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# RARE KAON DECAYS[a]

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The current status of the study of rare kaon decays is reviewed. Future prospects are discussed.

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## 1 Introduction

In recent years motivations for studying the rare decays of kaons have been threefold. First is the search for physics beyond the Standard Model (SM). Virtually every attempt to redress the theoretical shortcomings of the SM predicts some degree of lepton flavor violation (LFV). This can be manifested in kaon decays such as $K_L \rightarrow \mu^{\pm}e^{\mp}$. Such decays have very good experimental signatures and can therefore be pursued to remarkable sensitivities. These sensitivities in turn correspond to extremely high energy scales in models where the only suppression is that of the mass of the exchanged field.

There are also decays which are allowed by the SM, but which are extremely suppressed. In the the most interesting of these the leading contribution is a G.I.M.-suppressed [1] one-loop process quite sensitive to fundamental SM parameters such as $V_{td}$ and $m_t$. These decays are also potentially very sensitive to new physics.

Finally there are a number of long-distance-dominated decays which serve as a testing ground for theoretical techniques such as chiral lagrangians that seek to account for the low-energy behavior of QCD. Knowledge of certain of these decays is also needed to extract more fundamental information from some of the one-loop processes.

This field has been quite active in the last few years, so one has to be selective in a short review. This can be established by inspection of Table 1 which lists the decays for which results have been announced in the last couple of years and those that I happen to know are under analysis.

Table 1: Rare $K$ decay modes under recent or on-going study.

| | | | |
|---|---|---|---|
| $K^+ \rightarrow \pi^+\nu\bar{\nu}$ | $K_L \rightarrow \pi^0\nu\bar{\nu}$ | $K_L \rightarrow \pi^0\mu^+\mu^-$ | $K_L \rightarrow \pi^0e^+e^-$ |
| $K^+ \rightarrow \pi^+\mu^+\mu^-$ | $K^+ \rightarrow \pi^+e^+e^-$ | $K_L \rightarrow \mu^+\mu^-$ | $K_L \rightarrow e^+e^-$ |
| $K^+ \rightarrow \pi^+e^+e^-\gamma$ | $K^+ \rightarrow \pi^+\pi^0\nu\bar{\nu}$ | $K_L \rightarrow e^{\pm}e^{\mp}\mu^{\pm}\mu^{\mp}$ | $K^+ \rightarrow \pi^+\pi^0\gamma$ |
| $K_L \rightarrow \pi^+\pi^-\gamma$ | $K_L \rightarrow \pi^+\pi^-e^+e^-$ | $K^+ \rightarrow \pi^+\pi^0e^+e^-$ | $K^+ \rightarrow \pi^0\mu^+\nu\gamma$ |
| $K_L \rightarrow \pi^0\gamma\gamma$ | $K^+ \rightarrow \pi^+\gamma\gamma$ | $K^+ \rightarrow \mu^+\nu\gamma$ | $K^+ \rightarrow e^+\nu e^+e^-$ |
| $K^+ \rightarrow \mu^+\nu e^+e^-$ | $K^+ \rightarrow e^+\nu\mu^+\mu^-$ | $K_L \rightarrow e^+e^-\gamma$ | $K_L \rightarrow \mu^+\mu^-\gamma$ |
| $K_L \rightarrow e^+e^-\gamma\gamma$ | $K_L \rightarrow \mu^+\mu^-\gamma\gamma$ | $K_L \rightarrow e^+e^-e^+e^-$ | $K_L \rightarrow \pi^0e^+e^-\gamma$ |
| $K^+ \rightarrow \pi^+\mu^+e^-$ | $K_L \rightarrow \pi^0\mu^{\pm}e^{\mp}$ | $K_L \rightarrow \mu^{\pm}e^{\mp}$ | $K^+ \rightarrow \pi^-\mu^+e^+$ |
| $K^+ \rightarrow \pi^-e^+e^+$ | $K^+ \rightarrow \pi^-\mu^+\mu^+$ | $K^+ \rightarrow \pi^+X^0$ | $K_L \rightarrow e^{\pm}e^{\pm}\mu^{\mp}\mu^{\mp}$ |

For those who want a more extensive discussion of this subject, a new review by Barker and Kettell is now available [2].

## 2 Beyond the Standard Model

There were a series of dedicated $K$ decay experiments on the subject of lepton flavor violation at the Brookhaven AGS during the 1980's and 90's. These advanced the sensitivity to this sort of phenomenon by many orders of magnitude. In addition there were "by-product" results on this and other BSM topics from most of the other $K$ experiments in business during this period.

AGS E871, the most sensitive $K$ decay experiment ever mounted, was designed to detect $K_L \rightarrow e^{\pm}\mu^{\mp}$, $K_L \rightarrow e^+e^-$, and $K_L \rightarrow \mu^+\mu^-$. The final result on $K_L \rightarrow \mu e$ was recently published [3]. Fig.1 shows the distribution in $m_{\mu e}$ vs $p_T^2$ for events passing all other cuts. The rectangular contour bounds the region excluded from cut optimization. The inner contour bounds the actual signal region. Fig. 2 shows the projection onto the $m_{\mu e}$ axis of those events in Fig. 1 with $p_T^2 < 20(\text{MeV/c})^2$. Also shown

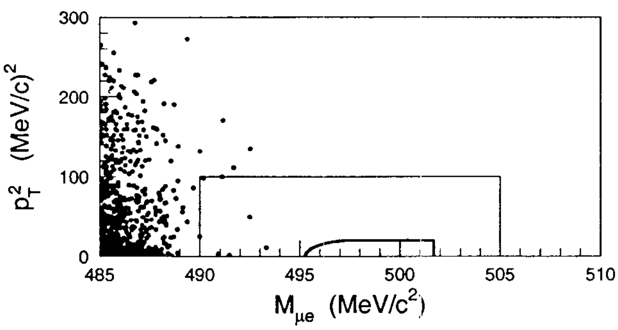


Figure 1: Distribution of $m_{\mu e}$ vs $p_T^2$ for events passing all other cuts (AGS E871). Rectangular contour bounds the "blind" region not interrogated until all cuts were finalized. Inner contour bounds the signal region.

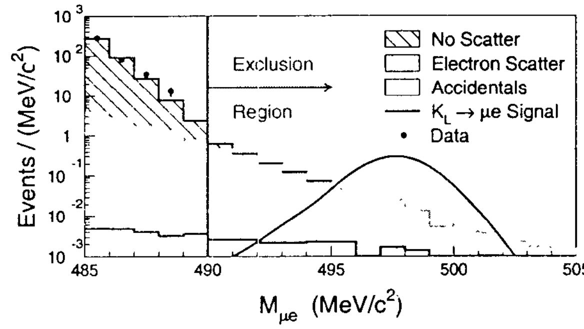


Figure 2: Spectrum of $m_{\mu e}$ for events passing all other cuts (AGS E871). The distribution of the three types of significant background are shown as is the expected signal shape.

are the calculated backgrounds, which total $\leq 0.1$ event in the signal region. Since no events were seen, a 90% CL upper limit was extracted: $B(K_L \to \mu e) < 4.7 \times 10^{-12}$. This is an 8-fold improvement on the previous limit and corresponds to a remarkable mass scale of $\sim 150\,\mathrm{TeV}$ for a hypothetical horizontal gauge boson (assuming standard electroweak coupling strength). This experiment was motivated largely by attempts to explain the electroweak scale via dynamical symmetry breaking, and results such as this one put this approach out of business. The experimenters believe that it would be possible to push their technique another order of magnitude before the background due to Mott scattering in chambers becomes intractable but there is no current plan to continue this work.

AGS E865 searched for the related decay $K^+ \to \pi^+\mu^+e^-$. While $K_L \to \mu e$ is sensitive only to new pseudoscalar or axial currents, the three body decay is sensitive to scalar or vector currents. Moreover if LFV is observed in $K$ decay, the three body decay will potentially be sensitive to details of the new interaction through a study of the Dalitz Plot. E865 has not completed analyzing all its data. However, they have recently released a result based on the 1996 data [4]. Fig. 3 is a scatterplot of the log-likelihood of the reconstructed events under the $K^+ \to \pi^+\mu^+e^-$ hypothesis versus the effective mass of the detected particles. A fit to the likelihood shapes of the signal and background yields a 90% CL upper limit of $B(K^+ \to \pi^+\mu^+e^-) < 3.9 \times 10^{-11}$. This is roughly five times better than the previous limit, and when combined with previous results from this series of experiments yields $B(K^+ \to \pi^+\mu^+e^-) < 2.8 \times 10^{-11}$. A further four-fold improvement in sensitivity is expected when all the data presently on tape is analyzed. In addition, E865 has new results [5] on $K^+ \to \pi^+\mu^-e^+$ and $K^+ \to \pi^-\mu^+e^+$, as well as on $K^+ \to \pi^-\mu^+\mu^+$ and $K^+ \to \pi^-e^+e^+$. This experiment also has many positive results on interesting kaon decays that I unfortunately don't have time to review.

Table 2 summarizes the current state of searches for $K$ decays that violate lepton flavor, lepton number, or both. It's noticeable that even the "by-product" results have in general reached sensitivities below $10^{-9}$. Limits on some of these processes can be related to results on $\nu$-oscillation, neutrinoless double $\beta$ decay and $\mu^- \to e^+$ conversion in the field of a nucleus [7].

## 3 One Loop Decays

As the activity in lepton flavor-violating kaon decays ramps down, that in SM-allowed "one-loop" decays is ramping up. These GIM-suppressed [1] decays are dominated by, or at least have measurable contributions from, loops involving weak bosons and heavy quarks. They include $K_L \to \pi^0\nu\bar{\nu}$, $K^+ \to \pi^+\nu\bar{\nu}$, $K_L \to \mu^+\mu^-$, $K_L \to \pi^0e^+e^-$ and $K_L \to \pi^0\mu^-\mu^-$. In some cases such as $K_L \to \pi^0\nu\bar{\nu}$, these contributions violate CP. The most interesting ones are those where the loops dominate. Fig 4 shows

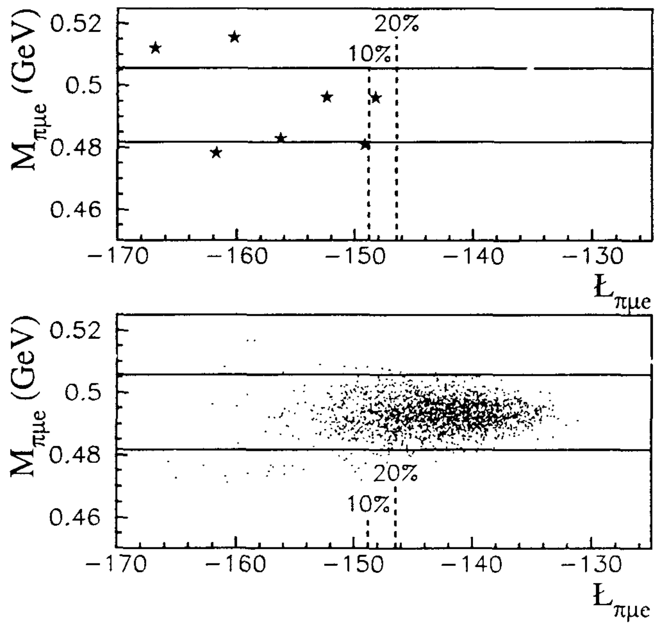


Figure 3: Scatter plot of E865 1996 data (top) and Monte Carlo (bottom). The horizontal lines demark the $3\sigma$ mass region.

Table 2: *Summary of searches for lepton flavor violating $K$ decays.*

| Decay Mode | Branching Ratio | Experiment | Pub. Date | Reference |
|---|---|---|---|---|
| $K_L \to \mu e$ | $< 4.7 \times 10^{-12}$ | E871 | 1998 | [3] |
| $K^+ \to \pi^+ \mu^+ e^-$ | $< 2.8 \times 10^{-11}$ | E865 | 2000 | [4] |
| $K_L \to \pi^0 \mu^\pm e^\mp$ | $< 4.4 \times 10^{-10}$ | KTeV | 2000 | [6] |
| $K^+ \to \pi^- \mu^+ \mu^+$ | $< 3 \times 10^{-9}$ | E865 | 2000 | [5] |
| $K^+ \to \pi^- e^+ e^+$ | $< 6.4 \times 10^{-10}$ | E865 | 2000 | [5] |
| $K^+ \to \pi^+ \mu^- e^+$ | $< 5.2 \times 10^{-10}$ | E865 | 2000 | [5] |
| $K^+ \to \pi^- \mu^+ e^+$ | $< 5.0 \times 10^{-10}$ | E865 | 2000 | [5] |
| $K_L \to e^\pm e^\pm \mu^\mp \mu^\mp$ | $< 6.1 \times 10^{-9}$ | E799-I | 1996 | [8] |

the Feynman diagrams for these loops. In general the processes are dominated by top quark loops, which makes them directly sensitive to the quantity $\lambda_t \equiv V_{ts}^* V_{td}$. In the past, it was common to assume CKM unitarity and discuss these processes in terms of their sensitivity to $\rho$ and $\eta$ or $|V_{td}|$. This is handy for comparison with information obtained from the $B$ system, but it is perhaps unfair to $K$'s. I will discuss an alternative a little later, but for now I will keep to this convention.

One can organize the CKM matrix information that can be obtained from $K$ decays around the popular unitarity plane construction, as shown in Fig. 5. The lighter triangle is usual one, whereas the darker triangle indicates the information available from rare kaon decays. Note that the "unitarity point", $(\rho, \eta)$ is determined from either triangle, and any disagreement between the $K$ and $B$ determinations indicates physics beyond the SM. The branching ratios closest to each line can determine the length of that line. The arrows from those branching ratios point to processes that need to be studied either because they potentially constitute backgrounds to the others, or because knowledge of them is useful in connecting the branching ratio measurements with fundamental parameters.

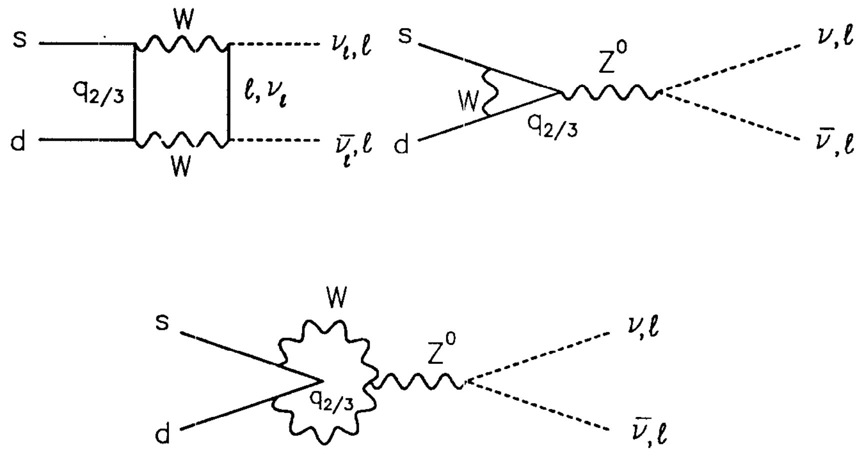


Figure 4: One loop contributions to $K$ decay.

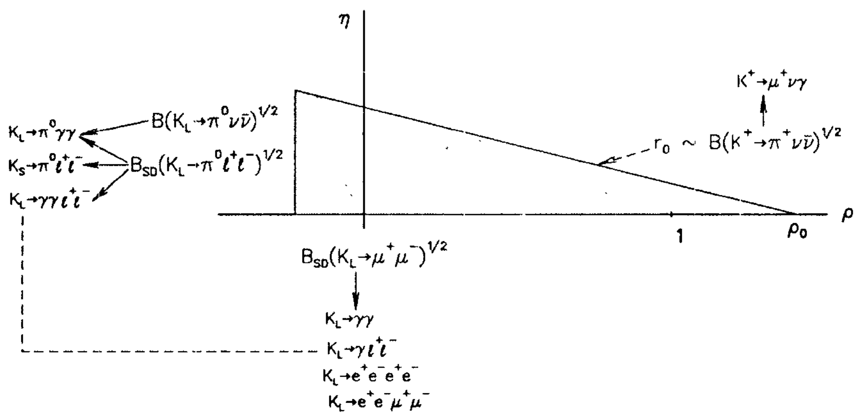


Figure 5: $K$ decays and the unitarity plane.

### *3.1* $K_L \to \mu^+\mu^-$

$K_L \to \mu^+\mu^-$ played a major role in the history of weak interactions, where the observation of a surprisingly small rate for this process was one of the inspirations for proposing the GIM mechanism [1]. There's a short distance contribution to the amplitude that is rather reliably calculable in the SM [9]. This contribution is proportional to the quantity $\rho_0 - \rho$, where $\rho_0$ is a function of CKM $A$, $m_t$ and $m_c$, and the QCD scale. The QCD corrections to this amplitude have been calculated to NLLA [9] and the residual uncertainty in $\rho_0$ due to this source is $< 10\%$. Numerically $\rho_0 \approx 1.2$. Thus a measurement of the rate for this process can potentially determine $\rho$, as indicated in Fig. 5. Unfortunately, its usefulness in this respect is limited by large long-distance effects stemming from the $\gamma\gamma$ intermediate state. This is dominated by the absorptive contribution:

$$\frac{\Gamma(K_L \to \gamma\gamma \to \mu\mu)_{abs}}{\Gamma(K_L \to \gamma\gamma)} = \alpha^2 (\frac{m_\mu}{m_K})^2 \frac{1}{2\beta} \left( \log \frac{1+\beta}{1-\beta} \right)^2 = 1.195 \times 10^{-5} \tag{1}$$

The resulting "unitarity bound" almost totally saturates the observed branching ratio. Now this contribution can be calculated rather precisely, so that with a sufficiently precise experiment, useful information on $\rho$ could still be obtained. Unfortunately there is also a dispersive contribution that is **much** more difficult to calculate [10] and that can interfere with the short-distance amplitude.

The most recent study of this decay is that of AGS E871 which observed a sample of approximately 6200 events [11]. Fig. 6 shows the $\mu\mu$ effective mass spectrum for $K_L \to \mu^+\mu^-$ candidates.

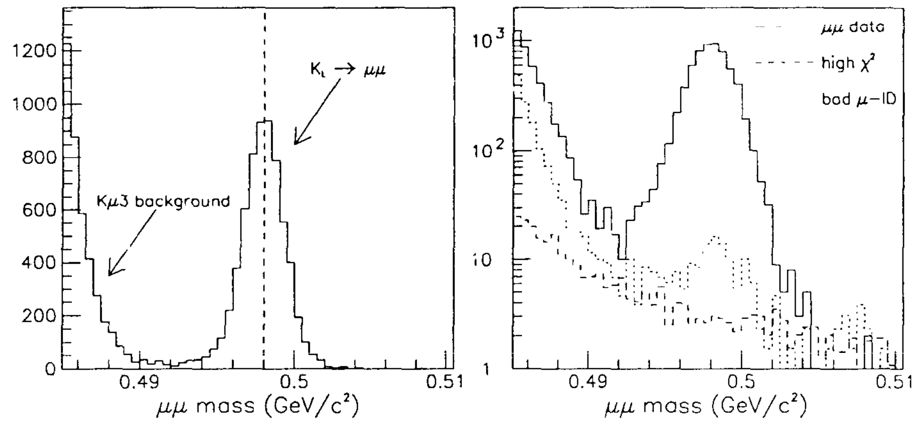


Figure 6: Two body effective mass distribution for $K_L \to \mu^+\mu^-$ candidates from AGS E871.

This data yields a branching ratio, $B(K_L \to \mu^+\mu^-) = (7.18 \pm 0.17) \times 10^{-9}$, which is to be compared with the unitarity bound from Eq. 3.1 of $(7.07 \pm 0.18) \times 10^{-9}$. The difference yields a 90% CL upper limit on the total dispersive contribution to $(K_L \to \mu^+\mu^-)$ of $0.37 \times 10^{-9}$. The E871 collaborators attempted to properly take the long distance contribution to the dispersive amplitude into account in extracting $\rho > -0.33$ at 90% CL from this limit. I will take a somewhat different approach below in exploiting this result. One can't leave the discussion of this experiment without mentioning the tour-de-force measurement [12], $B(K_L \to e^+e^-) = (8.7^{+5.7}_{-4.1}) \times 10^{-12}$, which is the smallest particle branching ratio ever reported.

There is no near-term plan for another experimental study of $K_L \to \mu^+\mu^-$. Further progress in the extraction of $\rho$ will depend on developments in theory. However it is thought that these developments can be advanced by the results of experiments on processes of the form $K_L \to \gamma\gamma$ in which one or both of the gammas is virtual. This hope is easy to understand, since it is off-shell intermediate states that contribute to the dispersive amplitude for $K_L \to \mu^+\mu^-$. The processes involved include (1) $K_L \to \gamma e^+e^-$, (2) $K_L \to \gamma\mu^+\mu^-$, (3) $K_L \to e^+e^-e^+e^-$, (4) $K_L \to e^+e^-\mu^+\mu^-$, and, in principle (5) $K_L \to \mu^+\mu^-\mu^+\mu^-$. There has been recent data on (1) [13], (2) [14], (3) [14], and (4) [14]. I would say that the jury is still out on how well theory copes with the dispersive amplitude.

### 3.2 $K^+ \to \pi^+\nu\bar{\nu}$

$K^+ \to \pi^+\nu\bar{\nu}$ is another process sensitive to short distance physics, that has none of the theoretical problems of $K_L \to \mu^+\mu^-$, but unfortunately is much more difficult to study. There are no significant long-distance contributions to this decay, and the usually problematic hadronic matrix element can be calculated via an isospin transformation from that of the well-measured $K_{e3}$ decay [15]. This decay is usually discussed in terms of its sensitivity to $V_{td}$. The amplitude for this decay is proportional to the dark slanted line at the right in Fig. 5. This is equal to the vector sum of the line proportional to $|V_{td}|/A\lambda^3$ and that from 1 to the point marked $\rho_0$. This length along the real axis is proportional to amplitude for the charm contribution to $K^+ \to \pi^+\nu\bar{\nu}$. The QCD corrections to this amplitude are the source of the largest theoretical uncertainty in the calculation of $B(K^+ \to \pi^+\nu\bar{\nu})$. These have been calculated to NLLA [9], and the residual uncertainty in the charm amplitude is estimated to be $\sim 15\%$. This results in a $\sim 6\%$ uncertainty [16] in extracting $|V_{td}|$ from $B(K^+ \to \pi^+\nu\bar{\nu})$. Experiment is still far from this level. In 1997, AGS E787 published evidence for one event of $K^+ \to \pi^+\nu\bar{\nu}$ [17]. Recently an analysis of a larger data set has been published [18]. Fig. 7 shows the results: no further events were seen, leading to a branching ratio $B(K^+ \to \pi^+\nu\bar{\nu})= (1.5^{+3.4}_{-1.2}) \times 10^{-10}$. This is to be compared to the expectation of $(0.82 \pm 0.32) \times 10^{-10}$ from fits to the CKM phenomenology [16]. E787 has a further sample under analysis of sensitivity about equal to the sum of all its previous runs. It is notable that E787 has established methods to reduce the residual background to $\sim 10\%$ of the the signal branching ratio predicted by the SM.

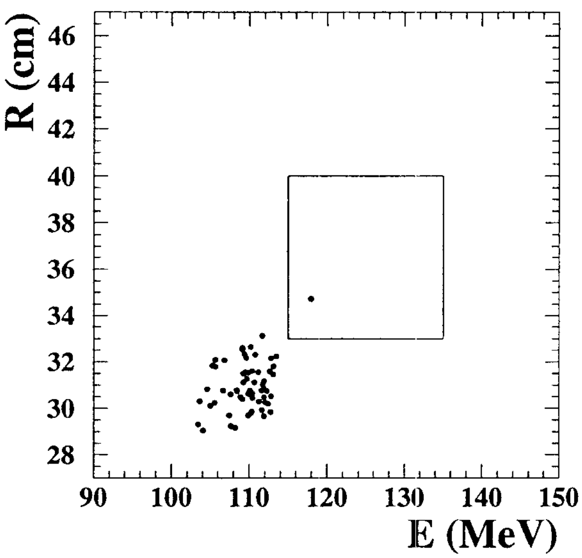


Figure 7: The $\pi^+$ range vs kinetic energy for events passing all other cuts from AGS E787. Box indicates $K^+ \to \pi^+\nu\bar{\nu}$ signal region. Events at lower left are residual $K^+ \to \pi^+\pi^0$.

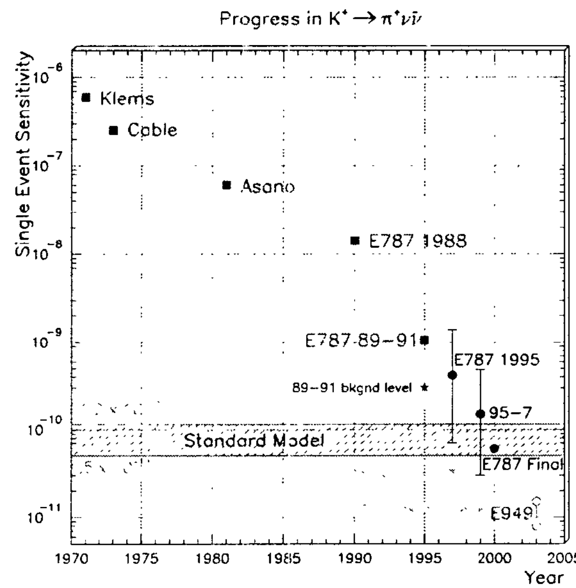


Figure 8: The history and projected future progress of the study of $K^+ \to \pi^+\nu\bar{\nu}$ compared to Standard Model expectation. AGS E949 will extend this study to the $10^{-11}$/event level by around 2004.

A new experiment, AGS E949 [19], based on an upgrade of the E787 detector, is in preparation and scheduled to run in 2001-3. Using the entire flux of the AGS, it is expected to reach a sensitivity of $\sim 10^{-11}$/event. Fig. 8 shows the history and near term expectations of the progress in studying this decay. All the experiments in Fig. 8 used stopped-$K^+$ beams. In the longer term, the CKM experiment [20] at Fermilab proposes to use an in-flight technique to reach $10^{-12}$/event sensitivity. If CKM fits to the SM phenomenology are correct, this will result in some 70 $K^+ \to \pi^+\nu\bar{\nu}$ events.

### *3.3* $K_L \to \pi^0\nu\bar{\nu}$

$K_L \to \pi^0\nu\bar{\nu}$ is the holy grail of the $K$ system. It is direct CP-violating to a very good approximation [21,22], like $K^+ \to \pi^+\nu\bar{\nu}$ it has a hadronic matrix element which can be determined from that of $K_{e3}$, but, unlike $K^+ \to \pi^+\nu\bar{\nu}$, it has no significant contribution from charm. Thus the intrinsic theoretical uncertainty on the connection between $B(K_L \to \pi^0\nu\bar{\nu})$ and the fundamental SM parameters is only about 2% [b].

Our best knowledge of $B(K_L \to \pi^0\nu\bar{\nu})$ comes indirectly from E787's measurement of $B(K^+ \to \pi^+\nu\bar{\nu})$ through a model independent relationship pointed out by Grossman and Nir [23]: $B(K_L \to$

[b] Note $B(K_L \to \pi^0\nu\bar{\nu}) \propto A^4\eta^2$.

$\pi^0\nu\bar{\nu}$)< $4.4\,B(K^+ \rightarrow \pi^+\nu\bar{\nu})$. In the SM this is equivalent to the statement that the imaginary part of an amplitude cannot be larger than its modulus. The E787 result cited above then yields $B(K_L \rightarrow \pi^0\nu\bar{\nu}) < 2.6 \times 10^{-9}$ at 90% CL. This is much tighter than the current direct experimental limit, $5.9 \times 10^{-7}$, which comes from the KTeV experiment at Fermilab[24]. However to actually measure $B(K_L \rightarrow \pi^0\nu\bar{\nu})$, it will be necessary to improve the reach of direct experiment by some five orders of magnitude. The KEK E391a experiment[25] does not quite propose to bridge this gap, but to achieve a sensitivity of $\sim 10^{-10}$/event, which would at least better the indirect limit by an order of magnitude. It will serve as a test for a future much more sensitive experiment to be performed at the Japanese Hadron Facility.

At Fermilab, the KaMI[26] proponents plan to use the high proton current of the Main Injector to make a $K_L$ beam sufficiently intense that a sensitivity of $< 10^{-12}$/event for $K_L \rightarrow \pi^0\nu\bar{\nu}$ can, in principle, be achieved. This experiment is similar to KEK E391a, with the major exception that the energy scale is a factor of ten higher. Both feature a pencil beam, a crystal spectrometer and an hermetic photon veto, all operating within an evacuated enclosure. KaMI would also incorporate a scintillating fiber charged particle spectrometer, greatly enhancing the physics menu of the experiment.

A completely different approach is taken by the KOPIO experiment[27] which will exploit the intensity and flexibility of the BNL AGS to make a high-flux, low-energy, microbunched $K_L$ beam. This allows time-of-flight determination of the $K_L$ velocity. In addition, the direction as well as energy of the final state photons will be measured, so that a well-defined vertex can be found. This provides a measurement of the $K_L$ 3-momentum so that kinematic constraints as well as photon vetoing are available to fight backgrounds. The leading expected background is $K_L \rightarrow \pi^0\pi^0$, which is some eight orders of magnitude larger than the predicted signal. Since $\pi^0$'s from $K_L \rightarrow \pi^0\pi^0$ have a unique energy in the $K_L$ center of mass, a very effective kinematic cut can be applied. This reduces the burden on the photon veto system to the point where the techniques proven in E787 are sufficient. This experiment will avoid having to operate most of its apparatus in a vacuum at the cost of having a thin vacuum enclosure around the beam. KOPIO aims to collect 65 $K_L \rightarrow \pi^0\nu\bar{\nu}$ events with a signal to background ratio of 2:1. This will permit $\eta$ to be determined to < 10%, given expected progress in measuring $m_t$ and $V_{cb}$. KOPIO will run during the ~20 hours/day the AGS is not needed for injection into RHIC.

### 3.4 *A different way of using rare kaon decay results*

Many theorists advocate comparing the results of rare kaon decay experiments with those from the $B$ system when both are available. Obviously $|V_{td}|$ derived from $B(K^+ \rightarrow \pi^+\nu\bar{\nu})$ can be checked against $|V_{td}|$ extracted from the ratio of $\bar{B}_s - B_s$ and $\bar{B}_d - B_d$ mixing. It has also been emphasized that results from $K^+ \rightarrow \pi^+\nu\bar{\nu}$ and $K_L \rightarrow \pi^0\nu\bar{\nu}$ can be combined to yield $sin2\beta$[28,29] or a quantity closely related to it[23,30,31]. A number of other useful relations have also been pointed out[16,29].

However, I favor a different approach, in which $\rho$ and $\eta$ are de-emphasized in favor of the real and imaginary part of $\lambda_t$. $Im(\lambda_t)$ is closely related to the Jarlskog invariant, $\mathcal{J}_{CP}$[32], and thus to the area of the unitarity triangle,

$$\mathcal{J}_{CP} = Im(V_{ud}^* V_{us} V_{ts}^* V_{td}) = \lambda(1 - \frac{\lambda^2}{2}) Im(\lambda_t) \tag{2}$$

In this approach, the branching ratio for $K_L \rightarrow \pi^0\nu\bar{\nu}$ can be written:

$$B(K_L \rightarrow \pi^0\nu\bar{\nu}) = \frac{3\tau_L \alpha^2 r_L B_{K^+e3}}{\tau_+ V_{us}^2 2\pi^2 sin^4\theta_W} (Im\lambda_t X(x_t))^2 \tag{3}$$

where we take $\alpha = 1/129$ and $sin^2\theta_W = 0.23$, $r_L$ is an isospin-breaking and phase space correction [15], which for this process = 0.944. $X(x_t)$ is an Inami-Lim [33] function of $x_t \equiv m_t^2/m_W^2$ which $\approx 0.65x_t^{0.59}$. The product $r_L B_{K^+e3}$ accounts for the hadronic matrix element, and the $\tau$'s are the $K_L$ and $K^+$ lifetimes. Since all these factors are well determined,[c] a measurement of $B(K_L \to \pi^0\nu\bar{\nu})$ gives a direct measurement of the area of the unitarity triangle. This can then be compared with any of a number of different indirect determinations of the unitarity triangle area from studies of $B$'s.

It is instructive to write the branching ratio for $K^+ \to \pi^+\nu\bar{\nu}$ in a form equivalent to Eq. 3:

$$B(K^+ \to \pi^+\nu\bar{\nu}) = \frac{r_+\alpha^2 B_{K^+e3}}{V_{us}^2 2\pi^2 sin^4\theta_W} \sum_i |\lambda_c X_{NL}^i + \lambda_t X(x_t)|^2 \tag{4}$$

where $r_+ = 0.901$, $\lambda_c \equiv V_{cs}^* V_{cd}$, $i = e, \mu, \tau$, and $X_{NL}^i$ is given in Table 1 of Ref. [16].

Now whereas there are no useful limits on $\lambda_t$ from Eq. 3, one can use Eq. 4 to extract limits from the latest result of E787. One can obtain [18]:

$$|Im(\lambda_t)| < 1.22 \times 10^{-3} \tag{5}$$
$$-1.10 \times 10^{-3} < Re(\lambda_t) < 1.39 \times 10^{-3} \tag{6}$$
$$1.07 \times 10^{-4} < |\lambda_t| < 1.39 \times 10^{-3}. \tag{7}$$

Note that the actual regions of the $\lambda_t$ plane constrained by $K^+ \to \pi^+\nu\bar{\nu}$ are (approximately) annuli to which the above one-dimensional limits are tangent. The charm contribution displaces the center of the limiting circles from the origin. To get conservative "outer" limits one must minimize $m_t$.

One can write down a similar form for the short-distance part of the $K_L \to \mu^+\mu^-$ branching ratio.

$$B^{SD}(K_L \to \mu\mu) = \frac{\tau_{K_L}\alpha^2 B_{K^+\mu\nu}}{\tau_{K^+} V_{us}^2 \pi^2 sin^4\theta_W}[Re(\lambda_c)Y_{NL} + Re(\lambda_t)Y(x_t)]^2 \tag{8}$$

where $Y(x_t) \approx 0.32x_t^{0.78}$ and $Y_{NL}$ is given in Table 3 of Ref. [16]. To make use of the current experimental result on $B(K_L \to \mu^+\mu^-)$, one must face the problem of the long-distance dispersive amplitude. A choice that has sometimes been made in the experimental literature is to limit the possible absolute size of this amplitude by the 90% CL limit of a recent calculation [34]: $|ReA_{LD}| < 2.9 \times 10^{-5}$. With some trepidation I use it, partly on the ground that considerably less conservative assumptions have sometimes been made in the same literature. For example, in extracting the limit on $\rho$ mentioned above, the E871 collaboration used the probability distribution of $|ReA_{LD}|$ given in Ref. [34], rather than imposing the 90% CL limit [35]. Having made the more conservative choice, and taking $m_t$ and $Y_{NL}$ to the limits recommended in Ref. [16] I obtain

$$-5.85 \times 10^{-4} < Re(\lambda_t) < 7.24 \times 10^{-4} \tag{9}$$

Fig. 9 shows the region of the $\lambda_t$ plane constrained by $K^+ \to \pi^+\nu\bar{\nu}$ and $K_L \to \mu^+\mu^-$, compared to results from recent CKM fits [36]. The allowed region is that both between the circles and between the vertical lines. These limits are weaker than those that can be obtained from the full available CKM phenomenology but they are independent of information obtained from the $B$ system, or from CP-violation measurements in $K \to \pi\pi$, thus the observed agreement is quite meaningful.

[c] Already the *current* uncertainty on $m_t$ would lead to only a 3.5% contribution to the uncertainty in $Im(\lambda_t)$ measured from $B(K_L \to \pi^0\nu\bar{\nu})$.

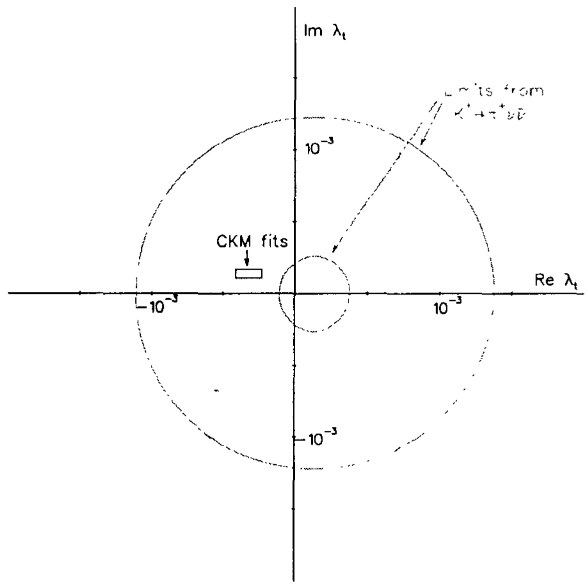


Figure 9: Region allowed by the two rare K decays, compared with current CKM fits.

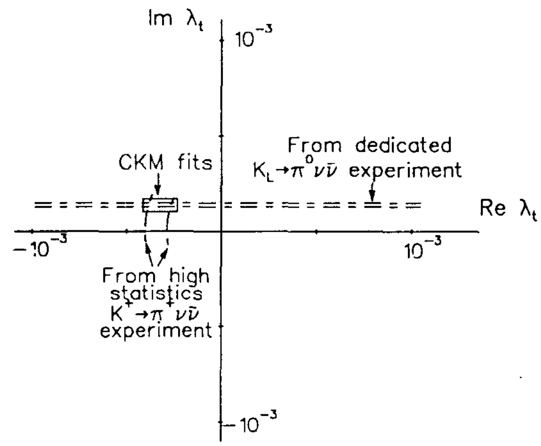


Figure 10: Comparison between present CKM phenomenology and results from future experiments on charged and neutral $K \to \pi\nu\bar{\nu}$.

### *3.5 Conclusions*

Although I couldn't cover much of this, the kaon decay sections of the Particle Data Book are being largely rewritten by the results of current and recent experiments.

Lepton flavor violation experiments have been pushed to remarkable sensitivities, corresponding to mass scales of well over 100 TeV. But this success has killed most models predicting LFV in kaon decay. Barring developments in theory, this subject will probably advance only as a by-product of other studies.

The recent precision measurement of $K_L \to \mu^+\mu^-$ will be very useful if theorists can nail down the size of the dispersive long-distance amplitude.

$K^+ \to \pi^+\nu\bar{\nu}$ has been seen and can clearly be further exploited. There are two coordinated initiatives devoted to this: a $10^{-11}$/event experiment is being prepared at the BNL AGS and a $10^{-12}$/event experiment at the FNAL Main Injector is in the R&D phase. The first dedicated experiment to seek $K_L \to \pi^0\nu\bar{\nu}$ is proceeding and two other initiatives are in progress with the goal of making a $\sim 10\%$ measurement of $Im(\lambda_t)$.

The motivation for pursuing $K \to \pi\nu\bar{\nu}$ is stronger than ever. An "alternative" unitarity triangle can be constructed from this data, and it will be invaluable for comparison with results from the $B$ system if new physics is at work in the flavor sector. By and large the effects of such new physics will be quite different in the $K$ and $B$ systems. Fig. 10 shows what $K \to \pi\nu\bar{\nu}$ experiments might offer in the next few years. This figure assumes the two sectors will agree, but it is important to notice the large area available for them *not* to do so!


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