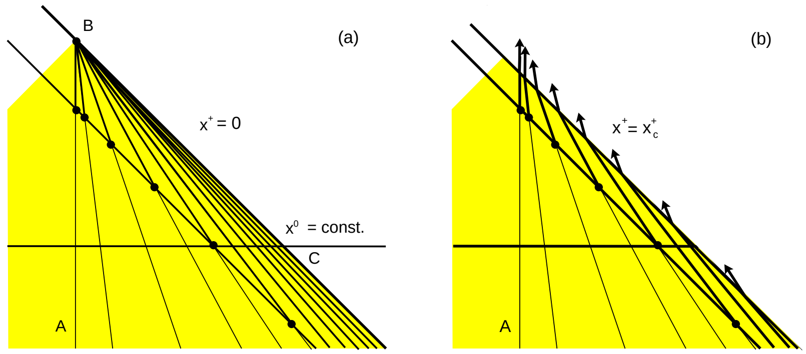

Figure 1.1 (a) A schematic picture of the parabolic orbifold, showing only coordinates x^+ and x^-. Close to the singularity, images of any particle become infinitely dense. (b) If one cuts off the spacetime at some finite $x_c^+ < 0$ and replaces it with an orbifold of a plane wave where the circle expands again, the images never come too close to each other, and the resulting spacetime is stable. The part of the geometry with $x^+ > x_c^+$, not shown in this figure, has a non-zero curvature.

5. SMOOTHING OUT THE PARABOLIC ORBIFOLD

Constructing the null-brane is not the only way to make a smooth manifold whose limit is the parabolic orbifold. Another possibility is to cut off the parabolic orbifold at some $x_c^+ < 0$ and replace it with an orbifold of a plane wave (see figure 1.1). The spacetime constructed in this way is well-behaved, and in particular, it is stable, as opposed to the parabolic orbifold itself. The details of this construction, and a brief discussion of its stability will be left to the references [3, 7].

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