An Optimal t-Δv Guidance Law for Intercepting a Boosting Target

L.C. Ng, E. Breitfeller, A.G. Ledebuhr

June 30, 2002

U.S. Department of Energy

Lawrence
Livermore
National
Laboratory

An Optimal t-Δv Guidance Law for Intercepting a Boosting Target

Lawrence C. Ng*, Eric Breitfeller[†], and Arno G. Ledebuhr[‡]

Lawrence Livermore National Laboratory

P.O. Box 808, L-278, Livermore, CA 94551

Abstract

We at Lawrence Livermore National Laboratory (LLNL) have developed a new missile guidance law for intercepting a missile during boost phase. Unlike other known missile guidance laws being used today, the new t-Δv guidance law optimally trades an interceptor's onboard fuel capacity against time-to-go before impact. In particular, this guidance law allows a missile designer to program the interceptor to maximally impact a boosting missile before burnout or burn termination and thus negating its ability to achieve the maximum kinetic velocity. For an intercontinental range ballistic missile (ICBM), it can be shown that for every second of earlier intercept prior to burnout, the ICBM ground range is reduced by 350 km. Therefore, intercepting a mere 15 seconds earlier would result in a miss of 5,250 km from the intended target or approximately a distance across the continental United States. This paper also shows how the t-Δv guidance law can incorporate uncertainties in target burnout time, predicted intercept point (PIP) error, time-to-go error, and other track estimation errors.

We believe that the t-Δv guidance law is a step toward the development of a new and smart missile guidance law that would enhance the probability of achieving a boost phase intercept

Introduction

This paper describes a new missile guidance law designed specifically for a boost phase intercept mission. It takes maximum advantage of a kinetic kill vehicle (KV) capable of thrust-on-demand, axial/lateral divert, propulsion system such as the Advanced Technology Kill Vehicle (ATKV) currently under exploratory development at LLNL. For reasons to be explained later, this new guidance law is called the optimal t-Δv guidance law. The key benefits of this guidance law are: (1) the KV would attempt to intercept a boosting target as early as possible in its boost phase, and (2) the KV would also attempt to minimize the total Δv (propellant)

consumption throughout the engagement. In short, the t-Δv guidance law is designed to maximize the time before target burnout and minimize the overall propellant usage.

The key advantage of this new guidance law is to increase the probability and effectiveness of boost phase intercept. The t-Δv guidance law, when applied to the boost phase intercept mission, accomplishes this by constantly choosing a vehicle acceleration command to achieve a compromise between earliest intercept and minimum Δv expenditure.

The t-Δv guidance is significantly different from the traditionally well known guidance laws such a Augmented Proportional Navigation (APNAV) and Zero-effort-miss (ZEM) [1]. The key difference, of course, is that neither APNAV nor ZEM has specifically taken the need to intercept the target before burnout into account.

The significance of intercepting early in boost phase can not be overemphasized. For example, intercepting a boosting target just a moment before burnout does very little to alter the throw weight velocity and therefore its impact point. On the other hand, an early intercept (10-15s before burnout) will significantly shorten the impact point (by 3500-5250 km) from its intended target location.

Statement of the Problem

Assuming the availability of a lightweight, high mass fraction kill vehicle, such as the ATKV with its flexible, thrust-on-demand, axial/divert and ACS (ADACS) propulsion system, we want to explore the relative advantages of such a system and in particularly how a missile guidance law might take advantage of this new capability to improve the overall system performance such as longer standoff range and greater intercept battle space.

Fig. 1 illustrates a possible boost/ascent phase intercept (BPI) scenario and the potential advantage of ADACS. Using the Navy Standard Missile as an example, the kill vehicle is sitting on top of a three stage booster stack. Each stage uses a solid propellant engine with the third stage capable of firing two separate pulses. In Fig. 1 we assumed that each pulse

is preceded by an IFTU for appropriate course correction. After the 3rd stage burn, the kill vehicle's axial velocity is fixed. For our example, we assumed a burnout velocity of 4.5 km/s, a burnout time of 80s, and a burnout altitude of about 100 km. The kill vehicle then coasts until target acquisition. A target is successfully acquired if it appears within the narrow field of view of the KV seeker which has been pointing toward the PIP. The endgame, lasting approximately 5 to 10 seconds, allows the KV to home to a target using it's divert engine. Note in the figure the KV reachability envelope (in altitude versus ground range) is described by a set of flyout curves marked by a constant time profile (in minutes) and at every 2° pitch over angle.

The ICBM target, launched at 1200 km downrange, is also shown with stage burnout time marked in minutes in the trajectory profile. We assumed intercept occurs at the 4 minute mark, just prior to deployment of the re-entry vehicle and decoys. Now comparing the the flyout time for the interceptor (3 minutes) and the target (4 minutes), one can deduce that there is less than one minute of launch delay. The intercept basket is defined by the blue cone. On the other hand, a KV with an ADACS such as the ATKV, the intercept basket is significantly larger. With a longer range and wider field of view acquisition sensor, the ATKV can burn axially to effectively increase the burnout velocity, resulting in a larger reachability basket. Thus the ATKV can reach the target before burnout at the 3 minute mark with about 30s of launch delay. We assumed that the ATKV can add more than 2 km/s of axial velocity or an equivalent V_{bo} of 6.5 km/s.
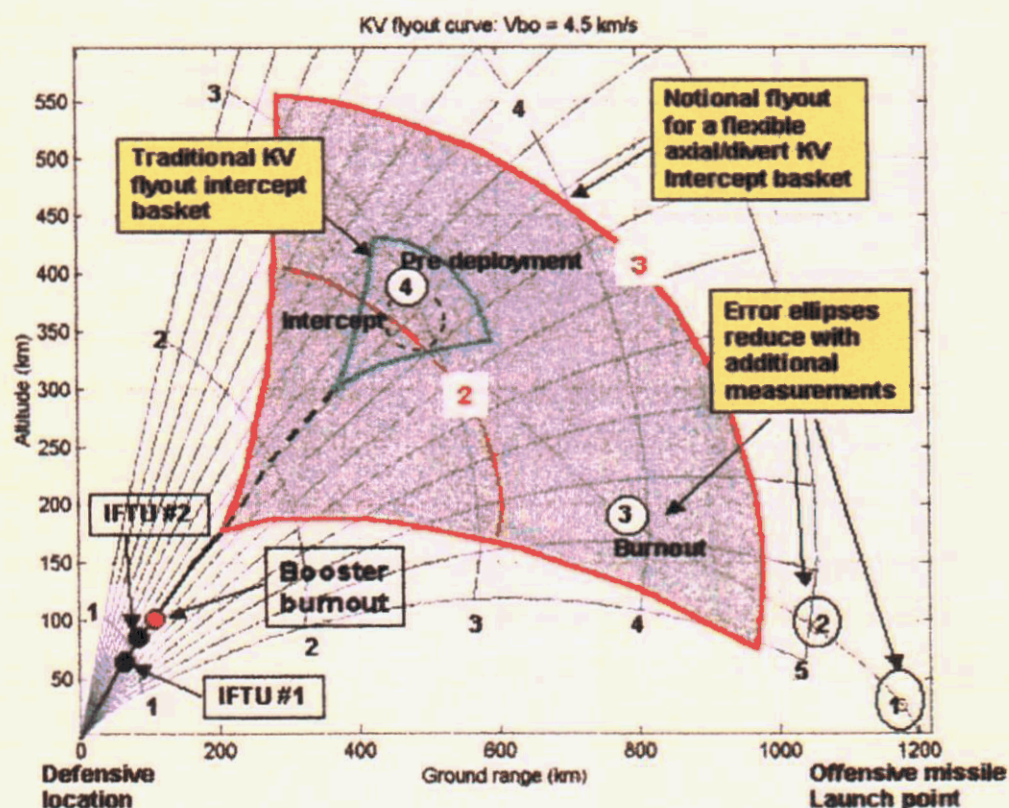
Increasing Vbo for Intercept Depth

For a successful BPI mission, it is important to intercept prior to target burnout. In fact it is advantageous to do so, as mentioned previously, since for every second of cut off before burnout, the target range reduces by approximately 350 km for an ICBM class (~10,000km range) missile [2]. We are interested in exploring the acceleration and burnout velocity (V_{bo}) requirement for intercepting the target at earlier times in flight. Using a simple kinematic model and ignoring the atmospheric drag, Fig. 2 shows potential intercept points against a hypothetical threat launched 1000km downrange that has approximately a 200s burnout time at an altitude of 250 km. We assumed an ideal interceptor missile has a 60s launch delay, a burnout time of 60s but with variable acceleration capability. It can be seen that the V_{bo} increases rapidly with increasing intercept depth (earlier boost phase). The increasing V_{bo} is a result of two factors: increasing intercept range and decreasing flight time. Since a typical booster provides a fixed V_{bo}, the ADACS as envisioned by the ATKV can provide the desired variable axial Δv capability.

Increasing Δv for Target Maneuvers

One of the well known and effective countermeasures against a BPI interceptor is target maneuvering. A maneuvering target degrades the track estimate resulting in a significant increase in PIP error. This translates into a larger divert Δv and greater acceleration requirements for the KV for a given miss distance specification.

Since increasing intercept depth (more axial Δv) and overcoming potential target maneuvers (more divert Δv) are both competing for greater fuel usage and that the Δv can not be pre-determined a priori (i.e., we do not know whether and when the target would maneuver), a proper balance between these two competing factors is needed. A KV with an ADACS is capable of achieving this balance. What is needed is the development of a guidance law which optimally determines the proper fuel expenditure between axial and divert guidance modes in real time for a given BPI scenario.
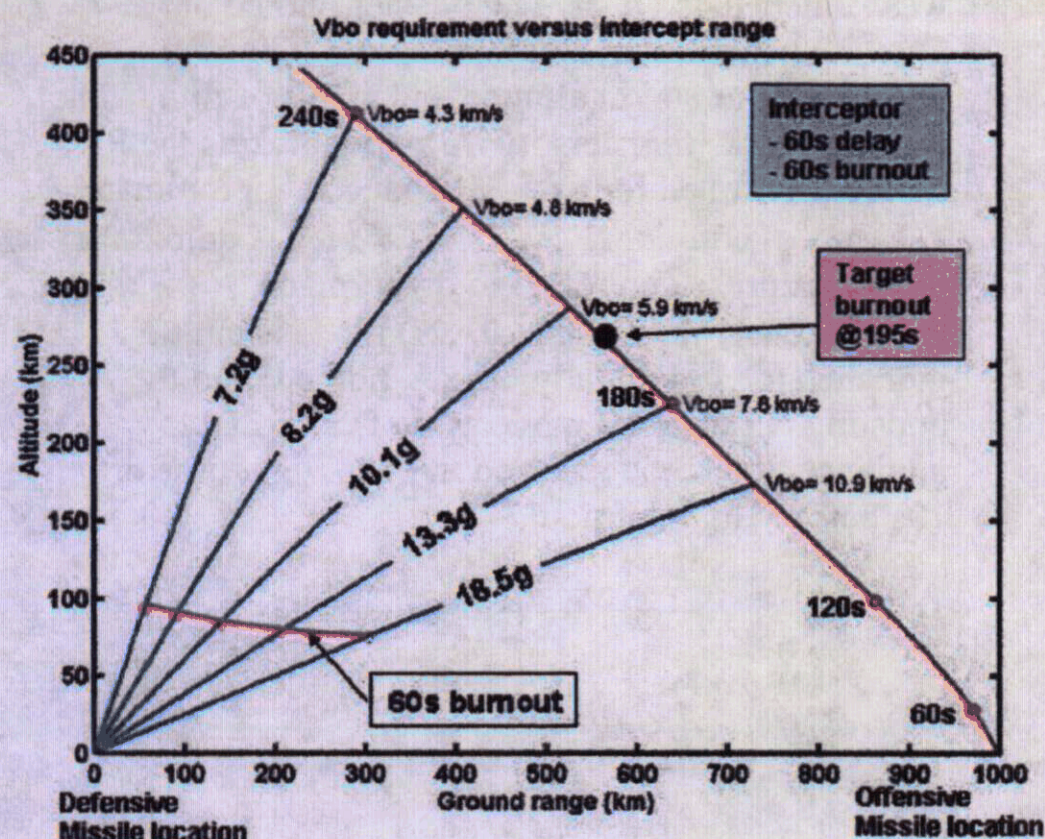

Fig. 1 A flexible axial/divert propulsion system expands the intercept battle space.

Fig. 2 Increasing Δv requirement for earlier intercept time (model neglects aerodynamic drag)

Formulation of the t-Δv Guidance Law

To help visualize how a KV can have flexible axial and divert capability, Fig. 3 shows the ATKV concept vehicle currently under exploratory research and development at LLNL. Utilizing a lightweight pumped propulsion approach [3], the ATKV has a mass budget of 30kg, a total Δv of 2.5 km/s distributable flexibly between axial and divert mode via a swivel thruster design. A fixed axial thruster approach can also be considered. The optimum choice will depend on in process performance trades with vehicle mass, size, volume and mechanical reliability. For the swivel thruster approach there are three operational modes: 4A, 2A2D, and 4D that denotes 4 axial thrusters operating, 2 axial and 2 divert thrusters operating, and 4 divert thrusters capable of thrusting respectively. The ATKV also has an Integrated Multi-spectral NFOV Seeker and two WFOV sensors for long range plume acquisition and tracking.

ATKV Guidance, Navigation and Control

The ATKV employs an integrated guidance and control strategy in which the KV guides the missile from launch to intercept, utilizing as many (or as few) IFTUs as available and onboard sensor collected information. Using a multi-aperture approach, the WFOV sensors allow the ATKV to operate autonomously with early target acquisition and KV guidance. The key strategies are summarized in Fig. 4.
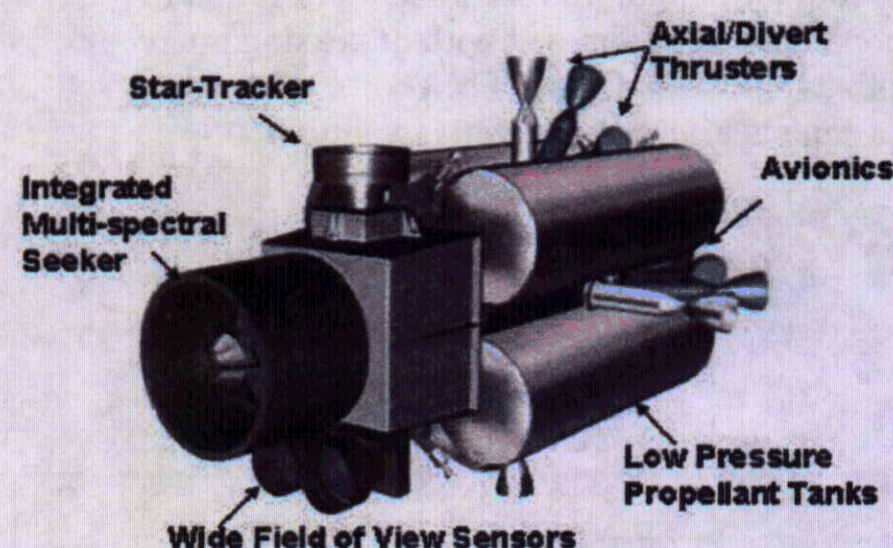

Fig. 3 LLNL's ATKV concept vehicle

Now suppose the KV is heading toward a PIP corresponding to the target burnout location and we are interested in switching the PIP to earlier times, say at points A, B, or C. Point A may be too close to booster burnout and therefore has minimal impact on target burnout velocity. Point C has maximum intercept depth but may be in danger of running out of fuel. Then point B may be the desired point which minimizes the Δv consumption and simultaneously maximizes the time before booster burnout (tbbo).

Since the proposed guidance approach optimizes over both time and Δv space, we have accordingly named it the t-Δv guidance law.
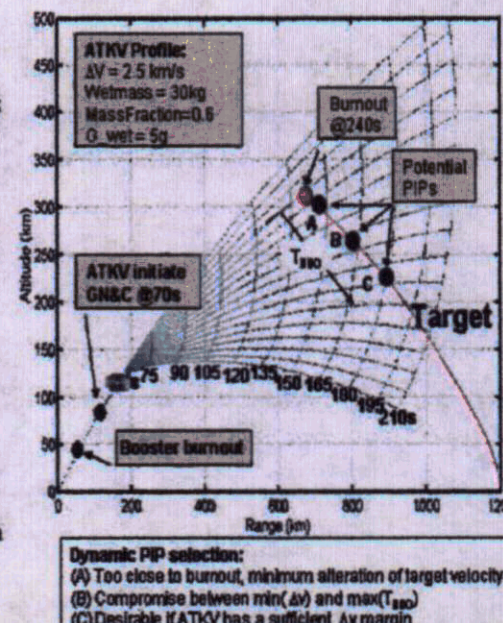

Fig. 4 Basic t-Δv guidance and control strategy is designed to intercept before target boost phase burnout

Derivation of the t-Δv Guidance Law

Defining tbbo as time before target burnout (assuming we have perfect knowledge of it for now) to a desired PIP, and Δv is the corresponding fuel usage to reach it, then one can form a weighted cost

function as shown in Eq. (1) and seek its minimum. The first term is minimized at zero effort and the second term is minimized with increasing intercept depth as shown in Fig. 5. The choice of the weighted coefficients affects the optimal solution.

$$J\left(a_{axial},\ a_{divert}\right) = \alpha\ \Delta v^2 + \frac{\beta}{t_{bbo}^2} \qquad (1)$$

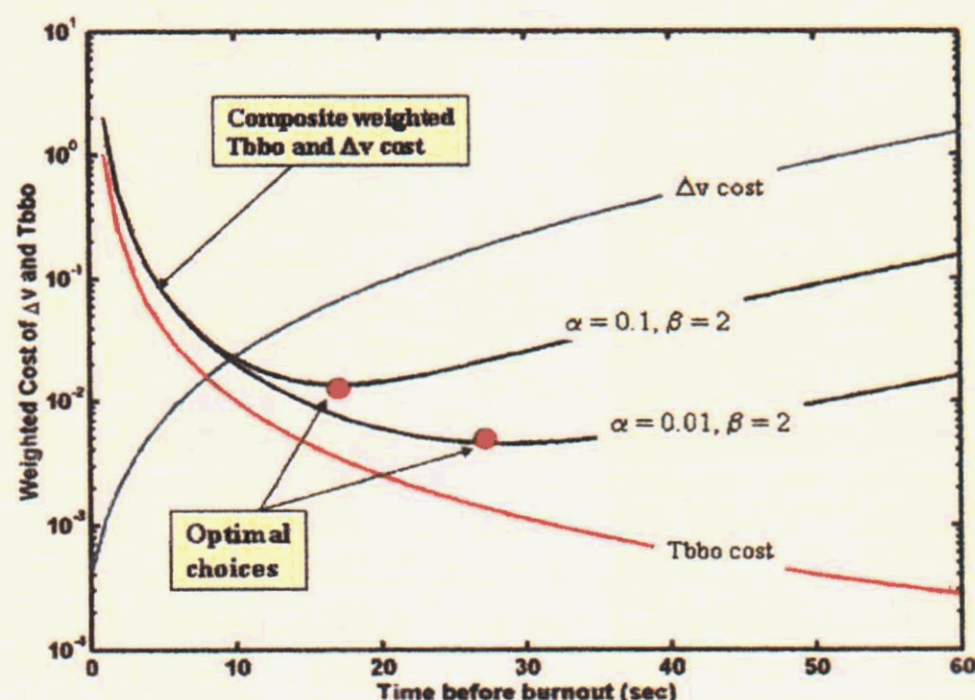**Fig. 5 Deterministic mathematical formulation of t-Δv guidance law showing optimal t_{bbo} choices for different set of coefficients**

Let Δv be the optimal solution, and let tgo be the time-to-go computed using Eq. (5), the acceleration vector command is then given by:

$$a = \frac{ZEM}{t_{go}^2} \qquad (2)$$

where ZEM, the zero effort miss vector, is related to Δv as:

$$\Delta v = \frac{|ZEM|}{t_{go}} \qquad (3)$$

6DoF Simulation of a Sea-based BPI Mission using the t-Δv Guidance Law

In order to demonstrate the applicability of the t-Δv guidance law, we conducted 6DoF BPI simulation studies. Example of a successful intercept scenario is shown in Fig. 6. The target is an ICBM class missile launched at 950km down range with a burnout time of 195s and a burnout altitude of 270 km. An ATKV

was launched 45s later. The booster burned out at 48 km with a burnout velocity of 5 km/s. The KV unshrouded at 80 km altitude and at 70s of flight. The KV aligned its thrusters in 4A mode towards the PIP within 2s, burned for 6s, and produced approximately 1.2 km/s of additional Δv. The KV then acquired the target within 2s and rolled to divert plane, autonomously thrusting in 2A2D mode using t-Δv guidance for 10s. Finally the KV homed onto the boosting target in 4D mode using 50s of ZEM guidance. Intercept occurred in 140s flight time or 10s before target burnout.
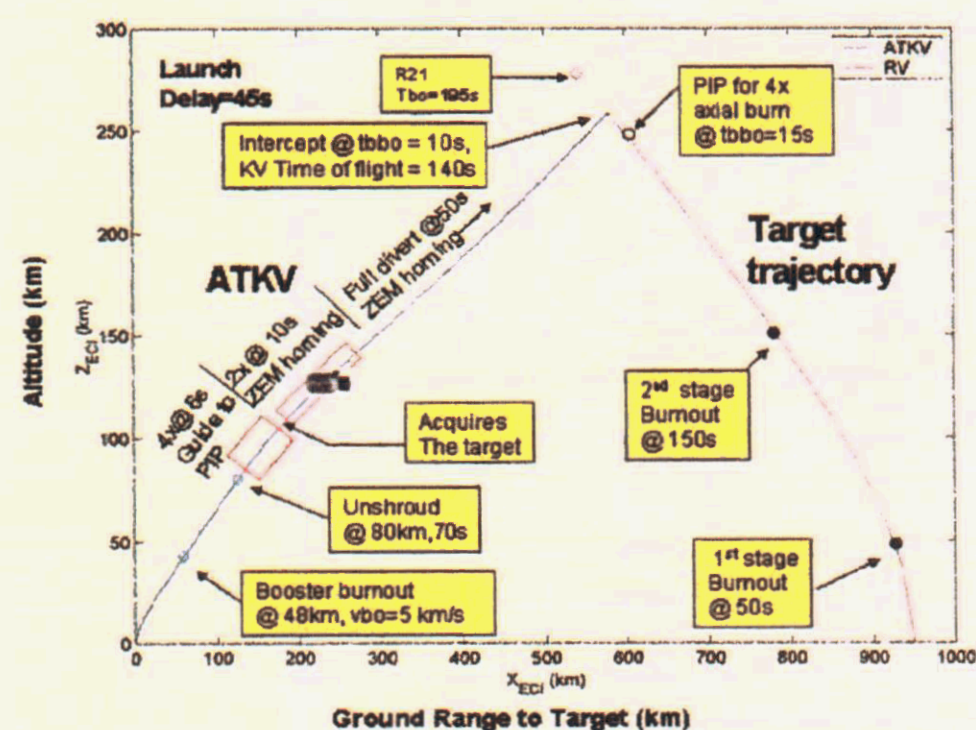

Fig. 6 Example of 6 DoF simulation of intercept time history and ATKV flyout engagement strategy

Fig. 7 shows the out of plane ZEM and acceleration time history. Note that the axial accelerations, shown in red, are used to extend the intercept range.
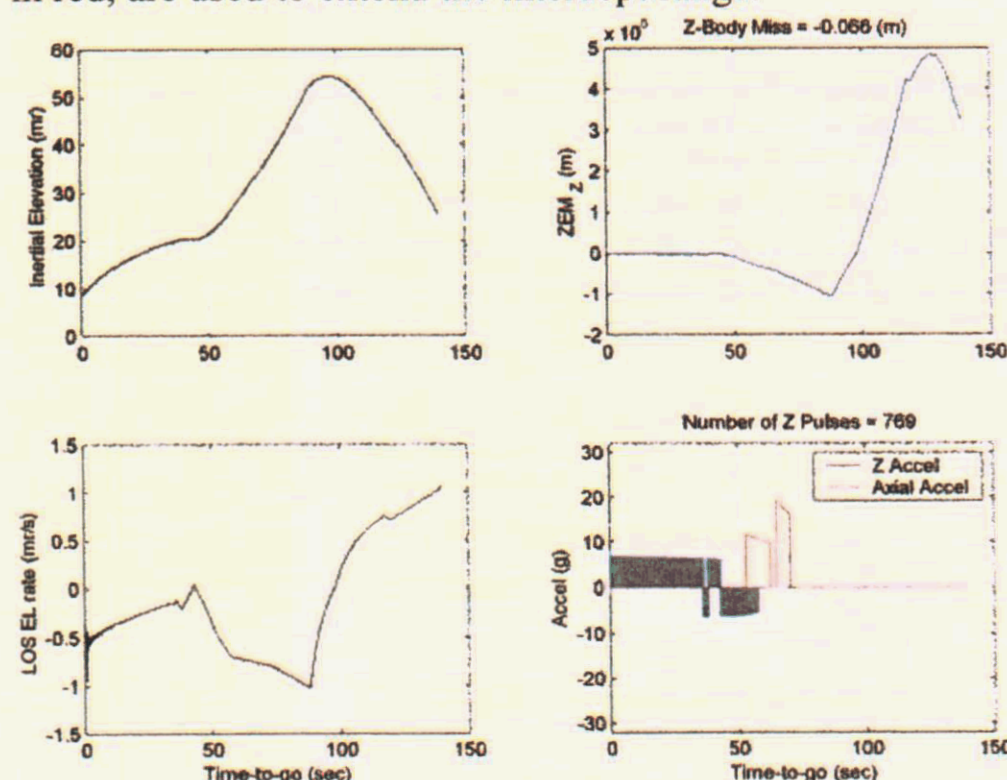

Fig. 7 In-plane axial and divert actions showing zero effort miss distance being driven to near zero.

Also note the simultaneous operation of thrusting in the 2 axial and 2 divert mode.

Performance Comparison of ZEM and t-Δv Guidance

For the same BPI scenario, we compare three different guidance strategies after KV has been unshrouded: (1) guide with ZEM only (no axial thrusting), (2) guide with 6s in 4A mode, 30s in 2A2D ZEM mode, and 30s in 4D ZEM mode; and (3) 6s of 4A mode, 30s in 2A2D t-Δv mode, follow with 30s in 4D ZEM mode.

Fig. 8 shows the simulation results assuming the intercept was launched within 30s of the target launch. The PIP was chosen to coincide with the target burnout position. We observed that the ZEM guidance intercepted the target at 1.5s after target burnout but used only 1.4 km/s of Δv. Since the KV has a total Δv of 2.5 km/s, 1.1 km/s of Δv are unspent. This brings out one of the drawbacks of ZEM, it achieves intercept by nulling the ZEM without taking into account the KV fuel reserve and the desire to intercept before target burnout. Since it intercepts at a later time, the Δv requirement is less as shown in Fig. 2.

Next let's examine case 2. Since the axial Δv is increased by 1.2 km/s, the ZEM guidance intercepted at 7.5s before target burnout and spent 1.5 km/s. Thus at intercept the KV still has 1.0 km/s in reserve. We can draw two conclusions: (1) increasing axial Δv will increase the intercept depth, and (2) ZEM guidance does not take into account KV fuel reserve and the depth of intercept. It just happens in this case that the intercept occurred before target burnout.

Finally let's examine case 3, which differs from case 2 only in the 30s 2A2D mode, where t-Δv guidance was used. The t-Δv guidance law continuously adding axial acceleration to increase the depth of intercept by taking into account of the reserve Δv available and the earliest intercept or minimum time-to-go. It intercepted at 13s before target burnout and spent 2.24 km/s of Δv or a reserve of just 0.26 km/s.

One can not overestimate the advantages for an earlier intercept. Not only it does it reduce the impact range at a rate of about 350 km per each second of intercept before burnout, it also makes the intercept problem easier because the closing velocity is smaller. A typical ICBM target gains 15% of its final burnout velocity in the last 10 seconds.
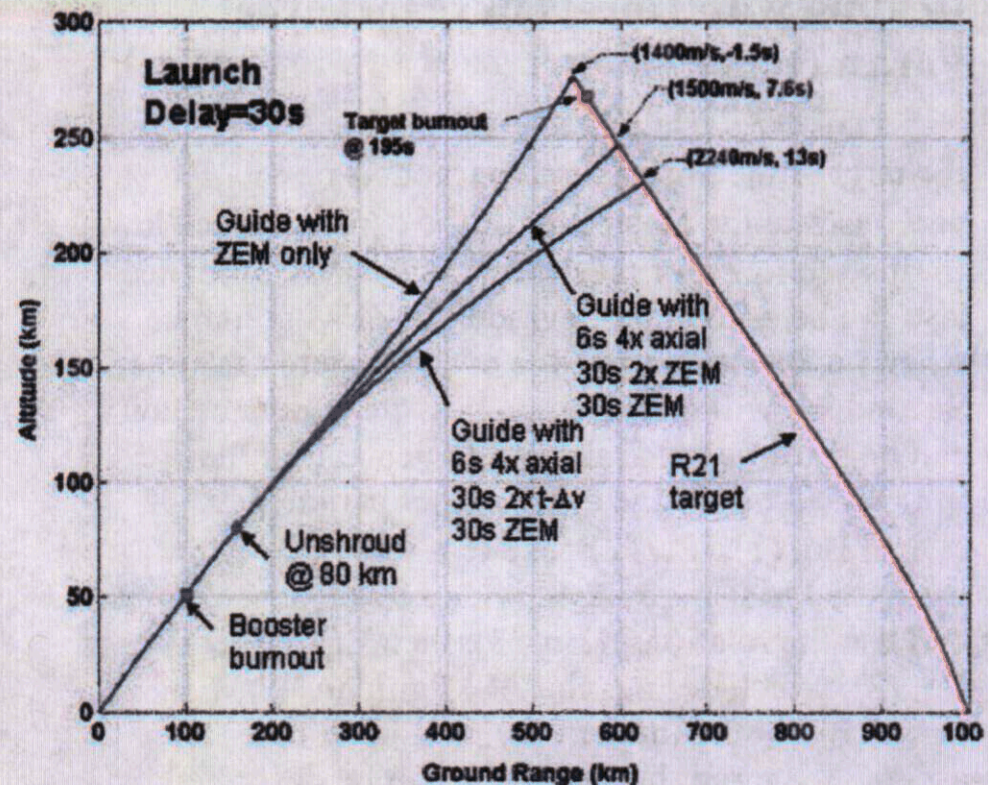
Fig. 8 Simulation results showing how t-Δv guidance law is able to intercept 13 s prior to booster burnout, while ZEM intercepted later than the burnout

To further gain a better understanding of the t-Δv guidance law, let's compare the well known missile guidance laws and examine their design criteria. Referring to Fig. 9, we see that proportional navigation (PNAV) achieves intercept by nulling the line-of-sight (LOS) rate as illustrated in the figure. Augmented proportional navigation (APNAV), on the other hand, nulls the combined LOS rate and the estimated target acceleration across the LOS angle. ZEM reduces the predicted miss distance to zero at the estimated tgo. Lambert guidance determines the velocity that solves the hit equation with tgo as a free parameter. Finally the t-Δv guidance law optimizes over time before target burnout and the onboard reserve Δv, it maximizes the probability of a boost phase intercept.

Fig. 9 Comparisons of different guidance laws and their optimization criteria.

Dealing with Uncertainty in Target Parameters

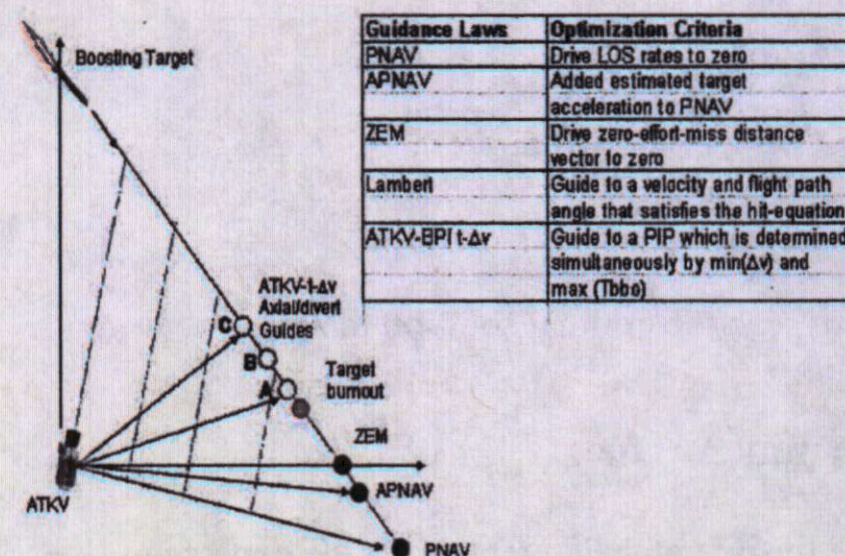Thus far we have assumed perfect knowledge of the target trajectory and its burnout time to demonstrate the feasibility of the t-Δv guidance law. In this section we first show that the t-Δv guidance law can be gracefully degraded to ZEM guidance when no apriori knowledge of the target is assumed. Second we will demonstrate how the guidance law can be reformulated taking into account of the imprecise knowledge of the target parameters.

In Eq. (1), if we choose $\alpha = 1/zem$, $\beta=0$, compute tgo using range divided by closing velocity and use the relation in Eq. (3), then Eq. (1) reduces to Eq. (2), which defines the ZEM guidance law.

Now for the uncertainty parameter case, let's assume the target burnout time can be described by a probability density function (pdf), say Guassian with prescribed mean and standard deviation. Therefore for a given tbbo (a deterministic parameter), the location of the target is simply given by the same pdf but shifted down by tbbo seconds as shown in Fig. 10. Now suppose at time t, the interceptor and the target are located at positions shown in Fig. 10. For a given tgo, one can compute the predicted position of both the interceptor and the target with the appropriate uncertainty ellipses. The predicted miss distance vector (or zero effort miss) Pi(tgo)-Pt(tgo) is also random with known statistics. Therefore we can rewrite Eq. (1) as:

$$J(a) = \alpha\left\{\frac{|P_i(tgo) - P_t(tgo)|^2}{tgo^2}\right\} + \frac{\beta}{t_{bbo}^2} \quad (4)$$

Where

$$t_{go} = t_{bo} - t_{bbo} - t \quad (5)$$

$$t_{bo} = Gauss(\bar{t}_{bo}, \sigma_{bo}^2)$$

and we seek the optimal solution that derives from

$$a_{opt}^* = min(\Delta v)\, max(t_{bbo})\left\{E[J(a)]\right\} \quad (6)$$

where the operator E[ ] represents the expected value. We are seeking a solution for the acceleration command that yields the minimum value of Δv usage for a maximum tbbo or intercept depth.

Since the cost function involves the square of the ratio of two random variables, the resulting probability density function can be shown to be related to the Cauchy distribution [4]. Once the pdf of J(a) is found, one can proceed to carry out the minimization as shown in Eq. (6). The resulting pdf is quite complex. A much simpler and useful approximation can be found as follows:
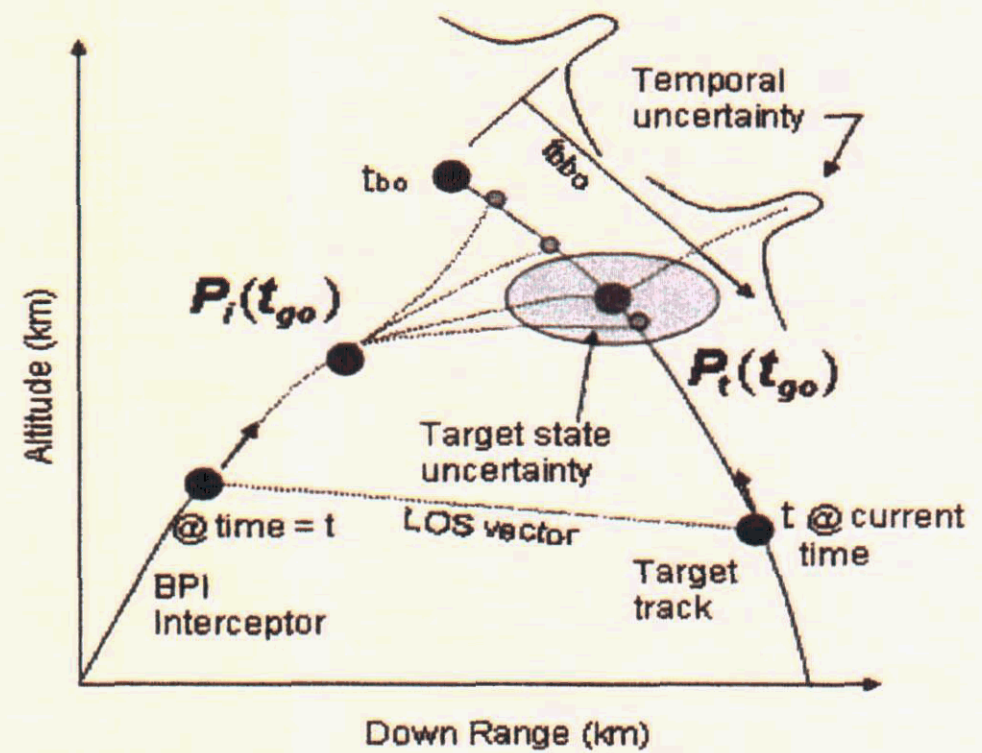

Fig. 10 Probabilistic formulation of the t-Δv guidance law

Let random variables X and Y represent the predicted miss distance and tgo respectively, we can then rewrite Eq. (4) as:

$$J(a) \cong \alpha\left(\frac{X}{Y}\right)^2 + \frac{\beta}{t_{bbo}^2}$$

$$\cong \alpha\left(\frac{\bar{X}}{\bar{Y}}\right)^2\left(\frac{1+x}{1+y}\right)^2 + \frac{\beta}{t_{bbo}^2} \quad (7)$$

$$\cong \alpha\,\Delta V^2\,(1+x)^2\left(1-2y+3y^2-...\right) + \frac{\beta}{t_{bbo}^2}$$

where $\bar{X}, \bar{Y}$ are the means and x,y are the corresponding zero mean random variables. Note that both x and y are very much less than 1 since uncertainty in the miss or tgo is significantly less than the mean. Assuming the random variables are statistically independent, we can take the expectation of Eq. (7) and obtain the desired result in Eq. (8).

$$E[J(a)]=\alpha\left(1+\frac{\sigma_x^2}{\bar{X}^2}+\frac{3\sigma_y^2}{\bar{Y}^2}+\frac{3\sigma_x^2\sigma_y^2}{\bar{X}^2\bar{Y}^2}+...\right)\Delta V^2 + \frac{\beta}{t_{bbo}^2} \quad (8)$$

Comparing Eq. (8) to (4), we see that uncertainty in the predicted miss distance and tgo results in an

effective increased weight on the Δv term. Since Δv increases for increasing intercept depth or greater tbbo, the increased weight causes the minimum to shift to the left or to intercept at a later time. This is intuitively correct since uncertainty in target parameters and the PIP error should favor a strategy to intercept at a later time in order to conserve Δv.

The validity of this observation is borne out in Fig. 11. For a given value of α and β, the deterministic solution yields a minimum at tbbo=16s. Now adding the uncertatinty in tbo, or time of burnout, with a standard deviation of 40s, the analytical solution and the ensemble average from 50 Monte-Carlo simulation runs yields the same optimal tbbo value. Using Eq. (8) instead of (4) significantly reduces the computational load for the implementation of the t-Δv guidance law.
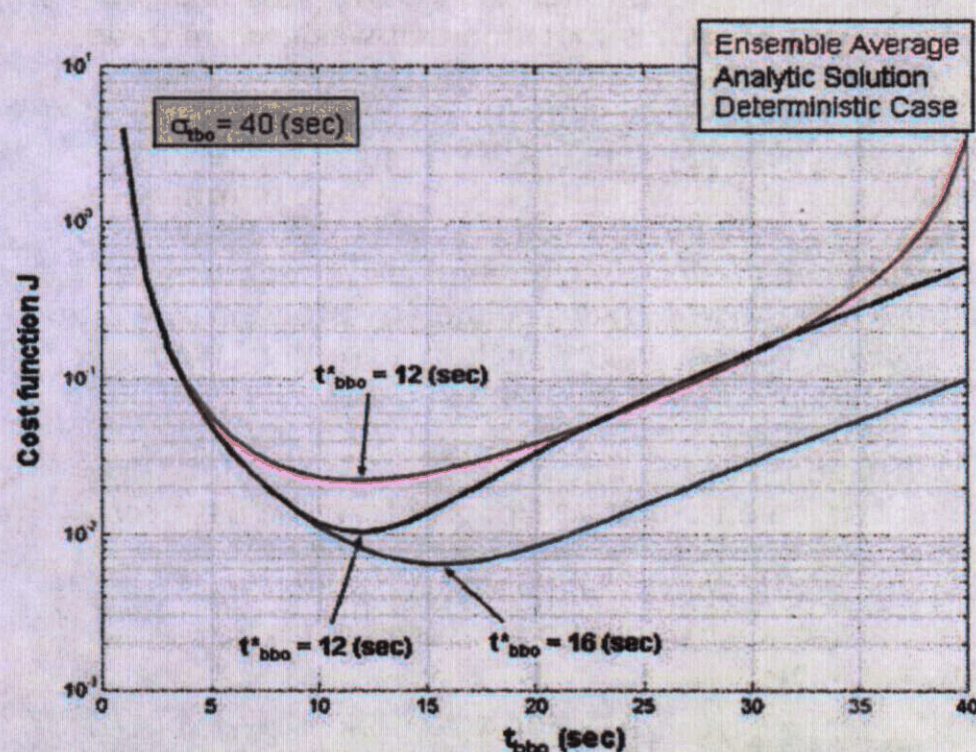

Fig. 11 Minimum of analytical solution matches almost exactly with the Monte-Carlo ensemble average calculation

In deriving the stochastic t-Δv guidance law, we only assume a probabilistic model of the target burnout time. Without loss of generality, the target burnout time could be replaced by the desired target intercept time. For example, one could specify that the desired target intercept time is say 100s after missile launch with a standard deviation of 20s. Thus the t-Δv guidance law does not require apriori knowledge of target burnout time or the trajectory for its implementation.

Finally we investigated the feasibility of applying the t-Δv guidance law from interceptor missile launch instead of just guiding the KV after 3rd stage burnout. Fig. 12 shows the results from a 6DoF simulation run incorporating target uncertainty and

track estimation error. Earlier guidance resulted in an additional gain in intercept time at only a modest increase in Δv expenditure.
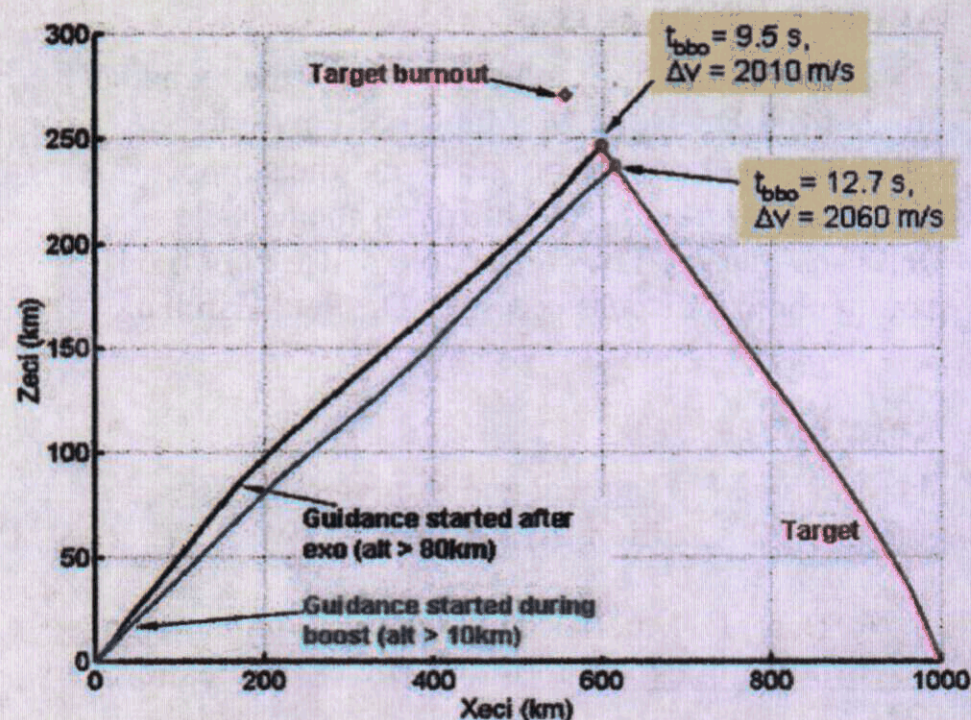

Fig. 12 Extending the t-Δv guidance all the way to interceptor missile launch gains additional interceptor depth with little additional cost in Δv

Summary, Conclusions, and Recommendations

We have derived and examined in detail a new missile guidance law called t-Δv that minimizes the total fuel usage (total Δv) against the desire to achieve earlier intercept time for the boost phase intercept mission. We demonstrated that the t-Δv guidance law can be degraded gracefully into the conventional ZEM guidance when no a priori knowledge of the target is assumed. However when statistics of the PIP (predicted Intercept point) and other target parameters are utilized, we demonstrated via 6 DoF simulations that the t-Δv guidance law can optimally trade earlier intercept time for minimum Δv consumption. With flexible axial and divert thrusting, the t-Δv guidance law can optimally distribute the propellant usage to achieve maximum intercept depth and overcome target maneuvers.

We have also developed an analytical solution to the complex minimization of a stochastic cost function, as required for the derivation of the guidance law, resulting in a significant reduction in the computational requirements.

To make maximum utilization of the t-Δv guidance law, the kill vehicle must have a flexible axial/divert (ADACS) burn capability such as the ATKV currently being explored at LLNL.

Finally we found that it is advantageous to apply the t-Δv guidance law as soon as possible and preferably at as early as interceptor missile launch.

Acknowledgments

The authors wish to acknowledge the technical inputs from Dr. Donn McMahon of Lawrence Livermore National Laboratory and the support from Drs. Walt Dyer and Jim Mulroy of the Missile Defense Agency. The authors were also benefited from technical discussions with Dr. Paul Zarchan.

References

[1] Zarchan, P., Tactical and Strategic Missile Guidance, 2nd Edition, AIAA Press, 1994.

[2] R. Bate, D. Mueller and J. White, Fundamentals of Astrodynamics, pp.300-305, Dover publication, 1971.

[3] Whitehead, J., "Hydrogen Peroxide Gas Generator Cycle with a Reciprocating Pump," 38th AIAA/ASME/SAE/ASEE Joint Propulsion Conference and Exhibit, 7-10 July 2002. Paper AIAA 2002-3702.

[4] Papoulis, A., Probability, Random Process, and Stochastic Processes, McGraw-Hill, 1965.

BIOGRAPHY

Dr. Lawrence C. Ng received his B.S. and M.S. degrees in Aeronautics and Astronautics from the Massachusetts Institute of Technology in 1973, and a PhD degree in Electrical Engineering and Computer Sciences from the University of Connecticut in 1983 under a Naval Undersea Warfare Center (NUWC) Fellowship. In addition, Dr. Ng received his commission as an Air Force officer in 1973 and served at the Hanscom Air Force Base in Bedford, MA. His work experience includes: four years with General Dynamics Electric Boat Division in Groton, CT, responsible for the development of the TRIDENT submarine digital control systems; seven years at the NUWC where he led the development of the advanced sonar signal processing for the Seawolf submarine. Since 1986, he joined the Lawrence Livermore National Laboratory where he was the group leader of the signal/image processing and control group and is currently the Chief Scientist for the Advanced Interceptor Technology Program. Dr. Ng is focusing his research in micro-spacecraft guidance and control, integrated ground testing, and ballistic missile defense systems analysis. In addition Dr. Ng is a member of several professional societies, including honorary memberships in Sigma Xi, Tau Beta Pi, and the National Research Council. He has published numerous papers in signal estimation and precision vehicle guidance and control.
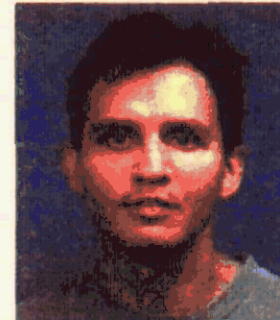
Eric F. Breitfeller has been an electrical engineer at Lawrence Livermore National Laboratory (LLNL) from 1989-1996 and 1997-present. From 1996-1997 he was at Hughes Missile Systems Company. He received his M.S.E.E. in 1988, from Ohio State University, with emphasis on control systems. From 1990-1992 he worked on guidance, navigation, and control (GNC) 6DOF simulations related to the Brilliant Pebbles program. Subsequently, he supported BMDO through the POET, where he developed a 6DOF simulation that was used in trade studies as they related to missile intercept scenarios. While at Hughes (Raytheon) he developed the attitude control system (ACS) for the Exo-atmospheric Kill Vehicle (EKV). Upon returning to LLNL in 1997 he assumed the lead GNC engineering position on the former Clementine-II program (currently the MicroSat Technology program). Precision control and estimation algorithms were designed in a 6DOF environment, and then applied to the fully-functional 5DOF (3DOF ACS + 2DOF translation) hot-gas micro-satellite. While not working on missile intercept problems, he has been involved in robotic applications related to the alignment of optical fibers to wave guides, and to beam steering of single-beam linear particle accelerators.

Dr. Arno G. Ledebuhr earned an undergraduate degree in Physics and Math in 1976 from the University of Wisconsin and masters and doctorate degrees in Physics from Michigan State University in 1982. Dr. Ledebuhr spent the following four years at the Hughes Aircraft Company and earned 15 patents in projection display technology. He has been at Lawrence Livermore National Laboratory since 1986 and led the development of advanced sensors for the Brilliant Pebbles interceptor program and the design of the Clementine sensor payload. In 1996 he was the Clementine II Program Leader and is currently the Program Leader for the Advanced Interceptor Technology Program. His interests include micro-spacecraft and kinetic kill vehicle technologies, including sensors and propulsion systems.