

# Damage Mechanics Based Fatigue Life Prediction for 63Sn-37Pb Solder Material\*

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## ABSTRACT

This paper presents a method of TMF analysis based on the theory of damage mechanics to examine the fatigue damage accumulation in 63Sn-37Pb solder. The method is developed by extending a viscoplastic damage model proposed earlier by the authors (Wei, et al 1999, 2000). A computer simulation is carried out to calculate hysteresis loops at three different strain ranges. The damage-coupled fatigue damage model is applied to predict the cyclic softening behavior of the material and the prediction is found to agree well with the experiment. With a proposed failure criterion based on the concept of damage accumulation, the TMF model is also found to predict successfully the fatigue life of 63Sn-37Pb solder.

## 1. Introduction

Failures in electronic packages under service often result from thermomechanical fatigue (TMF) in solder materials, especially under low cycle fatigue. Many studies have been performed in two fields: one is the constitutive modeling under cyclic loading (Lau, 1991, Frear, et al., 1994, Ishikawa, et al., 1996, Desai, et al., 1997), and another is the fatigue failure criterion (Solomon, et al., 1986, 1995, 1996, Wen, et al., 1995, Shi, et al., 2000). Previous studies showed that fatigue data could not fit into a single Coffin-Manson line for thermomechanical fatigue. Solomon (1995 & 1996) also observed that the fatigue life could not be formulated as a single valued function of the hysteresis energy. The hysteresis energy was thus corrected for the strain rate, temperature and type of wave form being utilized before it could be used to correlate the fatigue data.

Recently, several researchers have presented their work based on the theory of damage mechanics (Basaran, et al., 1998, Wei, et al., 1999). Wei, et al. (1999) proposed a damage-coupled viscoplastic constitutive model taking into account the effects of change in grain or phase size and damage for solder material. The model was used successfully to examine the behavior of solder material under creep loading and monotonic tensile loading under different

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temperatures (Wei, et al., 2000). In this paper, the model is further extended to characterize hysteresis response and fatigue life under cyclic loading. The fatigue damage evolution equations are developed to calculate damage accumulation under cyclic loading. A fatigue failure criterion is also established based on the concept of equivalent damage. A computer simulation with the TMF model is carried out to predict hysteresis loops for different strain ranges and successfully predictions are found to compare well with test results. Finally the model is applied for fatigue life prediction under strain-controlled cyclic loading. The predicted results are also found to agree well with test data (Guo, et al., 1992).

## 2. Concepts of Damage Mechanics

The general concepts of damage mechanics that have been applied to develop a damage model have been reported elsewhere to which interested readers may refer (Chow and Wei, 1999). However several basic equations needed for the development of the proposed TMF damage model will be described under this section. In essence, the model introduces two scalar variables,  $D$  and  $\mu$  to characterize damage accumulation in a material under load. The relationship between the effective stress and Cauchy stress is established as: (Lemaître and Chaboche, 1990)

$$\bar{\sigma} = \mathbf{M} : \sigma \quad (1)$$

where  $\sigma$  is the true stress tensor,  $\mathbf{M}$  is the damage effect tensor expressed as (Chow and Wei, 1999)

$$\mathbf{M} = \frac{1}{1-D} \begin{bmatrix} 1 & \mu & \mu & 0 & 0 & 0 \\ \mu & 1 & \mu & 0 & 0 & 0 \\ \mu & \mu & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1-\mu & 0 & 0 \\ 0 & 0 & 0 & 0 & 1-\mu & 0 \\ 0 & 0 & 0 & 0 & 0 & 1-\mu \end{bmatrix} \quad (2)$$

Based on the concept of energy equivalence (Chow and Wang, 1987), the free energy for a damaged material is established as

$$\begin{aligned} \Psi &= \frac{1}{2} \bar{\sigma}^T : \mathbf{C}_0^{-1} : \bar{\sigma} + \frac{3}{4} C_{k0}^{-1} \mathbf{X}^T : \mathbf{X} + \Psi^i \\ &= \frac{1}{2} \sigma^T : \mathbf{M}^T : \mathbf{C}_0^{-1} : \mathbf{M} : \sigma + \frac{3}{4} C_{k0}^{-1} \mathbf{X}^T : \mathbf{X} + \Psi^i \\ &= \frac{1}{2} \sigma^T : \mathbf{C}^{-1} : \sigma + \frac{3}{4} C_{k0}^{-1} \mathbf{X}^T : \mathbf{X} + \Psi^i \end{aligned} \quad (3)$$

where  $\Psi^i$  is the isotropic hardening part of free energy,  $\mathbf{X}$  is the back stress tensor,  $\mathbf{C}_0^{-1}$  is the fourth order elastic tensor without damage,  $C_{k0}$  is the material parameter for kinematic hardening, and  $\mathbf{C}^{-1}$  is the effective elastic tensor taking into account the effects of damage for a damaged material. Accordingly, the damage-coupled elastic equations are derived as

$$\begin{aligned} \varepsilon^e &= \frac{\partial \Psi}{\partial \sigma} = \mathbf{C}^{-1} : \sigma \\ \sigma &= \mathbf{C} : \varepsilon^e \end{aligned} \quad (4)$$

### 3. Damage-Coupled Constitutive Equations for TMF

In order to establish inelastic constitutive equations and damage evolution equations, a dissipation potential function  $\phi$  is introduced consisting of two independent processes, i.e. a deformation process  $\phi^{in}$  and a damage process  $\phi^d$ . Accordingly, the potential is expressed as

$$\phi = \phi^{in}(\sigma, \mathbf{X}, R_i) + \phi^d(Y_{ind}) \quad (5)$$

where  $R_i$  is the isotropic hardening variable and  $Y_{ind}$ , the equivalent damage energy release rate, is defined as

$$Y_{ind} = \left[ \frac{1}{2} (Y_D^2 + Y_\mu^2) \right]^{1/2} \quad (6)$$

$\gamma$  is the damage-related material constant which is associated with the change of Poisson's ratio, and  $Y_D$ ,  $Y_\mu$  are the thermodynamic conjugate forces of the damage variables that can be derived as

$$Y_D = -\frac{\partial \Psi}{\partial D} = -\frac{1}{1-D} \sigma^T : \mathbf{C}^{-1} : \sigma \quad Y_\mu = -\frac{\partial \Psi}{\partial \mu} = -\frac{1}{1-D} \sigma^T : \mathbf{Z} : \sigma \quad (7)$$

$$\mathbf{Z} = \frac{1}{E_0(1-D)} \begin{bmatrix} \mathbf{z}_1 & \mathbf{z}_2 & \mathbf{z}_2 & 0 & 0 & 0 \\ \mathbf{z}_2 & \mathbf{z}_1 & \mathbf{z}_2 & 0 & 0 & 0 \\ \mathbf{z}_2 & \mathbf{z}_2 & \mathbf{z}_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & (z_1 - z_2) & 0 & 0 \\ 0 & 0 & 0 & 0 & (z_1 - z_2) & 0 \\ 0 & 0 & 0 & 0 & 0 & (z_1 - z_2) \end{bmatrix} \quad (8)$$

$$z_1 = 2\mu(1-\nu_0) - 2\nu_0 \quad z_2 = (1+\mu)(1-\nu_0) - 2\mu\nu_0$$

Then, the damage-coupled inelastic constitutive equation can be derived as

$$\dot{\epsilon}^{in} = \dot{\lambda}_{in} \frac{\partial \phi}{\partial \sigma} = \dot{\lambda}_{in} \frac{\partial \phi^{in}}{\partial \sigma} \quad (9)$$

$\lambda_{in}$  is a multiplier. The damage evolution equations can be derived as

$$\dot{D} = -\dot{\lambda}_{in} \frac{\partial \phi}{\partial \mathbf{Y}_D} = -\dot{\lambda}_{in} \frac{\partial \phi^d}{\partial \mathbf{Y}_D} \quad \dot{\mu} = -\dot{\lambda}_{in} \frac{\partial \phi}{\partial \mathbf{Y}_\mu} = -\dot{\lambda}_{in} \frac{\partial \phi^d}{\partial \mathbf{Y}_\mu} \quad (10)$$

For eutectic materials, the deformation part of the dissipation potential is postulated as

$$\phi^{in} = J_2(\mathbf{S} - \mathbf{X}) \quad (11)$$

where  $J_2$  is a second invariant of the stress difference and defined as

$$J_2 = \left\{ \frac{3}{2} (\mathbf{S} - \mathbf{X})^T : (\mathbf{S} - \mathbf{X}) \right\}^{\frac{1}{2}} \quad (12)$$

$\mathbf{S}$  is the deviatoric stress. The damage part of the dissipation potential is formulated as

$$\phi^d = \frac{Y_h}{B_1 + 1} \left( \frac{Y_{ind}}{Y_h} \right)^{B_1+1} \quad (13)$$

where  $B_1$  is the damage-related material constant,  $Y_h$  is the damage hardening variable. Therefore, Equations (9) and (10) can be expressed as

$$\dot{\varepsilon}^{in} = \dot{\lambda}_{in} \frac{3\mathbf{S} - \mathbf{X}}{2J_2} = \dot{p}^{in} \frac{3\mathbf{S} - \mathbf{X}}{2J_2} \quad (14)$$

$$\dot{D} = -\dot{p}^{in} \left( \frac{Y_{ind}}{Y_h} \right)^{B_1} \frac{\partial Y_{ind}}{\partial Y_D} = -\dot{w} \frac{Y_D}{2Y_{ind}} \quad \dot{\mu} = -\dot{p}^{in} \left( \frac{Y_{ind}}{Y_h} \right)^{B_1} \frac{\partial Y_{ind}}{\partial Y_\mu} = -\dot{w} \frac{Y_\mu}{2Y_{ind}} \quad (15)$$

where the equivalent inelastic strain rate  $\dot{p}^{in}$  is expressed as (Wei, et al., 2000)

$$\dot{\lambda}_{in} = \dot{p}^{in} = \frac{1-\mu}{1-D} f \exp\left(\frac{-Q}{RT}\right) \left(\frac{\lambda_0}{\lambda}\right)^p \sinh^m\left(\frac{1-\mu}{1-D} \frac{J_2}{\alpha(c+\hat{c})}\right) \quad (16)$$

$f$ ,  $p$ ,  $m$  and  $Q$  are material parameters,  $R$  is the gas constant,  $T$  is the absolute temperature,  $\lambda$  is the current grain diameter,  $\lambda_0$  is the initial grain diameter,  $\alpha$  is a scalar function of the absolute temperature,  $c$  and  $\hat{c}$  are state variables. The equivalent damage rate is expressed as

$$\dot{w} = \dot{p}^{in} \left( \frac{Y_{ind}}{Y_h} \right)^{B_1} \quad (17)$$

The damage hardening variable  $Y_h$  may be expressed in terms of the equivalent damage  $w$  and the absolute temperature  $T$  as

$$Y_h(w, T) = Y_0 e^{(B_2 w + \frac{B_3}{T})} \quad (18)$$

where  $Y_0$ ,  $B_1$ ,  $B_2$  and  $B_3$  are damage-related material constants.

In general, a loading process may induce both the viscoplastic damage and the fatigue damage. It is postulated that both the viscoplastic damage and the fatigue damage evolution laws follow Equations (15) and (17). Their material constants should be determined separately to characterize the viscoplastic damage and fatigue damage accumulation. Therefore, the viscoplastic damage evolution equations are expressed as

$$\dot{D}_{in} = -\dot{w}_{in} \frac{Y_D}{2Y_{ind}} \quad \dot{\mu}_{in} = -\dot{w}_{in} \frac{Y_\mu}{2Y_{ind}} \quad (19)$$

$$\dot{w}_{in} = \dot{p}^{in} \left( \frac{Y_{ind}}{Y_{hin}} \right)^{B_1} \quad Y_{hin} = Y_0 e^{(B_2 w_{in} + \frac{B_3}{T})} \quad (20)$$

while the fatigue damage evolution equations are expressed as

$$\dot{D}_f = -\dot{w}_f \frac{Y_D}{2Y_{ind}} \quad \dot{\mu}_f = -\dot{w}_f \frac{Y_\mu}{2Y_{ind}} \quad (21)$$

$$\dot{w}_f = \dot{p}^{in} \frac{Y_{ind}}{Y_{hf}} \quad Y_{hf} = Y_{0f} e^{\frac{B_3}{T}} \quad (22)$$

where  $D_{in}$ ,  $\mu_{in}$  and  $w_{in}$  are viscoplastic damage variables,  $D_f$ ,  $\mu_f$  and  $w_f$  are fatigue damage variables. It is worth noting that the material constants taking into account the strain rate effects on damage evolution need to be determined experimentally through damage hardening variables

$Y_{\text{hin}}$  and  $Y_{\text{hf}}$ . The fatigue damage accumulation per cycle is then calculated by integrating Equations (21) and (22) with respect to time  $t$  over a loading cycle as

$$\frac{\Delta D_f}{\Delta N} = \int \dot{D}_f dt \quad \frac{\Delta \mu_f}{\Delta N} = \int \dot{\mu}_f dt \quad \frac{\Delta w_f}{\Delta N} = \int \dot{w}_f dt \quad (23)$$

The total damage is defined as the sum of viscoplastic damage and fatigue damage

$$D = D_{\text{in}} + D_f \quad \mu = \mu_{\text{in}} + \mu_f \quad w = w_{\text{in}} + w_f \quad (24)$$

A failure criterion developed is based on the postulation that *a material element is said to have ruptured when the total equivalent damage  $w$  reaches the critical value  $w_c$  of the material*.

The proposed constitutive model has been implemented in ABAQUS (version 5.8) through its user-defined material subroutine UMAT. Therefore, the model can be readily employed to simulate fatigue crack initiation and propagation in solder joints under TMF loading.

#### 4. Applications

The proposed model was used for the first instance to predict hysteresis loops of 63Sn-37Pb solder under strain-controlled cyclic loading at room temperature ( $25^0\text{C}$ ). A set of material constants required for the constitutive model has been described by the authors (Wei, et al. 2000). The measured material constants for the damage model are presented in Table 1. The fatigue test were performed at three strain ranges,  $\pm 0.3\%$ ,  $\pm 1.0\%$  and  $\pm 2.0\%$ , under the strain rate  $10^3/\text{s}$ . The predicted hysteresis loops at the first few cycles are compared with the test results shown in Fig.1. Fig.2 depicts the simulated softening behavior as the increasing number of cycles. The predicted hysteresis loops and material softening behavior agree well with the experimental observations. The damage accumulation contributes in part to the reduction in load or material softening behavior.

The fatigue life prediction for 63Sn-37Pb solder was verified with test data reported by Guo (Guo, et al., 1992). The test was performed under uniaxial total strain control over the total strain range of 0.3 to 3 percent with 120-second ramp time at  $25^0\text{C}$ . The predicted fatigue life and test results are compared well as shown in Fig.3

The test results and predictions depicted in Figs 1-4 demonstrate the validity of the proposed TMF model that has been used successfully to predict hysteresis response and fatigue life prediction of 63Sn-37Pb solder. Further research is in progress to extend the model to include the effects of strain rate, temperature and wave form of cyclic loading.

#### 5. Conclusions

The proposed TMF model presents the development of damage evolution equations for the solder material and the damage coupled constitutive equations. The model is able to quantify damage accumulation and its effects on fatigue life prediction. In addition, the model can also predict successfully the material softening behavior in the form of decreasing hysteresis loops.

Further research is in progress to examine the applicability of the proposed model under a range of TMF loading. This includes the effects of wave form fatigue loading based on Equation (23). The fatigue damage accumulation over a cycle can then be calculated by integrating a loading path, rather than governed by the mode of loading, such as strain range or hysteresis energy in a cycle.

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Table 1 Damage-Related Material Constants for 63Sn-37Pb

B <sub>1</sub>	B <sub>2</sub>	B <sub>3</sub> (K)	γ	Y <sub>0</sub> (MPa)	Y <sub>0f</sub> (MPa)	w <sub>c</sub>
0.4	5.0	3.34E+3	- 0.2	3.77E-8	3.40E-6	0.58

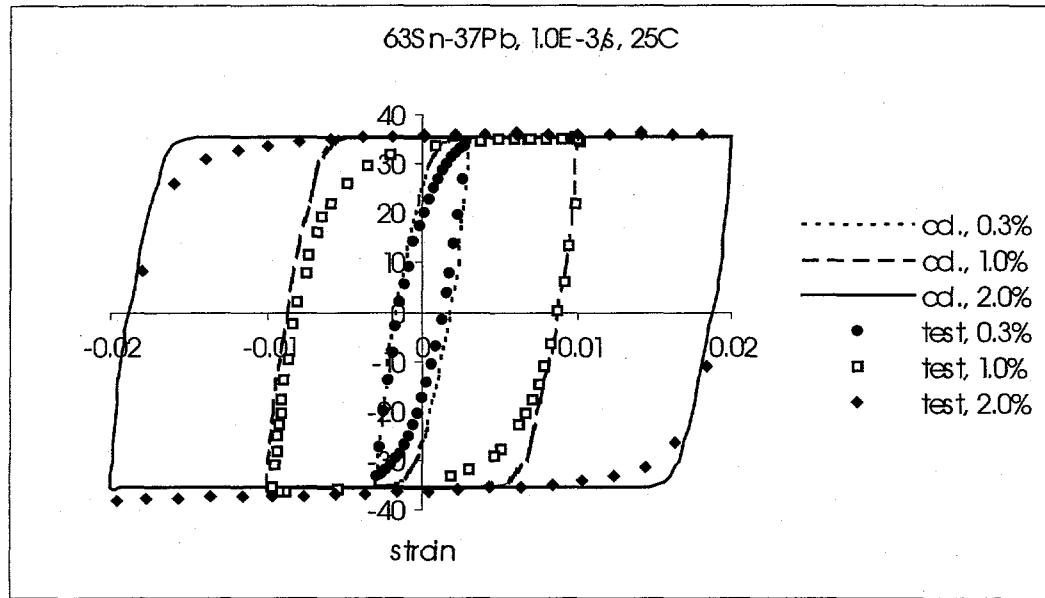


Fig.1 Hysteresis loops at strain rate  $10^{-3}/s$

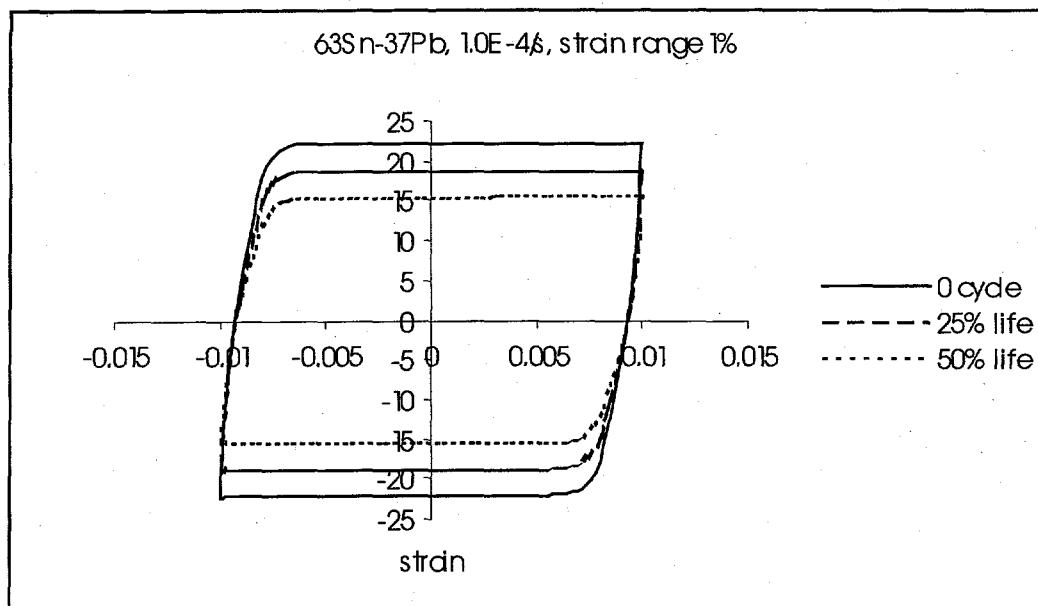


Fig.2 Soften behavior for strain range  $\pm 1.0\%$  at strain rate  $10^{-4}$ /s

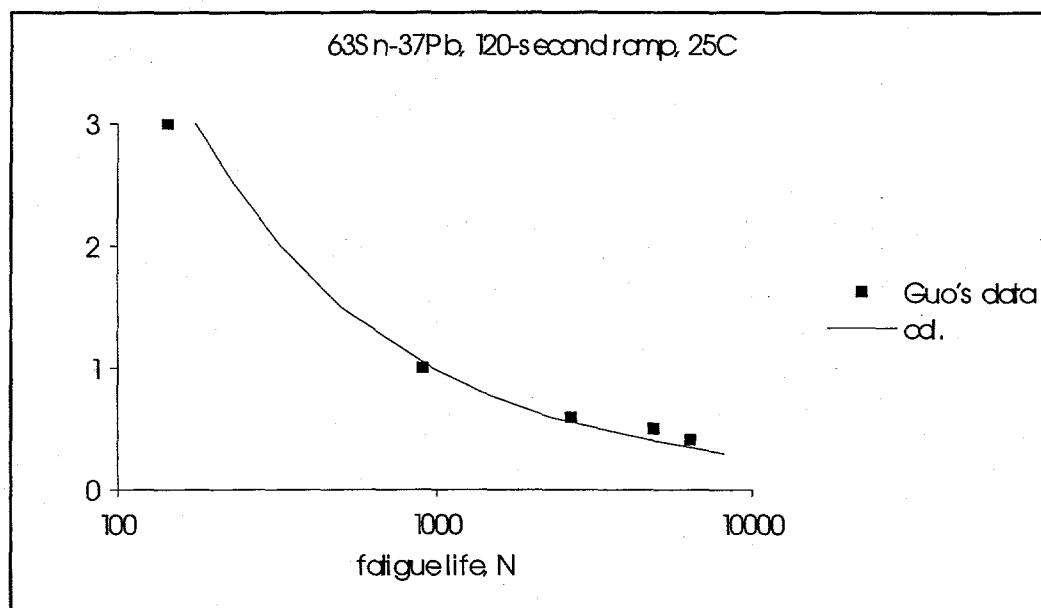


Fig.3 Fatigue life against strain amplitude