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Recent Work on Renormalization Group for the Colored Glass Condensate

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Abstract

In this brief report, I summarize the recent work which I have been doing with E. Iancu and A. Leonidov to get explicit formulae for the renormalization group equations which describe the Colored Glass Condensate.

1 The Problem

The Colored Glass Condensate is a theory of colored gluons in the presence of a light cone source $J\mu(x^-, x^+) = \delta^{\mu +}\rho(x^-, x_T)$, where one averages over all possible orientations of ρ[1]- [12]. The functional measure for this theory is

$$Z = \int [d\rho] \int [dA] exp^{iS(A,\rho)} \tag{1.1}$$

This system in weak coupling describe a bose condensed state of gluons. The averaging over sources is how one describes glasses. Hence the name, a colored glass condensate.

This theory is an effective theory. If we let p^- be a light cone momentum for any degree of freedom for the theory, then we require that $p^- >> p_0^-$.[13]

The effective action is a generalization of that for a system in an external current

$$S = \frac{1}{4}F^2 + \frac{i}{N_c}\int d^3x \; Tr\rho(x) P e^{i\int_{-\infty}^{+\infty} dx^+ A^-} \tag{1.2}$$

Here the field A^- within the path ordered phase is in an adjoint representation of the color group. This provides a gauge invariant generalization for the usual source action for a fixed external current.

Due to the mass shell constraint, particles in this theory would satisfy $p^+p^- = p_T^2/2$, so that the constraint for mass shell and near mass shell modes is equivalent to $p^+ < p_T^2/2p_0^-$.

The theory therefore should describe small x modes. Iancu, Leonidov and I have shown that the leading terms in the small x effective action are in fact generated by such almost on shell modes.[13]

The classical equations for this theory,

$$D_\mu F^{\mu\nu} = \delta^{\nu +} \rho(x^-, x_T) \tag{1.3}$$

have a simple solution with $A^- = A^+ = 0$ in light cone gauge. To see this solution it is first convenient to write the solution in covariant gauge and then rotate back to light cone gauge. In covariant gauge,

$$A^i = A^- = 0 \tag{1.4}$$

and

$$-\nabla_T^2 A^+ = \rho \tag{1.5}$$

Returning to light cone gauge involves the gauge rotation $A^\mu \rightarrow iU\partial^\mu U^\dagger + UA^\mu U^\dagger$, where U solves

$$-i\partial^+ U^\dagger = A^+ U^\dagger \tag{1.6}$$

The solution to this equation is only specified if we impose a boundary condition

$$lim_{x^- \rightarrow -\infty} U^\dagger = 1 \tag{1.7}$$

so that

$$U^\dagger = Pexp\left\{ \int_{-\infty}^{x^-} dx^{-\prime} \frac{1}{-\nabla_T^2} \rho \right\} \tag{1.8}$$

This boundary condition is associated with our gauge fixing prescription It is essentially a retarded prescription for defining the inverse of ∂^+

The gauge rotated light cone field is of the form

$$A^i = iU\nabla_T^i U^\dagger \tag{1.9}$$

This is in the 2-dimensional transverse subspace, a pure gauge, so we have

$$F^{ij} = 0 \tag{1.10}$$

It has no x^+ dependence so

$$F^{i-} = 0 \tag{1.11}$$

It has a non-zero F^{i+}. These equations imply that $F^{i0} = F^{iz}$ so that the field has $\mid \vec{E} \mid = \mid \vec{B} \mid$ and

$$\vec{z} \perp \vec{E} \perp \vec{B} \tag{1.12}$$

These fields are therefore the generalization of the Lienard-Wiechart potentials to electrodynamics from QCD. They describe almost on mass shell gluons which are plane polarized to their direction of motion (like massless spin one particles).

2 The Renormalization Group

The results one can derive from the above classical considerations are true if p^- is not too much greater than p_0^-. How do the classical considerations breakdown?

If one computes

$$< AA >=< A_{cl}A_{cl} > + < A_{cl}A_q > + < A_qA_{cl} > + < A_qA_q > \tag{2.13}$$

where A_{cl} is the classical field and A_q is the quantum, then one gets a correction both to the quantites $< A_q >$ and $< A_qA_q >$. The first is associated with vacuum polarization and we will call it a virtual piece, and the second is associated with computing the quantum propogators, ie the connected piece of the propogators, and will be called the real piece. Both terms involve corrections which are of order $\alpha_s ln(1/x)$ Therefore if

$$e^{-1/\alpha_s} x_{upper} << x << x_{upper} \tag{2.14}$$

where x_{upper} is the upper-cutoff in x associated with the p^- cutoff described in the first section.

This presents a problem if we are to use this theory to describe dynamics at very small x. The way around this is to use the fact that in a small interval of x where the quantum corrections are small, one may integrate out the quantum fluctuations. This can be done consistently in weak coupling. This generates a theory at a smaller cutoff scale. One can iterate this procedure. The iteration is essentially the renormalization group.

One can show that the only effect of integrating out the quantum fluctuations at an intermediate scale is to change

$$F[\rho] \rightarrow F'[\rho] \tag{2.15}$$

This is a functional transformation of the theory. The renormalization group gives functional differential equations for F.

The coefficients of this functional renormalization group equation involve loop integrals over propogators to all orders in the background field generated by the source. It appears to be a very difficult task to compute these terms. In fact, to define them one must be extremely careful to both completely specify the gauge and to smear out the field in x^-. After much work, Iancu, Leonidov and I have computed these terms explicitly. We found no way to make sense of them unless we had them smeared out, and worked in a gauge where the inverse of ∂^+ was defined by a retarded prescription. (Advanced would also have worked.) We can demonstrate the gauge invariance of the effective action which results from our prescription.[13]

Since fixing a gauge is so essential, we must ask whether we can express the objects of interest in a gauge invariant manner. In fact one can always write down light cone gauge operators in a gauge invariant way by inserting line ordered phases between the coordinates of interest in the operators. The problem with this procedure is that it then appears that the gauge invariant operators depend upon the path used to define them.

The operators of interest here involve measurements on scales much larger than the extent of the sources. Outside the source, the field is a pure two dimensional gauge. The expectation values of operators therefore do not depend upon the way one joins points together in the two dimensional subspace. This is sufficient to prove that for the operators of interest, they are both gauge invariant and path independent as a consequence of the special form of the classical fields.

We have used our result to compare with the results of that of Balitsky and Kovchegov for the large N_c limit for the correlation function of two line ordered phases. It can be argued that these line ordered phases generate F_2 in deep inelastic scattering. They argue that the equation for F_2 is a closed non-linear integral equation which can be solved numerically. Our results agree with the form of the equation by Balitsky and Kovchegov,[14]-[15] and disagree with the analysis of Kovner et. al.[16] We believe the problem is in the way that Kovner et. al. fix their gauge.

If the large N_c limit is simple for the two point function, then there may be some hope that one can solve the functional renormalization group equations for F at least in the large N limit. That is, the small x limit is exactly solvable in large N_c.

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