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Modular Approach to Achieving the Next-Generation X-Ray Light Source

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Abstract

A modular approach to the next-generation light source is described. The "modules" include photocathode, radio-frequency, electron guns and their associated drive-laser systems, linear accelerators, bunch-compression systems, seed laser systems, planar undulators, two-undulator harmonic generation schemes, high-gain harmonic generation systems (each composed of a modulative section, a dispersive section, and a radiative section), nonlinear higher harmonics, and wavelength shifting. These modules will be helpful in distributing the next-generation light source to many more laboratories than the current single-pass, high-gain free-electron laser designs permit, due to both monetary and/or physical space constraints.

I. Introduction

It is the desire of the light source community to design and build a next-generation light source capable of producing pulses that have ultrashort pulse lengths (< 10 femtoseconds) exhibiting temporal coherence unachievable by the existing third-generation machines [1,2,3].

After first describing the necessary modules, we present five simple examples that use these modules to build toward the fourth-generation light source, one of which will be examined more fully. In these examples, the electron beams never exceed 6 GeV, attempts are made to use the lowest number of "fresh bunches" (new electron beams) and the shortest possible radiation production lines (undulators, etc.), while applying the methods of single-pass, high-gain free-electron lasers (FELs).

II. The Modular Approach

Just as the user stations in today's third-generation x-ray machines are rapidly being occupied and additional machines are being built to try to accommodate the high demands, a similar trend is expected after the completion of the Linac Coherent Light Source (LCLS)-like [4] and TESLA [5] x-ray facilities. In this presentation, we use combinations of "modules" to produce next-generation x-ray light sources that may be more flexible and have better properties than currently expected. The types of modules include the following: photocathode; radio-frequency (rf) guns and associated drive-lasers; linear accelerators; bunch compressors; seed lasers; planar undulators; two-undulator harmonic generation schemes (TUHGS) [6]; high-gain harmonic generation

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(HGHG), consisting of a modulative section, a dispersive section, and a radiative section [7-12]; nonlinear higher harmonics [13-20]; and wavelength shifting.

The modular approach alone, less the nonlinear higher harmonics, represents a powerful tool; but with these higher harmonics, shorter wavelengths can be reached beyond those available from the fundamental in single-pass, high-gain FELs. In a self-amplified spontaneous emission (SASE) or amplifier system, the nonlinear harmonics appear to be substantial and quite useful [15-17,20]. In TUHGS and HGHG schemes, however, the downstream undulator is tuned to a higher harmonic than the input seed laser, generating coherent output at the fundamental *and* at the higher harmonics of this second undulator. In other words, in TUHGS and HGHG schemes, the shorter wavelengths are attainable more readily than in the SASE and amplifier schemes [19]. One final point is that as the system approaches saturation, harmonics develop from the bunching and grow very rapidly as compared to the fundamental. In fact, their "gain lengths" grow in inverse proportion to the harmonic number. These nonlinear harmonics are discussed further in the next section.

There are numerous reasons for assuming a modular approach, the most obvious are mentioned here. First, the modular approach makes the frequency up-conversion easier while maintaining the optical quality of the initial, input seed laser. Second, it allows one to reduce the number of portions of the machine that may lead to instabilities and radiation output jitter (i.e., rf systems). Finally, the modular approach enables a slow build-up to the next-generation machine, allowing one to get the first section working before adding the next and so on.

Many combinations of the modular path toward the next-generation light source exist, hence, after a brief review of the various types of modules, we will present five examples. Along with the most necessary modules of linear accelerators, seed lasers, bunch compressors, and nonlinear harmonic generation, the following best exemplify the general techniques employed using module combinations:

- multiple amplifier modules
- multiple TUHGS modules or multiple HGHG modules
- combinations of amplifier, TUHGS, and/or HGHG modules
- tabletop soft x-ray seed laser with amplifier, TUHGS, and/or HGHG modules
- wavelength shifting coupled with amplifier, TUHGS, and/or HGHG modules.

III. Theory Review

Here, we intend to review the FEL portion of the necessary modules and not the fundamentals of photocathode, radio-frequency gun and associated drive-laser, linear accelerator, bunch compression, or seed laser systems. The electron beam with the desired energy and beam parameters is derived from known design capabilities of the above systems. Also, the seed laser parameters presented here are taken from "off-the-shelf" systems.

A. Amplifier

In SASE [21-26] and amplifier systems, the incoming electron beam is tuned to the desired output wavelength via the well-known resonance condition, $\gamma^2 = \frac{\lambda_u}{2\lambda_r}(1+\frac{K^2}{2})$, where λ_u is the undulator period, λ_r is the radiation wavelength, γ is the relativistic energy parameter, and K is the undulator parameter. In the SASE case, the spontaneous radiation couples to the transverse electron motion, creates microbunching on the order of the radiation wavelength from the above resonance condition, and eventually, saturation occurs wherein no excess energy may be removed from the electron beam. In an amplifier, a traditional laser, which is also at this resonant wavelength, serves to seed the system and imparts full longitudinal coherence on the output. The FEL process is the same, except that this system does not start up from noise.

B. Two-Undulator Harmonic Generation Scheme

In the two-undulator harmonic generation scheme [6], a seed laser and electron beam enter a first undulator, where the seed laser is introduced to place full longitudinal coherence on the output radiation. The first undulator is tuned in resonance to the input seed wavelength and serves to impart a $\vec{v} \cdot \vec{E}$ energy modulation on the electron beam. This energy modulation is then converted into spatial bunching while further traversing this undulator. Eventually, the favorable FEL instability reaches saturation and laser-like radiation output occurs. Note that the electron beam now is fully microbunched at the resonant wavelength and the microbunching contains higher harmonic content. The same electron beam then enters a second undulator whose fundamental is tuned to a higher harmonic than the original seed laser. The fundamental of this second section is the "new" fundamental.

C. High-Gain Harmonic Generation

Similar to the TUHGS is the method of high-gain harmonic generation. In HGHG, a seed laser and electron beam enter a first undulator. Again, the seed laser is introduced to place full longitudinal coherence on the output radiation. This first undulator, referred to as the modulative section, is tuned in resonance to the input seed wavelength and serves to impart a $\vec{v} \cdot \vec{E}$ energy modulation on the electron beam. This wavelength is referred to as the "original" fundamental. The energy modulation is then converted into spatial bunching while traversing a dispersive section. The electron beam enters a second undulator, referred to as the radiative section, which is tuned to the desired output harmonic. The fundamental of the radiative section is the "new" fundamental.

D. Nonlinear Harmonic Generation

Nonlinear harmonics are generated in all single-pass, high-gain free-electron lasers based on planar undulators [13-20]. They arise from the electron beam bunching at the fundamental coupling to both the fundamental and natural harmonic radiation near saturation. The odd harmonics are favored in planar undulators due to the natural motion of the electron beam. Consequently, the even harmonics grow in a smaller proportion because the electron beam has a finite size and is not purely on the undulator axis. In addition, the electron beam feels a gradient effect due to the beam physically bending through the undulator. (At higher energies, the even harmonic growth is reduced.)

Since the use of the nonlinear harmonics greatly reduces the required electron beam energy and quality in comparison with trying to generate the same wavelength as the fundamental while still achieving significant power levels, they are of great importance toward generating a fourth-generation x-ray light source. These harmonics arise in all single-pass FELs based on the planar undulator designs and have been recently measured on the joint BNL/APS HGHG experiment [12] and plan to be measured in the APS SASE FEL at Argonne [15].

IV. The Five Conceptual Examples

A. Example I: Four Amplifier Modules

This scheme is composed of four amplifier modules (AMP I-IV) tuned to the fundamental resonance with four fresh electron bunches. Here, a λ_{seed} = 266 nm, P_{pk} = 100 MW seed laser is used. The schematic is laid out in Figure 1. The fifth nonlinear harmonic of the output radiation from AMP I-III each seed the next respective module. The final wavelength is 4.256 Å with an electron beam energy of ~4.3 GeV.

B. Example II: Two HGHG Modules

Although either the TUHGS or the HGHG schemes could represent this sort of combination, the case of multiple HGHG modules is shown here. This example is related to L.H. Yu's Cascading Stages of High-Gain Harmonic Generation [27] and further extends its usefulness by utilizing the higher nonlinear harmonic in the system. Of our five examples, this scheme provides the shortest wavelength via at a relatively low electron beam energy with the least number of "fresh" electron bunches as compared to the current designs.

As seen in Figure 2, a laser of λ_{seed} = 266 nm with P_{pk} = 1 MW is used as the seed for the first HGHG module (HGHG I). This seed is the "original" fundamental wavelength that drives the entire resultant system. Recall that each HGHG module consists of modulative, dispersive, and radiative sections where the radiative section is tuned to the desired harmonic. Along with the seed laser, a ~700-MeV electron beam enters HGHG I, where the energy modulation and spatial bunching are induced in the modulative and radiative sections, respectively. Here, radiative section I (RAD I) is tuned to the seventh harmonic (38 nm) of the "original" fundamental (266 nm). The fifth nonlinear harmonic (7.6 nm) of the output from RAD I (which is the 35^{th} harmonic to the "original" fundamental) is used to seed the second HGHG section (HGHG II). The first electron beam is bent into a dump and a second, ~4.8-GeV electron beam enters HGHG II along with this 7.6-nm seed. This grows in an amplifier mode of this second modulative section (MOD II) until enough energy modulation is imparted on the electron beam. The beam then passes through the dispersive section and through radiative section II (RAD II), tuned to the seventh harmonic of the 7.6-nm seed, which is 1.086 nm. Here, the longitudinally coherent output radiation in the fifth nonlinear harmonic has a wavelength of 2.1 Å.

C. Example III: Two Amplifier Modules and One HGHG Module

This third scheme employs the same seed laser described in Example II and uses three fresh electron bunches in two amplifier modules (AMP I and AMP II) and one HGHG module (HGHG), as seen in Figure 3. An alternative to an HGHG module is to use a TUHGS module.

As seen in Example I, the fifth nonlinear harmonic output from AMP I and AMP II serve as seeds for their following modules. The modulative section in the HGHG module is long enough to induce the desired energy modulation on the electron beam. The radiative section in HGHG is tuned to the seventh harmonic of the input seed from AMP II. The final wavelength in the fifth nonlinear harmonic emitted is 3.04 Å with an electron beam energy of ~4 GeV.

D. Example IV: Soft X-Ray Seed Laser Amplifier and One HGHG Module

In this example, a tabletop, Ni-like molybdenum soft x-ray laser with λ_{seed} = 18.9 nm, P_{pk} = 5 GW is used as the seed amplifier module (AMP), of which, the coherent power of the correct polarization is ~0.1 MW. This tabletop, soft x-ray laser would be identical to the "COMET" laser that is currently operational at Lawrence Livermore National Laboratory (LLNL) [18]. The fifth nonlinear harmonic output from AMP would serve to seed an HGHG module (HGHG), where the radiator is tuned to the seventh harmonic of the input seed, requiring an electron beam energy of ~6 GeV. This scheme may be visualized in Figure 4. Utilizing the fifth nonlinear harmonic of the output radiation yields 1.0 Å radiation.

E. Example V: Wavelength Shifting Coupled with Amplifier, TUHGS, and/or HGHG Modules

In the wavelength shifting case, the acceleration and radiation-producing modules are more integrated than in the previous examples. Here, the simplest case is to use modules in the following order, as seen in Figure 5. First, an electron beam is produced using a gun and a linear accelerator. A seed laser is introduced to the electron beam in an undulator, whose fundamental is tuned to the seed laser for the electron beam energy. This is performed to induce a specified amount of energy modulation. Then, the electron beam is over-rotated in phase space using a bunch compressor and further introduced to an accelerating section whose phase is slightly off-crest to induce an energy chirp. Next, the electron beam is compressed through a second bunch compressor and the chirp is removed in an additional accelerating section. After further acceleration, the electron beam is injected into an undulator, TUGHS, or HGHG module, where, since it is prebunched to a wavelength tuned by the compression process, the electron beam radiates in a fully coherent fashion. The resultant coherent output can be further introduced to more of the same wavelength-shifting modules or other modules.

As an aside, wavelength shifting allows one to generate arbitrary wavelengths independent of the seed after already imprinting its quality on an electron bunch. It can also be used to shift wavelengths up or down and so can be used for final wavelength tuning by passing wavelength-shifted, "saturated" beams through undulators tuned to the microbunch spacing.

V. Simulation Code

MEDUSA [28] is a 3D, nonlinear polychromatic code based on the source-dependent expansion [29] of the Gauss-Hermite waveguide modes. It has been benchmarked at the fundamental against four other simulation codes and demonstrates good agreement [30,31]. It is capable of simulating TUHGS, HGHG, and nonlinear harmonic generation [6]. The power in the third harmonic has also been compared with the 3D analytical model and is in good agreement [20]. For the following modular cases, the output radiation at the desired nonlinear harmonic was fed into the next section with the "fresh" electron bunch in consecutive computer runs.

VI. Numerical Example: Example II

Using MEDUSA, Example II was treated in numerical simulation. The output power of the fifth harmonic from the HGHG I radiative section is 9×10^4 and serves to seed HGHG II along with a fresh electron bunch. The fifth harmonic to the radiative section of HGHG II yields 1×10^7. The output power as a function of distance through HGHG I and HGHG II for the odd nonlinear harmonics are shown in Figure 6 (a) - (d) and Figure 7 (a) – (d), respectively. These powers are further summarized in Table 1.

We now further analyze the specific case that each linear accelerator module is composed of 3-m, SLAC-type accelerating structures, each capable of 50-MeV acceleration (assuming two SLED cavities and four 3-m structures per each modulator and klystron assembly). This translates into 1 GeV/60 m of linear accelerator. This conservative estimate is used to compare the "amount" of linear accelerator required for each of the five examples specified above. Table 2 lists of the required electron beam energy, radiation wavelength, and the length of the accelerator and radiation production sections (all-inclusive; undulators, drift, and dispersive sections) for Example II are provided.

VII. Conclusions

The modular approach to achieving the next-generation x-ray light source (or any light source) allows:

- imparting full longitudinal coherence on the output radiation by seeding at a much lower wavelength than that of the desired output wavelength with a coherent source,
- the use of multiple seeding configurations to work toward an ultrashort wavelength based on substantial frequency up-conversion,
- utilizing a much lower electron beam energy to produce the desired wavelengths, and
- the option of building toward shorter radiation wavelengths by first starting with a modest system and then adding additional modules, as time, money, and space permit.

It was the intent to demonstrate the importance of the modular methods as well as harmonic generation for building toward the next generation light source. Although many other modular combinations do indeed exist, producing both longer and shorter wavelengths of varying powers, these five examples were chosen for discussion to promote a type of source that was attainable, within monetary and/or physical space constraints, by many more institutions then currently expected. In particular, "fresh bunches" are not fully necessary, as multiple bunches can be generated in the linear accelerator and fast kicker magnets could simply gate the bunches into specific modules. A thorough design review of each of the five examples described is underway involving the electron beam and undulator tolerances.

VIII. Acknowledgements

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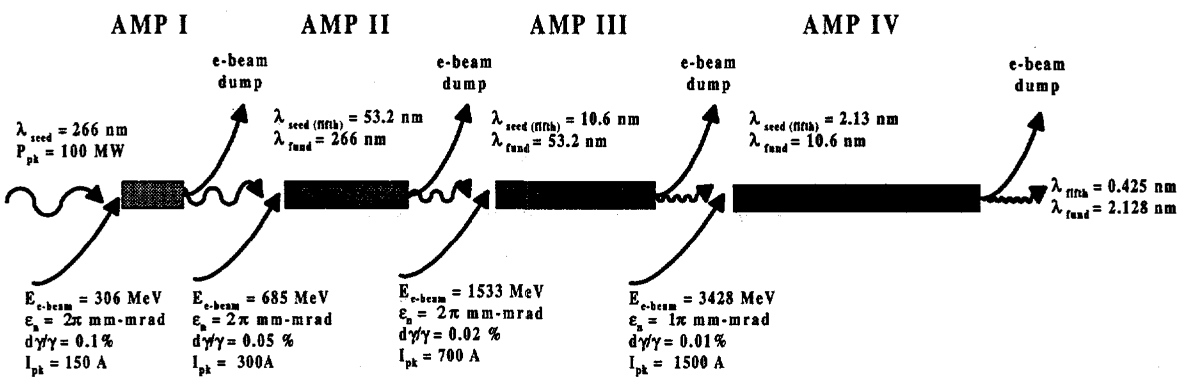

Figure 1: Example I – Four amplifier modules.

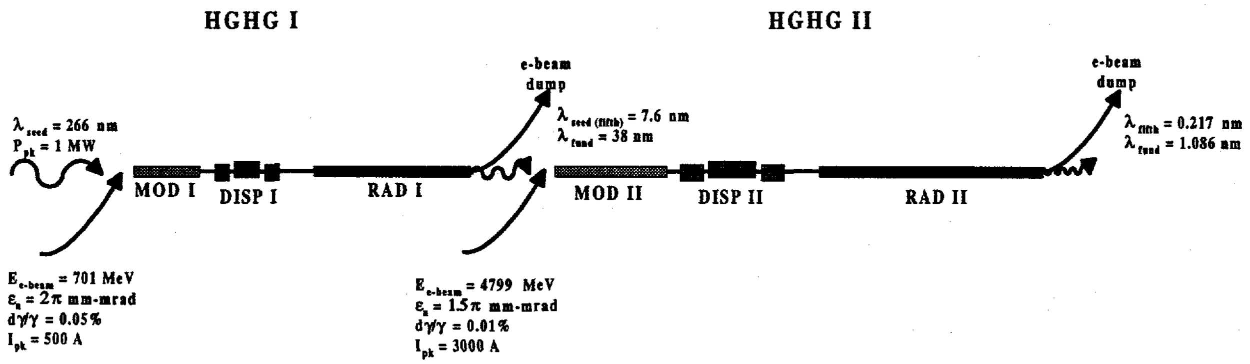

Figure 2: Example II – Two HGHG modules.

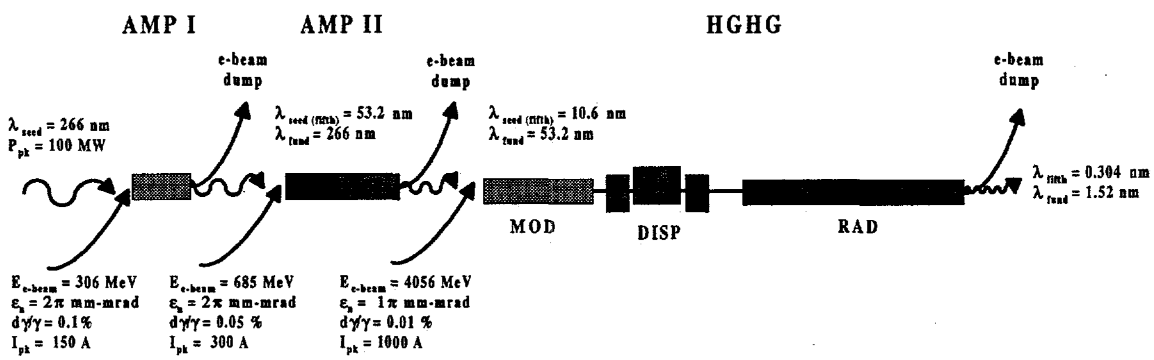

Figure 3: Example III – Two amplifier modules and one HGHG module.

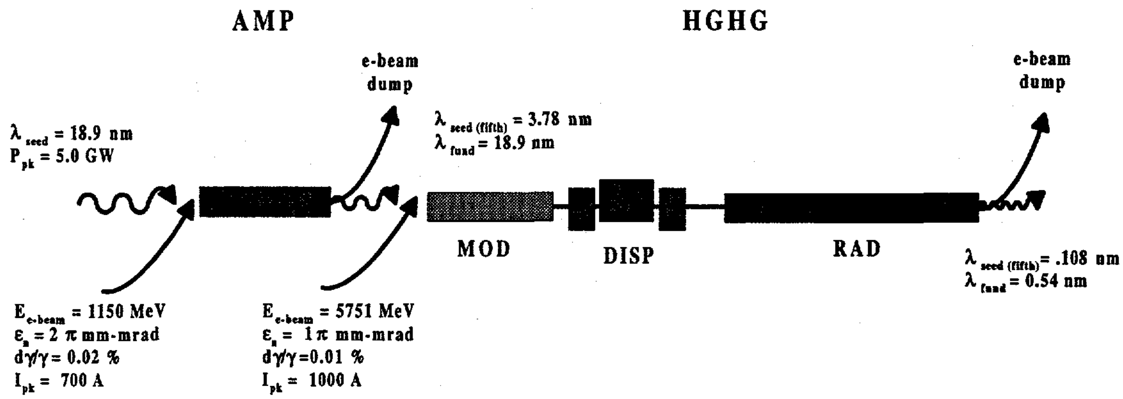

Figure 4: Example IV – One amplifier and one HGHG module.

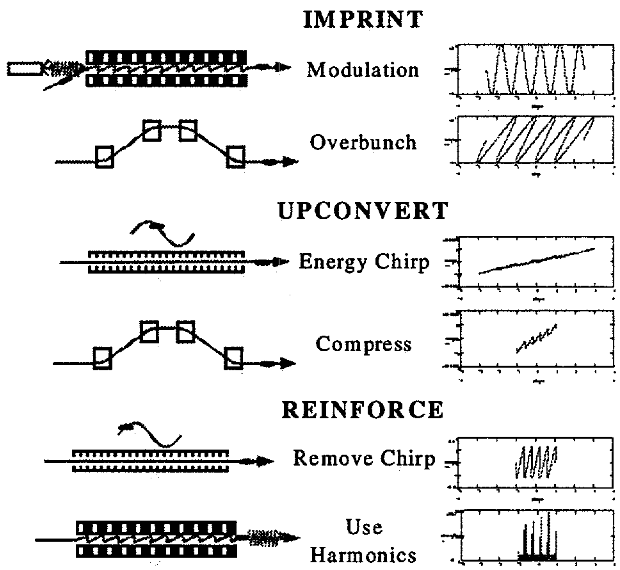

Figure 5: Example V – Wavelength shifting concept.

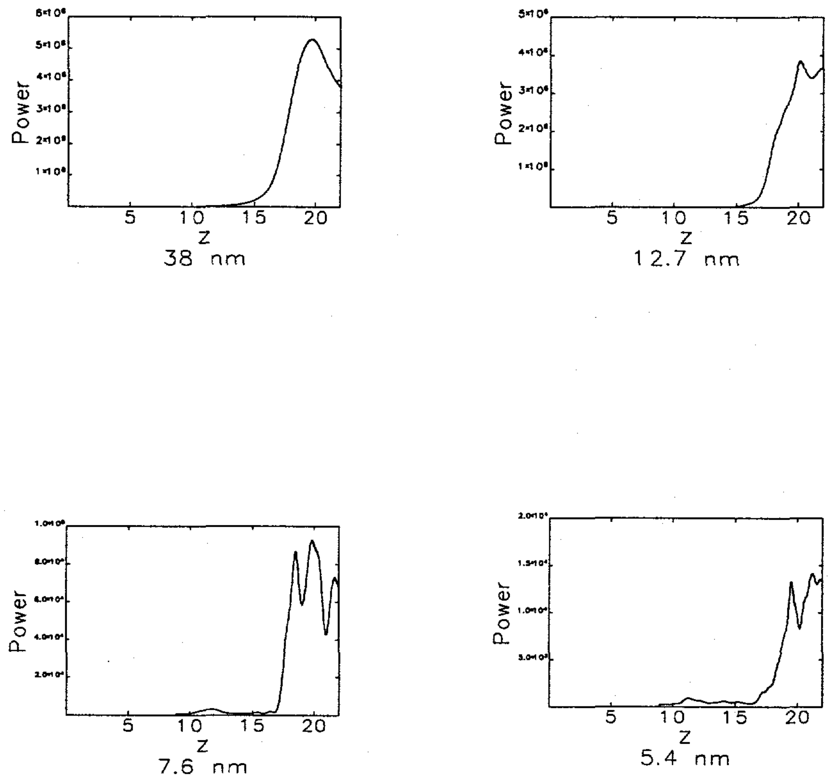

Figure 6: Power (W) versus distance (m) for the fundamental and odd nonlinear harmonics in HGHG I – 38 nm, 12.7 nm, 7.6 nm, and 5.4 nm.

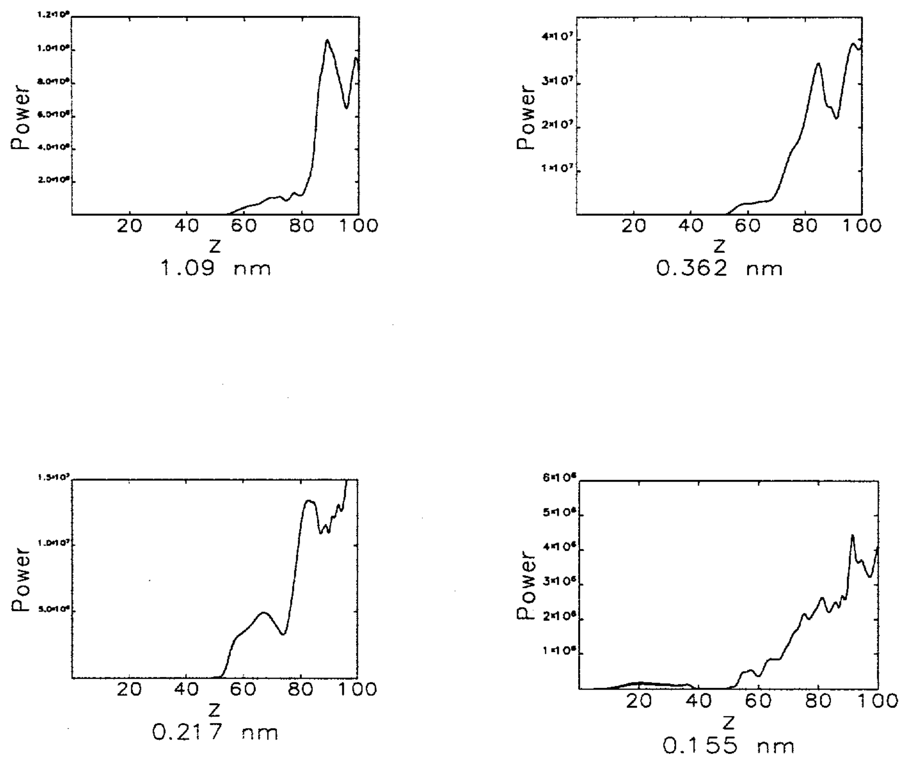

Figure 7: Power (W) versus distance (m) for the fundamental and odd nonlinear harmonics in HGHG II – 1.09 nm, 0.362 nm, 0.217 nm, and 0.155 nm.

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Table 1: Power after HGHG module in Numerical Example II for the odd harmonics up to h = 7.

	Harmonic Number	Radiation Wavelength (nm)	Power (W)
HGHG I	1	38	6×10^8
	3	12.67	3×10^6
	5	7.6	9×10^4
	7	5.4	1×10^4
HGHG II	1	1.09	1×10^9
	3	0.36	4×10^7
	5	0.218	1×10^7
	7	0.156	3×10^6

Table 2: Length Review of Numerical Example II.

	Electron Beam Energy (GeV)	Total Length of Linear Accelerator Required (m)	Radiation Wavelength (nm) of 5th harmonic to HGHG section	Total Length of Radiation Production Section Required (m)
HGHG I	0.7	42	7.6	21
HGHG II	4.8	288	0.217	85
Total Lengths	- na -	330	- na -	106