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LIMITS ON DEFENSES IN INTERACTIONS BETWEEN DISPARATE FORCES

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LIMITS ON DEFENSES IN INTERACTIONS BETWEEN DISPARATE FORCES

Gregory H. Canavan

Strong sides can deploy modest defenses without loss of stability, if they have strong preferences for the survival of high value targets and the weak has some survivable weapons. The maximum stable defense is largely determined by the weak side's survivable forces and the strong sides high value targets and preference for their survival.

This note studies the sensitivity of strike incentives to defenses for interactions between a strong side with defenses and a weak side with none using the exchange and cost models derived and discussed in a companion report[1] and force levels derived for trilateral interaction with the small weak forces of greatest concern from a stability perspective discussed in an earlier paper.[2] The conclusion is that the strong side can deploy modest defenses without loss of stability, if it strongly prefers the survival of its own high value targets and the weak side has some survivable weapons. The maximum number of defenses that can be deployed without exchange is largely determined by the weak side's survivable forces and the strong sides high value targets and preference for their survival. Allowable defenses could be increased by larger preferences for survival of high value, fewer high value targets, and more survivable weak force. The first act by making the strong side more reluctant to accept a given level of retaliation.

Parameters. The principal inputs to the calculations are forces and preferences. It is assumed that the strong side U has START III-like forces of 2,000 weapons, of which half are survivable, and from 0 to 16 ideal defensive interceptors. The weak side T has 15 targetable single weapon missiles and 5 non-targetable single weapon missiles for a total of 20 strategic weapons.

U's objectives are represented by three preferences (L, K, V). L is U's usual preference for damage to T's military relative to preventing damage to its own military targets. K is U's preference for survival of its own high value targets relative to the survival of its own military targets. V is U's preference for damaging P's high value targets relative to the survival of its own military targets. Thus, V/L is U's relative preference for damaging high value rather than military targets.

None of these parameters are known with precision. It is generally assumed that a non-aggressive strong side can be characterized by $L \approx 0.1$ to 0.25 and that a more aggressive sides is represented by larger L. If U places high value on the survival of its high value or urban targets, it should have $K >> 1$. If U is not interested in damaging the other's high value, V should be small. For nominal conditions, it is assumed below that U has preferences (0.25, 20, 0.1), i.e., that it is relatively non-aggressive, much more concerned about the survival of its own high value, and not interested in destroying T's. T is taken to be more aggressive, have less concern for survival of its high value targets, and be more interested in inflicting maximum damage on U's high value; thus, it is assumed to have preferences (0.5,1,1). The effect of variations about these nominal parameters is discussed below.

Through exchanges and the minimization of first strike costs, these preferences determine the two sides' allocations of first and second strikes to missiles, military, and high value targets. That process and typical results are discussed in the earlier report; here it suffices to recall that (f, g, h) = fraction of U's (1st strike on missiles, second strike on military targets, 1st strike on military targets).

Results. Figure 1 shows the allocations of weapons to target sets as a function of I, the number of U interceptors. U's allocation to missiles is $f \approx 0.7$ for small I, falling to ≈ 0.25 by $I = 8$ as it shifts form counterforce to countervalue targeting when its defenses can address T's restrike with less prior suppression. U's allocation of its second strike to military targets is $g \approx 0.84$ for all I, which means that U's 2nd strike S is primarily on military facilities. U's allocation of its first strike to military targets, h, increases from $\approx$ 0.75 to about 0.85 by $I = 8$, where U's strong defenses allow it to shift more of its 1st strike to T's missiles.

T's allocations to military targets are essentially zero throughout: f' is zero because attempting to suppress U's missiles is not effective; $g' \approx 0$ throughout, as T does not find it useful to allocate any of its 2nd strike on military targets; and h' is always 0, as T does not find it effective to allocate any of its 1st strike to military targets. Thus, T's strikes are essentially all on high value targets throughout.

Figure 2 shows their strikes if U strikes first. The top curve is U's first strike on military targets, which is about 680 weapons for I small and increases to about 1,920 for I

> 8. The second curve is U's first strike on T's high value targets, which increases from about 210 to 330 weapons. The third is T's second strike on U's high value, which is T's whole delivered strike, as f' = g' = 0. It is about 5.3 weapons at I = 0, falling to 0 by I = 6. T's second strike on U's military value is essentially zero.

Figure 3 shows their strikes if T strikes first. The top curve is U's 2nd strike on military targets, which is about 2,500 weapons for all I. The second curve is U's 2nd strike on T's high value, which is about 480 weapons. U's strike on military value is larger than that on high value targets because U can saturate the latter with a modest number of weapons for this $1/v' \approx 100$ high value target set. U's 2nd strikes are constant because the optimal allocation problem U faces is not changed by its own defenses. The third curve is T's 1st strike on U's high value, which is T's whole 20 weapons at I = 0 and drops monotonically to ≈ 4 at I = 16. T's 1st strike on U's military value is ≈ 0.

Figure 4 shows the components of U's cost for striking first, C_1, which is the top curve. For small I, it is dominated by cost of damage to its high value targets, C_{1v}, which in turn is mostly due to retaliatory damage by T's 5.3 surviving missiles to U's high value targets, C_{1vs}, which contributes ≈ 1.03 to cost. There is also a 0.12 contribution from incomplete damage to T's $1/k' \approx 1{,}000$ military targets, which are more numerous and difficult to saturate. There is a ten-fold smaller contribution from incomplete damage to T's high value targets, which can be saturated with fewer weapons, and a thousand-fold contribution from damage to U's military targets, which are targeted only by leakage. At large I, the contribution of the two dominant terms is reversed, with incomplete damage to T's military contributing about 0.036 and damage to U's high value contributing about 0.024, although their sum $C_1 \approx 0.06$ is small.

U's cost for striking second drops from ≈ 3.7 at I = 0 to 0.8 at I = 16, largely do to the decrease in the cost of damage to U's own high value targets, C_{2vs}, as T's second strikes are strongly suppressed by large defenses.

Figure 5 shows the components of the cost to T for striking first, C_1', which is the top curve. It increases with I largely due to the increase in the cost of incomplete damage to U's high value targets, which in turn is due to T's reduced retaliatory capability against U's improving defenses. Since $C_1' \approx 3.3 >> L' + V' = 1.5$ for all I, T would see no incentive to strike unless provoked. In $4 < I < 8$, T's second strike costs due to damage to its own military and high value targets increase strongly due the the shifts discussed above; overall, C_2' increases from 2.8 to 3.3.

Figure 6 shows the costs of nodes 1 & 2, where T is the decision maker. Node 1 is a strike by U followed by a restrike by T, for which U's cost decreases strongly and T's increase $\approx$ 10% as I increases from 0 to 8. The cost to U of striking first at node 1 falls below $C_1 = L + V = 0.25 + 0.1 = 0.35$ at $I \approx 4$, which gives it an incentive to strike for larger I whenever it can. T does not have an incentive to strike at node 2, which produces the costs of inaction (L + V, L' + V') = (0.35, 1.5) seen. Figure 7 shows that the consequence of C_1 falling below $L + V = 0.35$ at $I > 4$ is that it gives U an incentive to strike at node 5, which decreases U's cost by about a factor of six but roughly doubles T's already much larger cost.

Figure 8 shows the costs of nodes 3 & 4, where P is the decision maker. Node 3 is a strike by T followed by a restrike by U, for which T's 1st strike cost increases slightly and U's 2nd strike cost decreases strongly with I, although it is not a primary decision variable. At node 4, U has an incentive to strike first for $I > 4$ for the reasons shown in the previous figure. Note that T's cost for striking first is less than that for restriking only for but I = 8 to 10 interceptors, although this difference is slight and subject to details of the calculation.

Figure 9 shows the costs at node 6, where T decides by choosing the minimum of its costs from nodes 3 & 4. For $I \leq 4$, Fig. 8 shows that the minimum of C_1' and L' + V' is L' + V', so T does not strike. However, for $I > 4$, U's incentive to strike forces T to preemption. At I = 6, T's cost for striking first at node 3 is greater than its cost for restriking after node 4, so it does not act at node 6, which allows U to strike at node 4. For I from 8 to 10, T's cost for striking first at node 3 is less than its cost for being struck at node 4, so it preempts at node 6, which slightly reduces its cost but increases U's to C_2, which is very large. The jump in U's cost is $\approx C_2 \approx 2.6$. For larger I, C1' > L' + V', so T does not strike at node 4, which allows U to strike at node 4.

U's cost at node 6 is constant for $I \leq 4$, falls when it strikes at I = 6, increases as it is preempted at I = 8-10, and falls again as it strikes at larger I. T's costs have a simpler shape, being constant for $I \leq 4$ and rising monotonically for larger I, but they conceal the sequence of inaction, restrike, preemption, and restrike complementary to that discussed above for U.

Figure 10 shows the first strike probability u-weighted costs at node 7, assuming that the two sides have equal *a priori* probabilities of being able to strike first. Because both nodes 5 and 6 have strong interaction for I > 4, that is also reflected in the two side's costs after averaging for any first strike probability u. U striking first at I = 6 decreases its cost but greatly increases T's, while T's preemption in I = 8 to 10 slightly decreases its cost but greatly increasing U's. While U could reduce its expected costs significantly by deploying > 12 interceptors, T's ability to preempt in I = 8-10 presents a barrier to to such a deployment, unless U can jump discontinuously over the barrier. Even then, the cost reduction would be achieved by U striking first and using its defenses to negate P's restrike, which would involve strikes large on the scale of T's forces.

Analysis. U's strike incentive emerges at node 2, where C_1 falls below L + V at I slightly greater than 4, which produces U's strike incentive to strike and hence T's incentive to preempt. It is useful to examine the magnitudes of the terms in C_1 near I = 4. P does not strike military value, so $C_{1ms} \approx 0$. $C_{1vo} \approx 0.011$, and $C_{1mo} \approx 0.1$. At transition, where $C_1 = L + V$, $C_{1vs} \approx C_1 - C_{1mo} \approx 0.35 - 0.1 \approx 0.25$, so that $C_{1vs} \approx K(1 - e^{-vS'}) \approx KvS'$ $\approx Kv(nN - I_{tnx})$, since few vulnerable missile survive. Thus,

$$I_{tnx} \approx n'N' - 0.25/Kv \approx 5 - 0.25/(20 \times 0.01) \approx 5 - 1.25 \approx 4, \qquad (1)$$

in approximate accord with the transitional value in Figs. 4 and 6. I_{tnx} depends directly on the number of P's survivable weapons, n'N', so if T has no non-targetable weapons, no number of interceptors produces stability. I_{tnx} depends on U's preferences for damaging military and high value targets through the numerator of the second term. It also depends strongly on the denominator Kv. If K ≈ unity, i.e., U was less concerned with the survival of its high value targets, the second term would be on the order of 25, and I_{tnx} would be less than zero. Solving for the K that gives $I_{tnx} = 0$ gives

$$K = 0.25/v\, n'N' \approx 0.25/(0.01 \times 5) \approx 5 \qquad (2)$$

for the parameters used in the calculations above. Smaller values do not permit defenses; larger values do. These observations depend somewhat on the size of U's forces, as smaller forces alter the magnitudes of the terms ignored in deriving the approximate expressions of Eqns. (1) and (2).

Summary and conclusions. A strong side can deploy modest defenses without loss of stability if it has a strong preference for the survival of its own high value targets and the weaker side has some survivable weapons. The size of the allowable defenses could be increased by larger preferences for survival of high value targets, K, which cause U not to strike out of self deterrence. Larger values of v, i.e., fewer high value targets, make is possible for T to deter U with fewer penetrating weapons. More survivable T weapons would which directly raise the transitional number of intereptors.

References

[1] G. Canavan, "Cost of Addressing Targets of Unequal Value," Los Alamos National Laboratory Report LA-UR-01-draft, June 2001.

[2] G. Canavan, "Analysis of Decisions in Bi- and Tri-Lateral Engagements," U.S. *State Department Stability Workshop* (Institute for Defense Analysis, November 2000); Los Alamos National Laboratory Report LA-UR-00-5737, November 2000.

Fig. 1. Fractional allocations to target sets

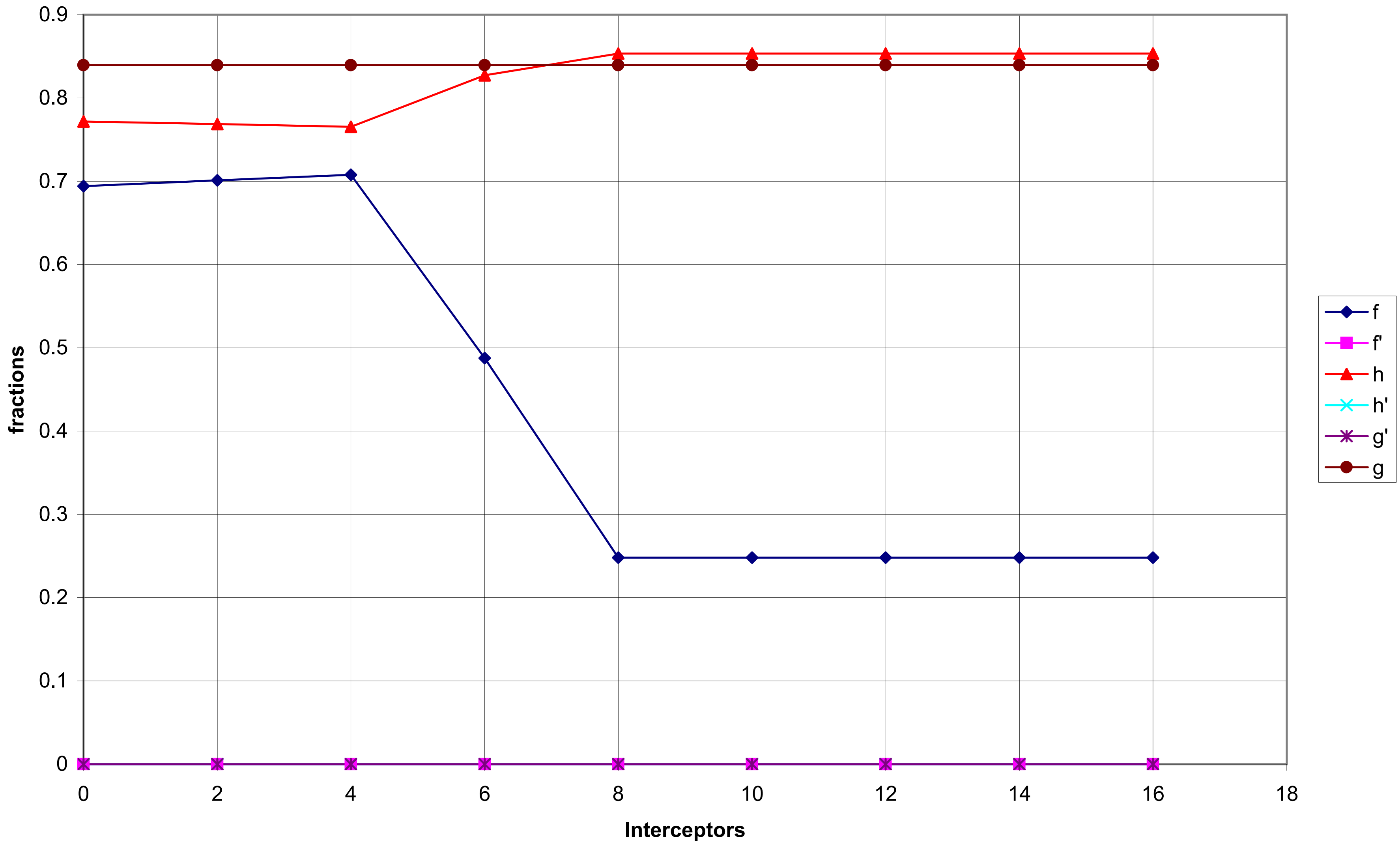


Fig. 2. First and second strikes if U strikes first

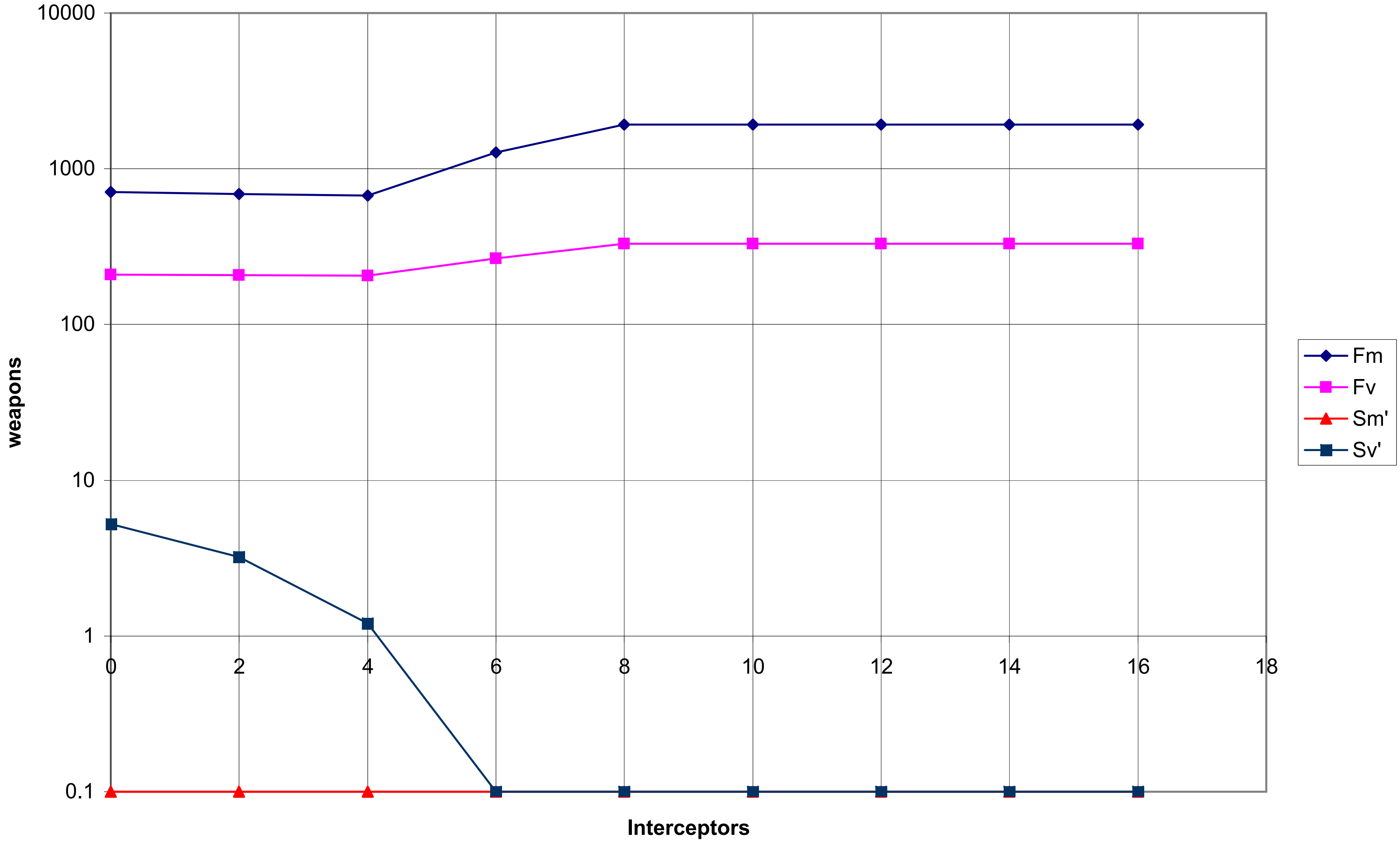


Fig. 3. First and second strikes if P strikes first

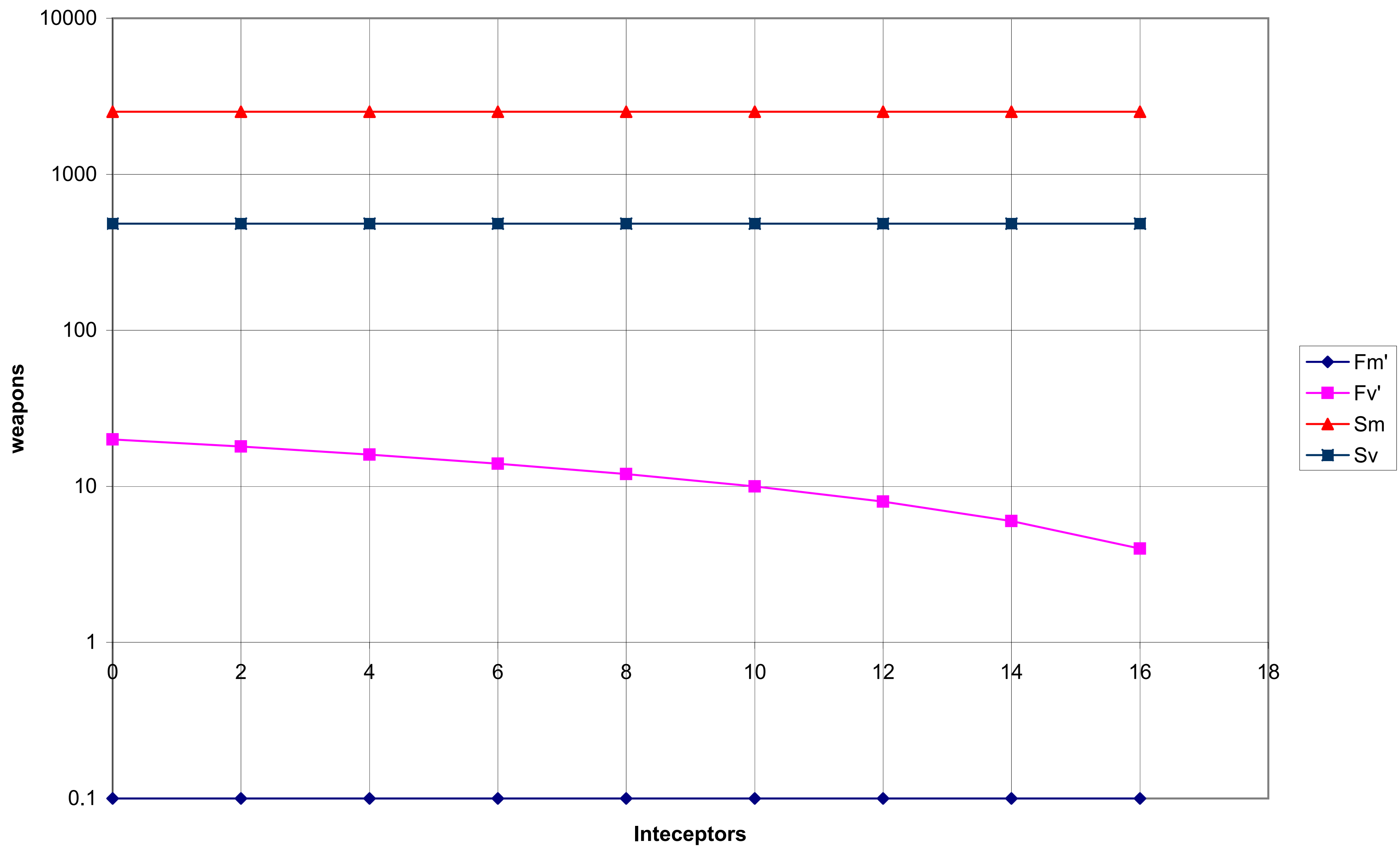
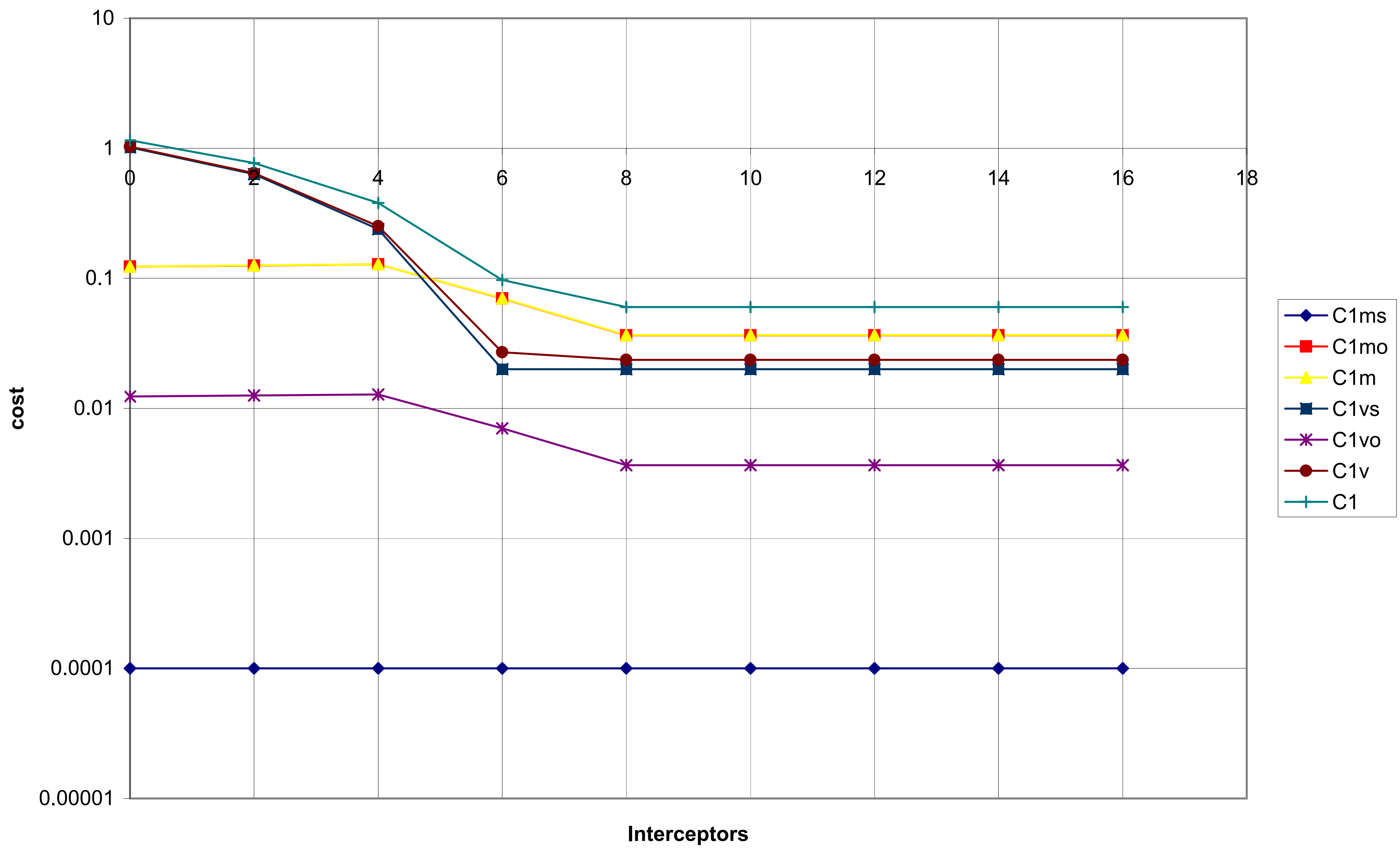
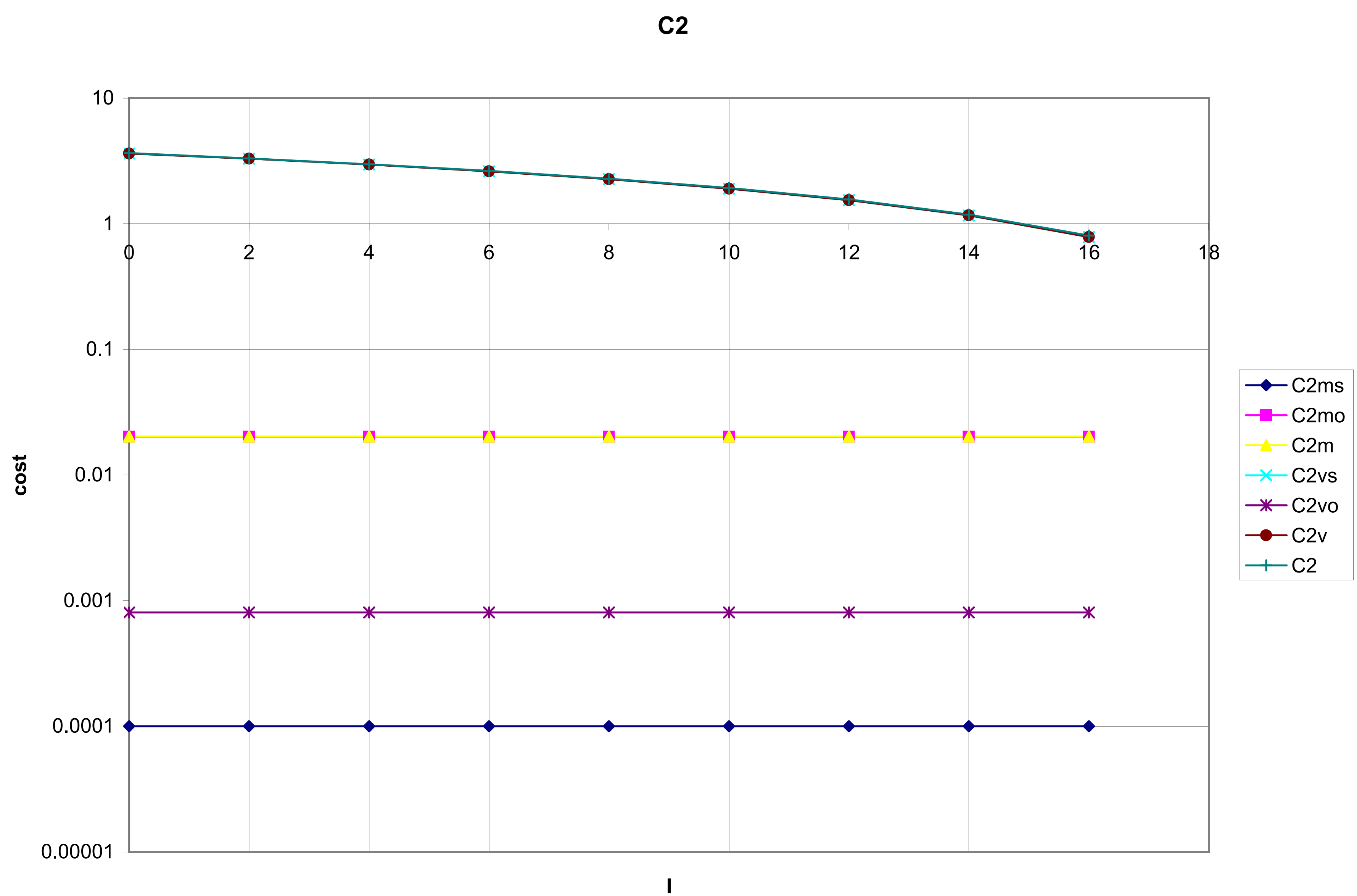


Fig. 4. Cost to U of striking first, C1





C2
10
1
0.1
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0.0001
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4
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18
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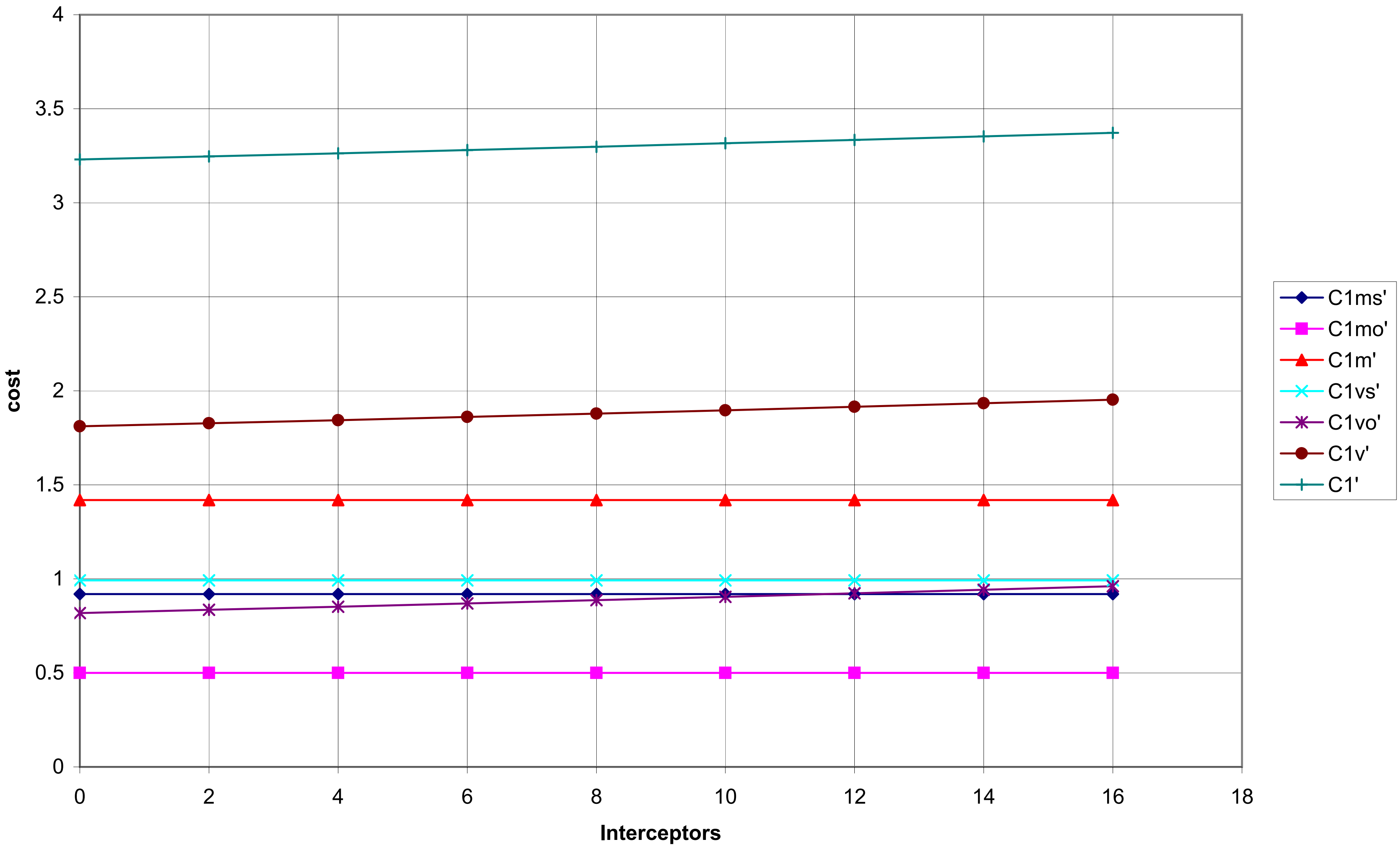
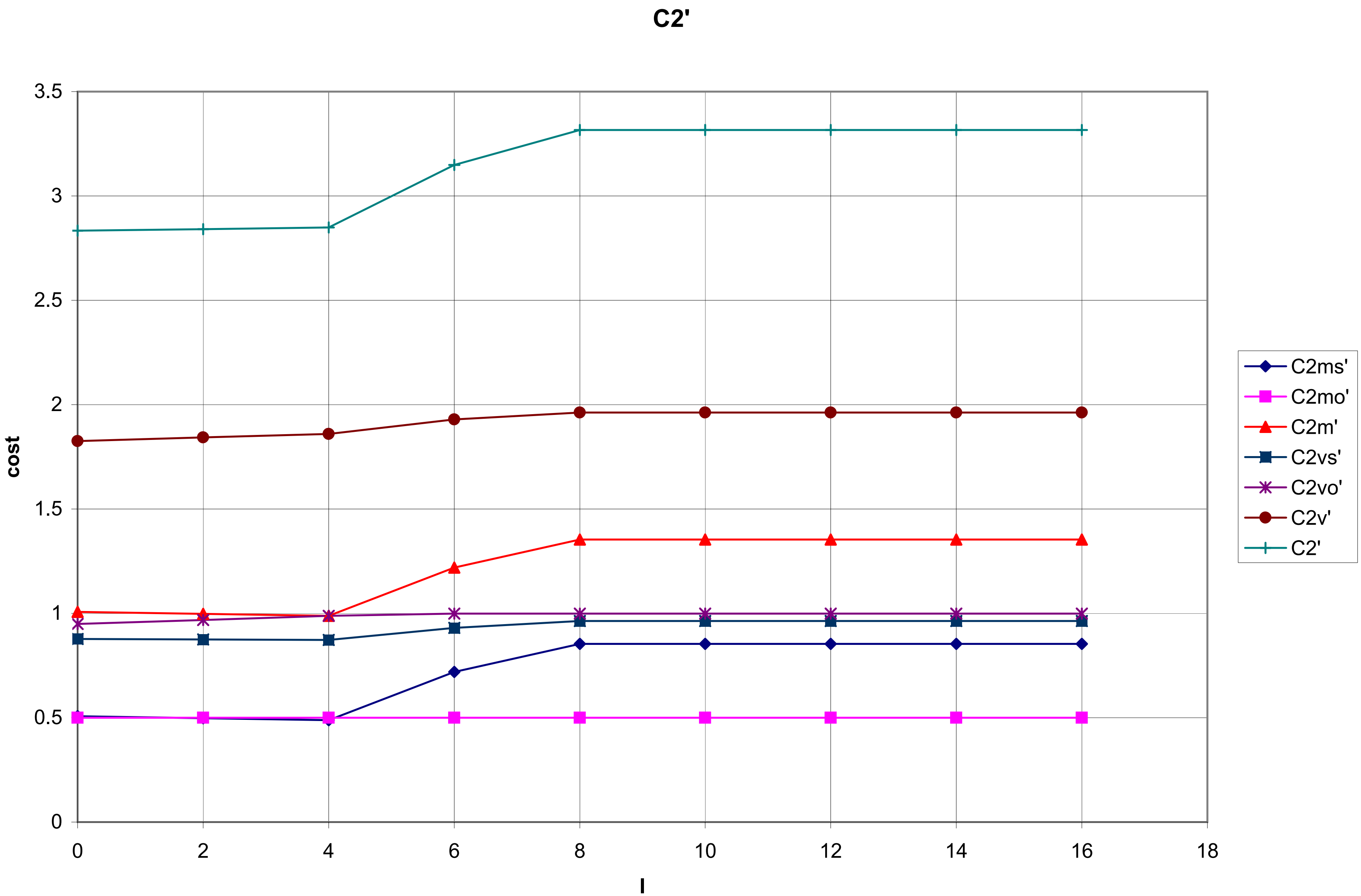
C2vo
C2v
C2

Fig. 5. Cost to P for striking first, C1'




C2'
cost
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1
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3.5
0
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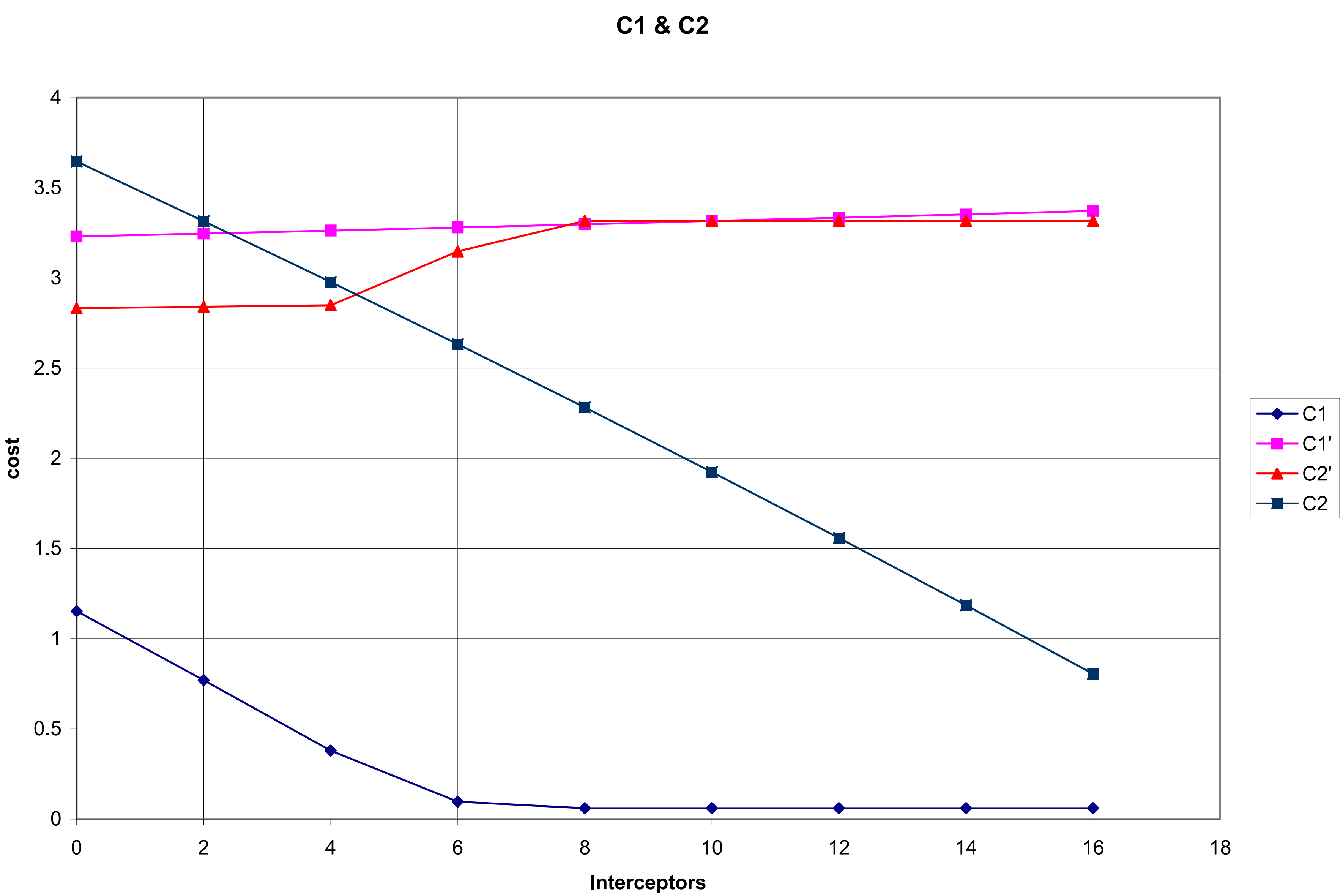
C2v'
C2'


C1 & C2
cost
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3
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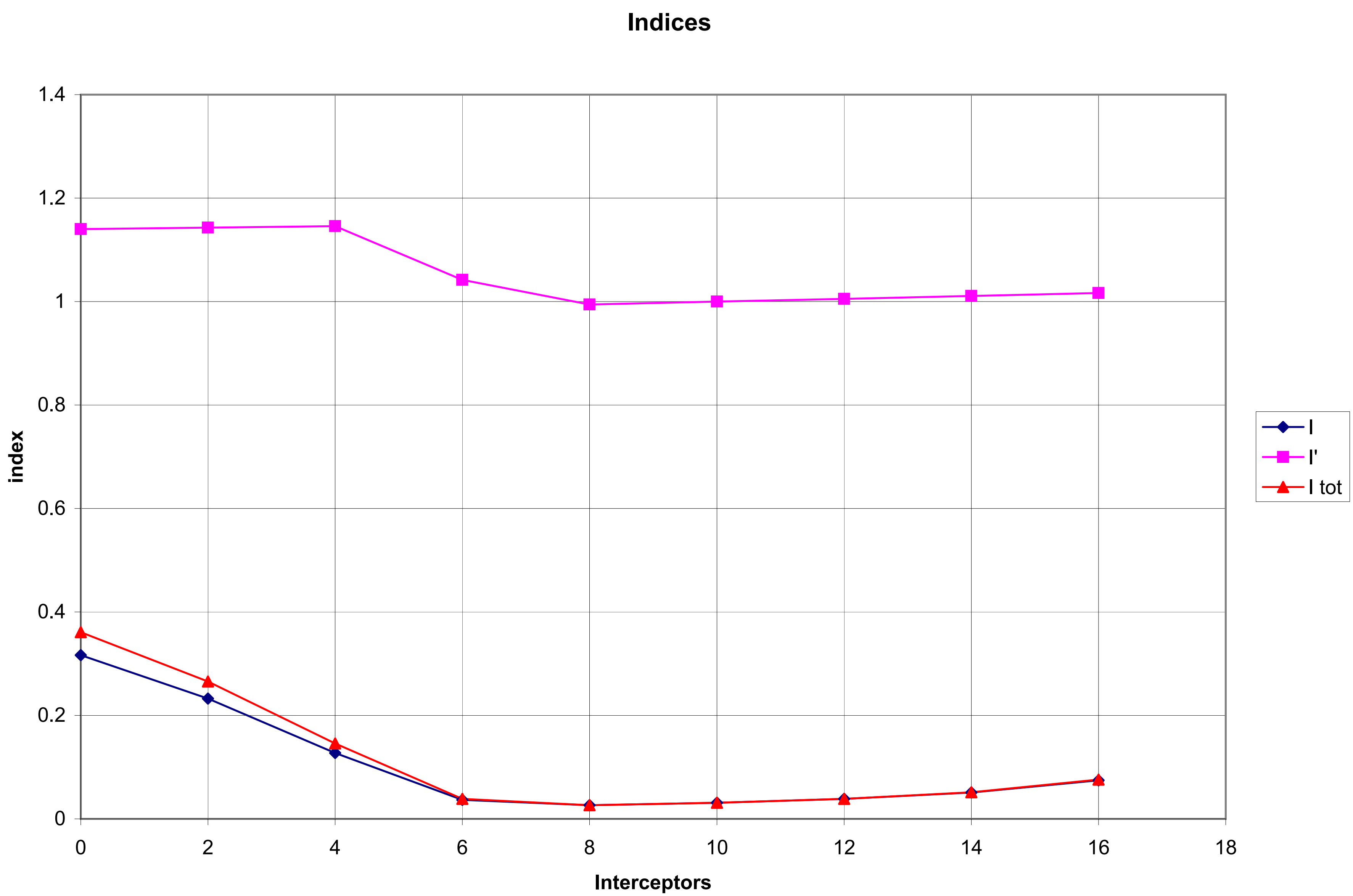
Interceptors
C1
C1'
C2'
C2



Indices
index
Interceptors
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1.2
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Fig. 6. Costs of nodes 1 & 2

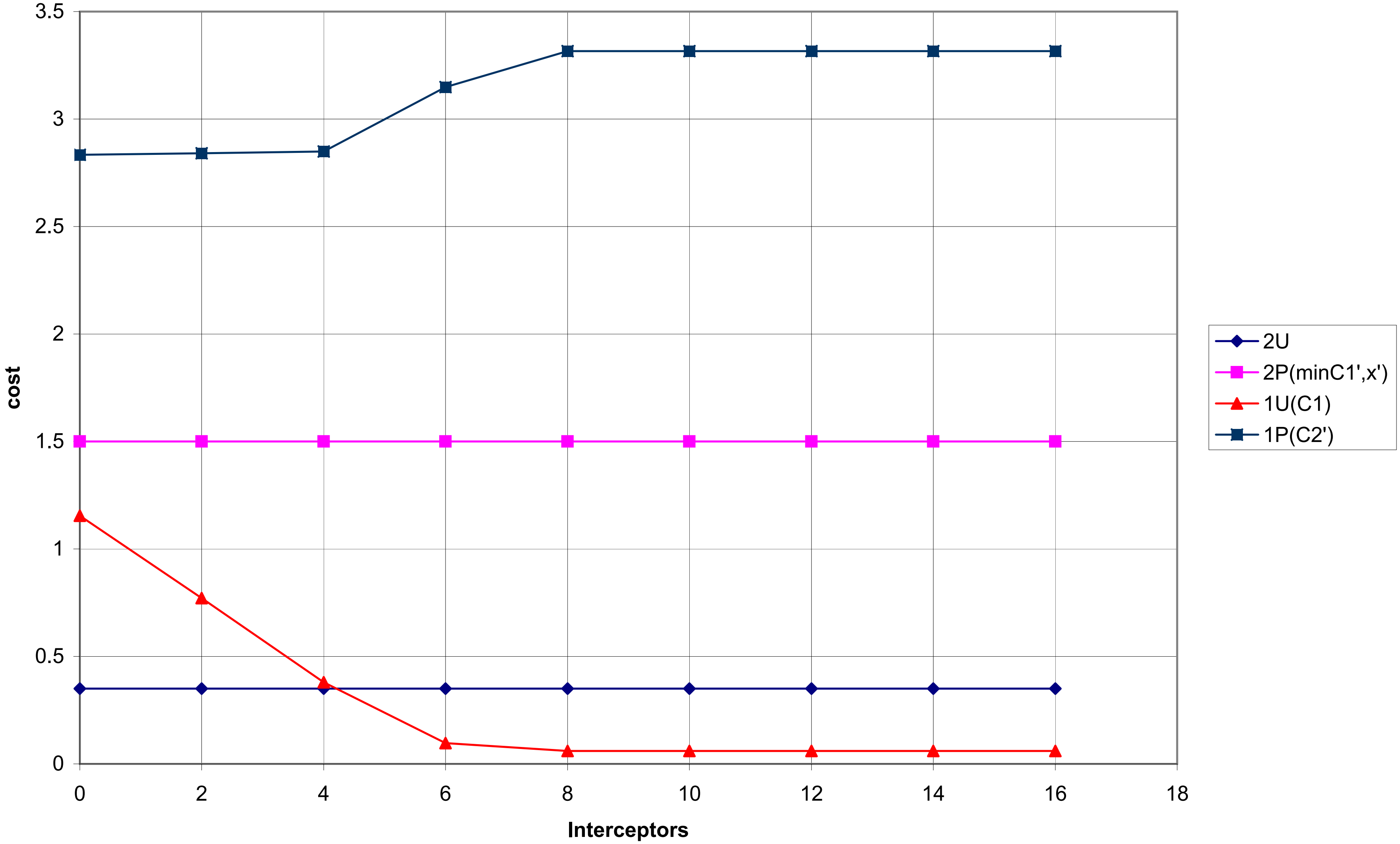


Fig. 7. Cost of node 5

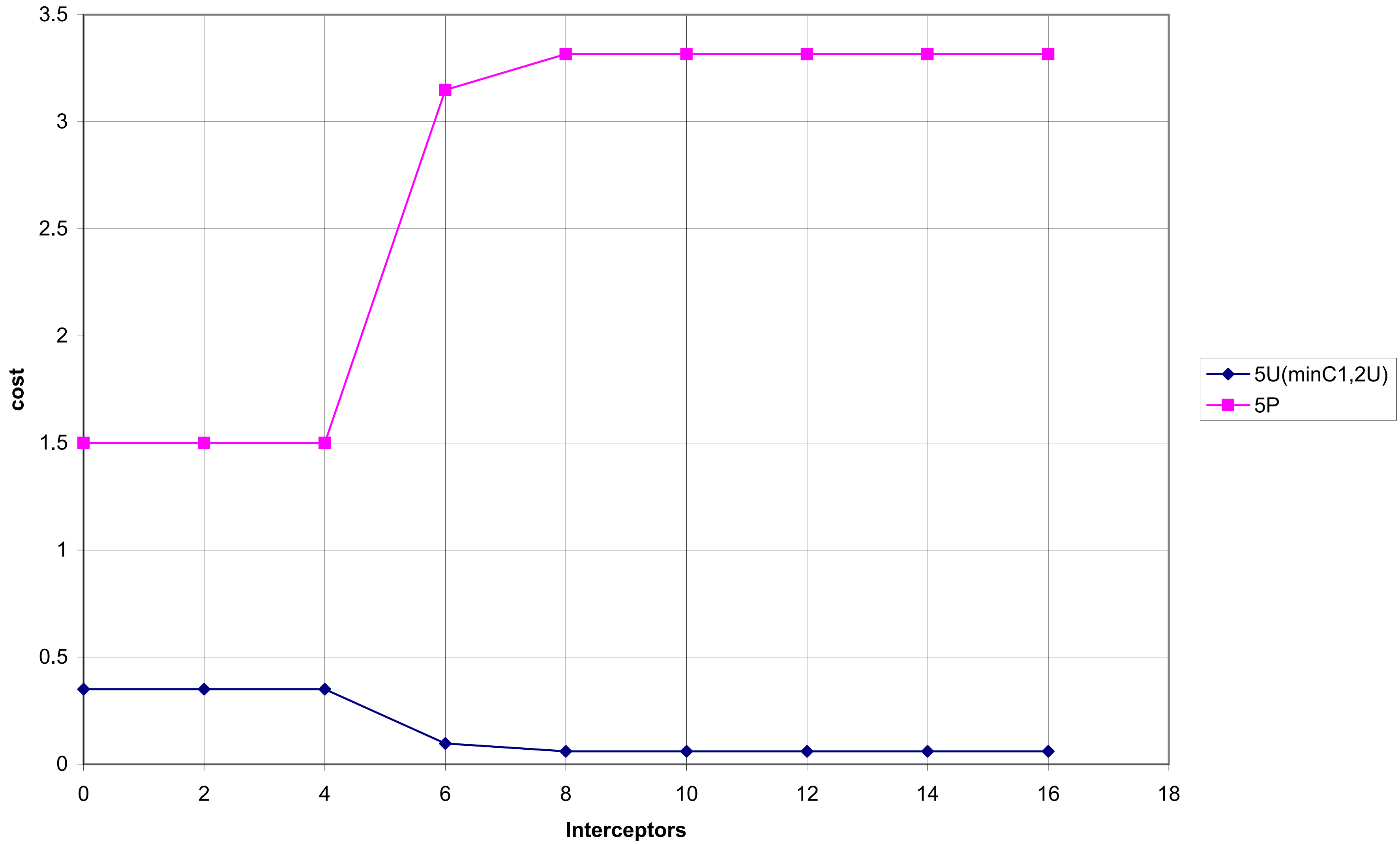


Fig. 8. Costs of nodes 3 & 4

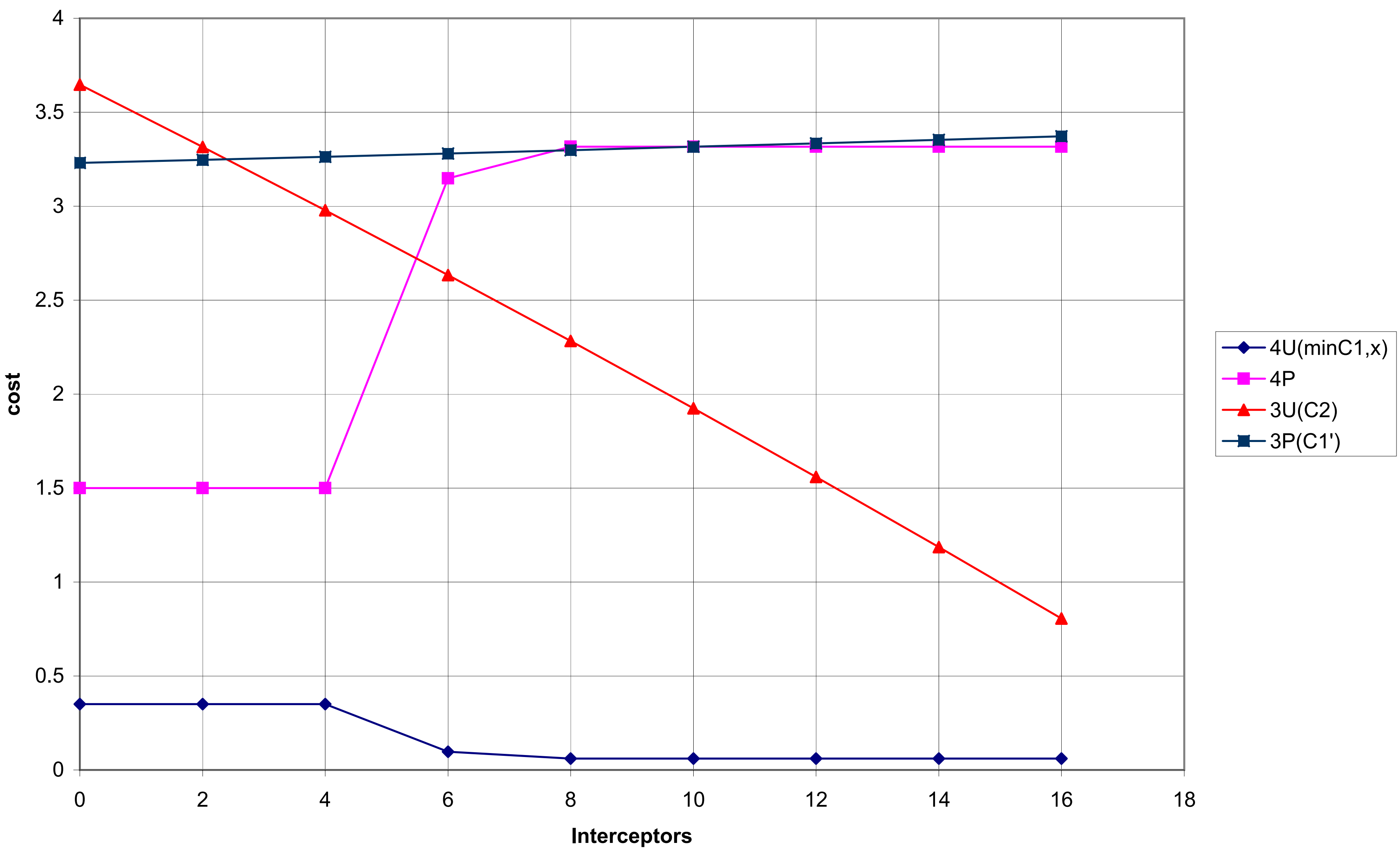


Fig. 9. Cost of node 6

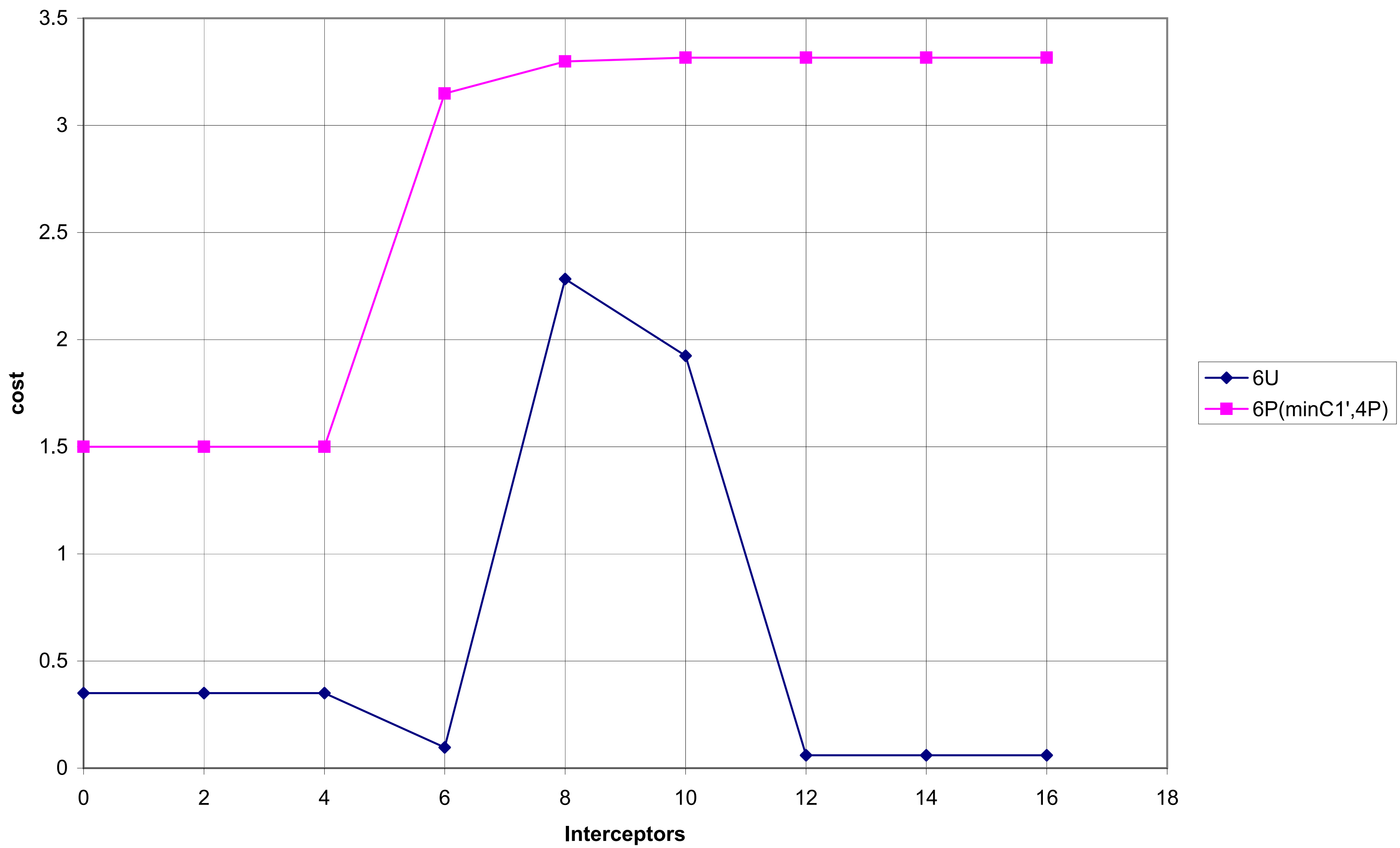
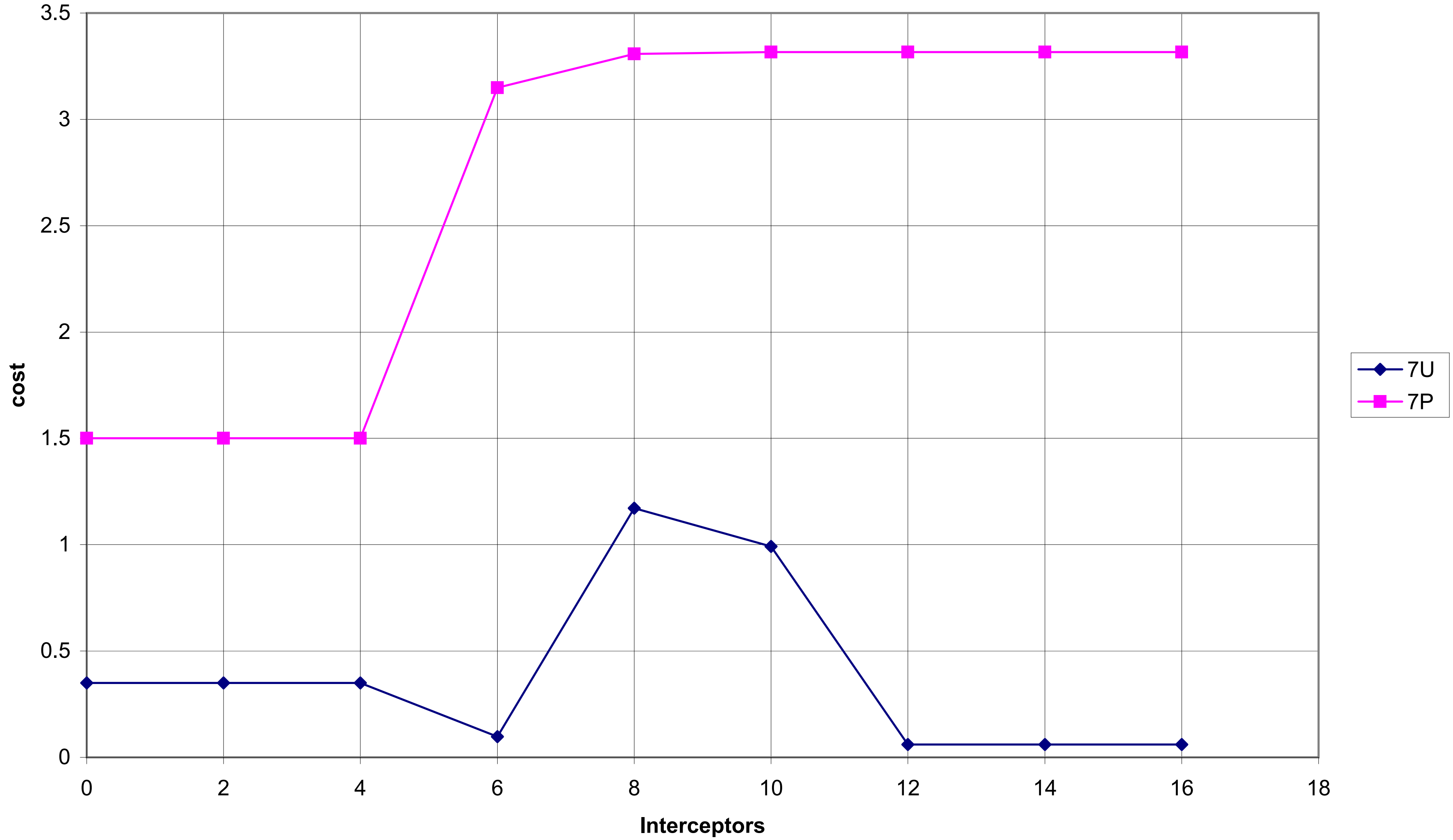


Fig. 10. Cost of node raching node 7 as function of interceptors allocated in tralateral interaction.



p	q	m	M	n	N	W	**l**	p'	q'	m'	M'	n'	N'	W'	l'	f	f'	F	F h h	h r	int ı	Q'	Q S'	S	g g'	g k	k'	v	v'	L	K	V	L'	K'	V'	Fm	Fv	Sm'	Sv'	C1ms	C1mo	C1m	C1vs	C1vo	C1v	C1	C1'	C2ms'	C2mo'	C2m'	C2vs'	C2vo'	C2v'	C2'	((((((C2	l	l'	l tot	l	u	L+V	C1	C2	L'+V'	C1'	C2'	7U	7P	6U	6P(mi	4U(mi	4P	3U(C	3P(C1	5U(m	5P	2U	2P(mi	1U(C1	1P(C2	d 8	8P
0.8	0	2	750	2	750	3000	0	0.8	0.2	1	**15**	1	5	20	0	0.694	0	918	0.77	2.8	2	0.02	5.23	#	0	0	0	0	0	**0.25**	**20**	**0.1**	**0.5**	**1**	**1**	708.2	209	0.1	5.2	1E-04	0.123	0.123	1.019	0.012	1.031	1.15	3.23	0.507	0.5	1.01	0.88	0.95	1.826	2.83	3.65	0.32	1.14	0.36	0	**0.5**	0.4	1.2	3.6	1.5	3.23	2.8	0.35	1.5	0.35	1.5	0.35	1.5	3.65	3.23	0.35	1.5	0.35	1.5	1.15	2.83		2
0.8	0	2	750	2	750	3000	**2**	0.8	0.2	1	15	1	5	20	0	0.701	0	897	0.77	2.8	2	0.01	3.21	#	0	0	0	0	0	0.25	20	0.1	0.5	1	1	689.5	208	0.1	3.2	1E-04	0.125	0.126	0.633	0.013	0.645	0.77	3.25	0.498	0.5	1	0.87	0.97	1.843	2.84	3.32	0.23	1.14	0.27	2	0.5	0.4	0.8	3.3	1.5	3.25	2.8	0.35	1.5	0.35	1.5	0.35	1.5	3.32	3.25	0.35	1.5	0.35	1.5	0.77	2.84		2
0.8	0	2	750	2	750	3000	4	0.8	0.2	1	15	1	5	20	0	0.708	0	876	0.77	2.8	2	0.01	1.2	#	0	0	0	0	0	0.25	20	0.1	0.5	1	1	670.7	206	0.1	1.2	1E-04	0.128	0.128	0.239	0.013	0.252	0.38	3.26	0.489	0.5	0.99	0.87	0.99	1.86	2.85	2.98	0.13	1.15	0.15	4	0.5	0.4	0.4	3	1.5	3.26	2.8	0.35	1.5	0.35	1.5	0.35	1.5	2.98	3.26	0.35	1.5	0.35	1.5	0.38	2.85		2
0.8	0	2	750	2	750	3000	6	0.8	0.2	1	15	1	5	20	0	0.488	0	1537	0.83	2	1	0.05	0.1	#	0	0	0	0	0	0.25	20	0.1	0.5	1	1	1272	266	0.1	0.1	1E-04	0.07	0.07	0.02	0.007	0.027	0.1	3.28	0.72	0.5	1.22	0.93	1	1.929	3.15	2.63	0.04	1.04	0.04	6	0.5	0.4	0.1	2.6	1.5	3.28	3.1	0.1	3.15	0.1	3.15	0.1	3.148	2.63	3.28	0.1	3.15	0.35	1.5	0.1	3.15		3
0.8	0	2	750	2	750	3000	8	0.8	0.2	1	15	1	5	20	0	0.248	0	2256	0.85	1	0	0.21	0.1	#	0	0	0	0	0	0.25	20	0.1	0.5	1	1	1925	331	0.1	0.1	1E-04	0.036	0.037	0.02	0.004	0.024	0.06	3.3	0.854	0.5	1.35	0.96	1	1.963	3.32	2.28	0.03	0.99	0.03	8	0.5	0.4	0.1	2.3	1.5	3.3	3.3	1.17	3.31	2.28	3.3	0.06	3.317	2.28	3.3	0.06	3.32	0.35	1.5	0.06	3.32		3
0.8	0	2	750	2	750	3000	10	0.8	0.2	1	15	1	5	20	0	0.248	0	2256	0.85	1	0	0.21	0.1	#	0	0	0	0	0	0.25	20	0.1	0.5	1	1	1925	331	0.1	0.1	1E-04	0.036	0.037	0.02	0.004	0.024	0.06	3.32	0.854	0.5	1.35	0.96	1	1.963	3.32	1.92	0.03	1	0.03	10	0.5	0.4	0.1	1.9	1.5	3.32	3.3	0.99	3.32	1.92	3.32	0.06	3.317	1.92	3.32	0.06	3.32	0.35	1.5	0.06	3.32		3
0.8	0	2	750	2	750	3000	12	0.8	0.2	1	15	1	5	20	0	0.248	0	2256	0.85	1	0	0.21	0.1	#	0	0	0	0	0	0.25	20	0.1	0.5	1	1	1925	331	0.1	0.1	1E-04	0.036	0.037	0.02	0.004	0.024	0.06	3.33	0.854	0.5	1.35	0.96	1	1.963	3.32	1.56	0.04	1.01	0.04	12	0.5	0.4	0.1	1.6	1.5	3.33	3.3	0.06	3.32	0.06	3.32	0.06	3.317	1.56	3.33	0.06	3.32	0.35	1.5	0.06	3.32		3
0.8	0	2	750	2	750	3000	14	0.8	0.2	1	15	1	5	20	0	0.248	0	2256	0.85	1	0	0.21	0.1	#	0	0	0	0	0	0.25	20	0.1	0.5	1	1	1925	331	0.1	0.1	1E-04	0.036	0.037	0.02	0.004	0.024	0.06	3.35	0.854	0.5	1.35	0.96	1	1.963	3.32	1.19	0.05	1.01	0.05	14	0.5	0.4	0.1	1.2	1.5	3.35	3.3	0.06	3.32	0.06	3.32	0.06	3.317	1.19	3.35	0.06	3.32	0.35	1.5	0.06	3.32		3
0.8	0	2	750	2	750	3000	16	0.8	0.2	1	15	1	5	20	0	0.248	0	2256	0.85	1	0	0.21	0.1	#	0	0	0	0	0	0.25	20	0.1	0.5	1	1	1925	331	0.1	0.1	1E-04	0.036	0.037	0.02	0.004	0.024	0.06	3.37	0.854	0.5	1.35	0.96	1	1.963	3.32	0.81	0.07	1.02	0.08	16	0.5	0.4	0.1	0.8	1.5	3.37	3.3	0.06	3.32	0.06	3.32	0.06	3.317	0.81	3.37	0.06	3.32	0.35	1.5	0.06	3.32		3

p'	q'	m'	M'	n'	N'	W'	l'	p	q	m	M	n	N	W	l	0.1	F'	h h'	r'	int ı	Q'	S	g g	k'	k	v'	v	L'	K'	V'	L	K	V	Fm'	Fv'	Sm	Sv	C1ms'	C1mo'	C1m'	C1vs'	C1vo'	C1v'	C1'	C2ms	C2mo	C2m	C2vs	C2vo	C2v	C2
0.8	0	1	15	1	5	20	0	0.8	0.2	2	750	2	750	3000	0	0	20	0	0	0	1	3000	0.84	0	0	0	0	0.5	1	1	0.25	20	0.1	0.1	20	2518	482	0.919	0.5	1.419	0.992	0.819	1.811	3.23	1E-04	0.02	0.02	3.63	0	3.626	3.65
0.8	0	1	15	1	5	20	0	0.8	0.2	2	750	2	750	3000	2	0	18	0	0	0	1	3000	0.84	0	0	0	0	0.5	1	1	0.25	20	0.1	0.1	18	2518	482	0.919	0.5	1.419	0.992	0.835	1.827	3.25	1E-04	0.02	0.02	3.29	0	3.295	3.32
0.8	0	1	15	1	5	20	0	0.8	0.2	2	750	2	750	3000	4	0	16	0	0	0	1	3000	0.84	0	0	0	0	0.5	1	1	0.25	20	0.1	0.1	16	2518	482	0.919	0.5	1.419	0.992	0.852	1.844	3.26	1E-04	0.02	0.02	2.96	0	2.958	2.98
0.8	0	1	15	1	5	20	0	0.8	0.2	2	750	2	750	3000	6	0	14	0	0	0	1	3000	0.84	0	0	0	0	0.5	1	1	0.25	20	0.1	0.1	14	2518	482	0.919	0.5	1.419	0.992	0.869	1.861	3.28	1E-04	0.02	0.02	2.61	0	2.614	2.63
0.8	0	1	15	1	5	20	0	0.8	0.2	2	750	2	750	3000	8	0	12	0	0	0	1	3000	0.84	0	0	0	0	0.5	1	1	0.25	20	0.1	0.1	12	2518	482	0.919	0.5	1.419	0.992	0.887	1.879	3.3	1E-04	0.02	0.02	2.26	0	2.262	2.28
0.8	0	1	15	1	5	20	0	0.8	0.2	2	750	2	750	3000	10	0	10	0	0	0	1	3000	0.84	0	0	0	0	0.5	1	1	0.25	20	0.1	0.1	10	2518	482	0.919	0.5	1.419	0.992	0.905	1.897	3.32	1E-04	0.02	0.02	1.9	0	1.904	1.92
0.8	0	1	15	1	5	20	0	0.8	0.2	2	750	2	750	3000	12	0	8	0	0	0	1	3000	0.84	0	0	0	0	0.5	1	1	0.25	20	0.1	0.1	8	2518	482	0.919	0.5	1.419	0.992	0.923	1.915	3.33	1E-04	0.02	0.02	1.54	0	1.538	1.56
0.8	0	1	15	1	5	20	0	0.8	0.2	2	750	2	750	3000	14	0	6	0	0	0	1	3000	0.84	0	0	0	0	0.5	1	1	0.25	20	0.1	0.1	6	2518	482	0.919	0.5	1.419	0.992	0.942	1.934	3.35	1E-04	0.02	0.02	1.16	0	1.166	1.19
0.8	0	1	15	1	5	20	0	0.8	0.2	2	750	2	750	3000	16	0	4	0	0	0	1	3000	0.84	0	0	0	0	0.5	1	1	0.25	20	0.1	0.1	4	2518	482	0.919	0.5	1.419	0.992	0.961	1.953	3.37	1E-04	0.02	0.02	0.78	0	0.785	0.81
																					m	0.1											min=	0.1	min=	**0.1**								32.4							