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Longitudinal Reference Particle Motion in Nearly Isochronous FFAG Recirculating Accelerators

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Longitudinal Reference Particle Motion in Nearly Isochronous FFAG Recirculating Accelerators

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A Fixed Field Alternating Gradient (FFAG) arc can be used to reduce the cost of a recirculating accelerator. Path length variation with energy in such an arc can limit its usefulness, however, due to phase offset at the linac. This paper examines the dynamics of the reference particle in an FFAG recirculating accelerator, and describes the limitations on the design because of path length variation with energy.

I. INTRODUCTION

From a cost as well as a dynamical perspective, the cost of a recirculating accelerator can be divided into arcs and linacs. Linacs tend to be must more costly per unit length than arcs, while the requirement for multiple arcs to allow the beam to recirculate gives a large total length of arc. The cost balances out at approximately the point where the arc costs equal the linac costs, since the costs can be approximated to lowest order as

$$\left(n - \frac{1}{2}\right) C_A + \frac{C_L}{2n}, \tag{1}$$

where C_A is the cost of 360° of arc, C_L is the cost of the linac required to accelerate to the full desired energy. The minimum of this function is when $nC_A \doteq C_L/(2n)$, and for large n this gives my earlier conclusion.

If one accelerates for a large number of turns in a multiple-arc recirculating accelerator, the highest energy particles from one pass may have a greater energy than the lowest energy particles from the next pass. If this is the case, it is impossible to passively switch the beam from one arc to another. At least at lower energies, a kicker magnet is impractical. Furthermore, due to space requirements for magnet coils, one must have an even greater separation in the energies of subsequent passes. Thus, the number of turns in a multiple-arc recirculating accelerator is limited, and it may not be possible to achieve the costs minimum described above.

An FFAG-based recirculating accelerator is intended to reduce both sources of cost, as well as address the switchyard problem. By having a single arc for all passes, it reduces the total length of arc, although cost per unit length of this arc is greater than a standard arc. Since it is no longer necessary to guide the beam into a different arc on each pass, the switchyard separation problem is nonexistent, and one can in principle recirculate for an arbitrary number of turns. The arc costs is independent of the number of passes, and the linac cost decreases as one increases the number of turns. Thus, if one is only concerned with cost (and not, for instance, with muon decays), one can reduce the cost to be only slightly above that of the FFAG arc.

The problem comes when one considers that the particles must arrive at the same RF phase (approximately) on each turn, otherwise the particles will eventually be decelerated. The arc has a path length which depends on energy, and since for an FFAG arc that energy range is rather large, path length differences can cause a problem. This problem is exacerbated by the fact that the phase slip is cumulative: a 5 cm path length error will give a total path length error of 15 cm after 3 turns, for instance.

One can try to make the arc approximately isochronous. If the arc were perfectly isochronous, there would be no problem with path length. Imperfect isochronosity would be expected to limit the number of turns: one can recirculate until the cumulative path length error causes the reference particle to no longer be at an accelerating phase.

Surprisingly, this expectation is wrong. This paper will demonstrate that it is wrong, and will describe what the reference particle actually does. It will also address what limitations the path length variation does impose on the design.

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II. BASIC EQUATIONS

The motion of a given particle can be described by the equations

$$E_n = E_{n-1} + Vc(\omega\tau_{n-1}) \qquad \tau_n = \tau_{n-1} + T(E_n), \tag{2}$$

where τ_n is the arrival time relative to the reference RF phase after n passes through linacs, E_n is the particle energy after n passes through linacs, V is the energy gain of a particle going through one linac on the RF crest, c is a function giving a relationship between voltage and phase (e.g., for a single frequency this would be cos), and $T(E)$ is the transit time through the arc relative to the transit time of a reference particle which has the same RF phase on both sides of the arc.

For an FFAG arc, to lowest order one can write [1]

$$T(E) = T_0 + \Delta T\left(\frac{E-E_0}{\Delta E}\right)^2. \tag{3}$$

All parameters in this equation are determined by the lattice design, with the exception of T_0 which is in principle a free parameter, which corresponds to lengthening or shortening the arc. Ideally, one makes E_0 the center of the energy range of the accelerator, ΔE half the width of the range, and then minimizes ΔT.

The design of a recirculating accelerator then involves minimizing V while treating T_0 and τ_0 as parameters.

One can also make a continuous approximation to the discrete system. This becomes a good approximation for a large number of turns, or if the RF is distributed uniformly throughout the recirculating accelerator. The equations of motion can be written

$$\frac{dE}{dn} = Vc(\omega\tau(n)) \qquad \frac{d\tau}{dn} = T(E(n)) \tag{4}$$

One can eliminate n from the continuous equations, getting

$$\frac{d\tau}{dE} = \frac{T(E)}{Vc(\omega\tau)} \tag{5}$$

Using (3), the assumption that E_0 is the central energy, and that $\tau = \tau_0$ when $E = E_0 - \Delta E$, this can be solved, giving

$$\frac{V}{\Delta E}[s(\omega\tau) - s(\omega\tau_0)] = \omega\left[\frac{1}{3}\Delta T\left(\frac{E-E_0}{\Delta E}\right)^3 + T_0\left(\frac{E-E_0}{\Delta E}\right) + \frac{1}{3}\Delta T + T_0\right] \qquad s(\phi) = \int_0^\phi c(\psi)d\psi. \tag{6}$$

The right hand side of this has extrema when $(E-E_0)/\Delta E = \pm\sqrt{-T_0/\Delta T}$. Call τ_1 the value of τ corresponding to the first extrema, τ_2 the second extrema, and τ_3 the value when $E = E_0 + \Delta E$, the maximum energy. We then get the relations

$$\frac{V}{\Delta E}[s(\omega\tau_1) - s(\omega\tau_0)] = \omega\Delta T\left[\frac{2}{3}\left(-\frac{T_0}{\Delta T}\right)^{3/2} - \left(-\frac{T_0}{\Delta T}\right) + \frac{1}{3}\right] \tag{7}$$

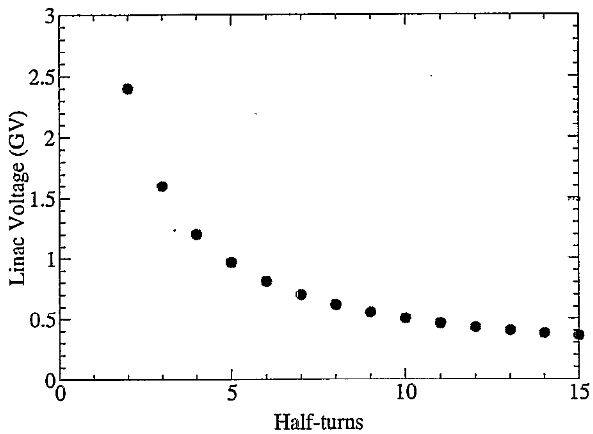
$$\frac{V}{\Delta E}[s(\omega\tau_2) - s(\omega\tau_0)] = \omega\Delta T\left[-\frac{2}{3}\left(-\frac{T_0}{\Delta T}\right)^{3/2} - \left(-\frac{T_0}{\Delta T}\right) + \frac{1}{3}\right] \tag{8}$$

$$\frac{V}{\Delta E}[s(\omega\tau_3) - s(\omega\tau_0)] = 2\omega\left(T_0 + \frac{1}{3}\Delta T\right). \tag{9}$$

The difference in the s functions on the left hand side of these equations takes on some maximum absolute value (if c were cos, it would be 2). To determine the minimum V, first find the value of T_0 which minimizes the largest of the right hand sides of Eqs. (7–9). This turns out to be $T_0 = -\Delta T/4$. Then the minimum V is $\omega\Delta T\Delta E/(6S_{\text{max}})$, where S_{max} is the maximum value that the difference of s at two different phases can take. For $c = \cos$, this is $\omega\Delta T E_{\text{gain}}/24$, where $E_{\text{gain}} = 2\Delta E$ is the energy gain in the accelerator.

III. EXAMPLE AND ANALYSIS

Let's begin by solving the discrete equations (2) with the quadratic path length variation (3), with $c = \cos$. This is done by guessing a τ_0 and T_0, finding the V that gives the desired energy gain in the desired number of



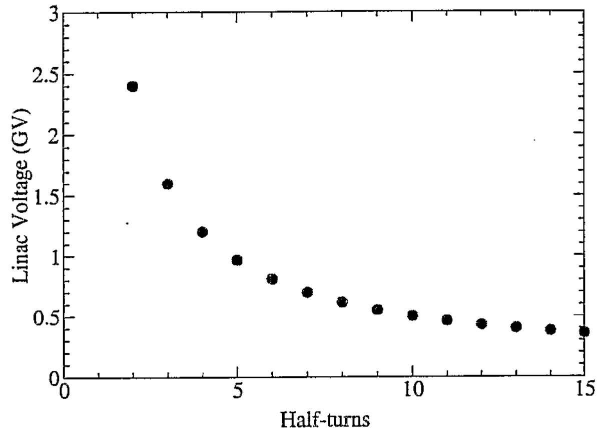


FIG. 1: Left, RF voltage required as a function of the number of half-turns in an FFAG recirculating accelerator. Right, the phase as a function of half-turn number for a 15-turn recirculating accelerator. The RF frequency is 200 MHz, the total path length swing is $\Delta T = 30$ cm/c, and the total energy gain is 4.8 GV.

turns, and then varying τ_0 and T_0 until V is minimized. As the number of turns gets large, this turns out to be very difficult, since the solution for V oscillates very rapidly as a function of τ_0 and T_0 when one is not close to the correct solution. Fig. 1 shows the results.

Note that as the number of turns gets large, the linac voltage does seem to approach the value calculated above, which would be 0.25 GV in this case. This is because for a large number of turns, the system behaves like a continuous system due to the small energy gain per turn. Secondly, note the variation of phase with turn number: the reference particle crosses the crest three times, the the initial and final phases having the same absolute value, and the intermediate maximum taking on those same values. As the number of turns gets larger, that maximum phase gets closer to $\pi/2$. While one can have an arbitrary number of turns, the linac voltage has a nonzero minimum value, so at some point there is little advantage in going to more turns. The limiting voltage is equivalent to $12S_{\text{max}}/(\omega\Delta T)$ half-turns on-crest in a multiple-arc recirculating accelerator (19 in this example).

The basic dynamics are as follows: the reference particle initially gains very little energy, being far off crest, but the path length deviation is large, and so the particle quickly reaches the crest. As the particle crosses the crest, its energy quickly approaches the energy where the path length is synchronized with the RF. This is the maximum in the phase oscillation. The path length error then takes on the opposite sign, and the particle turns around. This is the minimum of the parabola, and it turns out that (since in the large-turn limit $T_0 = -\Delta T/4$) this path length error is significantly less than the initial path length error. Thus, the particle crosses over the crest in the opposite direction more slowly than it crossed the first time. It eventually gains sufficient energy that the path length error returns to zero again, giving the minimum phase. As the particle continues to gain energy, it then quickly crosses the crest a final time.

One can estimate the effect of flattening out the RF crest with third harmonic cavities. In this case, if V is the installed fundamental voltage RF, S_{max} turns out to be 56/27, as opposed to 2 for the single-frequency case. Thus, in the limit of a large number of turns, the third harmonic cavities only reduced the required fundamental-mode cavity voltage by 3.7%, while requiring 1/9 the fundamental voltage in third-harmonic cavities. For fewer turns, the effect of the third-harmonic cavities may be greater; this issue needs further study.

One needs to study how large a beam can be accelerated by this system, and the effect of the number of turns on that acceptance. Furthermore, the emittance growth of the accelerated beam should be studied.

IV. CONCLUSIONS

I have shown that in an FFAG recirculating accelerator, one can circulate the reference particle for an arbitrary number of turns, but the required linac voltage has a non-zero minimum value even for a a large number of turns. The limiting linac voltage can be easily calculated.

[1] Carol Johnstone, to appear in the proceedings of Cyclotrons 2001, East Lansing, MI, 12–17 May 2001.