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Consolidation of Modal Parameters from Several Extraction Sets

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ABSTRACT

Experienced experimentalists have gone through the process of attempting to identify a final set of modal parameters from several different sets of extracted parameters. Usually, this is done by visually examining the mode shapes. With the advent of automated modal parameter extraction algorithms such as SMAC (Synthesize Modes and Correlate), very accurate extractions can be made to high frequencies. However, this process may generate several hundred modes that then must be consolidated into a final set of modal information. This has motivated the authors to generate a set of tools to speed the process of consolidating modal parameters by mathematical (instead of visual) means. These tools help quickly identify the best modal parameter extraction associated with several extractions of the same mode. The tools also indicate how many different modes have been extracted in a nominal frequency range and from which references. The mathematics are presented to achieve the best modal extraction of multiple modes at the same nominal frequency. Improvements in the SMAC graphical user interface and database are discussed that speed and improve the entire extraction process.

NOMENCLATURE

FRF:	frequency response function
SVD:	Singular Value Decomposition
φ_i:	i^{th} mode shape
ω_i:	i^{th} natural frequency
U_i:	i^{th} left singular vector of SVD
MIF:	Mode Indicator Function
NMIF:	Normal Mode Indicator Function
CMIF:	Complex Mode Indicator Function
MAC_{ij}:	Modal Assurance Criteria
$MACSV_{ij}$:	MAC using left singular vector U_i

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INTRODUCTION AND MOTIVATION

Many organizations are relying on innovative model validation techniques and higher fidelity models to assess the quality of a design. Under certain conditions, models are being used to qualify hardware. A process is required which includes validating the models so that one has confidence in the modeling approach and the predicted results. For a structural dynamics model, this puts high demands on experimental results and more specifically the modal parameter extraction process. The estimation accuracy and the modal parameter reduction efficiency are critical features in the success of the model validation process. Organizations desire accurate results delivered quickly that include higher frequency ranges than were previously obtained. When the frequency range is extended, typically the modal density is increased. To extract these modes, more force inputs are required to separate the modes in areas of high modal density. In addition, there is more data to reduce due to the number of modes required.

The ideal solution to this problem is an algorithm that automatically extracts modal parameters from large sets of experimental data. The SMAC (Synthesize Modes and Correlate) Modal Extraction Package has been developed over the past few years to meet these needs [1,2,3,4]. A recent enhancement to the SMAC algorithms has included a graphical user interface that enables the user to easily enter analysis parameters and evaluate the results of the modal extraction. In addition, a set of tools has been developed to speed the process of consolidating modal parameters from a multi-reference experiment based on a mathematical algorithm.

THEORY OF SMAC EXTRACTION PACKAGE

The SMAC algorithm is based upon the modal filtering approach rather than an assumed matrix polynomial form. In the strictest sense this means that there must be at least as many response measurements as there are active modes in the frequency band of interest. The sensors should be placed so that the associated experimental mode shape

DISCLAIMER

matrix is well conditioned for inversion. Since SMAC is not based on a matrix polynomial, there are no computational roots, eliminating a major set of decisions the analyst must make in deciding on the true system roots. Details of the SMAC theory can be found in the references [1,2,3,4].

ENHANCEMENTS TO SMAC EXTRACTION PACKAGE

There have been a number of recent enhancements to the SMAC Modal Extraction Package that have increased the speed and improved the overall extraction process. The SMAC algorithms have been implemented in MATLAB™ and take advantage of the IDEAS™ to MATLAB™ data translation software IMAT™ [5]. A new graphical user interface has been designed around the SMAC algorithms and is described in this section of the paper. The initial interface window for SMAC enables the user to select the type of input data that will be analyzed. Currently, the SMAC algorithms are able to process FRF data from IDEAS™ binary (*.afu), MATLAB™ binary (*.mat) and ASCII universal files using the SDRC format.

The graphical interface for SMAC provides the user with an easy means to input parameters. The first window that requires the input of analysis parameters is shown in Figure 1. At this interface the user will select parameters that will effect the execution of the pseudo-inverse and ultimately the entire modal analysis. The two major user selections at this interface are the solution method (i.e., real or complex modes) and the condensation of the experimental data that will be processed by SMAC. In Figure 1 the frequency range has been modified to include FRF data from 425 to 3250 Hz in the calculation of the pseudo-inverse. If all the mode shapes are independent, SMAC can process the entire frequency range, but if some mode shapes are not independent, reducing the frequency band is necessary. This frequency range specification enables the user to focus the analysis in a particular segment of the data.

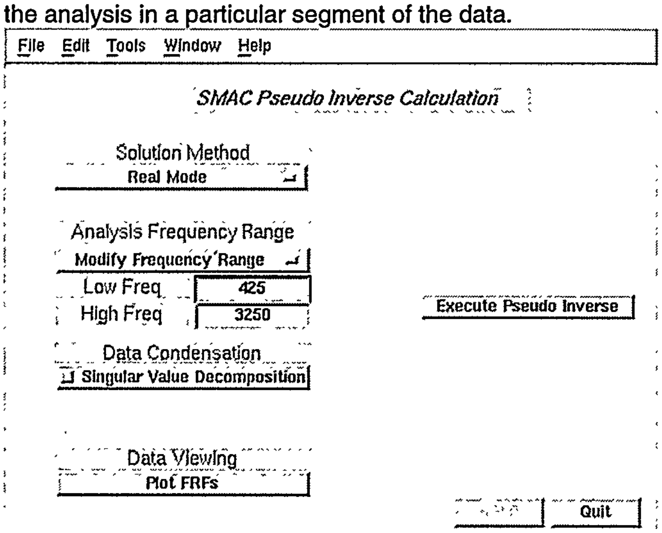

Figure 1. SMAC Pseudo-Inverse Interface

After the pseudo-inverse has been calculated on the selected set of experimental FRFs, the correlation coefficients are determined over a specified frequency band. Before executing the correlation calculation the user must specify an initial estimate of damping, the number of frequency lines used in the correlation process and the frequency band to analyze (Figure 2). It should be noted that the bandwidth of fit must lie within the modified frequency range that was selected in the calculation of the pseudo-inverse.

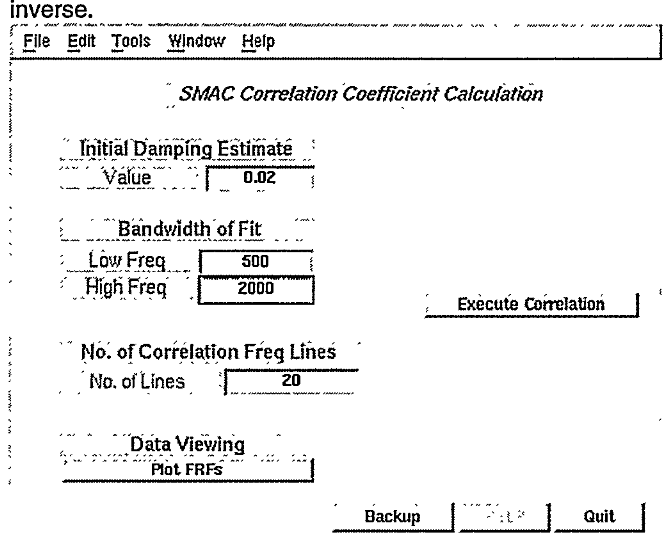

Figure 2. SMAC Correlation Coefficient Interface

The result of the correlation calculation is the curve displayed in Figure 3. A frequency at which there is a high correlation value is an indication of a mode of the system. The correlation value plot is typically generated with a frequency resolution similar to that of the experimental FRF data. The analyst may choose a threshold or minimum value in that interface below which no peaks are considered. In our experience only correlation coefficient peaks above 0.9 are worthy of investigation; however, this probably varies considerably from structure to structure. The frequency of each peak is saved and displayed in a table, and these become the starting points for the automated SMAC fitting algorithm.

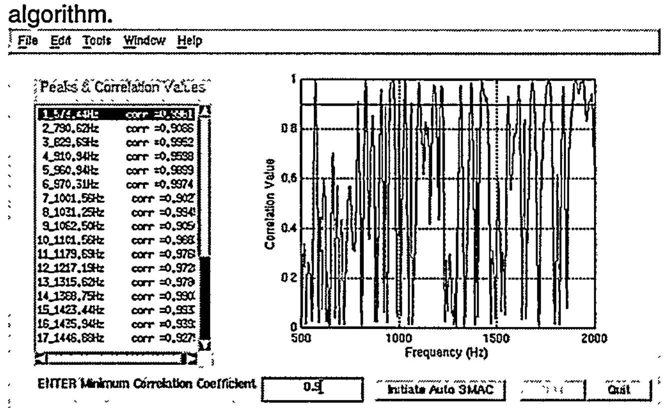

Figure 3. SMAC Frequency and Correlation Interface

Next the automated algorithms are executed. The algorithm starts with the first frequency from the table of peaks and the initial guess of the damping ratio. The user specifies the frequency band by selecting a percentage of the root frequency, typically 0.5 to 3 percent (Figure 4). Two routines, which operate in a similar manner, are used to converge on the root. In the first routine the correlation coefficients are calculated at ten equally spaced frequency points using the selected narrow frequency band and the assumed damping. Then a parabola is fit to the ten points, and the frequency at the maximum point of the parabola is calculated and becomes the first estimate of the natural frequency. Then the second routine is executed, which is exactly like the first except now the correlation coefficients are calculated at ten equally spaced damping points using the selected damping range, chosen by the user in Figure 4, and the frequency estimate just calculated above. A best estimate of damping is then obtained from the parabolic fit.

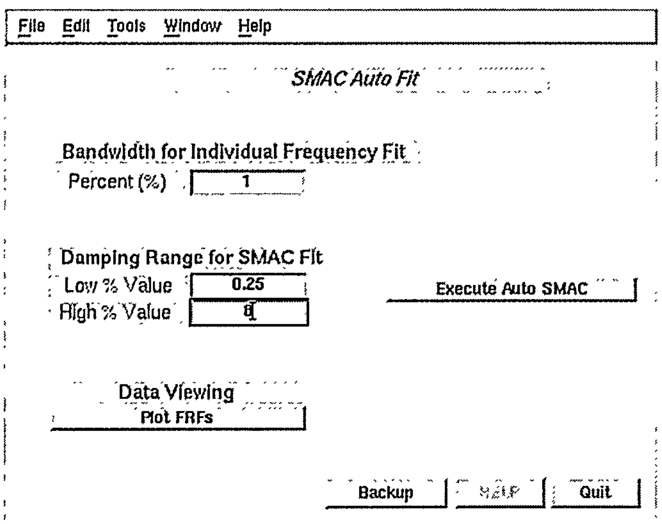

Figure 4. SMAC Automated Fit Parameters

The program switches back and forth between these two routines. With each switch the range of frequency and/or damping is reduced. When the damping ceases to change more than 0.5 percent of the damping value, the root is considered successfully converged and is saved. If the optimization process attempts to extend beyond the original frequency range, the root is rejected. Sometimes the same root is converged upon from two different starting points, so there is a built-in check to eliminate duplicate roots. Then the program repeats the process for the second frequency in the peak table and continues until all candidate roots have been converged upon or eliminated. Figure 5 displays the output of the automated SMAC fitting for a set of experimental data.

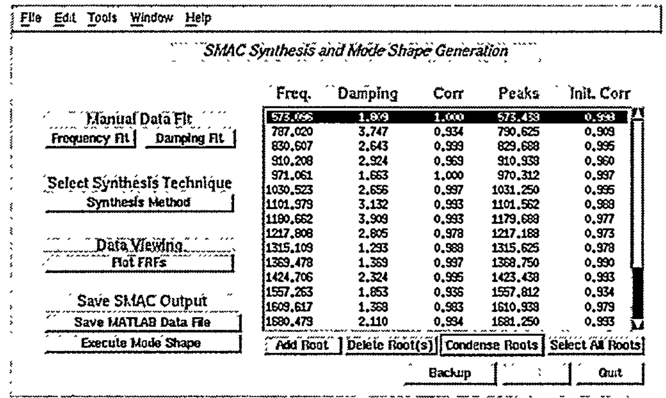

Figure 5. SMAC Roots for Synthesis and Mode Shape Generation

Typically, the automated SMAC algorithms can calculate 90 percent of the roots. It has been found that the automated process sometimes misses roots when two roots are within the initial 0.5 to 3 percent frequency band, or the damping ratio is very low or out of the range of the damping ratios that the analyst considered reasonable. A manual version of these algorithms can be used to converge on any roots that are missed (Figure 5 – Frequency Fit and Damping Fit). These omissions are discovered in the MIF quality comparisons, and the roots can be extracted through the manual process.

After the process of extracting modal parameters using SMAC, it is important to assess the quality of the extraction. Comparisons of the synthesized and actual FRF data are a typical method of checking whether all the modes of interest in a particular frequency band have been properly identified. A more global quality of fit is obtained by comparing synthesized MIFs with the MIFs calculated on the actual data. The MIF includes effects of the data from all the FRFs in a single curve to indicate the modes. If the synthesized MIF depicts modes in the same way as the MIF using the experimental data, then there is high confidence in the modal parameters. The Complex Mode Indicator Function (CMIF) [6] is a good tool for comparing the strength of the modes excited by a particular reference. However, the CMIF can obscure the effects of the weakly excited modes in the data. Because the SMAC algorithms have been shown to be extremely robust in extracting not only well excited but also weakly excited modes, the Normal Mode Indicator (NMIF) [7] is, typically, selected because it tends to be a more sensitive comparison for all the modes. Viewing both these MIFs comparisons is available through the graphical interface in Figure 5 (Synthesis Method).

Each of the major graphical interfaces that have been shown in the above figures provides direct access to the experimental data. Selecting the "Plot FRFs" button from any of these interfaces allows the user to view the experimental data during the analysis process. Additionally, all the modal parameters from an analysis are saved in a database for easy access and re-loading into SMAC for review or further analyses. The database of modal parameters also has direct application into the modal parameter condensation codes discussed below.

MODAL PARAMETER CONDENSATION APPROACH

With the advent of automated modal parameter extraction algorithms, extractions can be made to high frequencies[1,2,3,4,8]. Using the single-referenced SMAC approach may generate several hundred modes that then must be consolidated into a final set of modal information for validating a finite element model. Typically, the process to identify a final set of modal parameters from several different sets of extracted parameters has been done by visually examining the mode shapes. To speed the process of consolidating modal parameters, a set of mathematical (instead of visual) tools have been developed. These tools quickly help identify the best modal parameters associated with several extractions of the same mode. In general, the more extractions of the same mode (from separate references) in a condensation, the higher the confidence is in that mode. The mathematical tools also indicate how many different modes have been extracted in a nominal frequency range, and from which references. The mathematics are presented below.

The results of a SMAC analysis are stored in a database structure that can be directly accessed by the modal parameter condensation algorithms. Thus, after all references from a particular experiment have been analyzed using SMAC, each of their structures containing the specific modal information (frequency ranges, correlation values, MIF data and mode shapes) can be loaded into the modal condensation routines. During the loading process the user specifies an appropriate label for each of the modal references. Then a very narrow frequency range is selected to determine the best modal parameters associated with the modes in that range. The first mathematical indicator provided to the user is a modal fit quality factor. The quality factor, equation (1), is defined as

$$QF = \frac{\left(1 - NMIF_{\exp}(\omega_i)\right) - \left(\left|NMIF_{\exp}(\omega_i) - NMIF_{fit}(\omega_i)\right|\right)}{\left(1 - NMIF_{\exp}(\omega_i)\right)} * corr^2, \quad (1)$$

where ω_i is the natural frequency of the i^{th} mode and $NMIF(\omega_i)$ is the value of the Normal Mode Indicator Function at the natural frequency for either the experimental data (exp) or the synthesized analytical fit (fit). The first part of Equation (1) is a measure of how well the synthesized NMIF from the SMAC fit compares to the NMIF from the experimental data at the particular modal frequency of interest. That comparison value is multiplied by the square of the correlation coefficient for that mode from the SMAC fit. This gives an overall quality factor between 0 and 1; where 1 indicates a very high quality fit to the experimental data. It is important to use the quality factor in conjunction with the value from the NMIF in determining the best fit of the modes in the analysis band. A quality factor of 0.99 and a NMIF value of 0.8 would indicate a very good fit to a weakly excited mode. A better fit of the mode (assuming the modes are the same) might be a quality factor of 0.95 and a NMIF value of 0.25 since the mode would be much stronger in the data.

A second mathematical method for assessing the quality of modal fits from a set of SMAC extractions is the Singular Value Decomposition (SVD) [9]. The SVD is performed on the mode shapes from the user-selected frequency range. The singular values are used to indicate how many independent modes have been identified in the frequency range of interest. If there is large change (an order of magnitude or greater) between the first and second singular values, this is a strong indication that there is only one mode identified in the bandwidth analyzed. However, if the difference in the singular values is not significant, then there could be more than one independent mode in the analysis band.

Another mathematical method used in the modal parameter condensation is the Modal Assurance Criteria (MAC). Equation (2) gives the general definition of the MAC applied to analysis mode shapes

$$MAC_{ij} = \frac{(\varphi_i^T \bullet \varphi_j)^2}{(\varphi_i^T \bullet \varphi_i)(\varphi_j^T \bullet \varphi_j)}, \quad (2)$$

where φ_j is the mode shape of the j^{th} mode. The MAC values provide the user additional mathematical information about the linear independence of the modes from different references in the analysis bandwidth. MAC values are also calculated using the left singular vector U (from SVD calculation) and the analytical mode shapes, Equation (3)

$$MACSV_{ij} = \frac{(U_i^T \bullet \varphi_j)^2}{(U_i^T \bullet U_i)(\varphi_j^T \bullet \varphi_j)}. \quad (3)$$

The left singular vector U_i gives an estimate of the mode shapes in the analysis range. The MAC values generated from Equation (3) provide the user with information on which references extracted each mode or if the extraction is a mixture of multiple modes. The first singular vector should be the best estimate with accuracy decreasing for progressively higher singular values. Using all the mathematical tools described above helps in the process of determining the best modal fits of the experimental data. Combining all the mathematical information together makes the selection of an appropriate set of modal parameters attainable for an experimental program that has many references with several hundred extracted modes.

MODAL PARAMETER CONDENSATION APPLICATIONS

The following examples show the mathematical results from a modal parameter condensation of an electrical component experiment. An experiment, using both a modal hammer and shaker, was performed on the component. The SMAC extraction algorithms were used to fit all the experimental data up to 3 KHz and approximately 45 to 60 modes were identified for each reference. The data below shows the condensation process for six independent reference locations used to excite the component.

Figure 6 displays two plots of the modal data. The first plot shows the frequencies of the modes (asterisks) for each reference up to a frequency of 1200 Hz. By selecting any of the asterisks in the analysis band, the second plot displays the corresponding NMIF for the reference from which that mode was extracted. The dashed line is the NMIF synthesized from the SMAC extracted modal parameters. The first analysis frequency band selected was a narrow band from 560 to 580 Hz. The output from the analysis is listed below. A mode was fit for each of the six references in that frequency band as evident in the first table. The calculated quality factor is listed in the far right column for each of the modes in the band. All the quality factors are high, however, the largest value and best fit is from the 400y-reference location. The results from the SVD calculation indicate only one dominant singular value in the band. Additionally, the MAC analysis confirms that the six mode shapes are essentially identical (off-diagonal MAC terms equal to 0.999 and greater). For this analysis band the decision on which mode to select as the "best fit" is rather easy. As the analysis bands increase and move higher in frequency the decisions do become more difficult.

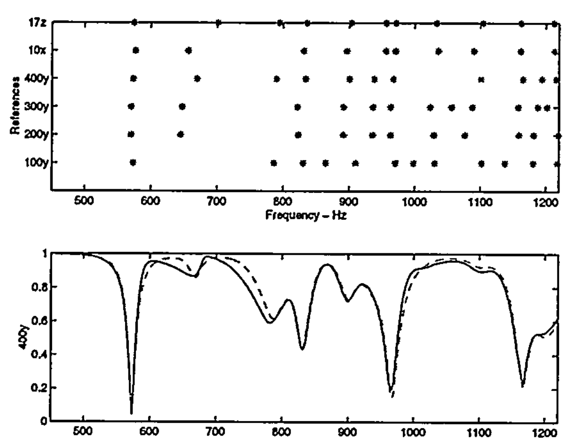

Figure 6. Modal Parameter Condensation at 570 Hz

ref	frequency	damping	corcoeff	NMIF	NMIFfit	quality
100y	573.094	1.814	0.9995	0.0988	0.0532	0.9485
200y	570.094	1.956	0.9874	0.0551	0.0196	0.9385
300y	570.422	1.984	0.9877	0.0476	0.0356	0.9634
400y	573.392	2.026	0.9949	0.0490	0.0269	0.9669
10x	576.562	1.865	0.9940	0.1683	0.0416	0.8375
17z	574.044	1.679	0.9998	0.0893	0.0402	0.9456

Singular values =
301.2211
2.7247
2.5810
0.5636
0.4209
0.0886

MACSV =

0.9999	0.9998	0.9999	0.9999	0.9996	0.9998
0.0001	0.0000	0.0000	0.0001	0.0003	0.0001
0.0000	0.0001	0.0001	0.0000	0.0001	0.0002
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

MAC =

1.0000	0.9998	0.9998	1.0000	0.9993	0.9996
0.9998	1.0000	1.0000	0.9997	0.9996	0.9992
0.9998	1.0000	1.0000	0.9998	0.9996	0.9993
1.0000	0.9997	0.9998	1.0000	0.9993	0.9996
0.9993	0.9996	0.9996	0.9993	1.0000	0.9994
0.9996	0.9992	0.9993	0.9996	0.9994	1.0000

The next example shows the results of an analysis between 1975 to 2015 Hz for the same set of experiment data. In this case there were four modes fit in this band. Notice that three of the quality factors are above 0.94 and all of the NMIF values are above 0.5. This indicates that in general the modes in this frequency band are weakly excited from these reference locations. The first two singular values from this analysis are relatively close in magnitude to one another indicating that there might be a couple of independent modes in this band. The calculation between the mode shapes and the left singular (MACSV) shows that the primary singular vector U_1 is strongly correlated with the second mode shape (from reference 300y). The second left singular vector U_2 is most strongly related to the third mode shape (from reference 400y). The modes from reference 300y and 400y have very high quality factors, 0.986 and 0.996 respectively. Finally, the general MAC calculation indicates that these two mode shapes (from 300y and 400y) are independent of each other with an off-diagonal term equal to 0.0109. By viewing the NMIF (Figure 7) and using the mathematical data provided it was determined that in this frequency band there are two independent modes that were identified by SMAC. The modes were excited from different reference locations but are very close in frequency (2000.7 Hz versus 2001.7 Hz).

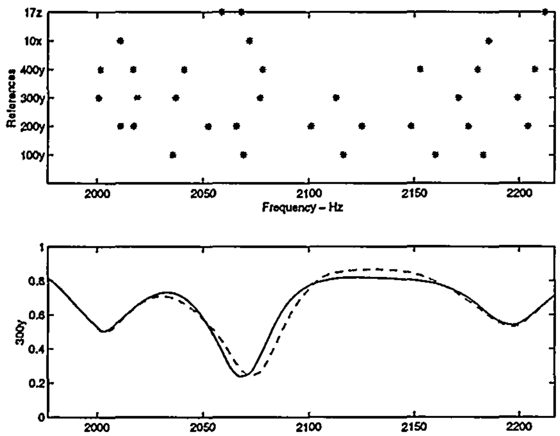

Figure 7. Modal Parameter Condensation at 2000 Hz

ref	frequency	damping	corcoeff	NMIF	NMIFfit	quality
200y	2011.027	1.030	0.9851	0.8462	0.8426	0.947
300y	2000.663	1.805	0.9994	0.5285	0.5228	0.986
400y	2001.719	1.853	0.9996	0.5294	0.5282	0.996
10x	2010.928	1.594	0.9446	0.9133	0.8635	0.380

singular values =

223.2719
122.5653
22.6849
16.9270

MACSV =

0.0330	0.9975	0.0237	0.0329
0.6647	0.0025	0.9691	0.8336
0.0639	0.0000	0.0046	0.1280
0.2384	0.0000	0.0026	0.0054

MAC =

1.0000	0.0197	0.6769	0.5226
0.0197	1.0000	0.0109	0.0183
0.6769	0.0109	1.0000	0.8076
0.5226	0.0183	0.8076	1.0000

UNCERTAINTY

The quality factor utilizing the NMIF provides the most objective quantification of uncertainty for this work. If there is a poor match of the synthesized MIF to the data, then there is high uncertainty in the estimated modal parameters. Measures such as the correlation coefficient provide additional information regarding the uncertainty of the modal extraction. Additionally, the authors have found that when there is a poor estimate of the driving point residue coefficient, specifically for weakly excited modes, then this can produce erroneously large noisy mode shapes. These large mode shapes will dominate the first singular vector and destroy the effectiveness of the MACSV.

CONCLUSIONS

The SMAC algorithms have now been automated, allowing the experimentalist to spend more time understanding system dynamics and significantly less time in extraction. These advances in modal extraction may generate several hundred modes from an experimental data set that then must be consolidated into a final set of modal information. Thus, a set of mathematical methods has been developed for condensing the modal parameters. These mathematical tools have greatly increased the speed and confidence in the modal extractions needed to support model validation programs.

REFERENCES

[1] Mayes, Randall L. and Johansen, David D., "A Modal Parameter Extraction Algorithm Using Best-Fit Reciprocal Vectors", *Proceedings of the 16th International Modal Analysis Conference,* pp.517-521, February 1998.

[2] Mayes, Randall L. and Klenke, Scott E., "The SMAC Modal Extraction Package", *Proceedings of the 17th International Modal Analysis Conference,* pp.812-818, February 1999.

[3] Mayes, Randall L. and Klenke, Scott E., "Automation and Other Extensions of the SMAC Modal Parameter Extraction Package", *Proceedings of the 18th International Modal Analysis Conference,* pp.779-785, February 2000.

[4] Mayes, Randall L., Dorrell, Larry R. and Klenke, Scott E., "Applications of the Automated SMAC Modal Parameter Extraction Package", *Proceedings of the 18th International Modal Analysis Conference,* February 2000.

[5] Deiters, Thomas A. and Smith, Kenneth S., "Faster, Easier Finite Element Model Reduction", *Proceedings of the 17th International Modal Analysis Conference,* pp. 924, February 1999.

[6] Shih, C.Y. and Brown, D.L., *"The Complex Mode Indicator Function Approach to Modal Analysis", Pre-IMAC 8 Symposium,* UC-Irvine, pp0-10, 1989.

[7] *User Manual for Modal Analysis 9.0,* General Electric CAE International, 1985, pp6-85.

[8] Pappa, R. S., James, G. H., and Zimmerman D. C., "Autonomous Modal Identification of the Space Shuttle Tail Rudder", *AIAA Journal of Spacecraft and Rockets,* Vol. 35, No. 2, March-April 1999, pp. 163-169.

[9] Golub, G.H. and Van Loan, C.F., *"Matrix Computations",* The John Hopkins University Press, 1989, pp. 427-437.