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# An Application of Game Theory: Funding Interdependent MC&A Upgrade Decisions

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## Abstract
Funding Material, Control and Accountability (MC&A) system upgrades has been identified as a partial solution for mitigating the diversion threat of weapons-grade nuclear material. Effective MC&A system upgrades are dependent on appropriate decisions based on based on funding, implementation, operation and oversight. Traditional MC&A upgrade decisions inherently assumed that all decision-makers possessed similar payoff vectors allowing for a fairly consistent and unified approach to MC&A system enhancements; however, MC&A upgrade projects in non-traditional environments may be required to take into account situations where the potential payoff vectors among decision-makers may be significantly different. Once a decision-maker is required to take into account the decisions of others, the process can be modeled as a game. Game theory has been previously be used to shed light on many aspects of social and economic behavior where a payoff from a set of strategies is dependent on the strategy of others. In this paper, the application of game theory in the context of MC&A upgrades is discussed. Various MC&A upgrades decision payoff matrices for relevant circumstances are evaluated for static (simultaneous) and dynamic (sequential decisions) games. Optimal strategies and equilibrium conditions for these payoff matrices are analyzed. Additional game factors (bargaining, uncertain outcomes, moral hazards) that may affect the outcome of the game are briefly discussed. By demonstrating the application of game theory to a nontraditional environment that may require MC&A upgrades, this work increases the understanding out how outcomes are logically connected to the respective value decision-makers assign to choices.

## Introduction
The funding of MC&A upgrades where the "funder" and the "fundee" have different interests, and different benefits (or penalties) for applying those interests can be analyzed in terms of a non-zero sum 2 by 2 matrix. Let Ruth be designated as the "Funder" and Charlie designated as the "Fundee". Ruth's choices, or pure strategies, will be listed in the rows of the matrix while Charlie's pure strategies will be designated in columns. A value in the matrix cell $c_{ij}$ represents Ruth's payoff for Ruth's choice $i$ and Charlie's choice $j$, while $-c_{ij}$ represents Charlie's payoff for the same choices. If two values are given in a cell, $a/b$, then Ruth payoff is represented as $a$ and Charlie's payoff is given as $b$. Payoffs are representation of gain or loss associated with a particular outcome. In some scenarios the designation of an associated value for a payoff may be relatively straight forward while in other cases the designation of a particular value may be uncertain. Payoff values may be associated with real-world values such as profits or prison terms. In other cases, the payoff values may only have relative association with other cell in the matrix. For example, the outcome $c_{ij}$ may be considered twice "as good" as outcome $c_{i'j'}$; therefore, $c_{ij}$ would be given a payoff twice as large as $c_{i'j'}$. In the case of relatively assigned payoff values, strategies can still be analyzed.

Ruth and Charlie both attempt to maximize their respective payoffs. The exact methods used by Ruth and Charlie to maximize their individual payoffs is dependent on their beliefs how to "play the game". For example, are the player confined to pure strategies? Do the players feel they can read each other? Do any implied social conventions enhance cooperation? Are guarantees important?

In the following examples the term "coordinated' and "uncoordinated" upgrades are considered from Ruth's point-of-view; therefore, a coordinated upgrade is when Ruth funds an upgrade and Charlie implements the upgrade from Ruth's perspective. Note that a coordinated upgrade does not necessarily mean a cooperative upgrade.

## Example 1: "coordinated" upgrades are twice as important as "uncoordinated" upgrades.

Ruth has a higher penalty for not funding potential "coordinated" upgrades (-2), than for funding "bad" upgrades (-1). In addition, Ruth has a higher gain for funding "coordinated" upgrades than for not funding potential "uncoordinated" upgrades. The payoff matrix is shown below.

| | MC&A Upgrades with Ruth's Interests | MC&A Upgrades in Charlie's Interests | Ruth's row minimum |
|---|---|---|---|
| Fund MC&A Upgrades | **2/1** | -1/2 | -1(maximum) |
| Do not fund MC&A Upgrades | -2/-2 | 1/-3 | -2 |
| Charlie's column minimum | -2 (maximum) | -3 | |

A possible method of "hunting" for an equilibrium position is described below. Both Ruth and Charlie are looking for a guarantee. Ruth decides that row one provides a minimum guarantee of –1, while Charlie selects column one based on the same rational.

The result is the initial choice is (1,1). Both players are initially pleased that their scores are higher than predicted. Ruth would not gain from changing her position; however, Charlie changes his choice to column two, resulting in a higher payoff for Charlie. Charlie's decision will cause Ruth to change her decision to row two, resulting in Charlie switching to column one, resulting in a cyclic pattern, where each side attempts to increase its gain: Charlie by implementing funded upgrades in his interests but willing to perform unfunded upgrades in Ruth's interests and Ruth by not funding upgrades in Charlie's interests and funding upgrades in her own interests. Any attempt to develop a predictable strategy from one side, can eventually be "read" by the other side, and subsequently countered with a resulting payoff for Ruth of zero and a payoff for Charlie of –2. At the same time, any attempt to find an equilibrium position will force Ruth to understand Charlie's position and Charlie, Ruth's.

It would appear difficult to break out of the above cyclic trap. If Ruth and/or Charlie consider making their respective choices on a random basis, then their moves could not be predicted.

If Ruth is agreeable to a random mixed strategy game, what strategy should she choose? The following model is used to devise a strategy for Ruth.

| | MC&A Upgrades with Ruth's Interests | MC&A Upgrades in Charlie's Interests |
|---|---|---|
| Fund MC&A Upgrades | $a_1, b_1$ | $a_1, b_2$ |
| Do not fund MC&A Upgrades | $a_2, b_1$ | $a_2, b_2$ |

If $[1\text{-}p, p]$ is Ruth's strategy for (row 1, row 2) and $[1\text{-q}, q]$ is Charlie's strategy for (column 1, column 2), it can be shown[1] that Ruth's expectation $e_R(p,q)$, for any $(p,q)$ combination is

$$e_R(p,q)=mp+c$$

$m=( a_1 - a_2 - a_3 + a_4)q + (a_3 - a_1)$ and $c=(a_2 - a_1)q+ a_1$

Ruth's expectation can be either increasing, constant or decreasing as a function of $p$. Ruth, of course, is attempting to maximize her expected payoff. The behavior of the function depends on the slope of the line.; therefore, the following decision criteria should apply to Ruth:

$$p=0 \text{ if } m<0;\ 0\le p\le 1 \text{ if m=0};\ p=1 \text{ if } m>0$$

For the above example, $m=(2-(-1)-(-2)+1)q+ (-2-2)=6q-4$, $c=(-1-2)q+2 = -3q+2$.

If Charlie plays a strategy of $q<2/3$, then Ruth would play $p=0$. Consider Charlie playing $q=0.5$; Ruth would play $p=0$, with an expected payoff of $e_R(p,q)= -3(0.5)+2= 0.5$.for Ruth.

Charlie's expected payoff for such a strategy would be, in general,

$$e_C(p,q)=m'q+c'$$

where $m' =( b_1 -b_2 - b_3 + b_4)p + (b_2 - b_1)$ and $c' =(b_3 - b_1)p+ b_1$.

Specifically, $m' =( 1 -2-(-2)-3)p + (2-1) = -2p+1$ and $c' =(-2-1)p+1= -3p+1$.

Charlie's payoff for the above parameters would be, $e_C(p,q)=(2-1)(0.5)+1= 1.5$. Could Charlie improve he strategy? Since $m'>0$, Charlie would benefit from playing $q=1$, for a payoff of 2. Of course, Charlie strategy of $q=0$ would cause Ruth the reevaluate her strategy.

Ruth can only stop Charlie from finding a better solution by playing the strategy that causes Charlie's slope of his expected payoff to be zero. Likewise, Charlie can only stop Ruth from finding a better solution by playing the strategy that causes Ruth's slope of her expected payoff to be zero. The equilibrium strategy is the point where the two decision graphs intersect: $p$=0.5, $q$=2/3.

The expected values would be: $e_R(0.5,.67)$= 0.5 and $e_C(0.5,.67)$=-0.5. While it is logical that Ruth must focus Charlie's payoff matrix in order to ensure an equilibrium, it may appear contrary to from Ruth's perspective that she can not focus on her own payoff matrix to ensure gain.

An important part of any game theory application is the use of sensitivity analysis. The below example illustrates the effect of simple changes to the payoff matrix and the repercussions to the players strategy.

## Example 2: "uncoordinated" upgrades receive higher penalty to Ruth and Charlie decisions are magnified if funding is not received.

Ruth, in this example, now has a lower penalty for not funding "coordinated" upgrades (-2) than for funding "uncoordinated" upgrades (-3). Charlie, when faced with Ruth's decision "not to fund" has a higher penalty for maintaining the upgrades are in Charlie's interest, than in Ruth's interest. This situation could arise if Charlie "regrets" the loss of possible funds from Ruth, if it is known that he is the cause of the "uncooperation". In addition, Charlie suffers less of a penalty if Ruth does not fund and he felt he was "cooperative".

| | MC&A Upgrades in Ruth's Interests | MC&A in Charlie's Interests | Ruth's Row minimum |
|---|---|---|---|
| Fund MC&A Upgrades | 2/1 | -3/2 | -3 |
| Do not fund MC&A Upgrades | **-2/-1** | 1/-4 | -2 (maximum) |
| Charlie's Column minimum | -1 (maximum) | **-3** | |

The initial outcome using a strategy of minimizing losses would be row 2, column 1 with the same cyclic behavior as described in Example 1.

Ruth's equilibrium strategy is given where Charlie's slope is zero at $m'$=0=(1-2+1-4)$p$+(2-1) and $p$=1/4. Compared to example 1, Ruth would now double the number of funded upgrade decisions. Note that Ruth, would change her strategy even if her own payoff matrix did not change. Charlie's equilibrium strategy would be $m'$=(2+3+2+1)$q$+(-2-2) and $q$= 0.5. Charlie's equilibrium has shifted to allow more upgrades to by in Rut's interests. The expected payoffs are $e_R(0.25,0.5)$= -0.5 and $e_C(0.25,0.5)$= 0.5. Ruth should not be surprised that larger penalties in on cell lead to an overall decrease in expected payoff. Charlie may be surprised that while his net sum of payoffs from the cells is unchanged at –2, he now has a positive expected payoff.

## Example 3 Smaller penalty for Charlie's non-cooperation:

Let's assume now that Charlie, when faced with Ruth's decision "not to fund" has a smaller penalty for not receiving funding for uncooperative upgrades. This situation could arise if the Charlie's feels "pushed-around" by conceding to Ruth's interest.

| | MC&A Upgrades in Ruth's interests | MC&A Upgrades in Charlie's interest | Ruth's Row minimum |
|---|---|---|---|
| Fund MC&A Upgrades | 2/1 | -3/2 | -3 |
| Do not fund MC&A Upgrades | -2/-2 | **1/-1** | **-2 (maximum)** |
| Charlie's column minimum | -2 | **-1 (maximum)** | |

The maximum value is row 1 column 2. Neither player will improve his position by changing his position. The game is considered stable with the decision "not to fund". Note that a slight change in the payoff matrix causes a totally polarized position. Although both sides will gain by cooperating, the funding of upgrades in Ruth's interests is clearly an unstable state.

## Example 4: Ruth has the same matrix as in example 1, Charlie has the same matrix as in example 3.

| | MC&A Upgrades in Ruth's interests | MC&A Upgrades in Charlie's Interests | Ruth's Row minimum |
|---|---|---|---|
| Fund MC&A Upgrades | 2/1 | -1/2 | -1 **(maximum)** |
| Do not fund MC&A Upgrades | -2/-2 | 1/-1 | **-**2 |
| Charlie's column minimum | -2 | **-1 (maximum)** | |

The result is an initial unstable position at (2,1), Ruth will move to a decision not to fund; the point (row 2,column2) will be a stable point. Note that a payoff matrix that previously worked for Ruth, not results in a polarized position with a larger negative payoff.

Obviously, both players could benefit from cooperating for a cell on the payoff matrix that benefits both sides.

## Conclusion:

The application of game theory to MC&A upgrades can provide predictions of the outcomes of the modeled games. Payoff matrices that are designed to reward and penalize decisions "appropriately" may drive outcomes that are not expected. Occasionally, these predictions do not coincide with the player's common sense, since players tend to view the outcome from their own perspective or payoff matrix. Sensitivity analysis can demonstrate in such games that the outcome can be fairly sensitive to relatively slight changes in the payoff matrix. In many scenarios, the decision "not to fund" will occur as a stable point or as part of a strategy using probabilities. The "stable-point" of a decision "not to fund" frequently arises if the player implementing the upgrades encounters a higher penalty for applying the funder's interest, even in the face of no funding.

**References**

[1] Stahl, Saul, "A Gentle Introduction to Game Theory", Mathematical World; v.13 1998

[2] Beirman, H. Scott, " Game Theory with Economic Applications",-2nd ed.; Addision –Wesley 1998