Preprint
UCRL-JC-139518

# Unstructured Polyhedral Mesh Thermal Radiation Diffusion

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This article was submitted to
2000 American Nuclear Society Winter Meeting
Washington, DC
November 12-16, 2000

**July 27, 2000**

*U.S. Department of Energy*

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## INTRODUCTION

Unstructured mesh particle transport and diffusion methods are gaining wider acceptance as mesh generation, scientific visualization and linear solvers improve. This paper describes an algorithm that is currently being used in the KULL[1] code at Lawrence Livermore National Laboratory to solve the radiative transfer equations. The algorithm employs a point-centered diffusion discretization[2] on arbitrary polyhedral meshes in 3D. We present the results of a few test problems to illustrate the capabilities of the radiation diffusion module.

## DESCRIPTION OF WORK

We are interested in solutions of the time-dependent equation of radiative transfer[3], which for monoenergetic photons is written as

$$\frac{1}{c}\frac{\partial E}{\partial t} + \vec{\nabla} \cdot \vec{F}(\vec{r},t) + \sigma(\vec{r},T)E(\vec{r},t) = \sigma(\vec{r},T)B(T). \tag{1}$$

Here $E(\vec{r},t)$ and $\vec{F}(\vec{r},t)$ are the radiation energy density and flux (akin to the scalar flux and current in neutronics notation), $T$ is the temperature of the matter or host material, $\sigma$ is the absorption cross-section or opacity, and $B$ is the Planck emission function for a material at temperature $T$. The matter temperature is coupled to the photon field by the energy balance equation,

$$C_v\frac{\partial T}{\partial t} = \sigma(\vec{r},T)\left(E(\vec{r},t) - B(T)\right). \tag{2}$$

Eqs. (1) and (2) are coupled non-linearly through the temperature. Numerical treatments of this system often involve a fully-implicit time discretization (for stability), which requires an evaluation of the Planck function at the end of time-step temperature. Numerically, we accomplish this by using a Newton-Raphson iteration on the temperature, which linearizes the Planck function at each iterate,

$$B(T^{k+1}) = B(T^k) + \left.\frac{\partial B}{\partial T}\right|_{T^k} \left(T^{k+1} - T^k\right) \tag{3}$$

where the superscripts $k$ and $k+1$ refer to previous and current Newton-Raphson iterates.

Applying a fully-implicit time discretization and substituting Eq. (3) into Eqs. (1) and (2) allows us to reduce the two coupled equations to a single, steady-state diffusion calculation for every Newton-Raphson iteration:

$$\vec{\nabla} \cdot \vec{F}^{k+1}(\vec{r}) + \sigma_a E^{k+1}(\vec{r}) = S, \tag{4}$$

where $\sigma_a$ is an effective absorption cross-section, and $S$ is an effective source. For details on the definitions of these effective terms, see Ref. 1.

The flux is related to the energy density through Fick's law. The standard material diffusion coefficient is modified by the Larsen n=2 flux-limiter[4], which limits super-luminal radiation flow.

All that remains is to apply a robust spatial differencing scheme to Eq. (4). We choose Palmer's point (or node) -centered scheme, which has been shown to have many attractive properties on unstructured polyhedral meshes[2]. This is a finite-volume method that has as its control volume the union of all *corners* surrounding a point. A corner is a subzone polyhedral volume defined by unique point and zone locations and an arbitrary number of edge locations. The diffusion equation is integrated over this control volume to generate a balance equation relating surface fluxes to volumetric source and absorption rates. The surface fluxes are then eliminated in favor of gradients of point intensities, which generates the global solution matrix. In general this matrix is asymmetric; however, this technique generates solutions which have many attractive properties: preservation of the homogeneous linear solution, second-order accuracy, convergence to the exact solution as the grid is refined regardless of mesh smoothness, and equivalency with the linear continuous finite elements method on tetrahedral meshes.

## RESULTS

Our first test problem is designed to show that our method can solve the steady-state diffusion equation on a general polyhedral mesh. On the 1 $cm^3$ cube shown in Fig. 1 we impose incident partial flux, vacuum, and reflecting boundary conditions on the top, bottom and other faces, respectively. The absorption cross-section is $10^{-6}$ $cm^{-1}$ and the total cross section is 500 $cm^{-1}$, which means that our solution will approach an exactly linear function of the $z$ coordinate. In Fig. 2 we plot the solution to this problem as a function of nodal $z$ coordinate and find that it exhibits the appropriate linear behavior.

Our second test problem demonstrates our ability to solve for the evolution of a time-dependent Marshak wave. Figure 3 shows the spatial distribution of the non-dimensional matter temperature at a number of different non-dimensionalized times ($\tau$) for the test problem solved on an orthogonal hexahedral mesh. Our solution agrees very well with the analytic solution[5]. Numerical solutions have been generated for many other test problems and our approach has produced solutions which are accurate and robust.

## CONCLUSION

Our 3D unstructured polyhedral mesh radiative diffusion methodology has been shown to be robust for the solution of a wide variety of interesting problems, two of which are discussed here. We are currently modifying the code to allow the solution of multifrequency problems. This will require an extra level of iteration, as the linearization procedure will produce a coupled set of diffusion equations which are similar to the multigroup neutron diffusion equations.

## ACKNOWLEDGEMENT

Portions of this work were performed under the auspices of the U.S. Department of Energy by Lawrence Livermore National Laboratory under contract No. W-7405-Eng-48.

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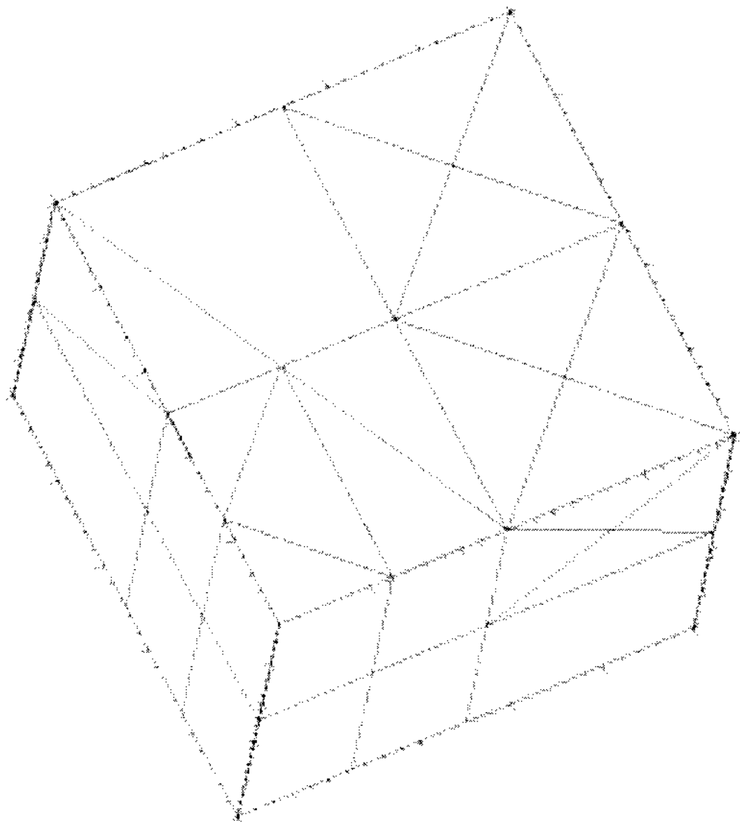

Figure 1: Polyhedral mesh, including type-5 polyhedron (7-point, 8-face volume element), used for "linear solution" problem.

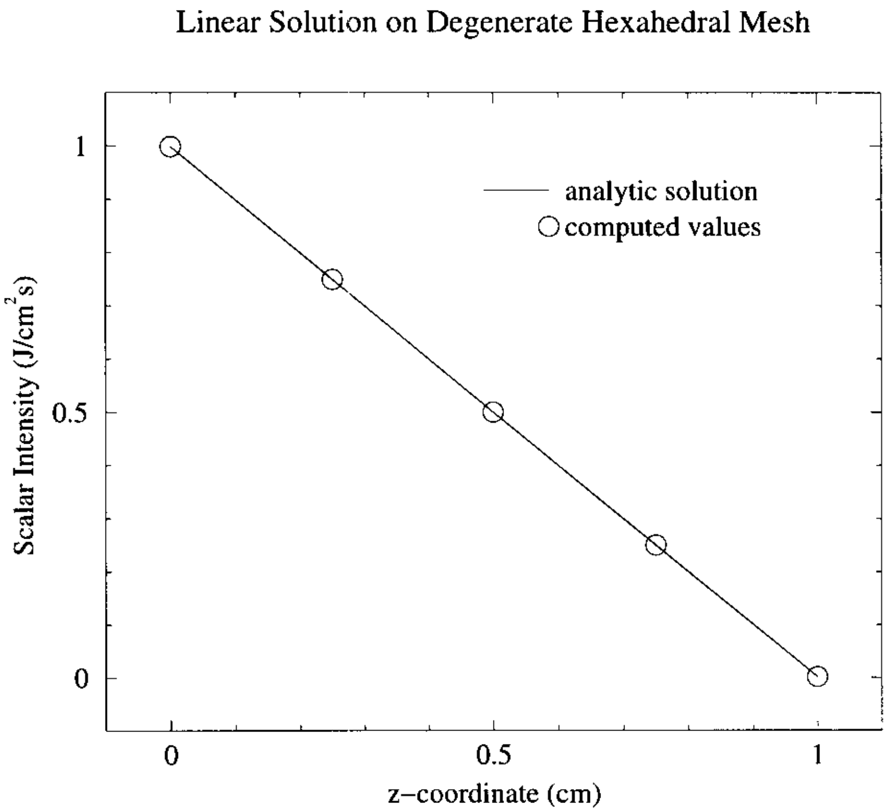


Figure 2: Solution of "linear solution" problem.

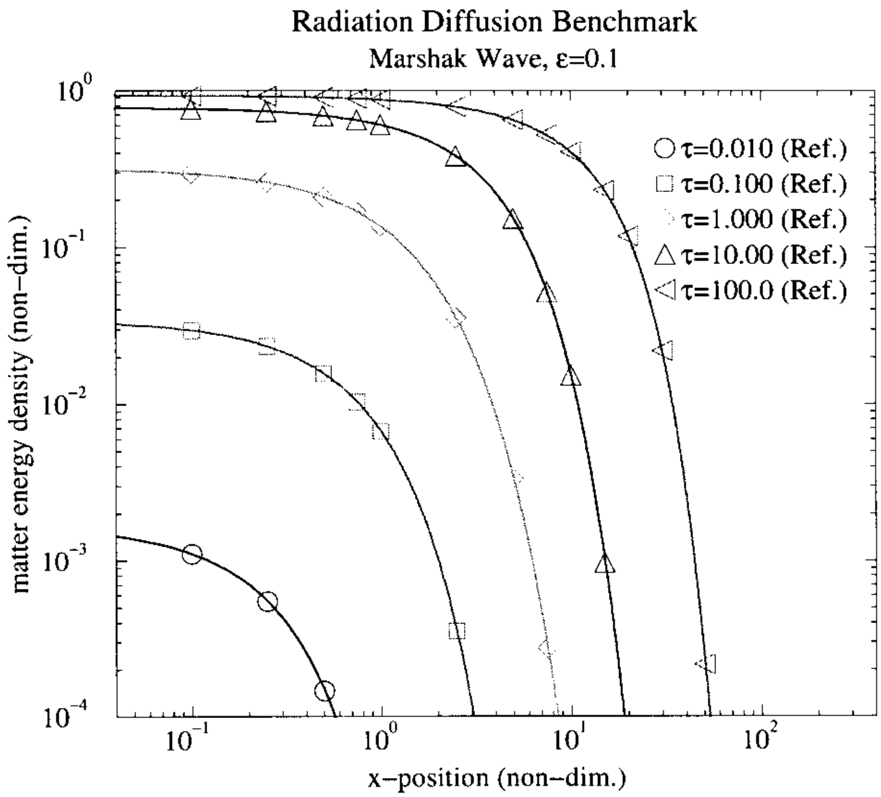


Figure 3: Solution to Marshak wave test problem.