Nuclear Energy Research Initiative (NERI) Program
DE-FG03-99SF21922

Quarterly Technical Progress Report
July 1 - September 30, 2000

An Innovative Reactor Analysis Methodology Based on a Quasidiffusion Nodal Core Model

Dmitriy Y. Anistratov, PI, Marvin L. Adams, Todd S. Palmer,
and Kord S. Smith, Co-PI's

RECEIVED
NOV 03 2000
OSTI

Tasks 1 & 3

We proceed working on one-dimensional models to develop theoretical foundation for procedures of homogenization, functionalization of group constants based on solutions of transport problems with albedo boundary conditions, and for solving homogenized (coarse-mesh) quasidiffusion (QD) low-order equations utilizing set of data parametrized by means of special functionals.

A code for solving multigroup eigenvalue one-dimensional neutron transport problems based on the QD method [1, 2] was developed. To discretize the transport equation, the characteristic scheme is used. The low-order QD equations are approximated by means of the finite-volume (integro-interpolation) approach [3]. This code is partially based on the code developed in [2]. Previously developed code for solving one-group transport problems was further modified to have an option for generation of data sets of cross sections averaged over spatial domains. The averaging of cross sections is performed according to the homegenization procedure developed earlier in this project [4].

In the current methods the assembly-level transport calculations are performed using reflective boundary conditions. Thus, the assembly averaged cross section data is prepared assuming that the given assembly is imbedded in an infinite sea of identical assemblies. We study the approach based on solving problems with albedo boundary conditions that can enable us to simulate interaction between different types of assemblies on assembly-level calculations. The research is carried out to determine the optimal and effective parameter that accounts for the leakage through boundaries of an assembly. This characteristic functional will be used to parametrize assembly group constants.

Another important issue of our current research is developing a method of solving the QD low-order equations when all necessary coefficient of these equations come from tabulated data of group constants instead of direct solving the transport equation. In this regard we work on definition of an efficient iteration process between parametrized data and the QD low-order equations. This iteration process involves the analysis of low-order solution obtained with the given set of coefficients (group constants) corresponding to a certain value of characteristic functional and determining which set of data to use on the next iteration in order to converge to the right solution of the given multi-region transport problem. To consider

DISCLAIMER

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency thereof, nor any of their employees, make any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

DISCLAIMER

Portions of this document may be illegible in electronic image products. Images are produced from the best available original document.

this type of iterative procedure, we analyzed the QD low-order equations in reformulated form in which the unknown function is the characteristic functional.

Tasks 2 & 4

During the last three months, we have been working on the application of the combination polynomial-analytic nodal treatment [5] to the two-dimensional QD low-order equations in the thermal energy group. To successfully apply this technique, the analytic solutions to three homogeneous (source-free) equations are necessary:

1. 1-d (x-direction) QD low-order equation with spatially constant cross-sections and QD tensor (E_{xx}) within the node,
2. 1-d (y-direction) QD low-order equation with spatially constant cross-sections and QD tensor (E_{yy}) within the node,
3. 2-d QD low-order equation with spatially constant cross-sections and QD tensor (E_{xx}, $E_{xy} = E_{yx}$, E_{yy}) within the node.

Upon inspection, it is clear that the one-dimensional QD low-order equations with constant nodal properties have fundamental solutions which are identical to those of the standard diffusion equation, with the important distinction that the diffusion lengths are now different in each direction due to the fact that E_{xx} and E_{yy} are not equal, in general. So, three of the basis functions used in our analytic nodal method are:

$$g_0(\xi) = 1, \tag{1}$$

$$g_1(\xi) = cosh(\kappa_{xx}^{i,j}\xi), \tag{2}$$

$$g_2(\xi) = sinh(\kappa_{yy}^{i,j}\xi), \tag{3}$$

where κ_{xx}^{ij} and κ_{yy}^{ij} are defined by:

$$\kappa_{xx}^{i,j} = \sqrt{\frac{h^2\Sigma_{t,2}^{i,j}\Sigma_2^{i,j}}{E_{xx}^{i,j}}}. \tag{4}$$

$$\kappa_{yy}^{i,j} = \sqrt{\frac{h^2\Sigma_{t,2}^{i,j}\Sigma_2^{i,j}}{E_{yy}^{i,j}}}. \tag{5}$$

The analytic solution of the 2-d QD low-order equations with constant nodal cross-sections and QD tensor is an area requiring research. We began by searching for solutions which are separable in the two spatial directions. The first step in this process is to classify the PDE, and then transform it to canonical form. We has completed this analysis, classifying the PDE as elliptic, and transforming the equation to canonical form. However, the solution in the original coordinate system (x,y) is non-separable. We continue to look for ways to incorporate the analytic solution to this system into the model.

Our code development, however, is proceeding with the assumption of a diagonalized QD tensor, $E_{xy} = E_{yx} = 0$. In this way, we can continue to build the machinery necessary to numerically solve the nodal equations as we search for analytic solutions to the 2-d QD low-order equations. When the QD tensor is diagonal, a separation of variables can be performed, which leads to these two additional basis functions in the analytic nodal treatment:

$$g_3(\xi) = cosh(\kappa_{xy}^{i,j}\xi), \tag{6}$$

$$g_4(\xi) = sinh(\kappa_{xy}^{i,j}\xi), \tag{7}$$

and κ_{xy}^{ij} is defined as:

$$\frac{1}{(\kappa_{xy}^{i,j})^2} = \frac{1}{(\kappa_{xx}^{i,j})^2} + \frac{1}{(\kappa_{yy}^{i,j})^2}. \tag{8}$$

While these equations appear very similar to those generated from the nodal diffusion procedure [5], there are significant differences. First, the components of the QD tensor are connected to the solution of the transport equation within the node. Second, the κ's are allowed to be different in the two spatial coordinates.

References

[1] V.Ya. Gol'din, "A Quasidiffusion Method for Solving the Kinetic Equation," *USSR Comp. Math. and Math. Phys.* **4**, 136-149 (1964).

[2] D. Y. Anistratov and V. Ya. Gol'din, "Solving the Multigroup Transport Equation by the Quasidiffusion Method," *Preprint of the Keldysh Institute for Applied Mathematics*, the USSR Academy of Sciences, No. 128 (1986) (in Russian).

[3] D.Y. Anistratov and V.Ya. Gol'din, "Nonlinear Methods for Solving Particle Transport Problems," *Transport Theory Statist. Phys.* **22** 42–77 (1993).

[4] D.Y. Anistratov and M.L. Adams, "Consistent Coarse-Mesh Discretization of the Low-Order Equations of the Quasidiffusion Method," *Trans. Am. Nucl. Soc.*, **83** (2000).

[5] Palmtag, S., **Advanced Nodal Methods for MOX Fuel Analysis**, Ph.D. Thesis, Massachusetts Institute of Technology (1997).

Nuclear Energy Research Initiative (NERI) Program
DE-FG03-99SF21922

Quarterly Technical Progress Report
July 1 - September 30, 2000

An Innovative Reactor Analysis Methodology Based on a Quasidiffusion Nodal Core Model

Status Summary of NERI Tasks

Task 1

The development of the following methods in 1D slab geometry:

1. Homogenization and definition of discontinuity factors,
2. Group constants functionalization using assembly transport solution of multigroup eigenvalue problem with albedo boundary conditions,
3. Solving coarse-mesh effective few-group 1D QD moment equations using tables of data parametrized with respect to the ratio $\vec{n} \cdot \vec{\tilde{J}}^G / \tilde{\phi}^G$ on boundaries.

Planned completion date: August 14, 2000

Task 2

Development of a numerical method for solving the 2D few-group moment QD equations:

1. Development of a nodal discretization method for 2D moment QD equations,
2. Development of an efficient iteration method for solving the system of equations of the nodal discretization method for 2D moment QD equations.

Planned completion date: August 14, 2000

Task 3

The development of the following methods in 2D X-Y geometry:

1. homogenization and definition of discontinuity factors,
2. group constants functionalization using assembly transport solution of multigroup eigenvalue problem with albedo boundary conditions,
3. solving coarse-mesh effective few-group QD moment equations using tables of data parametrized with respect to the ratio $\vec{n} \cdot \vec{\tilde{J}}^G / \tilde{\phi}^G$ on boundaries.

Planned completion date: August 14, 2001

Task 4

Development of a numerical method for solving the few-group moment QD equations in 3D geometry:

1. Development of a nodal method for discretization of 3D moment QD equations,
2. Development of an efficient iteration method for solving the system of nodal discretized equations of moment QD equations in 3D geometry.

Planned completion date: August 14, 2001