LA-UR-00-2640
*Approved for public release;
distribution is unlimited.*

Title:

STANDARD AND NON-STANDARD SOLAR MODELS

Author(s):

JOYCE A. GUZIK, CORINNE NEUFORGE-VERHEECKE, CODY YOUNG,
RICHARD I. EPSTEIN, FRANCIS M. POULIN
and JASON R. SCHISSEL

Submitted to:

http://lib-www.lanl.gov/la-pubs/00796328.pdf

STANDARD AND NON-STANDARD SOLAR MODELS

JOYCE A. GUZIK, CORINNE NEUFORGE-VERHEECKE, A. CODY YOUNG, RICHARD I. EPSTEIN, FRANCIS M. POULIN and JASON R. SCHISSEL
Los Alamos National Laboratory, Los Alamos, NM 87545-2345, U.S.A.

(Received 12 June 2000; accepted 13 September 2000)

Abstract. We summarize the physical input and assumptions commonly adopted in modern standard solar models that also produce good agreement with solar oscillation frequencies. We discuss two motivations for considering non-standard models: the solar neutrino problem and surface lithium abundance problem. We begin to explore the potential for mixed core models to solve the neutrino problem, and compare the structure, neutrino flux, and oscillation frequency predictions for several models in which the inner 25% of the radius is homogenized, taking into account the effects of non-local equilibrium abundances of ^{3}He. The results for the neutrino flux and helioseismic predictions are far from satisfactory, but such models have the potential to reduce the predicted ^{7}Be/^{8}B neutrino flux ratio, and further studies are warranted. Finally, we discuss how much the neutrino problem can be alleviated in the framework of the standard solar model by using reaction rates, abundances and neutrino capture cross-sections at the limits of their uncertainties, while still satisfying the constraints of helioseismology.

1. Introduction

Helioseimology provides a useful test of solar evolutionary models (see reviews by, e.g., Christensen-Dalsgaard *et al.*, 1996; Gough *et al.*, 1996). In this paper we summarize the assumptions and physical ingredients of a standard solar model, and discuss the motivation for non-standard models. We present new results for non-evolutionary mixed core models calibrated to the solar luminosity and radius that take into account local nonequilibrium burning of core ^{3}He. We also consider the extent to which predicted solar neutrino fluxes can be reduced by taking into account the extremes of uncertainties in nuclear reaction rates, solar abundances, and neutrino capture cross sections. Preliminary results of this paper were presented at the American Geophysical Union Special Session *Structure and Rotation of the Solar Core* held December 13, 1999.

2. Physical Ingredients of the Standard Solar Model

The usual procedure for modeling the evolution of the Sun begins by generating a one-dimensional one solar mass model of uniform composition, that has contracted gravitationally to the point where a significant amount of luminosity is produced by

Solar Physics **200:** 305–321, 2001.

core hydrogen burning (the zero-age main sequence). The modeling code solves the equations of stellar structure for successive timesteps, taking into account conditions on mass conservation, hydrostatic equilibrium, energy production, and energy transport by radiation, convection, and electron thermal conduction. Modern solar models also include a treatment for diffusive settling of helium and heavier elements relative to hydrogen. The initial mass fractions of hydrogen (X), helium (Y), and heavier elements (Z), and the mixing length to pressure scale-height ratio (α), are adjusted so that the model attains the observed solar luminosity, radius, and surface Z/X ratio (see below) at the present solar age.

The solution of these equations requires a calculational procedure or pre-calculated tables for opacities, equation of state, and nuclear reaction rates, as well as treatments of convective transport and surface boundary conditions. Here we list some widely used, but not necessarily the most sophisticated or refined, treatments for such data and processes.

The radiative opacities commonly used for solar modeling are the tables of the Livermore OPAL project (Iglesias and Rogers, 1996) or the Opacity Project (e.g., Seaton *et al.*, 1994). Estimates of the intrinsic uncertainties in radiative opacities are $\sim$5% for the solar core, where most elements are fully ionized, and electron scattering is the major contributor to the opacity. Opacity uncertainties are estimated at $\sim$ 10% below the convection zone where some elements are partially ionized. The exact opacity value at the convection zone base is critical to determining the convection zone depth, but because convection transports nearly all of the Sun's luminosity within the convection zone, exact opacities throughout the convection zone, where most elements are partially ionized, are fortunately not as important. It is crucial to accurately calculate the ionization state of elements within the convection zone, however, to determine the equation of state and adiabatic index which determine the temperature and sound speed profiles. For temperatures less than the photospheric temperature $\sim$ 6000 K, these radiative opacity tables must be supplemented by low-temperature atomic and molecular opacities (e.g., Kurucz, 1992; Alexander and Ferguson, 1994; Neuforge, 1993). Differences of factors of two to three exist between available low-temperature opacities for some temperature and density values relevant for the solar surface (Neuforge, 1993). Low-temperature opacities remain a major source of uncertainty, along with the treatment of the superadiabatic layers at the top of the convection zone, in calculating solar surface structure.

Possibilities for calculating the equation of state include tables, such as OPAL (Rogers, Swenson, and Iglesias, 1996), MHD (Hummer and Mihalas, 1988; Mihalas, Däppen, and Hummer, 1988; Däppen *et al.*, 1988); analytical procedures such as CEFF, an extension of the Eggleton, Faulkner, and Flannery (1973) EOS to include Coulomb corrections (Christensen-Dalsgaard and Däppen, 1992); or SIR-EFF, an extension of EFF by Swenson, Irwin, and Rogers (see Guzik and Swenson, 1997). The review by Däppen and Guzik (2000) summarizes available opacity and equation of state treatments, and lists additional references comparing solar and stellar models using these treatments.

The solar element mixture for which the opacity and EOS tables are computed is important. The 1996 OPAL tables are based on the Grevesse and Noels (1993, hereafter GN93) mixture. The recent Grevesse and Sauval (1998, hereafter GS98) surface abundance determination has lower mass fractions of C, N, and O, and a lower photospheric Z/X determination, which slightly decreases the opacities and convection zone depth, and gives poorer agreement with helioseismology (Turcotte and Christensen-Dalsgaard, 1998; Neuforge-Verheecke *et al.*, 2000). Modern solar models are calibrated to attain, after diffusive settling, the present surface mass fraction ratio of elements heavier than helium and hydrogen (Z), to hydrogen (X) : $Z/X = 0.0245 \pm 0.0015$ for the GN93 determination, and 0.0230 ± 0.0023 for the GS98 determination.

Modern solar models have incorporated nuclear reaction rates of Caughlan and Fowler, 1988, Bahcall *et al.*, 1995, Adelberger *et al.*, 1998, or the new European compilation NACRE (Angulo *et al.*, 1999). Including the diffusive settling of helium and heavier elements relative hydrogen turns out to be essential to achieving good agreement between helioseismic predictions and observations (Christensen-Dalsgaard *et al.*, 1996). Such treatments are discussed, for example, by Thoul, Bahcall, and Loeb (1994) and Cox, Guzik, and Kidman (1989). Convective energy transport has usually been treated using the standard mixing-length theory (Böhm-Vitense, 1958); some modelers have also applied with good results a more refined treatment allowing a spectrum of convective eddy length scales by Canuto and Mazzitelli (1991, 1992) and Canuto, Goldman, and Mazzitelli (1996).

Because of the extreme accuracy of solar oscillation frequency observations, even small differences in the adopted values for the solar radius, mass, luminosity, and age can have a noticeable effect on oscillation frequency predictions. The solar radius value traditionally used is 6.9599×10^{10} cm (Allen, 1973). Recent redeterminations of the photospheric radius suggest a decrease of 200–500 km (e.g., Brown and Christensen-Dalsgaard, 1998). A smaller solar radius appears to produce poorer agreement between the overall calculated and inferred sound speed in the solar interior (Basu, 1998). However, a 300 km decrease in solar radius improves the agreement between calculated and measured f-mode frequencies that sample the Sun's subsurface layers (Schou *et al.*, 1997). The usually adopted solar luminosity, mass, and age are, respectively, $3.846 \pm 0.005 \times 10^{33}$ erg s^{-1} (Willson *et al.*, 1986), $1.9891 \pm 0.0004 \times 10^{33}$ g, and 4.52 ± 0.04 Gyr (Guenther, 1989).

Additional assumptions of the standard model include negligible mass loss or accretion during the evolution; no exotic particle physics; negligible effect of magnetic fields or rotation on the structure or composition; and no convective overshooting below the convection zone base.

3. Motivation for Non-Standard Models

Such standard models have done remarkably well in attaining agreement with helioseismic inferences of the Sun's interior structure (see, e.g., Gough *et al.*, 1996; Christensen-Dalsgaard *et al.*, 1996; Basu, 1998; Guenther and Demarque, 1997; Guzik and Swenson, 1997; Gabriel, 1997; Brun, Turck-Chièze, and Morel, 1998). However, at least two discrepancies between standard model predictions and observations have motivated the calculation of non-standard models – the lithium problem and the neutrino problem.

3.1. THE SOLAR LITHIUM PROBLEM

The solar surface ^{7}Li abundance is observed to be lower by a factor of $\sim$160 than the meteoritic abundance, and presumed initial solar abundance (GS98). According to the predictions of standard evolution models, the Sun's convection zone base was never hot enough for long enough to allow the circulation of surface Li to temperatures where it could be depleted by nuclear processing. It was once thought that the Sun's ^{9}Be has also been depleted by a factor of 2–3, but recent abundance evaluations are consistent with no depletion (Balachandran and Bell, 1998). Two classes of non-standard model solutions have been proposed. The first invokes early mass loss of about 0.1 $M_\odot$, so that the Sun's present surface layers were once deep enough to deplete Li (e.g., Boothroyd, Sachmann, and Fowler, 1991; Swenson and Faulkner, 1992; Guzik and Cox, 1995; Morel, Provost, and Berthomieu, 1997). However, the mass loss phase must end within the first 0.2–0.3 Gyr of the Sun's evolution to avoid ruining the good agreement between the calculated and helioseismically inferred sound speed in the solar core. The luminosity of standard solar models at the beginning of the zero-age Main Sequence is about 70% of the present solar luminosity. Several lines of evidence including fossil records, sedimentary rocks, and oxygen isotope ratios suggest warmer Earth temperatures and the presence of liquid water earlier than 3.5 Gyr ago. While warmer early terrestrial temperatures can be attained by enhanced greenhouse warming (see, e.g., Gerard, Hauglustaine, and François, 1992), the higher luminosity of a more massive early Sun also can help to avoid this 'weak early Sun paradox' (Graedel, Sackmann, and Boothroyd, 1991).

A second class of solutions to the Li problem involves mixing below the convection zone induced by some mechanism such as differential rotation at the 'tachocline', the transition from differential to near solid-body rotation at the convection-zone base (Spiegel and Zahnn, 1992) or gravity waves (Schatzman, 1996; Fritts, Vadas, and Andreassen, 1998). Solar models including such mixing have been compared with helioseismic data and inferences by Gabriel (1997), Richard *et al.* (1996), Brun, Turck-Chièze, and Zahn (1999), and Morel, Provost, and Berthomieu (1997). Note that earlier conjectures that convective overshooting could produce the depletion have not been supported by the helioseismic determination of the convection zone depth, and a limit to the extent of the adiabatically stratified layer below the convection zone to less than 0.05 pressure scale heights (Basu, 1998). Both the mass loss and mixing mechanisms show promise for reducing the small sound speed discrepancy between the standard model with diffusion and the helioseismically-determined sound speed (Gabriel, 1997; Anderson *et al.*, 1996; Brun, Turck-Chièze, 1999; Gough, 1999).

3.2. The Solar Neutrino Problem

The solar neutrino problem, or the deficit in the observed flux of electron neutrinos relative to standard solar model (SSM) predictions, has been attacked since the first results of Davis, Harmer, and Hoffman (1968) from the ^{37}Cl experiment at the Homestake mine (see, e.g., reviews by Bahcall, 1996, and Haxton, 1998). While the only available observations were from this experiment that primarily detects the high-energy neutrino from the ^{8}B decay of the ppIII chain, and before the advent of helioseismology, many methods were proposed to effectively lower the central temperature of solar models, thereby reducing the predicted ^{8}B neutrino flux. One class of methods involves lowering the central opacity by some means (Christensen-Dalsgaard, 1996), for example by decreasing the core Z abundance (Guenther and Demarque, 1997), or condensing Fe out of the core (Cox, Guzik, and Raby, 1990). Other models included the effects of a central black hole (Clayton, Newman, and Talbot, 1975; Rouse, 2000), or exotic cosmological particles such as WIMPS (weakly-interacting massive particles) to increase energy transport from the core (Christensen-Dalsgaard, 1992; Gilliland *et al.*, 1986; Cox, Guzik, and Raby, 1990). Another class of models involves mixing a portion of the core (Richard and Vauclair, 1997; Morel and Schatzman, 1996; Guenther and Demarque, 1997; Brun, Turck-Chièze, and Morel, 1998). Other, perhaps even less plausible options involve reducing the initial mass of the Sun, or considerably reducing the present solar age.

However, as the more recent (post-1990) papers conclude, these non-standard model solutions do not withstand the test of helioseismology, and at best can only partially reduce the predicted neutrino flux. The helioseismically-inferred sound speed agrees very well with the standard solar model, to within a few tenths of a percent in the solar core (Gough *et al.*, 1996; Bahcall, Basu, and Kumar, 1997). Lowering the central temperature reduces the sound speed c_s which is proportional to $\sqrt{T/\mu}$, and must be offset by an *ad hoc* compensating change in the molecular weight gradient to restore the sound speed agreement. Likewise, mixing alters the molecular weight gradient, and must be offset by an *ad hoc* change in the temperature gradient to preserve the sound speed. Moreover, results from three new neutrino experiments revealed two additional solar neutrino problems (Bahcall, 1995). The combined results of the Homestake chlorine experiment and the Japanese Kamiokande experiments imply that the observed flux of ^{7}Be + CNO neutrinos must be less than 0.46 solar neutrino units (SNUs), whereas standard solar models predict a flux of 1.1 ± 0.1 SNUs for ^{7}Be neutrinos alone. The combined results of the SAGE and GALLEX gallium experiments indicate a flux of ^{7}Be + CNO neutrinos of less than 19 gallium SNUs, whereas the standard solar models predict a flux of 34 $\pm$ 4 SNUs for ^{7}Be neutrinos alone. The results are consistent with the absence of ^{7}Be neutrinos, and a 60% decrease in ^{8}B neutrinos, compared to SSM predictions. However, a lower central temperature would reduce the ^{8}B flux much more than the ^{7}Be flux, contrary to the neutrino observations (Bahcall, Basu, and Kumar, 1997; Antia, and Chitre, 1997). After these results, efforts to explain the neutrino problem turned away from solar model physics, and toward non-standard neutrino physics, such as vacuum or matter-enhanced (MSW-effect; Mikheyev and Smirnov, 1986; Wolfenstein, 1978, 1979) neutrino oscillations.

4. Another Approach to a Solar Model Solution to the Neutrino Problem

In 1996, Cumming and Haxton suggested a modification to solar models which might be consistent with recent neutrino experiments, in which ^{3}He is mixed into the core by a rapid downward flow in narrow plumes over a timescale of a few million years from a reservoir of higher ^{3}He equilibrium abundance that is built up farther out in radius. (The ^{3}He abundance of the standard solar model peaks at about 0.29 $R_\odot$; see Figure 1.) The ^{3}He is replenished during slow broader upward flows over a timescale of $\sim 10^7$ years. Mixing induced by the core ^{3}He composition gradient was first proposed by Dilke and Gough (1972). Such a steady-state, but out-of-local-equilibrium ^{3}He abundance, in which the H and ^{4}He are also mixed, would have the advantage of producing fewer ^{7}Be and ^{8}B neutrinos of the *pp*II and *pp*III network, due to an overall reduction in core temperature. This abundance gradient would also reduce the ^{7}Be to ^{8}B neutrino flux ratio, since ^{7}Be would be preferentially produced at smaller radius and higher temperature, favoring the *pp*III chain over *pp*II.

The helioseismology of static models with spherically-symmetric core mixing was discussed by Bahcall *et al.* (1997), and Guenther and Demarque (1997). Bahcall *et al.* estimate unacceptably large discrepancies of 7 to 10% between the helioseismically-inferred core sound speed and that of mixed-core models, much larger than the discrepancies of a few tenths of a percent for the SSM. Guenther and Demarque present a fairly thorough parameter study of evolutionary mixed models, varying the location and extent of the mixed region. They find little difference in neutrino predictions between the SSM and the mixed models, but a large discrepancy between observed and calculated frequency predictions. However, in these models, Guenther and Demarque assumed instantaneous mixing, and conclude that helioseismology cannot at this point rule out slow mixing of ^{3}He farther out in the core where the abundance of ^{4}He is nearly uniform (between $R \simeq 0.15$ and 0.25 $R_\odot$; see Figure 1.)

Here we present some first models of a parameter study to further investigate the Cumming and Haxton proposal. Our study is a little different from others, in that

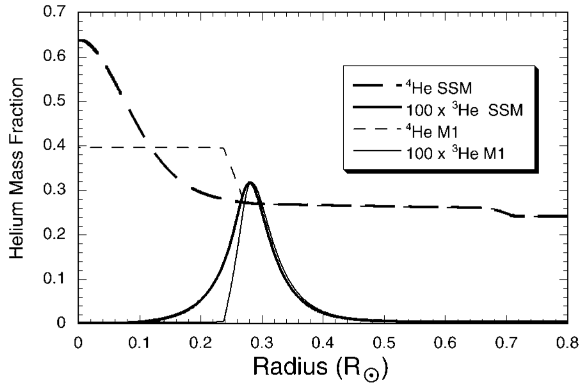

Figure 1. ^{4}He and ^{3}He mass fraction versus radius of our SSM and mixed core model M1. The ^{3}He mass fraction has been multiplied by a factor of 100 for presentation on the same scale.

we are not calculating an evolutionary model, but rather perturbing the core structure of a standard solar model. We begin with the composition versus mass fraction profile of an evolved standard solar model of Neuforge-Verheecke *et al.* (2000), which uses the GN93 solar mixture, OPAL (Iglesias and Rogers, 1996) opacities, Alexander and Ferguson (1995, private communication) low-temperature opacities, SIREFF (Guzik and Swenson, 1997) in-line EOS, and NACRE (Angulo *et al.*, 1999) nuclear reaction rates (see Table I). We integrate from the model surface to the center, with a prescribed mass zoning (1700 zones), enforcing boundary conditions of 1 $L_{\odot}$, 1 $M_{\odot}$, and 1 $R_{\odot}$ at the surface, and satisfying the conditions of hydrostatic equilibrium. We modify the standard composition profile to investigate mixing in the inner 25% of the radius, specifying the core abundance of ^{4}He (Y), ^{3}He, and heavier elements (Z), and adopting the central CNO abundance of the SSM for the mixed region. An outer radius of 0.25 $R_{\odot}$ for the mixed region was chosen because this was the mixing radius considered by Cumming and Haxton (1996), and because near this radius the ^{3}He begins to build up to higher equilibrium concentrations and becomes available to be mixed into the core (Figure 1). We iterate on this inward integration, varying the mixing length to pressure-scale-height ratio (α) and the mixed core Y abundance, until all of the mass is accounted for at zero radius, and nuclear energy generation produces one solar luminosity.

We first calculated a static model (M1eq, Table I) with a constant mixed core Y abundance, but assuming an equilibrium ^{3}He abundance profile for the energy production. In this model, the additional core hydrogen results in a lower central temperature and density, and significantly lowers the neutrino flux. The structure of this model is similar to evolutionary mixed core models in the literature (e.g., Guenther and Demarque, 1997; Morel and Schatzman, 1996) in which ^{3}He is allowed to equilibrate during the evolution, as it does very rapidly near the Sun's center. However, for these models, the predicted ^{7}Be/^{8}B neutrino flux ratio is higher

(3.96 for the gallium experiments) than for the SSM (2.69), as the lower central temperatures favor the *pp*II reaction network over *pp*III.

We also calculated several static models with constant mixed core ^{3}He abundances that differ from the local equilibrium values, and calculated the energy production using non-equilibrium rates. We find that the α and Y values needed to calibrate the models increase systematically with increasing core ^{3}He mass fraction; a core ^{3}He mass fraction near 4.0×10^{-5} produces a model with convection zone depth not too far from the helioseismically determined depth ($0.7135 \pm 0.0005\ R_\odot$; Christensen-Dalsgaard *et al.*, 1991; Basu, 1998). Table I summarizes the properties of this mixed-core model (M1). Note that if the inner 25% of the radius of the standard solar model was mixed instantaneously at the present solar age, the core ^{3}He abundance would be 4.35×10^{-4}, an order of magnitude larger. The core ^{3}He mass fraction chosen for model M1 also minimizes the predicted ^{7}Be/^{8}B neutrino flux ratio, reducing it to 1.62 (gallium experiments) compared to 2.69 for the SSM. However, the central temperature of this model is unrealistically high, higher than that of the SSM, for two reasons: first, the enhanced ^{3}He abundance produces a higher local instantaneous rate of the ^{3}He + ^{3}He reaction; the high local central abundance of ^{3}He would be consumed on a very short timescale, and would not be sustainable in a steady state (although a somewhat higher-than-local equilibrium abundance is expected to be sustained by rapid inward mixing in the Cumming and Haxton scenario). Second, for this model we suppressed core convection that is produced by such a high local nuclear reaction rate, whereas allowing convection would lower the core temperature gradient. A model true to the Cumming and Haxton idea would have a self- sustaining ^{3}He abundance gradient somewhere between our models M1eq and M1. ^{3}He would be injected into the core in a manner that results in a ^{3}He profile somewhat above the local equilibrium abundance in the inner core, and somewhat below the local equilibrium abundance farther out. The circulation would occur on a timescale of millions of years, perhaps 10^6 years for the localized downward flows, and 10^7 years for the broader upward flows. This timescale is slower than the convective timescale, but faster than the evolutionary timescale, and slow enough to allow ^{3}He to build up in the outer core to provide a source of ^{3}He to mix into the center.

The mixed models must of course be put to the test of helioseismology. We calculated the low-degree ($\ell = 0, 1, 2$, and 3) p-mode frequencies for these two mixed models using the nonradial nonadiabatic pulsation code of Pesnell (1990). Figure 2 shows the observed minus calculated (O−C) frequencies of our SSM and mixed core models M1 and M2 (to be described later). The observed low-degree frequencies are from Chaplin *et al.* (1996, 1998), and Schou and Tomczyk (1996). Some of the discrepancy, in particular an offset in O−C for M1, is due to the slightly deeper convection base radius compared to the helioseismically-determined radius. This offset could be removed by a small change in solar abundances or in opacities, easily accomodated by the uncertainties in opacity calculations, diffusion treatments, or abundance determinations. However, the much larger spread in O−C frequen-

TABLE I

Properties of SSM of Neuforge-Verbeecke *et al.* (2000), and mixed core models. In models M1, M1eq, M2, and M2eq, the composition is mixed from the center to 25% in radius. To obtain a mixed model in hydrostatic equilibrium, the ^{4}He abundance is the mixed region and the convection parameter (α) are adjusted. In Models M1 and M2, the ^{3}He abundance in the mixed region is fixed, whereas in models M1eq and M2eq, the local ^{3}He equilibrium abundance, calculated from the local temperature and pressure, is assumed. In Models M2 and M2eq, the opacity is multiplied by 0.7 for T $\geq 8 \times 10^6$ K. The $(\phi\sigma)_i$ are the predicted events rates for the chlorine and gallium experiments, and are calculated using the neutrino capture cross sections given in Bahcall and Ulrich (1988), Bahcall *et al.* (1996), and Bahcall (1997). The event rates are expressed in SNUs, one SNU being 10^{-36} interactions per target atom per second.

Model	SSM	M1eq	M1	M2eq	M2
$^3\mathrm{He}_{\text{mixed region}}$	–	equilibrium	4.0×10^{-5}	equilibrium	4.0×10^{-5}
Core opacity multiplier	no	no	no	0.7	0.7
$^4\mathrm{He}_{\text{mixed region}}$	–	0.3661	0.3955	0.3150	0.3230
α	1.7738	1.7201	1.8436	1.7072	1.6260
T_{central} (10^6 K)	15.66	14.94	16.28	14.28	15.63
ρ_{central} (g cm^{-3})	152.2	97.5	78.39	102.3	94.53
$R_{\text{convection zone base}}$ ($R_\odot$)	0.7135	0.7187	0.7055	0.7203	0.7314
Event rates for the chlorine experiment					
$\sigma\phi_{^7\mathrm{Be}}$	1.14	0.64	0.97	0.39	0.78
$\sigma\phi_{^8\mathrm{B}}$	6.04	2.31	8.49	0.80	3.99
$\sum(\phi\sigma)_i$ ^{37}Cl (3σ)	7.85	3.40	10.08	1.55	5.33
$\sigma\phi_{^7\mathrm{Be}}/\sigma\phi_{^8\mathrm{B}}$	0.189	0.277	0.114	0.488	0.195
Event rates for the gallium experiment					
$\sigma\phi_{pp}$	69.7	72.7	70.6	74.2	71.8
$\sigma\phi_{^7\mathrm{Be}}$	34.1	19.3	29.0	11.7	23.3
$\sigma\phi_{^8\mathrm{B}}$	12.7	4.87	17.9	1.69	8.39
$\sum(\phi\sigma)_i$ ^{71}Ga (3σ)	128.8	104.3	128.7	93.0	113.3
$\sigma\phi_{^7\mathrm{Be}}/\sigma\phi_{^8\mathrm{B}}$	2.69	3.96	1.62	6.92	2.78

cies, with the average O–C frequency generally increasing with increasing degree ℓ, indicates a difference between the sound speed gradient of the Sun and the mixed models that cannot be accomodated by uncertainties in p-mode observations.

Another, more sensitive, helioseismic test of the core structure is to examine the so-called 'small separations' between $\ell = 0$ and $\ell = 2$ p-modes separated by one radial order n (Figure 3). For this plot, a linear least squares fit to the observed small separations was subtracted from each curve to magnify the variations with increasing radial order n. Again, the SSM agrees very well with the observations,

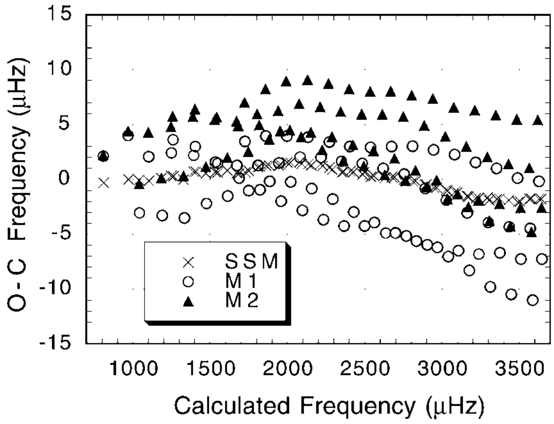

Figure 2. Observed minus calculated (O–C) low-degree (ℓ = 0, 1, 2, and 3) frequencies versus calculated frequency for standard solar model of Neuforge-Verheecke *et al.* (2000), and mixed core models M1 and M2 with non-local equilibrium core ^{3}He abundances. For the mixed core models, there is a spread in O–C frequency, with the average O–C frequency generally increasing with increasing degree ℓ. This spread does not occur for the SSM.

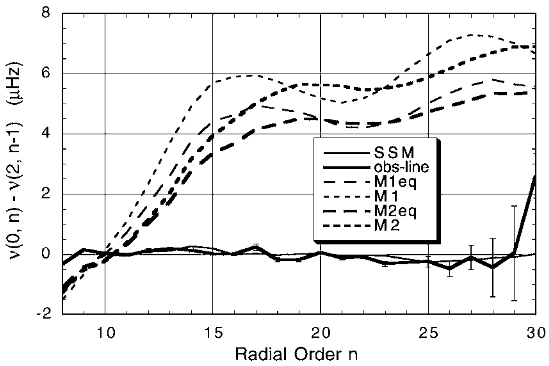

Figure 3. Small *p*-mode frequency differences between $\ell = 0$ and $\ell = 2$ modes separated by one radial order n. The SSM agrees very well with the observed small separations (thick solid line + error bars), while the small separations of the mixed core models of Table I are generally too high. A core opacity reduction of 30% for models M2eq and M2 has very little effect on reducing the small separations.

while the separations for the mixed core models lie far outside the observational error bars.

As discussed by Christensen-Dalsgaard (1996, 1998), the small separations of mixed core models, with shallower core molecular weight gradients, are too high, whereas the small separations of low-opacity models, with shallower core temperature gradients, are too low (Figure 4, reproduced from Christensen-Dalsgaard, 1996). We therefore decided to test whether the agreement with the small separations could be improved for the mixed core models by a simultaneous reduction in core opacity. For models M2eq and M2, we applied an opacity multiplier of

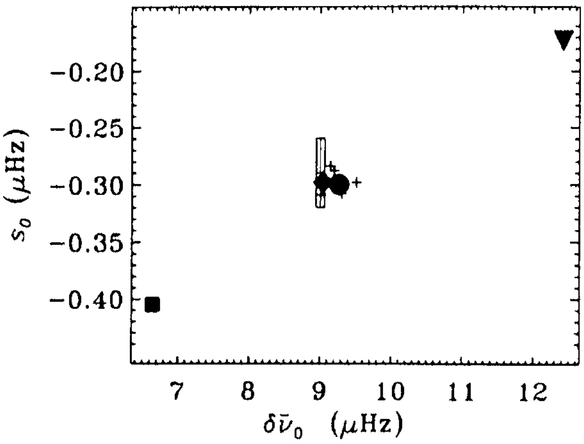

Figure 4. To slope (s_0) and intercept at $n = 21$ ($\delta\overline{\nu}$) of least-squares fits of lines to the small frequency separation versus n for various solar models. Standard solar models with and without diffusion lie in the center of the plot, in good agreement with observations (*box*). Models with reduced core opacity are located to the lower left (*filled box*), while models with a partially mixed core are located in the upper right (*solid triangle*). This figure is reproduced from Figure 16 of Christensen-Dalsgaard (1996).

0.7 for temperatures less than 8 million K (inside the radius of the mixed core at 0.25 $R_\odot$). The opacity multiplier further lowers the central temperature, and therefore further reduces the predicted neutrino flux (see Table I). Figure 2 also shows the small separations for models M2eq and M2; as can be seen, even the unrealistically high opacity multiplier only slightly improves the small separations (compare M1 and M2, and M1eq and M2eq). Opacity modifications are plausible only to make much smaller adjustments to the sound speed profile than required by these mixed models. Note that this result supports the conclusion of Christensen-Dalsgaard (1998) that even opacity reductions of a factor of eight are insufficient to compensate for the change in molecular weight gradient produced by the mixed core models.

Figure 5 shows the fractional difference in sound speed between our models of Table I and that inferred from helioseismic inversion by Basu, Pinsonneault, and Bahcall (2000). The sound speed profile of the standard model agrees very well, within a few tenths of a percent, with the sound speed derived from inversions. The sound speed discrepancies for the mixed core models become as large as 8% in the mixed region below $R = 0.25\ R_\odot$. It is interesting that the discrepancies also become significant in the radiative region between the convection zone base and the mixed core. All of the models have the same composition profile versus mass fraction in this region, but because of the widely different α values needed to calibrate the models to the solar radius, the amount of mass within the convection zone, and therefore the conditions at the convection zone base, are quite different.

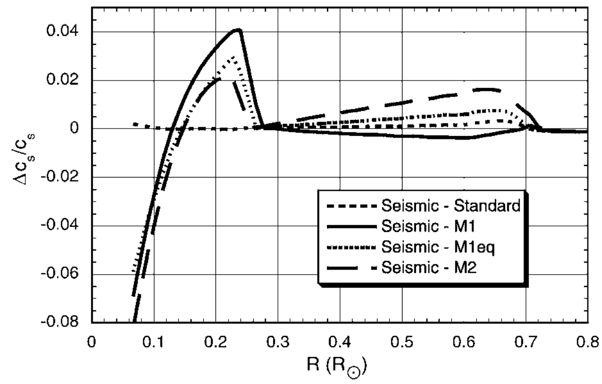

Figure 5. Fractional differences [(Sun – Model)/Sun] between sound speed profile inferred from helioseismic inversions by Basu, Pinsonneault, and Bahcall (2000) and those of models of Table I. The fractional sound speed difference for model M2eq is not shown, as it is nearly identical to that of model M1eq.

Additional parameter studies are in progress, in which we are varying the location and extent of the mixed region and the shape of the ^{3}He profile, and also including the temperature gradient modification due to a very small convective core (or possibly a convective shell) produced by high nonlocal equilibrium concentrations of ^{3}He. Note that, for some models, Cumming and Haxton (1996) excluded up to 4% of the inner radius from mixing, which would possibly turn off such convection. If a promising present-day model of the Sun is found, we then need to consider whether this model can be reached from an evolutionary sequence including slow core mixing. Such models may not solve the neutrino problem, but they may alleviate it somewhat, and widen the parameter space of neutrino mixing angle and mass differences required to account for the remainder of the missing neutrinos by matter-induced oscillations.

Ultimately, the Cumming and Haxton suggestion can be investigated only to a limited extent using one-dimensional spherically symmetric static models. The nonuniformity of the mixing, different time and spatial scales for the upward versus downward flows, and the consequences of differential rotation and angular momentum conservation during the mixing need to be taken into account before core mixing can be ruled out entirely (see, e.g., Gough, 1999). Perhaps a small amount of mixing, much less extensive than proposed by Cumming and Haxton, can still be accomodated while still satisfying the constraints of helioseismology.

5. Neutrino Flux Reductions in the Context of the Standard Solar Model

While it appears that non-standard solar model solutions are unlikely to solve the neutrino problem, exploiting uncertainties in input physics of the standard solar model and of neutrino capture cross sections can slightly alleviate the problem.

Of course, it is improbable that all of the uncertainties would conspire in the right direction to favor such a reduction in the neutrino flux.

Table II compares the results of four 'standard' models of Neuforge-Verheecke *et al.* (2000), evolved to examine the effects of such uncertainties on the predicted neutrino flux as well as on helioseismic predictions. The first column summarizes the properties of the SSM described above, using the GN93 solar mixture. The second column summarizes properties of an evolution model using the GS98 solar abundance mixture; this mixture has lower abundances of C, N, and O (by 7%, 12%, and 10%, respectively), and lower Z/X than GN93, which reduce the opacities. In the third model, the NACRE reaction rates were modified up to the limits of their uncertainties to bias toward the ppI chain and against the ppII and ppIII chains. The $p + p$ reaction rate was multipled by 1.05, ^{3}He+^{3}He by 1.06, ^{3}He+^{4}He by 0.84, and ^{7}Be + p by 0.895. In the fourth model, in addition to the modified reaction rates, Z/X was reduced from 0.023 to 0.0208, corresponding to the lower limit (-10%) of the uncertainties of the GS98 determination. Finally, in addition to modified reaction rates and lower Z/X, for the neutrino fluxes reported in parentheses, the lower limits of the neutrino capture rates were adopted (Bahcall and Ulrich, 1988; Bahcall *et al.*, 1996; Bahcall, 1997).

Note that if all of these uncertainties are exploited, the predicted neutrino flux of a SSM can be reduced to 4.70 SNUs for the ^{37}Cl experiment, and to 103 SNUs for the ^{71}Ga experiments, closer to the observed values of 2.55 ± 0.25 (Cleveland *et al.*, 1998) and 77.5 ± 6.2 (Hampel *et al.*, 1999). Neuforge-Verheecke *et al.* (2000) find that these changes do not affect the structure of the radiative interior enough to destroy the agreement with helioseismology. The sound speed differences between these four models of Table II are at most 0.4% at the convection zone base, and 0.2% in the core, much smaller than the 6–10 % discrepancies of the mixed core models presented above. Most of the larger sound speed discrepancy (0.3% out of 0.4%) at the convection zone base is due to the reduced CNO abundances of the GS98 mixture compared to the GN93 mixture, and could be compensated by modifications to opacities or diffusion treatments within the limits of uncertainties.

6. Conclusions

The standard solar model agrees quite well with helioseismic tests, leaving little necessity or margin for the changes in interior structure of non-standard models proposed to solve the neutrino problem. Minor changes to model physics within the scope of standard models (e.g., taking advantage of uncertainties in abundance determination, nuclear reaction rates, neutrino capture rates, and opacities) can significantly reduce the predicted neutrino flux without violating the constraints of helioseismology. However, these changes are not sufficient to solve the neutrino problem if consistency between solar neutrino experiment results is required, since

TABLE II

Properties of standard evolution models of Neuforge-Verheecke *et al.* (2000). ‘Center’ means that the nuclear reaction rates have been taken at the center of their error bars, whereas ‘limit’ means that the following rates have been stretched to the limit of these error bars: ^{1}H(p, $\nu_e e^+$)^{2}H × 1.05, ^{3}He(^{3}He, $2p$)^{4}He × 1.06, ^{3}He(^{4}He, γ)^{7}Be × 0.84, ^{7}Be(p, γ)^{8}B × 0.895. X_0, Y_0, and Z_0 are the initial hydrogen, helium and heavy element mass fraction. All the other quantities are for the present Sun. The values of the total event rates for the lower limits (3σ) of the neutrino capture cross sections are indicated in parentheses.

Model	1	2	3	4
mixture	GN93	GS98	GS98	GS98
Z/X	0.0245	0.0230	0.0230	0.0208
rates	center	center	limit	limit
X_0	0.7100	0.7067	0.7066	0.7134
Y_0	0.2703	0.2747	0.2750	0.2698
Z_0	0.0197	0.0186	0.0184	0.0168
α	1.7738	1.7346	1.7457	1.7144
Z/X	0.0245	0.0232	0.0230	0.0208
T_{central} (10^6 K)	15.66	15.68	15.56	15.48
ρ_{central} (g cm^{-3})	152.2	152.7	149.9	149.2
$R_{\mathrm{convection\ zone\ base}}$ ($R_\odot$)	0.7135	0.7158	0.7149	0.7175
$Y_{\mathrm{convection\ zone}}$	0.2408	0.2444	0.2454	0.2404
Event rates for the chlorine experiment				
$\sigma\phi_{^7\mathrm{Be}}$	1.14	1.17	0.93	0.89
$\sigma\phi_{^8\mathrm{B}}$	6.04	6.24	4.11	3.72
$\sum(\phi\sigma)_i$ ^{37}Cl (3σ)	7.85 (7.16)	8.05 (7.35)	5.63 (5.15)	5.15 (4.70)
Event rates for the gallium experiment				
$\sigma\phi_{pp}$	69.7	69.6	70.9	71.3
$\sigma\phi_{^7\mathrm{Be}}$	34.1	34.8	27.7	26.5
$\sigma\phi_{^8\mathrm{B}}$	12.7	13.1	8.66	7.84
$\sum(\phi\sigma)_i$ ^{71}Ga (3σ)	128.8 (113.4)	129.3 (113.7)	117.8 (104.8)	114.9 (102.6)

the larger deficiency of ^{7}Be neutrinos compared to ^{8}B neutrinos cannot be achieved by simply lowering the Sun’s central temperature.

Preliminary calculations of non-standard models with mixed cores and non-local equilibrium abundaces of ^{3}He can reduce the ^{7}Be/^{8}B neutrino flux ratio as well as the ^{8}B neutrino flux, but the agreement between calculated and observed low-degree oscillation frequencies is unsatisfactory. Additional parameter studies, varying the ^{3}He and ^{4}He core profile, and the spatial extent of mixing are in progress to determine to what extent core mixing can even partially alleviate the neutrino problem. Finding such a model would also have implications for the parameter space of neutrino mass difference and mixing angle of a matter-induced or vacuum neutrino oscillation solution.

Acknowledgements

The authors would like to acknowledge Wick Haxton, Douglas Gough, Sasha Kosovichev, Yvonne Elsworth, and Sylvaine Turck-Chièze for important discussions. We thank the referees and *Solar Physics* Editor Jack Harvey for helpful advice and suggestions. We gratefully acknowledge support from Los Alamos National Laboratory IGPP Grant #922, and NASA Astrophysics Theory Program Grant S-30934-F.

References

Adelberger, E. et al.: 1998, *Rev. Mod. Phys.* **70**, 1265.
Alexander, D. R. and Ferguson, J. W.: 1994, *Astrophys. J.* **437**, 879.
Allen, C. W.: 1973, *Astrophysical Quantities*, 3rd edition, Athlone Press, London, p. 169.
Anderson, E. *et al.*: 1996, in J. Provost and F.-X. Schmieder (eds.), *Sounding Solar and Stellar Interiors*, IAU, p. 151.
Angulo, C. et al.: 1999, *Nucl. Phys.* **A656**, 3.
Antia H. M. and Chitre, S. M.: 1997, *Monthly Notices Royal Astron. Soc.* **289**, L1.
Balachandran, S. C. and Bell, R. A.: 1998, *Nature* **392**, 791.
Bahcall, J. N.: 1995, *Nucl. Phys.* **B43**, 41.
Bahcall, J. N.: 1996, *Astrophys. J.* **467**, 475.
Bahcall, J. N.: 1997, *Phys. Rev.* **C56**, 3391.
Bahcall, J. N. and Ulrich, R. K.: 1998, *Rev. Mod. Phys* **60**, 297.
Bahcall, J. N., Basu, S., and Kumar, P.: 1997, *Astrophys. J.* **485**, L91.
Bahcall, J. N., Pinsonneault, M. H., and Wasserburg, G. J.: 1995, *Rev. Mod. Phys.* **67**, 781.
Bahcall, J. N. *et al.*: 1996, *Phys. Rev.* **C54**, 411.
Bahcall, J. N., Pinsonneault, M. H., Basu, S., and Christensen-Dalsgaard, J.: 1997, *Phys. Rev. Lett.* **78**, 171.
Basu, S.: 1998, *Monthly Notices Royal Astron. Soc.* **298**, 719.
Basu, S., Pinsonneault, M. H., and Bahcall, J. N.: 2000, *Astrophys. J.* **529**, 1084.
Böhm-Vitense, E.: 1958, *Z. Astrophys.* **45**, 114.
Boothroyd, A. I., Sackmann, I.-J., and Fowler, W. A.: 1991, *Astrophys. J.* **377**, 318.
Brown, T. M. and Christensen-Dalsgaard, J.: 1998, *Astrophys. J.* **500**, L195.
Brun, A. S., Turck-Chièze, S., and Morel, P.: 1998, *Astrophys. J.* **506**, 913.
Brun, A. S., Turck-Chièze, S., and Zahn, J.-P.: 1999, *Astrophys. J.* **525**, 1032.
Canuto, V. M. and Mazzitelli, I.: 1991, *Astrophys. J.* **370**, 295.
Canuto, V.M. and Mazzitelli, I.: 1992, *Astrophys. J.* **389**, 724.
Canuto, V. M., Goldman, I., and Mazzitelli, I.: 1996, *Astrophys. J.* **473**, 550.
Caughlan, G. R. and Fowler, W. A.: 1988, *Atomic Nucl. Data Tables* **40**, 283.

Chaplin, W. J., Elsworth, Y., Isaak, G. R., Lines, R., McLeod, C. P., Miller, B. A., and New, R.: 1996, *Monthly Notices Royal Astron. Soc.* **282**, L15.
Chaplin, W.J. *et al.*: 1998, *Monthly Notices Royal Astron. Soc.* **300**, 1077.
Christensen-Dalsgaard, J.: 1992, *Astrophys. J.* **385**, 354.
Christensen-Dalsgaard, J.: 1996, in T. Roca Cortes and F. Sanchez (eds.), *Proc. VI IAC Winter School, The Structure of the Sun*, Cambridge University Press, Cambridge, p. 47.
Christensen-Dalsgaard, J. 1998, in A. V., Olinto, J. E., Frieman, and D. N., Schramm (eds.), *Proc. 18th Texas Symposium on Relativistic Astrophysics*, World Scientific Press, Singapore, p. 694.
Christensen-Dalsgaard, J. and Däppen, W.: 1992, *Astron. Astrophys. Rev.* **4**, 267.
Christensen-Dalsgaard, J., Gough. D. O., and Thompson, M. J.: 1991, *Astrophys. J.* **378**, 413.
Christensen-Dalsgaard, J. *et al.*: 1996, *Science* **272**, 1286.
Clayton, D. D., Newman, M. J., and Talbot, R. J., Jr.: 1975, *Astrophys. J.* **201**, 489.
Cleveland, B. T. *et al.*: 1998, *Astrophys. J.* **496**, 505.
Cox, A. N., Guzik, J. A., and Kidman, R. B.: 1989, *Astrophys. J.* **342**, 1187.
Cox, A. N., Guzik, J. A., and Raby, S.: 1990, *Astrophys. J.* **353**, 698.
Cumming, A. and Haxton, W. C.: 1996, *Phys. Rev. Lett.* **77**, 4286.
Däppen, W. and Guzik, J. A.: 2000, in C. Ibanoglu (ed.), *Variable Stars as Essential Astrophysical Tools*, Kluwer Academic Publishers, Dordrecht, Holland, p. 177.
Däppen, W., Mihalas, D., Hummer, D. G., and Mihalas, B. W.: 1988, *Astrophys. J.* **332**, 261.
Davis, R., Jr., Harmer, D. S., and Hoffman, K. C.: 1968, *Phys. Rev. Lett.* **20**, 1205.
Dilke, F. W. W. and Gough, D. O.: 1972, *Nature* **240**, 262.
Eggleton, P. P., Faulkner, J., and Flannery, B. P.: 1973, *Astron. Astrophys.* **23**, 325.
Fritts, D. C., Vadas, S. L., and Andreassen, O.: 1998, *Astron. Astrophys.* **333**, 343.
Gabriel, M.: 1997, *Astron. Astrophys.* **327**, 771.
Gerard, J.-C., Hauglustaine, D. A. and Francois, L. M.: 1992, *Palaeogeography, Palaeoclimatology, Palaeoecology* **91**, 133.
Gilliland, R., Faulkner, J., Press, W. H., and Spergel, D. N.: 1986, *Astrophys. J.* **306**, 703.
Graedel, T. E., Sackmann, I.-J., and Boothroyd, A. I.: 1991, *Geophys. Res. Lett.* **18**, 1881.
Gough, D. O.: 1999, *Nucl. Phys. B* **77**, 81.
Gough, D. O. *et al.*: 1996, *Science* **272**, 1296.
Grevesse N. and Noels, A.: 1993, in N. Prantzos, E. Vangioni-Flam, and M. Cassé (eds.), *Origin and Evolution of the Elements*, Cambridge University Press, Cambridge, p. 15.
Grevesse, N. and Sauval, A. J.: 1998, *Space Sci. Rev.* **85**, 161.
Guenther, D. B.: 1989, *Astrophys. J.* **339**, 1156.
Guenther, D. B. and Demarque, P.: 1997, *Astrophys. J.* **484**, 937.
Guzik, J. A. and Cox, A. N.: 1995, *Astrophys. J.* **448**, 905.
Guzik, J. A. and Swenson, F. J.: 1997, *Astrophys. J.* **491**, 967.
Hampel, W. *et al.*: 1999, *Phys. Lett.* **B447**, 127.
Haxton, W. C.: 1998, *Prog. Particle Nucl. Phys.* **40**, 101.
Hummer, D. G. and Mihalas, D.: 1988, *Astrophys. J.* **331**, 794.
Iglesias, C.A. and Rogers, F. J.: 1996, *Astrophys. J.* **464**, 943.
Kurucz, R. L.: 1992, *Rev. Mex. Astron. Astrof.* **23**, 181.
Mihalas, D., Däppen, W., and Hummer, D. G.: 1988, *Astrophys. J.* **331**, 815.
Mikheyev, S. P. and Smirnov, A. Yu.: 1986, *Nuovo Cimento* **9C**, 17.
Mikheyev, S. P. and Smirnov, A. Yu.: 1986, *Prog. Part. Nucl. Phys.* **23**, 41.
Morel, P. and Schatzman, E.: 1996, *Astron. Astrophys.* **310**, 982.
Morel, P., Provost, J., and Berthomieu, G.: 1997, *Astron. Astrophys.* **327**, 349.
Neuforge, C.: 1993, *Astron. Astrophys.* **274**, 818.
Neuforge-Verheecke, C., Goriely, S., Guzik, J. A., Swenson, F. J., and Bradley, P. A.: 2000, *Astrophys. J.*, in press.
Pesnell, W. D.: 1990, *Astrophys. J.* **363**, 227.

Richard, O. and Vauclair, S.: 1997, *Astron. Astrophys.* **322**, 671.
Richard, O., Vauclair, S., Charbonnel, C., and Dziembowski, W. A.: 1996, *Astron. Astrophys.* **312**, 1000.
Rogers, F. J., Swenson, F. J., and Iglesias, C. A.: 1996, *Astrophys. J.* **456**, 902.
Rouse, C. A.: 2000, in O. Manuel (ed.), *The Origin of the Elements in the Solar System*, Kluwer Academic Publishers, Dordrecht, Holland, in press.
Schatzman, E.: 1996, *Solar Phys.* **169**, 245.
Schou, J. and Tomczyk, S.: 1996, m2 table, `http://www.hao.ucar.edu/public/research/mlso/LowL/data.html`
Schou, J., Kosovichev, A. G., Goode, P. R. and Dziembowski, W.: 1997, *Astrophys. J.* **489**, L197.
Seaton, M. J., Yan, Y., Mihalas, D., and Pradhan, A. K.: 1994, *Monthly Notices Royal Astron. Soc.* **266**, 805.
Spiegel, E. A. and Zahn, J.-P.: 1992, *Astron. Astrophys.* **265**, 106.
Swenson, F. J. and Faulkner, J.: 1992, *Astrophys. J.* **395**, 654.
Thoul, A. A., Bahcall, J. N., and Loeb, A.: 1994, *Astrophys. J.* **421**, 828.
Turcotte, S. and Christensen-Dalsgaard, J.: 1998, in *Proceedings of the SOHO6/GONG98 Workshop: Structure and Dynamics of the Interior of the Sun and Sun-like Stars*, ESA SP-418, p. 561.
Willson, R. C., Hudson, H. S., Frolich, C., and Brusa, R. W.: 1986, *Science* **234**, 1114.
Wolfenstein, L.: 1978, *Phys. Rev.* **D17**, 2369.
Wolfenstein, L.: 1979, *Phys. Rev.* **D20**, 2634.