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Neutrino masses in R-parity violating supersymmetric models

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Abstract

We study neutrino masses and mixing in R-parity violating supersymmetric models with generic soft supersymmetry breaking terms. Neutrinos acquire masses from various sources: Tree level neutrino–neutralino mixing and loop effects proportional to bilinear and/or trilinear R-parity violating parameters. Each of these contributions is controlled by different parameters and have different suppression or enhancement factors which we identified. Within an Abelian horizontal symmetry framework these factors are related and specific predictions can be made. We found that the main contributions to the neutrino masses are from the tree level and the bilinear loops and that the observed neutrino data can be accommodated once mild fine-tuning is allowed.

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I. INTRODUCTION

Neutrino oscillation experiments indicate that the neutrinos are massive [1]. The data is best explained with the following set of parameters [2]

$$\begin{aligned} &\Delta m_{23}^2 = 2.0 \times 10^{-3}\ \text{eV}^2, \qquad \Delta m_{12}^2 = 7.2 \times 10^{-5}\ \text{eV}^2, \\ &\sin^2\theta_{23} = 0.5, \qquad \sin^2\theta_{12} = 0.3, \qquad \sin^2\theta_{13} < 0.074, \end{aligned} \tag{1.1}$$

where $\Delta m_{ij}^2 \equiv m_i^2 - m_j^2$ and θ_{ij} are the leptonic mixing angles. Eq. (1.1) tells us that the neutrino masses exhibit a mild hierarchy and that there is one somewhat small mixing angle (θ_{13}) and two large mixing angles (θ_{12} and θ_{23}).

Any theory beyond the Standard Model (SM) needs to explain this neutrino mass structure. One of the challenges is to generate large mixing angles with hierarchical masses. Generally, small mixing angles are associated with mass hierarchies and vice versa. This situation is avoided when the determinant of the mass matrix is much smaller then its natural value, namely, when there are cancellations between different terms in the determinant. Such cancellations can arise naturally in models where different neutrinos acquire masses from different sources. One such a framework is R-parity Violating (RPV) supersymmetry [3], where generically a single neutrino acquires a mass at the tree level via mixing with the neutralinos while the other two neutrinos become massive by one-loop effects.

Neutrino masses in the framework of RPV supersymmetry have been widely studied [4]. In the earlier works, the only loop contributions that were considered are from the loops that depend on trilinear RPV couplings. Later, it was realized that the effect of sneutrino–antisneutrino mixing [5, 6] can be very important since it is related to loops that contribute to the neutrino masses and depend on bilinear RPV parameters [7, 8]. In Ref. [9] many more loop contributions besides the "traditional" trilinear ones were identified. These loops were also studied in [10, 11, 12, 13].

In generic RPV models there are too many free parameters and no specific predictions for the neutrino spectrum can be made. In general it is even not possible to identify the important contributions to the neutrino masses. In this paper we discuss the various contributions to neutrino masses and identify different suppression and enhancement factors in each of them. We also study one specific framework, that of Abelian horizontal symmetry, where specific predictions can be made. We found that the main contributions to the neutrino masses are from the tree level and the sneutrino–neutralino loops and that the model can accommodate the observed data once mild fine-tuning is allowed.

II. THE MODEL

We start by describing the RPV framework. We follow here the notation of [7] where the model is described in more details.

In order to avoid the bounds from proton stability, we consider the most general low-energy supersymmetric model consisting of the MSSM fields that conserves a $\mathbf{Z_3}$ baryon triality [14]. Such a theory possesses RPV-interactions that violate lepton number. Once R-parity is violated, there is no conserved quantum number that distinguishes between the lepton supermultiplets $\hat{L}_m$ ($m = 1, 2, 3$) and the down-type Higgs supermultiplet $\hat{H}_D$. It is therefore convenient to denote the four supermultiplets by one symbol $\hat{L}_\alpha$ ($\alpha = 0, 1, 2, 3$), with $\hat{L}_0 \equiv \hat{H}_D$. We use Greek indices to indicate the four dimensional extended lepton flavor space, and Latin ones for the usual three dimensional flavor spaces.

The most general renormalizable superpotential is given by:

$$W = \epsilon_{ij} \left[-\mu_\alpha \hat{L}^i_\alpha \hat{H}^j_U + \tfrac{1}{2} \lambda_{\alpha\beta m} \hat{L}^i_\alpha \hat{L}^j_\beta \hat{E}_m + \lambda'_{\alpha nm} \hat{L}^i_\alpha \hat{Q}^j_n \hat{D}_m - h_{nm} \hat{H}^i_U \hat{Q}^j_n \hat{U}_m \right] , \tag{2.1}$$

where $\hat{H}_U$ is the up-type Higgs supermultiplet, the $\hat{Q}_n$ are doublet quark supermultiplets, $\hat{U}_m$ [$\hat{D}_m$] are singlet up-type [down-type] quark supermultiplets and $\hat{E}_m$ are the singlet charged lepton supermultiplets. The coefficients $\lambda_{\alpha\beta m}$ are antisymmetric under the interchange of the indices α and β. Note that the μ-term of the MSSM [which corresponds to μ_0 in Eq. (2.1)] is now extended to a four-component vector, μ_α, and that the Yukawa matrices of the MSSM [which correspond to λ'_{0ij} and λ_{0ij} in Eq. (2.1)] are now extended into rank three tensors, $\lambda'_{\alpha nm}$ and $\lambda_{\alpha\beta m}$.

Next we consider the most general set of renormalizable soft supersymmetry breaking terms. In addition to the usual soft supersymmetry breaking terms of the R-parity Conserving (RPC) MSSM, one must also add new A and B terms corresponding to the RPV terms of the superpotential. In addition, new RPV scalar squared-mass terms also exist. As above, we extend the definitions of the RPC terms to allow indices of type α. Explicitly, the relevant terms are

$$V_{\rm soft} = (M^2_{\widetilde{L}})_{\alpha\beta}\, \widetilde{L}^{i*}_\alpha \widetilde{L}^i_\beta - (\epsilon_{ij} B_\alpha \widetilde{L}^i_\alpha H^j_U + \text{h.c.}) + \epsilon_{ij} [\tfrac{1}{2} A_{\alpha\beta m} \widetilde{L}^i_\alpha \widetilde{L}^j_\beta \widetilde{E}_m + A'_{\alpha nm} \widetilde{L}^i_\alpha \widetilde{Q}^j_n \widetilde{D}_m + \text{h.c.}], \tag{2.2}$$

and we do not present the terms that are unchanged from the RPC (that can be found, for example, in [7]). Note that the single B term of the MSSM is extended to a four-component vector, B_α, and that the single squared-mass term for the down-type Higgs boson and the 3×3 lepton scalar squared-mass matrix are now part of a 4×4 matrix, $(M^2_{\widetilde{L}})_{\alpha\beta}$. We further define

$$|\mu|^2 \equiv \sum_\alpha |\mu_\alpha|^2, \qquad \langle H_U \rangle \equiv \tfrac{1}{\sqrt{2}} v_u, \qquad \langle \tilde{\nu}_\alpha \rangle \equiv \tfrac{1}{\sqrt{2}} v_\alpha, \qquad v_d \equiv |v_\alpha|, \tag{2.3}$$

with

$$v \equiv (|v_u|^2 + |v_d|^2)^{1/2} = \frac{2m_W}{g} = 246 \text{ GeV} , \qquad \tan\beta \equiv \frac{v_u}{v_d}. \tag{2.4}$$

These vacuum expectation values are determined via the minimum equations [7].

From now on we will work in a specific basis in the space spanned by $\hat{L}_\alpha$ such that $v_m = 0$ and $v_0 = v_d$. The down-type quark and lepton mass matrices in this basis arise from the Yukawa couplings to $\hat{H}_D$, namely,

$$(m_d)_{nm} = \tfrac{1}{\sqrt{2}} v_d \lambda'_{0nm} , \qquad (m_\ell)_{nm} = \tfrac{1}{\sqrt{2}} v_d \lambda_{0nm} . \tag{2.5}$$

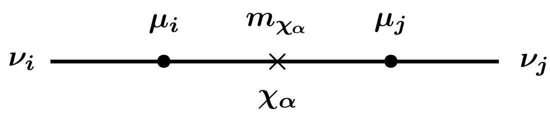

FIG. 1: Tree level neutrino mass in the mass insertion approximation. A blob represents mixing between the neutrino and the up-type Higgsino. The cross on the neutralino propagator signifies a Majorana mass term for the neutralino.

Note that due to the small RPV admixture with the charged Higgsinos, $(m_\ell)_{nm}$ is not precisely the charged lepton mass matrix [11]. This small effect is not important for our analysis.

In the literature one often finds other basis choices. The most common is the one where $\mu_0 = \mu$ and $\mu_m = 0$. Of course, the results for physical observables are independent of the basis choice. (For basis independent parameterizations of R-parity violation see [9, 15, 16, 17].)

III. NEUTRINO MASSES

The neutrino mass matrix receives contributions both at the tree level and from loops. In the following we review the various kinds of contributions and identify the leading factors that govern their magnitudes.

A. Tree level ($\mu\mu$) masses

At tree level the neutrino mass matrix receives contributions from RPV mixing between the neutrinos and the neutralinos, see Fig. 1. The masses are calculated from the neutral fermion (neutralinos and neutrinos) mass matrix. We work perturbatively, and thus at leading order we do not distinguish between μ and μ_0. Then, the tree level neutral fermion mass matrix, with rows and columns corresponding to $\{\widetilde{B}, \widetilde{W}^3, \widetilde{H}_U, \nu_\beta\}$, is given by [7, 18, 19]:

$$\begin{pmatrix} M_1 & 0 & m_Z s_W v_u/v & -m_Z s_W v_d/v & 0 & 0 & 0 \\ 0 & M_2 & -m_Z c_W v_u/v & m_Z c_W v_d/v & 0 & 0 & 0 \\ m_Z s_W v_u/v & -m_Z c_W v_u/v & 0 & \mu & \mu_1 & \mu_2 & \mu_3 \\ -m_Z s_W v_d/v & m_Z c_W v_d/v & \mu & 0 & 0 & 0 & 0 \\ 0 & 0 & \mu_1 & 0 & 0 & 0 & 0 \\ 0 & 0 & \mu_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & \mu_3 & 0 & 0 & 0 & 0 \end{pmatrix} \qquad (3.1)$$

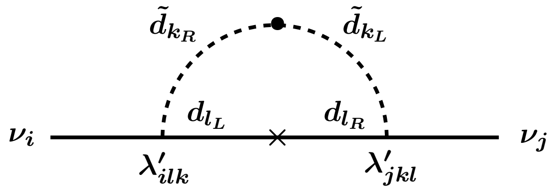

FIG. 2: Trilinear loop contribution to the neutrino mass matrix. The blob on the scalar line indicates mixing between the left-handed and the right-handed squarks. A mass insertion on the internal quark propagator is denoted by the cross.

where M_1 is the Bino mass, M_2 is the Wino mass, $c_W \equiv \cos\theta_W$ and $s_W \equiv \sin\theta_W$. Integrating out the four neutralinos we get the neutrino mass matrix

$$[m_\nu]_{ij}^{(\mu\mu)} = X_T \mu_i \mu_j \,, \tag{3.2}$$

where

$$X_T = \frac{m_Z^2 m_{\tilde{\gamma}} \cos^2\beta}{\mu(m_Z^2 m_{\tilde{\gamma}} \sin 2\beta - M_1 M_2 \mu)} \sim \frac{\cos^2\beta}{\tilde{m}} \,, \tag{3.3}$$

such that $m_{\tilde{\gamma}} \equiv c_W^2 M_1 + s_W^2 M_2$ and in the last step we assume that all the relevant masses are at the electroweak (or supersymmetry breaking) scale, $\tilde{m}$. The tree level neutrino masses are the eigenvalues of $[m_\nu]_{ij}^{(\mu\mu)}$

$$m_3^{(T)} = X_T(\mu_1^2 + \mu_2^2 + \mu_3^2) \,, \qquad m_1^{(T)} = m_2^{(T)} = 0 \,. \tag{3.4}$$

Here, and in what follows, we use $m_3 \geq m_2 \geq m_1$.

We see that at the tree level only one neutrino is massive. Its mass is proportional to the RPV parameter $\sum \mu_i^2$ and to $\cos^2\beta$. For large $\tan\beta$ the later is a suppression factor. As we discuss later, this suppression factor can be important.

B. Trilinear ($\lambda'\lambda'$ and $\lambda\lambda$) loops

The neutrino mass matrix receives contributions from loops that are proportional to trilinear RPV couplings, see Fig. 2. These kinds of loops received much attention in the literature. Here we only present approximated expressions which are sufficient for our study. Full results can be found, for example, in [7].

Neglecting quark flavor mixing, the contribution of the $\lambda'\lambda'$ loops is proportional to the internal fermion mass and to the mixing between left and right sfermions. Explicitly,

$$[m_\nu]_{ij}^{(\lambda'\lambda')} \approx \sum_{l,k} \frac{3}{8\pi^2} \lambda'_{ilk}\, \lambda'_{jkl}\, \frac{m_{d_l}\, \Delta m^2_{\tilde{d}_k}}{m^2_{\tilde{d}_k}} \sim \sum_{l,k} \frac{3}{8\pi^2} \lambda'_{ilk} \lambda'_{jkl} \frac{m_{d_l} m_{d_k}}{\tilde{m}}, \tag{3.5}$$

where $m_{\tilde{d}_k}$ is the average kth sfermion mass, $\Delta m^2_{\tilde{d}_k}$ is the squared mass splitting between the two kth sfermions, and in the last step we used $\Delta m^2_{\tilde{d}_k} \approx m_{d_k} \tilde{m}$ and $m_{\tilde{d}_k} \sim \tilde{m}$. There are

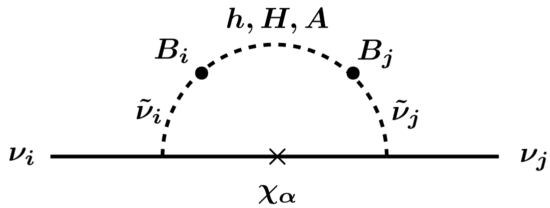

FIG. 3: The BB loop-generated neutrino mass. Here the blobs denote mixing of the sneutrinos with the neutral Higgs bosons. The cross on the internal neutralino propagator denotes a Majorana mass for the neutralino.

similar contributions from loops with intermediate charged leptons where λ' is replaced by λ and there is no color factor in the numerator.

We see that the trilinear loop-generated masses are suppressed by the RPV couplings λ'^2 [λ^2], by a loop factor, and by two down-type quark [charged lepton] masses. The later factor, which is absent in other types of loops, make the trilinear contribution irrelevant in most cases.

C. Bilinear (BB) loop induced masses

Neutrinos acquire masses from loops that are proportional to bilinear RPV couplings. Here we discuss the contributions that are proportional to two insertions of RPV B_i parameters, see Fig. 3. We also refer to the masses induced by these diagrams as the sneutrino splitting induced masses. The reason is that the two B insertions in the scalar line also generate splitting between the two sneutrino mass eigenstates. The contribution of the BB loop diagram is related to this sneutrino mass splitting [5, 9]. In particular, if the sneutrino splitting vanishes the neutrino is massless.

The one-loop contribution to the neutrino mass matrix from the BB loop is given by [9]

$$[m_\nu]_{ij}^{(BB)} = \sum_{\alpha,i,j} \frac{g^2 B_i B_j}{4\cos^2\beta} (Z_{\alpha 2} - Z_{\alpha 1} g'/g)^2 m_{\chi_\alpha} \Big\{ I_4(m_h, m_{\tilde{\nu}_i}, m_{\tilde{\nu}_j}, m_{\chi_\alpha}) \cos^2(\alpha-\beta)$$
$$+ I_4(m_H, m_{\tilde{\nu}_i}, m_{\tilde{\nu}_j}, m_{\chi_\alpha}) \sin^2(\alpha-\beta) - I_4(m_A, m_{\tilde{\nu}_i}, m_{\tilde{\nu}_j}, m_{\chi_\alpha}) \Big\} , \tag{3.6}$$

where $Z_{\alpha\beta}$ is the neutralino mixing matrix with $\alpha, \beta = 1, .., 4$ and

$$\begin{aligned} I_4(m_1, m_2, m_3, m_4) &= \frac{1}{m_1^2 - m_2^2} \left[I_3(m_1, m_3, m_4) - I_3(m_2, m_3, m_4) \right] , \\ I_3(m_1, m_2, m_3) &= \frac{1}{m_1^2 - m_2^2} \left[I_2(m_1, m_3) - I_2(m_2, m_3) \right] , \\ I_2(m_1, m_2) &= -\frac{1}{16\pi^2} \frac{m_1^2}{m_1^2 - m_2^2} \ln \frac{m_1^2}{m_2^2} . \end{aligned} \tag{3.7}$$

Assuming that all the masses in the RHS of Eq. (3.6) are of the order of the weak scale, we

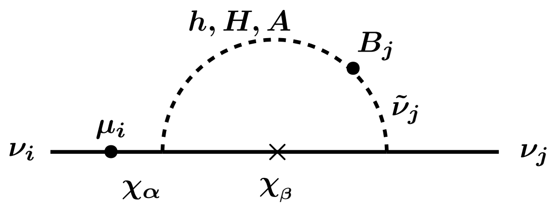

FIG. 4: Neutrino Majorana mass generated by μB loop. The blob on the external fermion line signifies mixing between a neutrino and a neutralino. The blob on the internal scalar line stands for mixing between the sneutrinos and the neutral Higgs bosons. The cross on the internal neutralino line denotes, as before, a Majorana mass term for the neutralino. There exists other diagrams with $i \leftrightarrow j$.

estimate

$$[m_\nu]_{ij}^{(BB)} \sim \frac{g^2}{64\pi^2 \cos^2\beta} \frac{B_i B_j}{\tilde{m}^3}. \tag{3.8}$$

In the above estimation, no cancellation between the different Higgs loops were assumed. We do, however, expect to have some degree of cancellation between these loops. To see it note that if the three I_4 functions in (3.6) were equal, $[m_\nu]_{ij}^{(BB)}$ would vanish. The remnant of this effect is a partial cancellation that becomes stronger in the decoupling limit. Then $\cos^2(\alpha - \beta) \to 0$ and $m_H \to m_A$ and from Eq. (3.6) we see that $[m_\nu]_{ij}^{(BB)} \to 0$. We discuss this cancellation in Appendix A.

Next we study the BB loop effect on the neutrino masses. For this we rewrite (3.6) as

$$[m_\nu]_{ij}^{(BB)} = C_{ij} B_i B_j. \tag{3.9}$$

If all the elements of the matrix C_{ij} were identical, $[m_\nu]_{ij}^{(BB)}$ would have only one non-vanishing eigenvalue. Using (3.6) we see that such a situation arises when the sneutrinos are degenerate. More generally, we conclude that the contribution to the light neutrinos receive potentially additional suppression by a factor proportional to the non-degeneracy in the sneutrino sector.

In general we expect that B_α is not proportional to μ_α. Then, one neutrino mass eigenstate acquires mass at tree level, and the other two from bilinear loops where the mass of the lightest neutrino is proportional to the amount of non-degeneracy of the sneutrinos. We elaborate more on this effect in Appendix B.

We conclude that the BB loop-generated masses are suppressed by the RPV couplings BB, by a loop factor and by a possible effect due to the cancellation between the three Higgs loops. For large $\tan\beta$ the BB loop is enhanced by $\tan^2\beta$. The third neutrino mass may get an extra suppression proportional to the non-degeneracy among the sneutrinos.

D. μB loops

Another type of diagrams which induce neutrino masses from mixing between the sneutrinos and the neutral Higgs bosons is given in Fig. 4. The contribution from this diagram to the neutrino mass matrix is given by [9][1]

$$
\begin{aligned}
[m_\nu]_{ij}^{(\mu B)} = & \sum_{\alpha,\beta} \frac{g^2}{4\cos\beta}\mu_i B_j \frac{m_{\chi_\beta}}{m_{\chi_\alpha}} Z_{\alpha 3}(Z_{\beta 2} - Z_{\beta 1}g'/g) \\
& \Big\{ - \Big[Z_{\alpha 4}(Z_{\beta 2} - Z_{\beta 1}g'/g)\sin\alpha + (Z_{\alpha 2} - Z_{\alpha 1}g'/g)Z_{\beta 3}\cos\alpha \\
& \quad +(Z_{\alpha 2} - Z_{\alpha 1}g'/g)Z_{\beta 4}\sin\alpha \Big] \cos(\alpha-\beta)\, I_3(m_h, m_{\chi_\beta}, m_{\tilde{\nu}_j}) \\
& + \Big[Z_{\alpha 4}(Z_{\beta 2} - Z_{\beta 1}g'/g)\cos\alpha - (Z_{\alpha 2} - Z_{\alpha 1}g'/g)Z_{\beta 3}\sin\alpha \\
& \quad +(Z_{\alpha 2} - Z_{\alpha 1}g'/g)Z_{\beta 4}\cos\alpha \Big] \sin(\alpha-\beta)\, I_3(m_H, m_{\chi_\beta}, m_{\tilde{\nu}_j}) \\
& + \Big[Z_{\alpha 4}(Z_{\beta 2} - Z_{\beta 1}g'/g)\sin\beta + (Z_{\alpha 2} - Z_{\alpha 1}g'/g)Z_{\beta 3}\cos\beta \\
& \quad +(Z_{\alpha 2} - Z_{\alpha 1}g'/g)Z_{\beta 4}\sin\beta \Big]\, I_3(m_A, m_{\chi_\beta}, m_{\tilde{\nu}_j}) \Big\} + (i \leftrightarrow j)
\end{aligned}
\tag{3.10}
$$

Assuming that all the masses are at the weak scale, this contribution to the neutrino mass matrix is given approximately by [9]

$$[m_\nu]_{ij}^{(\mu B)} \sim \frac{g^2}{64\pi^2\cos\beta}\,\frac{\mu_i B_j + \mu_j B_i}{\tilde{m}^2}. \tag{3.11}$$

In the flavor basis these diagrams are expected to yield similar contributions to the BB loops. Yet, as pointed out in [13], due to the dependence on μ_i, the μB loop contribution to the neutrino masses is sub-leading. See Appendix C for details.

Similar to the BB loops, also in the μB loop there is a partial cancellation between the different Higgs loops. In the decoupling limit this can be seen from Eq. (3.10). Since, as we just mentioned, the effect of the μB loop is sub-leading, we do not elaborate on the decoupling effect.

We conclude that the μB loop-generated masses are suppressed by the RPV couplings μB, by a loop factor and by a possible effect due to Higgs decoupling. In the case where the tree level contribution is dominant, their effect on the neutrino masses is second order in these small parameters.

E. Other loops

There are many other loops that contribute to the neutrino masses [9]. Almost all of them are suppressed by at least two Yukawa interactions, and are therefore likely to be negligible.

[1] Note that we disagree with [9] on the sign of the term that originate from the A loop.

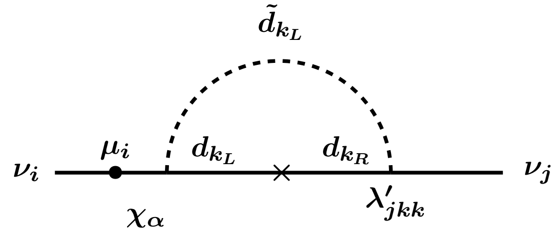

FIG. 5: Neutrino Majorana mass generated by $\mu\lambda'$ loop. The blob on the external fermion line signifies a mixing between a neutrino and a up-type Higgsino which is then converted to a gaugino. The cross on the internal fermion line stands for a Dirac mass insertion. There exists another diagram with $i \leftrightarrow j$.

There is only one contribution to the neutrino masses that depends on both bilinear and trilinear couplings and is suppressed by only one Yukawa coupling. This diagram is shown in Fig. 5. Neglecting squark flavor mixing, the $\mu\lambda'$ contribution to the neutrino mass matrix is [9]

$$[m_\nu]_{ij}^{(\mu\lambda')} \approx \sum_k \frac{3}{16\pi^2}\, g\, m_{d_k}\, \frac{\mu_i \lambda'_{jkk} + \mu_j \lambda'_{ikk}}{\tilde{m}}. \tag{3.12}$$

There are similar contributions from diagrams with λ instead of λ' couplings, where leptons and sleptons are running in the loop.

We see that the $\mu\lambda'$ diagrams are suppressed by the RPV couplings $\mu\lambda'$, by a loop factor and by one Yukawa coupling. Also here, similar to the case of the μB loops, once the tree level effect is taken into account, the $\mu\lambda'$ and $\mu\lambda$ contributions to the light mass eigenstates are second order in the above mentioned suppression factors.

F. Model independent considerations

As we discussed, there are many contributions to the neutrino masses that are suppressed by different small parameters. In general, the leading effects are model dependent. Nevertheless, here we make some general remarks.

One neutrino is massive at the tree level and unless $\tan\beta$ is very large, this is the dominant contribution to m_3. The other neutrinos get masses at the loop level. Despite the partial cancellation between different Higgs loops, we expect the BB loops to be the dominant one. All the other contributions are generically suppressed compared to it due to the following reasons:

- The μB loop contribution to the light neutrino masses is second order in the small ratio between the loop-induced mass and the tree level one.

- The $\lambda'\lambda'$ and $\lambda\lambda$ diagrams are doubly Yukawa suppressed.

- The $\mu\lambda'$ and $\mu\lambda$ diagrams are singly Yukawa suppressed, and similar to the situation with the μB loops, their contributions to the light neutrino masses are second order in the suppression factor.

Therefore, while there are several caveats as explained above, the situation is likely to be as follows: The heaviest neutrino mass, m_3, arises at the tree level. The major contribution to m_2 is from the BB loops. For non-degenerate sneutrinos, m_1 is also generated by the BB loops. For degenerate sneutrinos, however, m_1 is very small and the major contribution to it can arise from any of the other sources. Note that since neutrino oscillation data are not sensitive to the lightest neutrino mass, our ignorance of the mechanism that generate m_1 is not problematic.

In the following we consider a specific model where we can explicitly check the relevance of the different contributions.

IV. HORIZONTAL SYMMETRY

We work in the Abelian horizontal symmetry framework [20]. The horizontal symmetry, H, is explicitly broken by a small parameter λ to which we attribute charge -1. This can be viewed as the effective low energy theory that comes from a supersymmetric extension of the Froggatt–Nielsen mechanism at a high scale [21]. Then, the following selection rule apply: (a) Terms in the superpotential that carry charge $n \geq 0$ under H are suppressed by $O(\lambda^n)$, while those with $n < 0$ are forbidden by holomorphy; (b) Soft supersymmetry breaking terms that carry charge n under H are suppressed by $O(\lambda^{|n|})$. For simplicity, in the following we assume that the horizontal charges of all the MSSM superfields are non-negative.

We identify the down-type Higgs doublet with the doublet superfield that carries the smallest charge, which we choose to be $L_0 \equiv H_d$. (To simplify the notation what we denote before as $\hat{L}_0$ is now L_0.) We order the remaining doublets according to their charges:

$$H(L_1) \geq H(L_2) \geq H(L_3) \geq H(H_d) \geq 0\,. \tag{4.1}$$

Similar ordering is made for the three generations of the charged leptons and quarks.

Our methods of analyzing lepton and neutralino mass matrices are described in detail in [18] and [22], respectively. Specifically, we use the above mentioned selection rules to estimate the magnitude of the relevant parameters

$$\begin{aligned}
\mu_\alpha &\sim \tilde{\mu}\lambda^{H(L_\alpha)+H(H_u)}\,, \\
B_\alpha &\sim \tilde{m}^2\lambda^{H(L_\alpha)+H(H_u)}\,, \\
(M^2_{\tilde{L}})_{\alpha\beta} &\sim \tilde{m}^2\lambda^{|H(L_\beta)-H(L_\alpha)|}\,, \\
\lambda'_{\alpha jk} &\sim \lambda^{H(L_\alpha)+H(Q_j)+H(\bar{d}_k)}\,, \\
\lambda_{\alpha\beta k} &\sim \lambda^{H(L_\alpha)+H(L_\beta)+H(\bar{\ell}_k)}\,.
\end{aligned} \tag{4.2}$$

Here $\tilde{\mu}$ is the natural scale for the μ terms. We assume that $\tilde{\mu} = O(\tilde{m})$ and since μ_0 is phenomenologically required to be also of $O(\tilde{m})$, we take $H(H_d) = H(H_u) = 0$. Then we get

$$\frac{\mu_i}{\mu_0} \sim \frac{B_i}{B_0} \sim \frac{(M_{\tilde{L}}^2)_{0i}}{(M_{\tilde{L}}^2)_{\alpha\alpha}} \sim \frac{v\,\lambda_{ijk}}{(m_\ell)_{jk}} \sim \frac{v\,\lambda'_{ijk}}{(m_d)_{jk}} \sim \lambda^{H(L_i)}\,. \tag{4.3}$$

We see that all the RPV parameters are suppressed by a common factor compared to their corresponding RPC parameters. In particular, this implies that the RPV trilinear couplings are very small since they are related to the small RPC Yukawa couplings.

Several other parameters are expected to be of $O(1)$ in the horizontal symmetry framework: In particular,

$$\cos\beta, \qquad \epsilon_H, \qquad \epsilon_D, \tag{4.4}$$

such that ϵ_H is the suppression due to the effect of Higgs decoupling (defined in Eq. (A4)) and ϵ_D is the suppression due to sneutrino degeneracy (defined in Eq. (B6)). Yet, in the following we keep them in order to understand what parameters are needed to be fine-tuned in order to get a viable model.

Now we can estimate the order of magnitude of the different contributions to the neutrino mass matrix

$$\begin{aligned}
[m_\nu]_{ij}^{(\mu\mu)} &\sim m_0\,\cos^2\beta\,\lambda^{H(L_i)+H(L_j)}\,, \\
[m_\nu]_{ij}^{(BB)} &\sim \frac{m_0}{\cos^2\beta}\,\epsilon_L\epsilon_H\lambda^{H(L_i)+H(L_j)}\,, \\
[m_\nu]_{ij}^{(\mu B)} &\sim \frac{m_0}{\cos\beta}\,\epsilon_L\epsilon_H\lambda^{H(L_i)+H(L_j)}\,, \\
[m_\nu]_{ij}^{(\lambda'\lambda')} &\sim m_0\,\epsilon_L\left(\frac{m_b}{v}\right)^4\lambda^{H(L_i)+H(L_j)}\,, \\
[m_\nu]_{ij}^{(\mu\lambda')} &\sim m_0\,\epsilon_L\left(\frac{m_b}{v}\right)^2\lambda^{H(L_i)+H(L_j)}\,,
\end{aligned} \tag{4.5}$$

where $\epsilon_L \sim 10^{-2}$ is the loop suppression factor. While ϵ_H and ϵ_L are in general different for the different contributions, we expect them to be of the same order and thus we omit their identification indices. In our simple model the overall scale $m_0 \sim O(\tilde{m})$. It can be much smaller if there is a mechanism that generates a common suppression factor for all the RPV parameters.

In order to get the relative importance of the different contributions to the neutrino masses we note the following points:

- There is a common factor, $m_0\,\lambda^{H(L_i)+H(L_j)}$, to all the contributions.
- Within the horizontal symmetry framework $[m_\nu]_{ij}^{(BB)} \sim [m_\nu]_{ij}^{(\mu B)}$. As explained above, as long as the tree contribution is dominant, this implies that the effect of $[m_\nu]_{ij}^{(\mu B)}$ on the neutrino masses is negligible.

- Due to the extra Yukawa suppressions $[m_\nu]_{ij}^{(\mu\lambda')} > [m_\nu]_{ij}^{(\lambda'\lambda')}$. However, $[m_\nu]_{ij}^{(\mu\lambda')}$ contributes to the neutrino mass at second order in the suppression factors. Using Eq. (4.5) we get the ratio of these two contributions to the lightest neutrino mass

$$\frac{m_1^{(\mu\lambda')}}{m_1^{(\lambda'\lambda')}} \sim \frac{\epsilon_L}{\cos^2\beta}\,. \tag{4.6}$$

We conclude that unless $\cos\beta$ is very small, the contribution of the $\lambda'\lambda'$ loops to the neutrino masses is more important than that of the $\mu\lambda'$ ones.

We see that there are three possible important contributions to the neutrino masses, $[m_\nu]_{ij}^{(\mu\mu)}$, $[m_\nu]_{ij}^{(BB)}$ and $[m_\nu]_{ij}^{(\lambda'\lambda')}$. Their relative effects are controlled by $\cos\beta$, ϵ_H, and ϵ_L, see Eq. (4.5). We assume that $\cos\beta$, ϵ_H and ϵ_D are not very small. Then, the $\lambda'\lambda'$ loops can be neglected and we get the order of magnitude of the neutrino masses as

$$\begin{aligned} m_3 &\sim m_0\cos^2\beta\,\lambda^{2H(L_3)}, \\ m_2 &\sim \frac{m_0}{\cos^2\beta}\,\epsilon_L\epsilon_H\lambda^{2H(L_2)}, \\ m_1 &\sim \frac{m_0}{\cos^2\beta}\,\epsilon_L\epsilon_H\epsilon_D\lambda^{2H(L_1)}. \end{aligned} \tag{4.7}$$

If ϵ_D is very small then the lightest neutrino mass is dominated by the $\lambda'\lambda'$ loop contribution,

$$m_1 \sim m_0\lambda^{2H(L_1)}\epsilon_L\left(\frac{m_b}{v}\right)^4\,. \tag{4.8}$$

Next we check whether the neutrino data can be explained in our model. The mixing angles are given by [22]

$$\sin\theta_{ij} \sim \lambda^{|H(L_i)-H(L_j)|}. \tag{4.9}$$

The requirement that θ_{23} and θ_{12} are large [2] implies that

$$H(L_3) = H(L_2) = H(L_1). \tag{4.10}$$

A potential problem is that this choice of horizontal charges also predicts large θ_{13}. In order to generate a viable neutrino mass spectrum we require that $m_3 \sim 10^{-1}$ eV and $m_2 \sim 10^{-2}$ eV. This is the case when

$$m_0\lambda^{2H(L_3)}\cos^2\beta \sim 10^{-1}\text{ eV}\,, \tag{4.11}$$

and

$$\frac{\cos^4\beta}{\epsilon_H} \sim 10^{-1}, \tag{4.12}$$

where we used $\epsilon_L \sim 10^{-2}$.

The requirement in Eq. (4.11) can be met once appropriate input parameters (or charges) are chosen. The ratio in (4.12) as well as the requirement of small θ_{13}, however, required mild fine-tuning. Within our model both are predicted to be of order $O(1)$ while the data suggest that they are $O(10^{-1})$.

Here we consider only a simple model base on a $U(1)_H$ symmetry. In more elaborated models, like those with more complicated symmetry group, e.g. $U(1) \times U(1)$ or discrete symmetry group [23], one may be able to achieve a viable model with less fine-tuning.

V. CONCLUSIONS

RPV supersymmetric models provide an alternative to the see-saw mechanism. One virtue of RPV models is that they naturally provide a mechanism for large mixing with hierarchy, as indicated by the data.

We study the magnitudes of various sources of neutrino masses in RPV models. There are several parameters that determine them and therefore there are several suppression factors in each of them. Thus, their relative importance is model dependent. Due to the Yukawa suppression of the trilinear loops, it is generally likely that the tree level and the BB loops are the dominant contributions.

We study one specific model with an Abelian horizontal symmetry. In this model indeed the tree level and the BB one-loop contributions are the dominant ones. We find that such a model can describe the neutrino data as long as mild fine-tuning is permitted.

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APPENDIX A: HIGGS CANCELLATION IN THE BB LOOPS

Here we study the cancellation between the three BB loops of Fig 3.6. The weighted sum of the three Higgs propagators, before integrating over the internal momenta k, is

$$P_S = \frac{1}{k^2 - m_h^2} \cos^2(\alpha - \beta) + \frac{1}{k^2 - m_H^2} \sin^2(\alpha - \beta) - \frac{1}{k^2 - m_A^2}. \tag{A1}$$

For simplicity we use the tree level relations [24, 25]

$$\cos^2(\alpha - \beta) = \frac{m_h^2(m_Z^2 - m_h^2)}{m_A^2(m_H^2 - m_h^2)}, \qquad m_Z^2 - m_h^2 = m_H^2 - m_A^2, \tag{A2}$$

$\tan\beta$	m_h	m_A	m_H	$\epsilon_H \times 10^3$
4	92	184	190	9.1
2	81	426	430	4.7
20	106	285	284	1.1
14	106	294	294	0.3

TABLE I: Numerical values of the suppression factor due to the cancellation between different Higgs contributions in the BB loops. We used $m_{\tilde{\nu}_1} = 100$ GeV, $m_{\tilde{\nu}_2} = 200$ GeV and $m_{\chi_\alpha} = 300$ GeV.

and we obtain

$$P_S = \frac{-k^2(m_Z^2 - m_h^2)(m_A^2 - m_h^2)}{m_A^2(k^2 - m_H^2)(k^2 - m_A^2)(k^2 - m_h^2)}. \tag{A3}$$

Consider the decoupling limit where $m_H \sim m_A \gg m_h \sim m_Z$. In that limit the H and A propagators scale like one over their heavy mass squared. From Eq. (A3) we see that the weighted sum scales like one over the heavy mass to the fourth power.

While the partial cancellation is more severe in the decoupling limit, it also occurs far away from that limit. The reason is that the I_4 function, defined in Eq. (3.7), is not very sensitive to variation in one of its arguments as long as it is not the largest one.

We have checked the effect of the summation over the different Higgs mediated diagrams numerically. We define the following measure of the suppression factor

$$\epsilon_H \equiv \left| \frac{I(m_h)\cos^2(\alpha-\beta) + I(m_H)\sin^2(\alpha-\beta) - I(m_A)}{|I(m_h)|\cos^2(\alpha-\beta) + |I(m_H)|\sin^2(\alpha-\beta) + |I(m_A)|} \right| , \tag{A4}$$

where $I(x) \equiv I_4(x, m_{\tilde{\nu}_i}, m_{\tilde{\nu}_j}, m_{\chi_\alpha})$ [see Eq. (3.6)], and the i, j, α indices of ϵ_H are implicit. While the tree level relation is a good approximation of the effect, in the numerical calculation we use the two-loop spectrum for the Higgs boson masses and mixing angles [26]. Some representative numbers are presented in Table I.

We also checked the effect of $\tan\beta$. We found that ϵ_H decreases as $\tan\beta$ increases. Thus, the sensitivity of $[m_\nu]_{ij}^{(BB)}$ to $\tan\beta$ is reduced. On one hand it scales like $1/\cos^2\beta$ [see Eq. (3.6)], and on the other hand the cancellation between the different Higgs loops becomes stronger for large $\tan\beta$. In fact, using the tree level Higgs mass relations, we found that asymptotically $\epsilon_H \propto \cos^2\beta$. Thus, at the tree level in the $\tan\beta \to \infty$ limit, $[m_\nu]_{ij}^{(BB)}$ is independent of $\tan\beta$.

APPENDIX B: THE SUPPRESSION DUE TO SNEUTRINO DEGENERACY

Here we study the effect of the sneutrino degeneracy on the light mass eigenstate from the BB loops. We assume that the heaviest neutrino acquires large mass at the tree level. Then, for simplicity, we deal only with the loop contribution to the first two generations.

We define the mass-squares of the two sneutrinos as

$$(m^2_{\tilde\nu})_{1,2} \equiv m^2_{\tilde\nu}(1 \pm \Delta). \tag{B1}$$

Computing the BB one-loop contributions up to order Δ^2, we get a mass matrix of the following form:

$$f_1 \begin{pmatrix} B_1B_1 & B_1B_2 \\ B_2B_1 & B_2B_2 \end{pmatrix} + \Delta f_2 \begin{pmatrix} B_1B_1 & 0 \\ 0 & -B_2B_2 \end{pmatrix} + \Delta^2 f_3 \begin{pmatrix} 3B_1B_1 & B_1B_2 \\ B_2B_1 & 3B_2B_2 \end{pmatrix} + \mathcal{O}(\Delta^3) \tag{B2}$$

where

$$f_1 = m_{Deg}|_{\Delta\to 0}\,, \qquad f_2 = \left.\frac{\partial m_{Deg}}{\partial \Delta}\right|_{\Delta\to 0}, \qquad f_3 = \frac{1}{2}\left.\frac{\partial^2 m_{Deg}}{\partial \Delta^2}\right|_{\Delta\to 0}. \tag{B3}$$

and

$$\begin{aligned} m_{Deg}(\Delta) = &\sum_\alpha g^2 \frac{1}{4\cos^2\beta}(Z_{\alpha 2} - Z_{\alpha 1}g'/g)^2 m_{\chi\alpha}\Big[I_4(m_h, m, m, m_{\chi_\alpha})\cos^2(\alpha-\beta) \\ &+ I_4(m_H, m, m, m_{\chi_\alpha})\sin^2(\alpha-\beta) - I_4(m_A, m, m, m_{\chi_\alpha})\Big], \end{aligned} \tag{B4}$$

where $m^2 = m^2_{\tilde\nu}(1+\Delta)$. After diagonalization, we get the following masses:

$$\begin{aligned} m_2 &= (B_1^2 + B_2^2)\, f_1 + \mathcal{O}(\Delta), \\ m_1 &= \frac{B_1^2 B_2^2}{B_1^2 + B_2^2}(4f_1f_3 - f_2^2)\Delta^2 + \mathcal{O}(\Delta^3). \end{aligned} \tag{B5}$$

We see that the dominant contribution to m_2 is the same as that in the degenerate case. The leading contribution to m_1, on the other hand, is proportional to the square of the sneutrino mass splitting.

We define the following measure of the degeneracy suppression

$$\epsilon_D \equiv \frac{m_1}{m_2}, \tag{B6}$$

which is given by

$$\epsilon_D = f_c \frac{B_1^2 B_2^2}{(B_1^2 + B_2^2)^2}\Delta^2, \qquad f_c = \frac{4f_1f_3 - f_2^2}{f_1}. \tag{B7}$$

We have checked numerically and found that typically $f_c \sim 0.1$. Thus, in addition to the Δ^2 suppression, the lightest neutrino mass is also suppressed by f_c.

APPENDIX C: μ_i DEPENDENT ONE-LOOP CONTRIBUTIONS

Here we explain why when the tree level is the dominant contribution to the neutrino mass matrix, the effect of loops that have one μ_i insertion are small. They appear only at second order in the ratio between the loop contribution to the mass matrix and the tree level one. This effect was also discussed in [13].

We consider a two generation case with only one type of one-loop contribution at a time. First we assume that we have the following two contributions

$$[m_\nu]_{ij}^{(\mu\mu)} = C\mu_i\mu_j, \qquad [m_\nu]_{ij}^{(V\mu)} = C\varepsilon_L(\mu_i V_j + \mu_j V_i) \tag{C1}$$

where C is a constant and V is a normalized general vector in flavor space such that $|V| = |\mu|$. For example, in the case of the μB diagram, V_i corresponds to the product of B_i with the loop function. The fact that the tree level is dominant is encoded by the choice $\varepsilon_L \ll 1$. The mass matrix is then

$$m_\nu = C \begin{pmatrix} \mu_1^2 + 2\varepsilon_L V_1\mu_1 & \mu_1\mu_2 + \varepsilon_L(V_1\mu_2 + V_2\mu_1) \\ \mu_1\mu_2 + \varepsilon_L(V_1\mu_2 + V_2\mu_1) & \mu_2^2 + 2\varepsilon_L V_2\mu_2 \end{pmatrix} \tag{C2}$$

We see that the ratio of the two mass eigenstates is

$$\frac{m_1}{m_2} \sim O\left(\varepsilon_L^2\right). \tag{C3}$$

Next consider a case where the loop effect is generated without any μ_i insertion. For example

$$[m_\nu]_{ij}^{(\mu\mu)} = C\mu_i\mu_j, \qquad [m_\nu]_{ij}^{(VU)} = C_{ij}^{VU}\varepsilon_L(V_i U_j + V_j U_i), \tag{C4}$$

where U is another normalized vector and $C \sim C_{ij}^{VU}$ for any i and j. For $m_\nu = [m_\nu]^{(\mu\mu)} + [m_\nu]^{(VU)}$ we generally get

$$\frac{m_1}{m_2} \sim O(\varepsilon_L). \tag{C5}$$

Note that the above holds also for $U = V$.

Comparing Eqs. (C5) and (C3) we see that diagrams with one μ_i insertion are unlikely to affect the neutrino masses significantly.

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