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A Reduced Order, One Dimensional Model of Joint Response*

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Abstract

As a joint is loaded, the tangent stiffness of the joint reduces due to slip at interfaces. This stiffness reduction continues until the direction of the applied load is reversed or the total interface slips. Total interface slippage in joints is called macro-slip. For joints not undergoing macro-slip, when load reversal occurs the tangent stiffness immediately rebounds to its maximum value. This occurs due to stiction effects at the interface. Thus, for periodic loads, a softening and rebound hardening cycle is produced which defines a hysteretic, energy absorbing trajectory. For many jointed substructures, this hysteretic trajectory can be approximated using simple polynomial representations. This allows for complex joint substructures to be represented using simple non-linear models. In this paper a simple one dimensional model is discussed.

Introduction

In many structural systems, joints and interfaces are the primary subcomponents responsible for energy loss. Although the mechanisms which produce energy loss in joints and interfaces are highly non-linear, in many structural systems, the majority of all dynamics behave in a nearly linearly fashion, therefore, damping due to joints and interfaces are usually approximated in some linear form (i.e. Rayleigh or modal damping[1]).

The use of more predictive models of joints and interfaces in structural systems is important to the development of newer methodologies of design. In the past, structural design was performed by using prototypes and experimental analysis. Numerical dynamic analysis was usually only used after the initial design was complete when a better understanding of dynamic response was required. When using numerical analysis in this fashion, linearized methods of modeling damping developed from experimental data, such as Rayleigh or modal damping approximations, were sufficient. At present, newer methodologies of structural design are being developed. In these methodologies, predictions of the response of a system will be produced without the use or with little or no use of full system experimental data. These predictions will be made using numerical models developed primarily from design drawings and physics level experiments. To assure the accuracy of these predictions, better methods of modeling significant "unknowns" such as damping in joints and interfaces are needed.

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DISCLAIMER

Established methods for modeling damping are simply no longer acceptable for the development of newer methodologies of design. In particular Rayleigh and modal damping is limited in a number of respects.

- Rayleigh or modal damping parameters are usually approximated using experimental observations. Therefore, these method of modeling damping are inherently ad-hoc and empirical.
- Joints not only damp the structure but also produce variations in structural stiffnesses - variations which are not predicted using Rayleigh or modal damping models. These approximations usually produce over-stiff predictions of structural response.
- Rayleigh and modal approximations do not correctly represent variations in energy loss per cycle due to variations in excitation amplitude. These approximations predict that the energy loss per cycle in a joint is proportional to the amplitude of excitation raised to the second power; however, experiments show that for a real joint, the exponent is usually between two and three.

In order to predictively model the response of a jointed structure without using ad-hoc methods based on system level experimental analysis, a better understanding of the physics of a joint in a structure is required. From these physics, predictive models of joint and interface substructures can be developed.

In general, joint interface motion can be very complicated, however global joint response is usually not complex. Therefore, the response of most joints can be represented using relatively simple, low order models. These low order models are advantageous from a computational standpoint since they can quickly and efficiently determine the response of a jointed sub-structure when integrated into a complex, high order representation of the total structure. Of course, the parameters used in these reduced order models are highly dependent on the constitutive relationships describing interface motion. Therefore, various levels of modeling are required.

In the following sections, a high order, one dimensional joint model will be developed. This model will be constructed using the finite element method. From this high order model, a reduced order model of joint dynamics will be constructed. This low order model will allow for the simple and efficient solution of transient dynamics when integrated into a full system model.

A High Order One Dimensional Joint Model

Figure 1 is an illustration of a simple one dimensional joint. This joint consist of two bars held together by a uniform normal load, N. A load, $f(t)$, is applied to the end of this joint. As a result of this applied load, the upper bar displaces a distance $u_{upper}(x)$ and the lower bar displaces a distance $u_{lower}(x)$. The two bars undergo different displacements due to slippage at the interface. This slippage produces energy absorption in the joint.

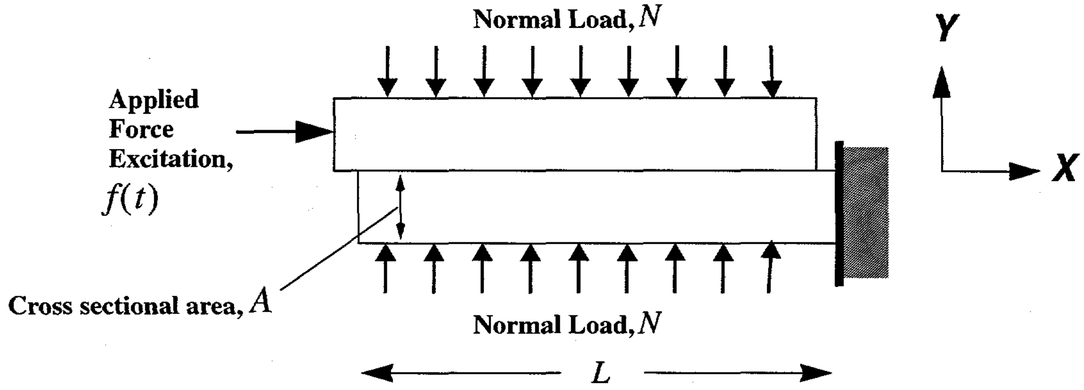

Figure 1: Model of a one dimensional joint

To understand the physics of the joint in the Figure 1 in greater detail, a finite element model of the joint was developed and used to simulate joint behavior. The description of this model is given in Appendix A. Model parameters were assumed to be those given in Table 1. The resulting model contained 160 physical degrees of freedom. Simulations were performed to determine the response of the joint to a sinusoidal excitation. Figure 2 shows a sampling of the results of these simulations.Other researchers have obtained similar results [2]. Results compared well to the closed form solution given by Goodman [3].

Parameter	Value
E elastic modulus	$30.0\times10^{6}\ lb_f/in^2$
A cross sectional area	$1.0\ in^2$
L length of joint	$20.0\ in$
μ coefficient of friction	0.25
N normal load	$1.0\ lb_f/in^2$

Table 1: Parameters for Example Problem

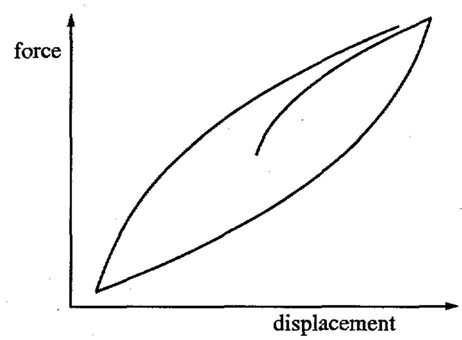

Figure 2: Hysteretic response of the joint to a harmonic load

A Low Order Joint Model

In the above section, a one dimensional joint was modeled and simulated using the finite element solution shown in Appendix A. This finite element model contained a (relatively) large number of degrees of freedom; however, the response of the joint to a harmonic excitation was relatively simple (see Figure 2). In many cases, for joints not undergoing macro-slip, as the geometric complexity of the joint become greater, the complexity of joint response increases very little; therefore, the added complexity of using a large number of degrees of freedom to represent the response of a joint in simulations does little to improve the fidelity of the total solution. Thus, a reduced order representation of joint response is highly desirable. In this section, a reduced order representation of the above one dimensional joint is discussed.

A reduced order representation of the above one dimensional joint can be derived by understanding the behavior of the joint. If the excitation force were not a harmonic excitation but a ramp excitation which started at zero and progressed to a large load level, the force displacement curve would look similar to that shown in Figure 3. The initial slope of this curve, k_{max}, is the maximum stiffness of the joint. This stiffness occurs at the beginning of the ramp since all points along the interface are in stuck together and the joint behaves as if its interfaces were welded. The minimum slope of this curve, k_{min}, is the stiffness which the joint obtains immediately before macro-slip. The maximum force which the joint can withstand, f_{max}, will occur at the point of minimum stiffness.

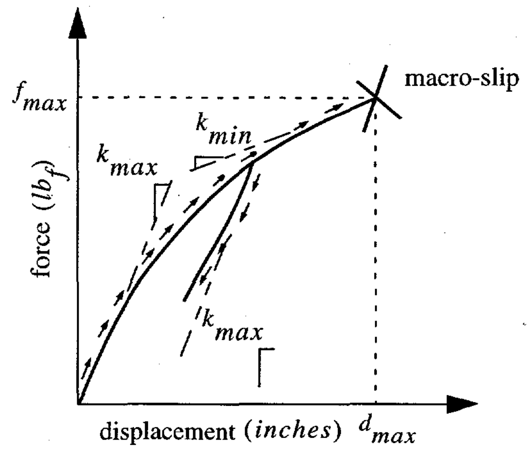

Figure 3: Element of joint behavior

If the input force in the above example is not ramped up to f_{max}, but is increased to some value less than f_{max} and then reversed, the stiffness of the system immediately returns to k_{max}. This is because points which are in slip must go into stick before they can go into slip in the opposite direction. Therefore, at reversal, all points stick and the stiffness goes to a maximum. After reversal a new trajectory can be *approximated* by knowing the displacement and force at reversal, (d_r, f_f), and by knowing the displacement, and force, and stiffness at macro-slip, $(d_{max}, f_{max}, k_{min})$.

Figure 4 illustrates how this approximation is derived. The trajectories of joint responses are often not complicated, and therefore, can be approximated using simple polynomial functions. The construction of a trajectory on reversal can be estimated by solving for a fourth order polynomial representation which passes through (d_r, f_f) with a slope of k_{max} and passes through $-d_{max}$, $-f_{max}$ and with a slope of k_{min}. This trajectory is recalculated each time load reversal occurs. The result is a simple, yet reasonable, reduced order approximation of joint response.

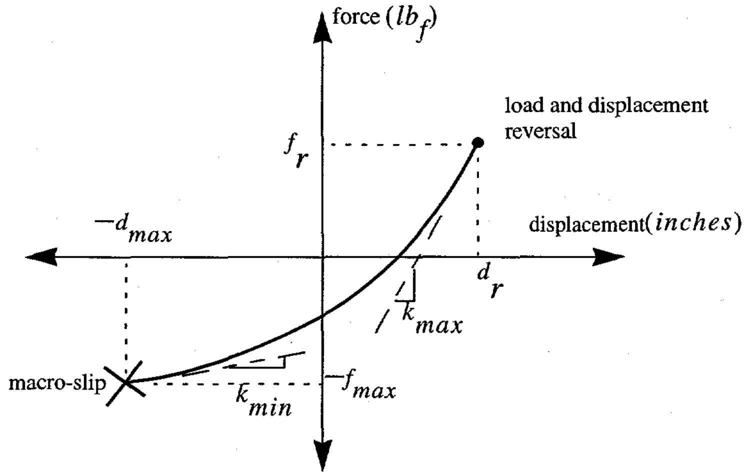

Figure 4: The construction of a new trajectory after reversal

The above reduced order modeling methodology was applied to the example problem in Figure 2. Trajectories from the two models where compared. Shown in Figure 5 is one of those trajectories. As can be seen in this figure, the trajectories for the full order finite element model and the low order model are very similar.

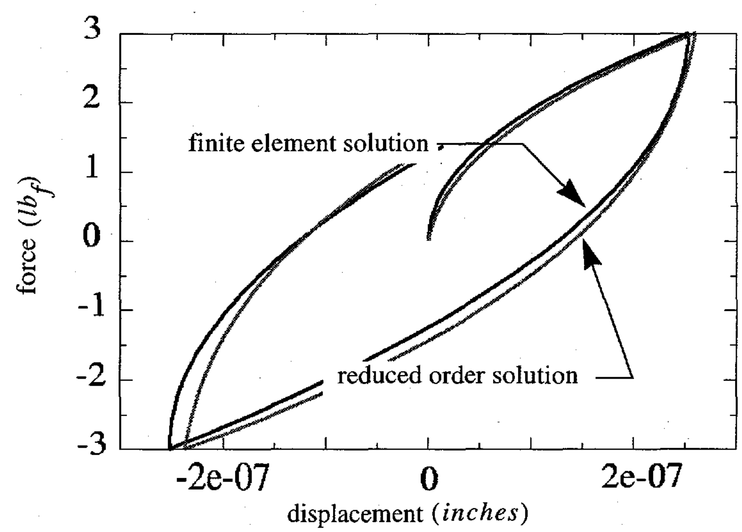

Figure 5: Reduced and full order model response

There are a number of benefits to using the low order model presented above.

- As the level of micro-slip in the joint decreased, the order of the finite element model has to be increased significantly to produce a reasonable response; however the order of the reduced order model remains constant.
- As the number of degrees of freedom in the finite element model are increased, the conditioning of the matrices in equation 4 (in Appendix A) becomes poorer. The low order model does not exhibit this behavior.

- As the number of degrees of freedom in the finite element model increases the time to solve equation 4 also increases. In general, the reduced order model runs more quickly than the finite element model.
- The low order model can also easily be integrated into a full body model in a structural dynamics solution. By using the reduced order model discussed in this paper, the time domain response of a 2084 lb_m mass attached to the Figure 1 joint was simulated. Figure 6 shows the results from this simulation. There is no damping matrix in this simulation, but, as expected, the hysteretic loss produces a finite amplitude response to a harmonic force excitation on the mass.

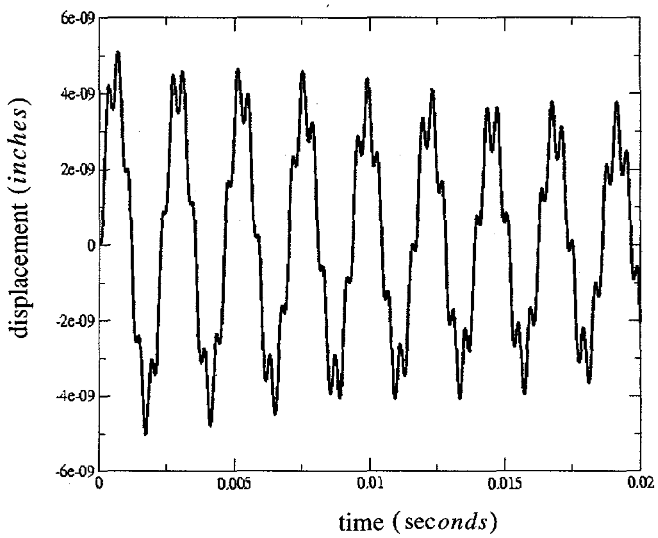

Figure 6: Time history response of jointed system to harmonic excitation

Conclusions and Future Work

In general, the simplified, reduced order model presented in this paper accurately represented the observed behavior of the higher over model. In the future, we propose to expand the above method to solve for the reduced order non-linear response of a multi-dimensional joint.

References

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Appendix #1: Finite Element Model of Figure #1 Type Joints

In this appendix, a finite element model for the joint shown in figure 1 is described. In developing this model, it is assumed that

- The bars are thin, and therefore; the normal stress through the thickness of the bars is a constant,
- The maximum shear force at the interface is given by,
$$\beta_{max} = \mu N.$$

The equation of motion for a bar is given by

$$EA\frac{\partial^2 u}{\partial x^2} = \beta \tag{1}$$

where

E = elastic modulus (lb_f/in^2), A = cross sectional area of bar (in^2)
$u = u_{upper}(x)$, or $u_{lower}(x)$ in (in), and β = shear force (lb_f).

For the upper bar, the finite element, approximation of (1) is given by

$$\mathbf{f} = \boldsymbol{a}\mathbf{T} + \mathbf{K}_{upper}\mathbf{U}_{upper} \tag{2a}$$

where $\mathbf{f}^T = \begin{bmatrix}0\ f(t)0\ \dots\ 0\end{bmatrix}^T$, $\mathbf{T}^T = \begin{bmatrix}\beta_1, \beta_2, \dots\beta_N\end{bmatrix}^T$,

$\mathbf{U}_{upper}{}^T = \begin{bmatrix}u^u{}_1, u^u{}_2 \dots u^u{}_N\end{bmatrix}^T$ is a vector of nodal displacements along the upper bar.

For the lower bar, the finite element approximation of (1) is given by

$$\mathbf{0} = -\boldsymbol{a}\mathbf{T} + \mathbf{K}_{lower}\mathbf{U}_{lower} \tag{2b}$$

where $\mathbf{0}^T = \begin{bmatrix}0\ 0\ \dots\ 0\end{bmatrix}^T$, $\mathbf{U}_{lower}{}^T = \begin{bmatrix}u^l{}_1, u^l{}_2 \dots u^l{}_N\end{bmatrix}^T$ is a vector of nodal displacements along the lower bar.

Combining equations 2a and 2b gives

$$\mathbf{F} = \boldsymbol{A}\mathbf{T} + \mathbf{KU} \tag{2c}$$

where $\mathbf{F} = \begin{bmatrix}\mathbf{f} \\ \mathbf{0}\end{bmatrix}$, $\mathbf{A} = \begin{bmatrix}\mathbf{a} \\ -\mathbf{a}\end{bmatrix}$, $\mathbf{K} = \begin{bmatrix}\mathbf{K}_{upper} & 0 \\ 0 & \mathbf{K}_{lower}\end{bmatrix}$, and $\mathbf{U} = \begin{bmatrix}\mathbf{U}_{upper} \\ \mathbf{U}_{lower}\end{bmatrix}$.

The vectors and matrices in 2c must be partitioned and rearranged into known and unknown quantities before unknowns can be solved for. At any node either the displacement or the shear force is known. If the displacement is known and the shear force is unknown, then the node is said to be a ***stick*** node. If the displacement is unknown and the shear force is known, then the node is said to be a ***slip*** node.

The matrices in 2c can be partition into ***stick*** and ***slip*** nodes by using transformation matrices $Q_1 \in \Re^{2Nx2N}$ and $Q_2 \in \Re^{NxN}$ where

$$\mathbf{F}_p = Q_1\mathbf{F} = \begin{bmatrix} \mathbf{f}_{slip} \\ \mathbf{f}_{stick} \end{bmatrix}, \mathbf{U}_p = Q_1\mathbf{U} = \begin{bmatrix} \mathbf{U}_{slip} \\ \mathbf{U}_{stick} \end{bmatrix}, \mathbf{T}_p = Q_2\mathbf{T} = \begin{bmatrix} \mathbf{T}_{slip} \\ \mathbf{T}_{stick} \end{bmatrix},$$

$$\mathbf{A}_p = Q_1\mathbf{A}Q_2^{-1} = \begin{bmatrix} \mathbf{A}_{11} & \mathbf{A}_{12} \\ \mathbf{A}_{21} & \mathbf{A}_{22} \end{bmatrix}, \mathbf{K}_p = Q_1\mathbf{K}Q_1^{-1} = \begin{bmatrix} \mathbf{K}_{11} & \mathbf{K}_{12} \\ \mathbf{K}_{21} & \mathbf{K}_{22} \end{bmatrix},$$

$$\mathbf{A}_{11} \in \Re^{2N_{slip}xN_{slip}}, \mathbf{A}_{12} \in \Re^{2N_{slip}xN_{stick}}, \mathbf{A}_{21} \in \Re^{2N_{stick}xN_{slip}}, \mathbf{A}_{22} \in \Re^{2N_{stick}xN_{stick}}$$

$$\mathbf{K}_{11} \in \Re^{2N_{slip}x2N_{slip}}, \mathbf{K}_{12} \in \Re^{2N_{slip}x2N_{stick}}, \mathbf{K}_{21} \in \Re^{2N_{stick}x2N_{slip}}, \mathbf{K}_{22} \in \Re^{2N_{stick}x2N_{stick}},$$

N_{slip} are the number of slip nodes, N_{stick} are the number of stick nodes, and the subscripts *slip* and *stick* define quantities associated with slip and stick nodes.

Therefore, in partitioned form, equation 2c becomes

$$\begin{bmatrix} \begin{bmatrix} \mathbf{f}^u_{slip} \\ \mathbf{f}^l_{slip} \end{bmatrix} \\ \begin{bmatrix} \mathbf{f}^u_{stick} \\ \mathbf{f}^l_{stick} \end{bmatrix} \end{bmatrix} = \begin{bmatrix} \mathbf{A}_{11} & \mathbf{A}_{12} \\ \mathbf{A}_{21} & \mathbf{A}_{22} \end{bmatrix} \begin{bmatrix} \mathbf{T}_{slip} \\ \mathbf{T}_{stick} \end{bmatrix} + \begin{bmatrix} \mathbf{K}_{11} & \mathbf{K}_{12} \\ \mathbf{K}_{21} & \mathbf{K}_{22} \end{bmatrix} \begin{bmatrix} \begin{bmatrix} \mathbf{U}^u_{slip} \\ \mathbf{U}^l_{slip} \end{bmatrix} \\ \begin{bmatrix} \mathbf{U}^u_{stick} \\ \mathbf{U}^l_{stick} \end{bmatrix} \end{bmatrix} \quad (2d)$$

where the superscripts u and l define quantities associated with the upper and lower bars, and the vectors $\mathbf{f}_{slip}$, $\mathbf{f}_{stick}$, $\mathbf{U}_{slip}$, and $\mathbf{U}_{stick}$ have been further partitioned form to show similar quantities.

In the above set of equations there are $2N + N_{stick}$ unknowns ($\mathbf{U}^u_{slip}$, $\mathbf{U}^l_{slip}$, $\mathbf{U}^u_{stick}$, $\mathbf{U}^l_{stick}$, and $\mathbf{T}_{stick}$); however there are only $2N$ equations. Therefore, an additional N_{stick} equations are needed to solve this problem. When nodes of the upper and lower bars are not slipping, they are a fixed distance apart. That is

$$\mathbf{d} = \mathbf{U}^u_{stick} - \mathbf{U}^l_{stick} \quad (3)$$

where $\mathbf{d}$ is a constant while the nodes are sticking. Equation 3 represents the additional equations needed to solve this problem.

Combining equations 2d and 3 gives

$$\begin{bmatrix} \begin{bmatrix} \mathbf{f}^u_{slip} \\ \mathbf{f}^l_{slip} \end{bmatrix} \\ \begin{bmatrix} \mathbf{f}^u_{stick} \\ \mathbf{f}^l_{stick} \end{bmatrix} \\ \mathbf{d} \end{bmatrix} - \begin{bmatrix} \mathbf{A}_{11} \\ \mathbf{A}_{21} \\ \mathbf{0} \end{bmatrix} \mathbf{T}_{slip} = \begin{bmatrix} \mathbf{K}_{11} & \mathbf{K}_{12} & \mathbf{A}_{12} \\ \mathbf{K}_{21} & \mathbf{K}_{22} & \mathbf{A}_{22} \\ \mathbf{0} & \begin{bmatrix} \mathbf{I} & -\mathbf{I} \end{bmatrix} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \begin{bmatrix} \mathbf{U}^u_{slip} \\ \mathbf{U}^l_{slip} \end{bmatrix} \\ \begin{bmatrix} \mathbf{U}^u_{stick} \\ \mathbf{U}^l_{stick} \end{bmatrix} \\ \mathbf{T}_{stick} \end{bmatrix} . \quad (4)$$

In equation 4, all known parameters are on the right side and all unknowns on the left. There are $2N + N_{stick}$ equations and $2N + N_{stick}$ unknowns. Therefore, this equation can be solved for by inversion once stick and slip nodes are defined.

Initially all nodes are stick nodes. Due to the applied load, stick nodes can then become slip nodes and slip nodes can return to being stick nodes. The conditions of going from a stick node to a slip node and visa versa is given below.

- If $|\beta_n| > \beta_{max}$ then $|\beta| = \beta_{max}$ and the node is moved from the stick set to the slip set.
- If Δu_n along its path goes to zero, the node is moved from the slip set to the stick set.

The above conditions along with equation 4 are all that are needed to solve for the response of the Figure 1 joint to an applied force, $f(t)$. This solution method works well when the number of degrees of freedom are small. As the order of the system is increased, the conditioning of the inverted matrix in equation 4 becomes poor.