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Formal Report

Proceedings of RIKEN BNL Research Center Workshop Volume 20

GAUGE-INVARIANT VARIABLES IN GAUGE THEORIES

May 25–29, 1999

Organizers

Pierre Van Baal, Peter Orland and Rob Pisarski

RIKEN BNL Research Center

Building 510, Brookhaven National Laboratory, Upton, NY 11973, USA

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Preface to the Series

The RIKEN BNL Research Center was established this April at Brookhaven National Laboratory. It is funded by the "Rikagaku Kenkysho" (Institute of Physical and Chemical Research) of Japan. The Center is dedicated to the study of strong interactions, including hard QCD/spin physics, lattice QCD and RHIC physics through nurturing of a new generation of young physicists.

For the first year, the Center will have only a Theory Group, with an Experimental Group to be structured later. The Theory Group will consist of about 12-15 Postdocs and Fellows, and plans to have an active Visiting Scientist program. A 0.6 teraflop parallel processor will be completed at the Center by the end of this year. In addition, the Center organizes workshops centered on specific problems in strong interactions.

Each workshop speaker is encouraged to select a few of the most important transparencies from his or her presentation, accompanied by a page of explanation. This material is collected at the end of the workshop by the organizer to form a proceedings, which can therefore be available within a short time.

T.D. Lee
July 4, 1997

CONTENTS

Introduction

This four-day workshop focused on the wide variety of approaches to the non-perturbative physics of QCD. The main topic was the formulation of non-Abelian gauge theory in orbit space, but some other ideas were discussed, in particular the possible extension of the Maldacena conjecture to nonsupersymmetric gauge theories. The idea was to involve most of the participants in general discussions on the problem. Panel discussions were organized to further encourage debate and understanding. Most of the talks roughly fell into three categories:

1. Variational methods in field theory.
2. Anti-de Sitter space ideas.
3. The fundamental domain, gauge fixing, Gribov copies and topological objects (both in the continuum and on a lattice).

In particular some remarkable progress in three-dimensional gauge theories was presented, from the analytic side by V.P. Nair and mostly from the numerical side by O. Philipsen. This work may ultimately have important implications for RHIC experiments on the high-temperature quark-gluon plasma.

Many thanks to all participants of the workshop for their contributions and especially for the high level of discussion which clarified many aspects of field theory. We sincerely wish to express our gratitude to Pam Esposito, the secretary of the RIKEN-BNL Research Center. Finally we thank Brookhaven National Laboratory and the U.S. Department of Energy for providing the facilities to hold this workshop.

Peter Orland (Baruch College, City University of New York)

Pierre van Baal (University of Leiden)

Robert Pisarski (BNL Physics)

Gauge fields and AdS/(L?)CFT

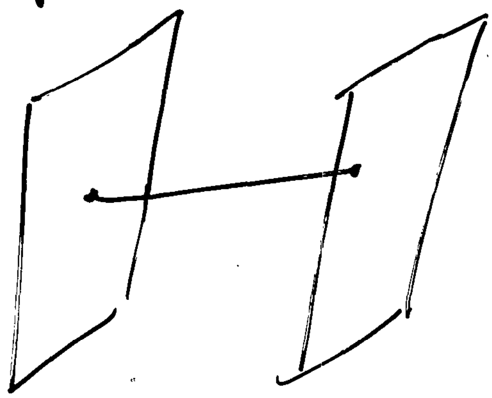
I. Kogan

"Gauge invariant variables in gauge theories"
RIKEN BNL May 25-29, 1999.

1) Gauge fields from D-branes
2). AdS/CFT conjecture
3). Instantons, monopoles and all that
4). Type 0 strings and asymptotic freedom
5). LCFT versus CFT (gluon ?)
6). Thermodynamics, Z_N and all that
7). Conclusion (if any)

Instanton moduli space — AdS!

Monopoles



1-3 system

How about running coupling constant

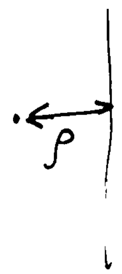
0 strings (Polyakov's conjecture, Klebanov, Tseytlin, Minahan)

No space-time fermions

2 RR sector — E and M 3 branes.

Tachyon in the bulk, but not on the boundary

$-m^2\phi^2$, $F_{\dots}^2\phi^2$



One can show that

$$e^{\Phi/2} \sim \frac{1}{\ln \rho}$$

Coupling constant $\frac{1}{g^2_{YM}} = e^{-\Phi} \sim (\ln \rho)^2$

$$ds^2 \sim \frac{c}{\ln \frac{1}{z}} \left[\frac{d\vec{x}^2 + dz^2}{z^2} + d\ell_5^2 \right]$$

$Q\bar{Q}$ potential (Kinahan)

$\leftarrow L \rightarrow$

$$E_{Q\bar{Q}} \sim \frac{c}{L} \frac{1}{\ln \frac{L_0}{L}}$$ $\leftarrow$ as it must be in AF theory

$M\bar{M}$ potential (IK, C.Luzon)

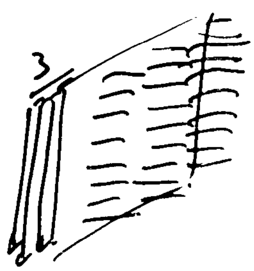
instead of F strings take D strings

$$E_{M\bar{M}} \sim \frac{c}{L} \ln \frac{L_0}{L} \rightarrow \frac{1}{g_m^2} \sim \frac{1}{\ln L_0/L}$$

$$g_e(L)\, g_m(L) = 1.$$ Dirac condition is scale independent

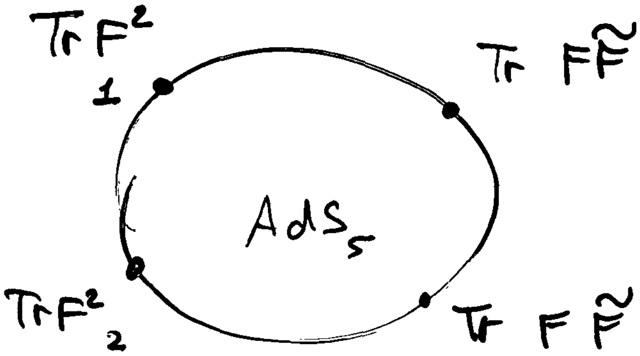
Screening of magnetic charge $\rightarrow$ monopole condensation.

Forest of 1 branes on 3 brane?

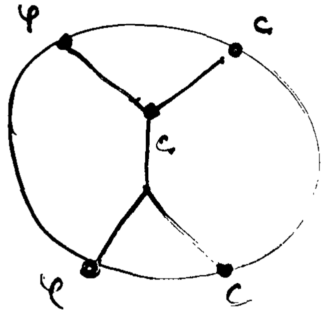


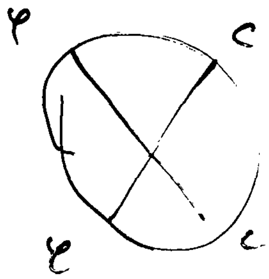
CFT or LCFT ?

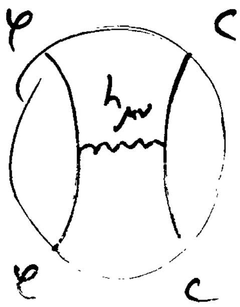
Witten
Gubser, Klebanov,
Polyakov



$Tr F^2$ – dilaton φ
$Tr F\tilde{F}$ – axion c







d'Hoker et al,
Liu, Tseytlin
Brodie, Gutperle
Chalmers ...

Logarithms in x space.

$$\boxed{\frac{1}{x_{13}^8 \, x_{24}^8} \ln \frac{x_{13} x_{24}}{x_{14} x_{23}}}$$

LCFT ? (Gurarie, Caux, Kogan, Tsvelik)

$$O_1 \cdot O_2 = \sum_\Delta \frac{1}{x_{12}^{\Delta_1+\Delta_2-\Delta}} O_\Delta$$

Logarithmic pair

$$\langle C(x)\, C(0)\rangle = 0$$

$$\langle C(x)\, D(0)\rangle = \frac{c}{x^{2\Delta}}$$

$$\langle D(x)\, D(0)\rangle = \frac{-2c \ln x + \text{const}}{x^{2\Delta}}$$

$$L_0\, C = \Delta C$$

$$L_0\, D = \Delta D + C$$

What in AdS ?

Singletons

$$\int d^{d+1}x \left(\partial_\mu A\, \partial_\mu B + m^2 AB + \mu^2 B^2\right) + \int A_B C + B_B D$$

$$\downarrow$$

$$\int A_B(\vec{x}) \frac{1}{(\vec{x}-\vec{y})^{2\Delta}} B_B(\vec{y}) + \int B_B(\vec{x}) \frac{\ln(\vec{x}-\vec{y})}{(\vec{x}-\vec{y})^{2\Delta}} B_B(\vec{y})$$

Conclusion

Questions

1). $U(1)$ and Z_N ?

2). what is a charge g^2_{YM} ?

$\underline{II}$ A,B versus 0 A,B

3). gluon ?

$$\langle F^a_{\mu\nu}(x)\, F^b_{\lambda\rho}(y)\rangle = \frac{\delta^{ab}}{(\bar{x}-\bar{y})^4}$$

4). collective degrees of freedom of D brane

5). condensation of monopoles

and personal)
can we say anything about

$\underline{\Psi_{\text{ground}}[\vec{A}]}$

YANG-MILLS THEORY IN (2+1) DIMENSIONS.

VACUUM WAVE FUNCTION, MASS GAP, STRING TENSION

KARABALI & NAIR (2 paper

KARABALI, KIM & NAIR (2 paper

WHY 2+1 DIMENSIONS?

1. INTERESTING IN ITS OWN RIGHT, MODEL FOR YM_{4D}

2. MAGNETIC SCREENING IN QUARK-GLUON PLASMA

   MASS GAP OF $YM_{2+1} \approx$ MAGNETIC MASS OF YM_{3+1} AT $T \neq 0$

3. USE RESULTS FROM CFT-2.

[illegible]

GAUGE $A_0 = 0$

$$\mathcal{H} = \underbrace{\frac{e^2}{2}\int E^2}_{T} + \underbrace{\int \frac{B^2}{2e^2}}_{V} = \int \frac{1}{2}\left(-e^2 \frac{\delta^2}{\delta A^2} + \frac{B^2}{e^2}\right)$$

$$\langle \mathcal{H} \rangle = \int \left[\int \frac{e^2}{2} \left|\frac{\delta \psi}{\delta A}\right|^2 + \int \frac{B^2}{2e^2} |\psi|^2\right] d\mu(\mathcal{C})$$

$$\mathcal{A} = \left\{ \text{SET OF ALL GAUGE POTENTIALS } A_i(\vec{x}) \right\}$$

$$\mathcal{G}_* = \left\{ \text{SET OF ALL } g(x) : \mathbb{R}^2 \to G,\ g \to 1 \text{ AS } |\vec{x}| \to \infty \right\}$$

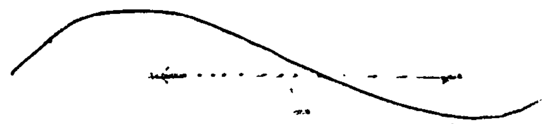
$$A_i^g = g^{-1} A_i\, g + g^{-1} \partial_i\, g$$

CONFIGURATIONS
PHYSICAL, GAUGE INVARIANT $\Big\} = \mathcal{C} = \mathcal{A}/\mathcal{G}_*$

ABOUT WAVE FUNCTIONALS

1. $\Psi_0[A]$ REAL, POSITIVE

2. $\Psi_{exc}[A]$: $\left|\frac{\delta \Psi}{\delta A}\right|^2 \sim \frac{1}{L^2}$



"L CANNOT BE MADE ARBITRARILY LARGE" [illegible]

$\rightarrow$ [illegible]

WHAT WE DO

1. GAUGE INVARIANT MATRIX VARIABLES

2. CALCULATE $d\mu(\mathcal{C})$, INNER PRODUCT (WZW MODEL)

3. CURRENT J AS "COLLECTIVE FIELD" VARIABLE

4. INTUITIVE ARGUMENT FOR MASS GAP

5. CALCULATE $\Psi_0[A]$, STRING TENSION

6. MASS GAP, EXCITED STATES, REMARKS

GAUGE-INVARIANT MATRIX VARIABLES

$$z = x_1 - i\, x_2 \qquad \bar{z} = x_1 + i\, x_2$$

$$A_z = \tfrac{1}{2}(A_1 + i\, A_2) \qquad A_{\bar{z}} = \tfrac{1}{2}(A_1 - i\, A_2)$$

$$\boxed{A_z = -\,\partial_z M\, M^{-1} \qquad A_{\bar{z}} = M^{\dagger -1}\,\partial_{\bar{z}} M^{\dagger}}$$

$M, M^\dagger$ = COMPLEX MATRICES $\underline{SL(N,\mathbb{C})}$

$$\partial_z M = -\,A_z M$$

$$M = 1 - \int G\, A_z\, M \qquad \partial_z G(z,z') = \delta^{(2)}(x-x')$$

$$G(z,z') = \frac{1}{\pi(\bar{z}-\bar{z}')}$$

$$= 1 - \int G A + \int G A G A + \cdots$$

GAUGE TRANSFORMATION

$$A_i \to A_i^g = g^{-1} A_i g + g^{-1}\partial_i g \qquad g : SU(N)$$

$$M \to g^{-1} M \qquad M^\dagger \to M^\dagger g$$

$$H = M^\dagger M \qquad \text{GAUGE-INVARIANT}$$

$$\frac{SL(N,\mathbb{C})}{SU(N)}$$

$$[dA\, d\bar{A}] = (\det D\bar{D})\, d\mu(H)\, d\mu(U)$$

$$\boxed{d\mu(\mathcal{C}) = \frac{[dA\, d\bar{A}]}{d\mu(U)} = \underbrace{\det D\bar{D}}\quad d\mu(H)}$$

$$\Gamma = \log \det D\bar{D}$$

$$\frac{\delta\Gamma}{\delta\bar{A}^a(x)} = -i\, \mathrm{Tr}\left[\left(\frac{1}{\bar{\partial}+\bar{A}}\right)_{xy} t^a\right]_{y\to x}$$

$$\bar{D}\phi = (\bar{\partial} + M^{\dagger -1}\bar{\partial}M^\dagger)\phi = M^{\dagger -1}\bar{\partial}(M^\dagger\phi)$$

$$\bar{D}^{-1}(x,y) = \frac{M^{\dagger -1}(x)\, M^\dagger(y)}{\pi(x-y)}$$

$$\simeq \frac{1}{\pi(x-y)} + \frac{M^{\dagger -1}(x)\,[\partial M^\dagger (y-x) + \cdots]}{\pi(x-y)}$$

$$\mathrm{Tr}\left[\bar{D}^{-1}(x,y)\, e^{A(x-y)\cdots}\, t^a\right]_{y\to x}$$

$$= \frac{N}{\pi}\left(A_z - M^{\dagger -1}\partial_z M^\dagger\right)^a$$

$$\Gamma = 2N\, S_{WZW}(H)$$

$$\mathcal{H} = T + V$$

$$T = \int \frac{e^2}{2} E^2$$

$$\boxed{T = \frac{e^2 N}{2\pi} \left[\int J^a \frac{\delta}{\delta J^a} + \int \Omega_{ab}(u,v) \frac{\delta}{\delta J^a} \frac{\delta}{\delta J^b} \right]}$$

$$\Omega_{ab} = \frac{N}{\pi^2} \frac{\delta_{ab}}{(u-v)^2} - \frac{i f_{abc} J_c(v)}{u-v}.$$

$$\boxed{V = \int \frac{B^2}{2e^2} = \frac{2\pi^2}{e^2 N^2} \int \bar{\partial}_z J^a \, \bar{\partial}_z J^a}$$

$\rightarrow$ EVERY J^a IN $\Psi(J)$ HAS MASS $m =$ [illegible]

WAVE [illegible]

IGNORE V FOR THE MOMENT.

VACUUM WAVE FUNCTION $\quad \Psi_0 = 1$

$$T \Psi_0 = 0$$

NORMALIZABLE $\quad \int \Psi_0^* \Psi_0 \, e^{2N S_{WZW}(H)} \, d\mu(H) < \infty.$

$$W_F(C) = \mathrm{Tr}\left[P \exp \oint_C A\, dx\right]$$

$$\langle W_F(C) \rangle = \text{CONST.}\ e^{-\sigma \mathcal{A}(C)}$$

$$\boxed{\sqrt{\sigma} = e^2 \sqrt{\frac{N^2-1}{8\pi}}}$$

$$\frac{\sqrt{\sigma}}{e^2} = 0\cdot 345,\quad 0\cdot 564,\quad 0\cdot 772,\quad 0\cdot 977$$

$$= 0\cdot 335,\quad 0\cdot 553,\quad 0\cdot 758,\quad 0\cdot 966$$

$$T\, J_a = m\, J_a \qquad m = \frac{e^2 N}{2\pi}$$

$$(T+V)\, J_a \approx \sqrt{k^2+m^2}\, J_a + \cdots$$

EXPECTED MASS GAP $\approx m \approx \dfrac{e^2 N}{2\pi}$
FOR "GLUONS"

Instantons and the QCD Vacuum Wavefunctional

William Brown - Rockefeller University

- Introduction to the variational approach. Summary of results so far.
- Hamiltonian portrait of an instanton.
- Small size instantons.
- Mass corrections - stable size.

Ansatz

$$\Psi[A] = \int DU(x) \exp\left[\frac{-1}{2}\int d^3x\, d^3y\, A_i^{aU}(x)\, G_{ij}^{-1\,ab}(x-y)\, A_j^{bU}(y)\right]$$

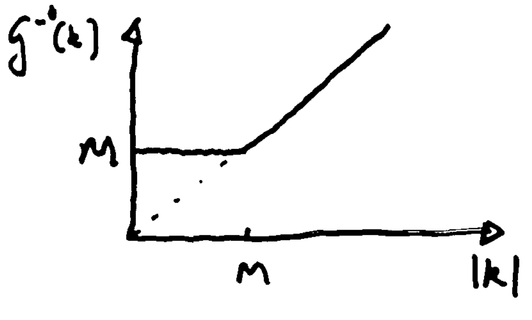
$$A_i^{aU}(x) = S^{ab}(x) A_i^b(x) + \lambda_i^a(x)$$

$$S^{ab}(x) = \frac{1}{2}\mathrm{tr}\left[\tau^a U^\dagger(x)\, \tau^b U(x)\right]$$

$$\lambda_i^a(x) = \frac{i}{2}\mathrm{tr}\left[\tau^a U^\dagger(x)\, \partial_i U(x)\right]$$

By Construction :-

- Correct UV limit $\quad G_{ij}^{-1\,ab}(k) = |k|\delta_{ij}^{ab}, \quad |k| \to \infty$
- Gaussian in A, manageable.
- G.I.
- $G_{ij}^{-1\,ab}(k) = M\delta_{ij}^{ab} \quad |k| \to 0$



$$G^{-1}(k) = \begin{cases} |k| & k^2 > M^2 \\ M & k^2 < M^2 \end{cases}$$

Summary of Results

- Gauge Invariant Ansatz with correct UV limit. (Kogan & Kovner)

- Energy minimized $M = 8.86\,\Lambda_{QCD} = 1.33\,\text{GeV}$ $(\alpha = 0.26)$

- $\frac{\alpha}{M}\left\langle F^a_{\mu\nu} F^a_{\mu\nu} \right\rangle = 0.008\,\text{GeV}^4$ (SVZ 0.012)

- $\beta_{var} = -\frac{g^3}{(4\pi)^2} C_2(G)\,4$, $\beta_{QCD} = \frac{-g^3}{(4\pi)^2} C_2(G)\left(4 - \frac{1}{3}\right)$ (B & Kogan)

  Difference due to ommision of some transverse gluon interactions in var. calc. (B)

- QED_3 – all known results reproduced (KCK)

  QED_3 with θ-term. (B & Kogan)

- May be a linear potential between static external quarks. (Diakonov) (Zarembo) (Kovner & Svetitsky)

  Confinement?

- Weak coupling; WKB approx good, QCD I reliable.
  Small overlap of tails of wf in H. Gaussian tails decay too fast.

  Strong coupling; WKB not so good. Tails do not dominate overlap in H formalism. Gaussian approx much better.

Small Size Instantons $\rho \ll \frac{1}{m}$ $m=0$

$$G^{-1}(k) = |k| \qquad \int d^3x\, G^{-1}(x) = 0$$

$$G^{-1}(x-y) = \frac{-1}{\pi^2}\left(\frac{\theta\left(|x-y| - \frac{1}{\Lambda}\right)}{|x-y|^4} - \Lambda^3 \delta\left(|x-y| - \frac{1}{\Lambda}\right) \right)$$

regulator $\Lambda \to \infty$ at end of calc.

$$U(x) = e^{i\tau^a \hat{x}^a f(r)} \qquad \tau^a \sim SU(2)$$

$SU(N>2)$ obtained from embedding $SU(2)$.

Action = finite difference between two infinite terms.

$$\Gamma = \frac{-1}{8\pi^2}\int d^3x\, d^3y \frac{1}{|x-y|^2}\left(\frac{\partial^2}{\partial x_j^2} + \frac{\partial^2}{\partial y_j^2} - 2\frac{\partial^2}{\partial x_j \partial y_j}\right)\left[\lambda_i^a(x)\,\lambda_i^a(y)\right]$$

$$+ \text{S.T.}$$

$$\text{S.T.} = \begin{cases} 0 & f(r) \le \frac{1}{r^2} \quad |r| \to \infty \\ \infty & f(r) > \frac{1}{r^2} \quad |r| \to \infty \end{cases}$$

- Perform all angular integration.
- Profile function $f \sim r^{-\alpha}, \quad r \to \infty.$

$f \sim \pi - r^{\beta}, \quad r \to 0.$

$$f_1(r) = \pi \left[\frac{\rho^{\alpha}}{\rho^{\alpha} + r^{\alpha}} \right]$$ motivated by U for I

$$f_2(r) = 2 \tan^{-1} \left(\frac{\rho}{r} \right)^{\alpha}$$ " " solitons.

$\Gamma \sim$ monotonically increasing function of $\alpha \Rightarrow \alpha = 2.$

$$f_1(r) = \pi \left[\frac{\rho^{\beta}}{\rho^{\beta} + r^{\beta}} \right]^{2/\beta} \qquad \beta = 1.1 \qquad \Gamma = 1.96 \frac{8\pi^2}{g^2}$$

$$f_2(r) = 2 \tan^{-1} \left[\frac{2}{\left(\frac{r}{\rho}\right)^2 + \left(\frac{r}{\rho}\right)^{\beta}} \right] \qquad \beta = 1.0 \qquad \Gamma = 2.07 \frac{8\pi^2}{g^2}$$

Dilatational Invariance $\Rightarrow \Gamma$ indep of ρ.

$$f_{AM} = \pi \left[1 - \frac{1}{\sqrt{1 + \frac{\rho^2}{r^2}}} \right] \qquad \begin{matrix} \alpha = 2 \\ \beta = 1 \end{matrix} \qquad \Gamma = 1.97 \frac{8\pi^2}{g^2}$$

Fig 1.

Mass Correction

- Scaling argument: running coupling ⇒ small I's → big.

$M \neq 0 \qquad \Gamma = \frac{1}{2}\frac{M}{g^2(M)} \,\mathrm{tr} \int d^3x\, \partial_i U^{+}(x) \partial_i U(x)$

$\Gamma\big(U(\lambda x)\big) = \lambda^{-1}\Gamma\big(U(x)\big) \qquad \lambda \to \infty$ I's shrink

(Derrick's collapse)

$\rho \sim \frac{1}{M}$ with what coeff?

- $G^{-1}(k) = \underset{|k|}{\overset{\shortparallel}{G_0^{-1}(k)}} + \Delta G^{-1}(k) \qquad , \quad \Delta G^{-1}(k) = \Theta(M-|k|)(M-|k|)$

$$\Delta G^{-1}(|x-y|) = \frac{-1}{\pi^2 |x-y|^4}\left(\cos M|x-y| - 1 + \frac{M|x-y|}{2}\sin M|x-y|\right)$$

$$\Delta\Gamma = \frac{1}{4}\int d^3x\, d^3y\, \lambda_i^a(x)\, \Delta G^{-1}(x-y)\, \lambda_i^a(y)$$

Action favours small I. Fig 2.

- Running coupling: $\beta_{vor}(g) = \frac{-g^3}{(4\pi)^2}\, 8$

$$\left.\begin{aligned} \frac{8\pi^2}{g^2(\mu > M)} &= 8\ln\left(\frac{\mu}{\Lambda_{QCD}}\right) \\ \frac{8\pi^2}{g^2(\mu < M)} &= 8\ln\left(\frac{aM}{\Lambda_{QCD}}\right) \end{aligned}\right\} \qquad \frac{8\pi^2}{g^2(\frac{1}{\rho})} = 4\ln\left(\frac{M^2}{\Lambda_{QCD}^2}\left(\frac{1}{M^2\rho^2} + a\right)\right)$$

$a \sim O(1)$

- Zero modes $dx_1 dx_2 dx_3 d\rho\, \rho^{-4}$ $\Gamma_{extra} = 4 \log \rho$

Fig 3

- Lowest action ~ f_1, f_{AM}, indistinguishable. f_2 larger.

- $\frac{1}{2} < a < 2$ $\frac{1.0}{M} < \rho_{stable} < \frac{1.5}{M}$ f_1, f_{AM}

- Dynamical mass generation stabilizes the instanton size. Large I problem finds a non-pert solution in the framework of the Gaussian variational approx.

- Implicitly, instantons already discussed: important role in energy minimization. MFA ~ E minimized in disordered phase of σ-model. Transition between ordered & disordered phases in a stat. mech. system usually described by condensation of top. defects. (Ising, XY) M≠0 driven by condensation of instantons.

- Numerically, Γ ~ 35 large. Small fugacity overcome by $I\bar{I}$ interactions, entropy. Important configs are multi-instanton. Entirely possible that important config ~ Instanton liquid.

$$\rho \left(\frac{\alpha}{\pi} \langle F^a_{\mu\nu} F^a_{\mu\nu} \rangle \right)^{1/4} \sim 0.4 \qquad \text{I-liquid}$$

$$\rho \left(\frac{\alpha}{\pi} \langle F^a_{\mu\nu} F^a_{\mu\nu} \rangle \right)^{1/4} \sim 0.2 - 0.3 \qquad \text{variational}$$

QCD from a 5-dimensional point of view

D. Zwanziger
New York University.

QCD is described from a 5-dimensional point of view where $t = x_5$ represents number of sweeps in a Monte Carlo simulation. Renormalizability controls both the static 4-D dimensional renormalization group and the dynamic renormalization-group that controls auto-correlation time of the numerical simulation. The 5-D action is topological = BRST exact. Ghosts decouple and the ghost determinant is unity.

QCD from a 5-dimensional point of view

D. Zwanziger (with L. Baulieu)

x_μ $\mu = 1 \cdots 4$ Euclidian

$t = x_5$ = no of sweeps in Monte Carlo simulation

Stochastic quantization

$$\dot{A}_\mu = D_\mu v - \frac{\delta S}{\delta A_\mu} + \eta_\mu$$

η_μ = Gaussian noise

$D_\mu v$ = Gauge-fixing $\quad v = \partial \cdot A$

$t = x_5 \qquad v = A_5 \qquad F_{5\mu} = \dot{A}_\mu - D_\mu A_5$

$$F_{5\mu} = D_\lambda F_{\lambda\mu} + \eta_\mu$$

5-dimensionally gauge-invariant.

gauge $\quad A_5 = \partial_\mu A_\mu$

Local action in 5 dimensions

$$\Sigma = \int dt\, d^4x\, \frac{1}{2}\left(F_{5\mu} - D_\lambda F_{\lambda\mu}\right)^2$$

$[A_\mu] = 1 \qquad [A_5] = 2$

$[x] = -1 \qquad [t] = -2$

$[\Sigma] = 0$

gauge $\quad A_5 = \partial_\mu A_\mu$

Renormalizable in 5-D.

$$(s+d)(A+c) + \frac{1}{2}[A+c, A+c]$$
$$= F + \psi_\mu dx^\mu + \psi_5 dx^5 + \phi$$

curvature

$$[A+c+(s+d)](F + \psi_\mu dx^\mu + \psi_5 dx^5 + \phi) = 0$$

Bianchi

$$\Sigma = s \int dt\, d^4x\, \mathrm{Tr}\left[\bar{\Psi}\left(F_{5\mu} - D_\lambda F_{\lambda\mu} - \frac{1}{2} b_\mu\right) + \text{gauge-fixing}\right.$$

$\uparrow$ topological action — BRST exact.

<u>Gluons decouple from ghosts</u>

$A_5 = \partial_\mu A_\mu$ like axial gauge

$s A_\mu = D_\mu c + \psi_\mu$ & $\mu \to 5$

$s \psi_\mu = - D_\mu \phi - [c, \psi_\mu]$ & $\mu \to 5$

$s c = \phi - \frac{1}{2} [c, c]$

$s \phi = - [c, \phi]$

$s(A_5 - \partial_\mu A_\mu) = D_5 c - \partial_\mu D_\mu c$

= ghost equation of motion.

PARABOLIC for all ghosts

$\det [D_5 - \partial_\mu D_\mu] = 1.$ etc.

Global Properties of Gauge-Fixing

$a A_5 = \partial_\mu A_\mu$ $a \to 0$ Landau

$a A_5 = \vec{\nabla} \cdot \vec{A} + b A_4$ $a \to 0$ $b \to 0$ Coulomb

Landau:

$$\dot{A}_\mu = \frac{1}{a} D_\mu \partial \cdot A$$

$$\frac{d}{dt} \int d^4x \; A_\mu^2 = -\frac{2}{a} \int d^4x \; (\partial \cdot A)^2$$

Restoring force with equilibrium at $\partial \cdot A = 0$

$$\frac{d}{dt} \partial \cdot A = \frac{1}{a} \partial \cdot D \; \partial \cdot A$$

Equilibrium is stable where

$\partial \cdot D > 0.$ only

Stable only inside Gribov Horizon

Confinement in the Coulomb Gauge: Theory & Numerical Results

Dan Zwanziger NYU

in collaboration with Attilio Cucchieri

(APE-Roma & Bielefeld)

1. Introduction

Simple idea: in (minimal) Coulomb gauge

$D_{11} = D_{22} = D_{33} \neq D_{44}$

$D_{44}(r, t) = V(r)\, \delta(t)$ + non-instantaneous

$V(r) \sim K r$ long-range color-confining coulomb potential

K — string tension

$D_{ij}(k)$ = 3-dimensionally transverse propagation of <u>would-be</u> physical gluons

$k_i D_{ij}(k) = 0$

suppressed at $k=0$ $D_{ij}(0) = 0$ (!!!) $\sim \frac{1}{k^2}$

so NO physical gluons.

PETER ORLAND

TOPOGRAPHY ON ORBIT SPACE

Gauge Theory:

- hep-th/9607134
- hep-th/9709031 (Zakopane) (w. Kudinov + Moreno)

O(n) Sigma Model (1+1):

- hep-th/9709031 (Zakopane) (w. Kudinov + Moreno)
- JHEP 9904 (1999) 002 (w. Moreno)

Work in progress on SU(N) gauge theory (w. G. Semenoff)

I. FIELD THEORY AS QUANTUM MECH.

II. 1+1 = d O(n) SIGMA MODEL

III. RIVER VALLEY EQ. FOR GAUGE THEORY

B. Distance $\rho[\varphi_1, \varphi_2]$

$$\langle \varphi_1 | e^{ie_0 T\epsilon} | \varphi_2 \rangle \sim \left(\frac{e_0}{2\pi i \epsilon} \right)^{\mathcal{N}} \exp \frac{ie_0}{2\epsilon} \rho[\varphi_1, \varphi_2]^2$$

$\mathcal{N}$ = U.V. regulate no. of deg. of freedom.

Only

$$d\rho^2 = \rho[\varphi, \varphi + \delta\varphi]$$
$$= \int d^D x \int d^D y \, G_{(x,A)(y,B)} \delta\varphi^A(x) \delta\varphi^B(y)$$

is unique.

C. Examples

i) Scalar f.t.: $\rho[\varphi_1, \varphi_2]^2 = \frac{1}{2} \int d^D x \, |\varphi_{b,1}(x) - \varphi_{b,2}(x)|^2$

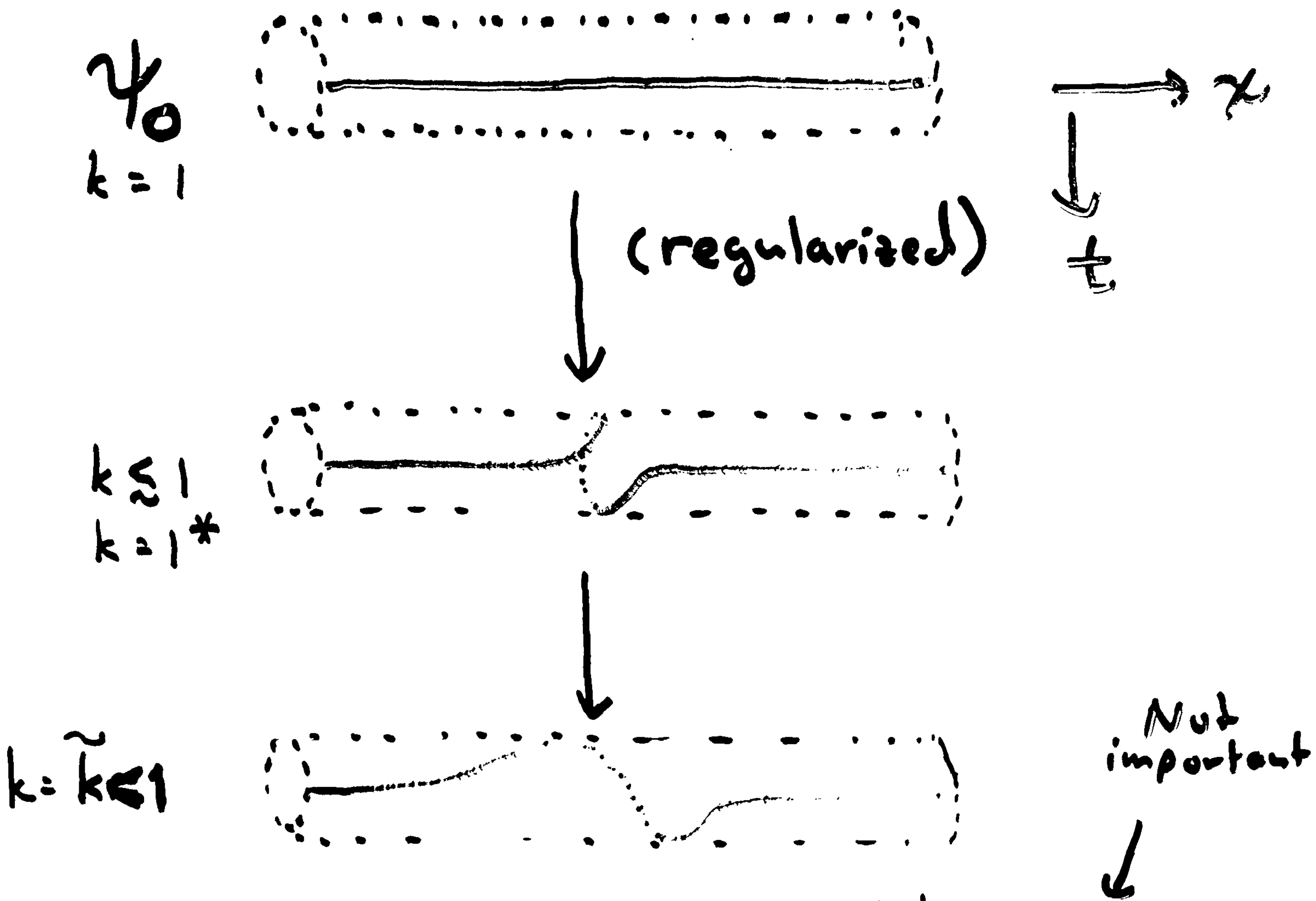
ii) O(n) Sigma model: i) is again physically correct, but there are advantages to considering

$$\rho[\{Rf\}, \{RS\}]^2 = \frac{1}{2} \inf_{R \in O(n)} \int d^D x (Rf - S)^2 .$$

(Feynman 198[redacted])

$\{Rf\}$ = "orbit", $R \in O(n)$.

ii) Barrier penetration
(α_1^{circ})



Derivative regularization $\left|\frac{ds}{dx}\right| \leq 2Q\Lambda$

Barrier-penetration amplitude:

$$\mathcal{T} = e^{-W}, \quad W = \int_{\tilde{k}}^{1^*} dk \, \frac{d\rho(k)}{dk} \sqrt{\frac{2}{e_0}\left[\frac{u(k)}{e_0} - \frac{u(\tilde{k})}{e_0}\right]}$$

$$\approx \frac{2\sqrt{2}}{e_0} \log(Q L \Lambda)$$

Kosterlitz-Thouless transition

$$T = \sum_x \mathcal{T} = \Lambda L \mathcal{T} \propto (\Lambda L)^{1 - 2\sqrt{2}/e_0}$$

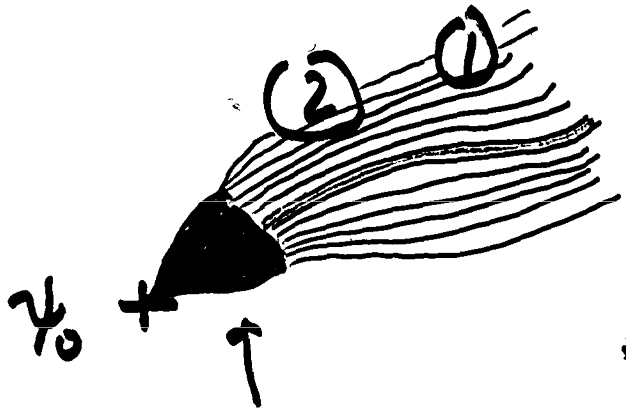
For $e_0 < 2\sqrt{2}$ this dies as $L \to \infty$. Few tunnelings occur.

For $e_0 > 2\sqrt{2}$, tunnelings (kinks) condense (proliferate).

F. O(3)-Model Potential Topography

MERONS?

River Delta



Barrier

= Circulating O(2) config. $\alpha_1^{\text{circ.}}$

(1) = spin wave

(2) = Bloch wall

Consider the topological charge

$$q = \frac{1}{4\pi}\int dt\, dx\, \epsilon^{abc}\, s_a\, \partial_t s_b\, \partial_x s_c$$

for any barrier penteration in the river delta $\psi_0 \to$ spin wave.

ANSWER: $q = 1/2$

No gap at $\Theta = \pi$.

(1987) Haldane / Affleck conjecture:

III. The River-Valley Equation for Gauge Theory

A. Play the same game as for sigma models:

$$Q[\alpha] = \frac{1}{4}\int d^D x \, \mathrm{tr}\, F_{ij}^2 + \lambda\left[\frac{1}{2}\inf_g \int \mathrm{tr}(A^g)^2 d^D x - \rho_0^2\right]$$

$$\delta Q[\alpha] = 0.$$

B. We can't find $\rho^2 = \frac{1}{2}\inf_g \int \mathrm{tr}(A^g)^2 d^D x$ from $1^{\underline{st}}$ principles. **However..** the <u>solution</u> A^g satisfies a differential equation.

$A^g \to A$

$$-[D_i, F_{ij}] + \lambda A_j = 0 \qquad (*)$$

(this implies $\partial \cdot A = 0$).

Not all solutions of (*) have $\delta Q = 0$

But $\delta Q = 0 \Longrightarrow (*)$.

Furthermore ρ is <u>calculable</u>.

Fujikawa

Gribov Problem and BRST Formulation

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1. Motivation

2. A BRST symmetric re-interpretation of the Faddeev-Popov formula
"BRST invariant summation over Gribov copies"

3. Gribov problem and gauge independence
"Possible breakdown of BRST symmetry due to Gribov problem"

4. BRST symmetric formulation of the soluble model of Friedberg et al.
"Agreement with the canonical analysis"
"Gribov horizon is not serious?"

5. Conclusion

2. BRST Symmetric Summation of Gribov Copies

Faddeev-Popov formula:

$$\langle +\infty | -\infty \rangle = \int \mathcal{D}A_\mu^\omega \mathcal{D}C \, \delta(\partial_\mu A_\mu^\omega - C) \Delta(A)$$

$$\times \exp\left[i S(A_\mu^\omega) - \frac{i}{2\alpha} \int C^2 dx \right]$$

$S(A_\mu^\omega)$: local gauge invariant action

$A_\mu^\omega \equiv$ gauge transform of A_μ with gauge parameter $\omega(x)$.

$$\Delta(A)^{-1} \equiv \int \mathcal{D}\omega \mathcal{D}C \, \delta(\partial_\mu A_\mu^\omega - C) \exp\left[-\frac{i}{2\alpha} \int C^2(x) dx \right]$$

$$= \sum_k \left| \det \frac{\partial}{\partial \omega^k} \partial^\mu A_\mu^{\omega_k} \right|^{-1}$$

Sum over Gribov copies of

$$\partial_\mu A_\mu^{\omega'(\omega, A, C)} \equiv \partial_\mu A_\mu^\omega - C = 0$$

$\Delta(A)$ is very complicated in the presence of Gribov copies.

Alternative formulation: K.F. P.T.P. 61 ('79) 627,

P. Hirschfeld, NP B157 ('79) 37

$$\langle +\infty | -\infty \rangle = \frac{1}{N} \int \mathcal{D}A_\mu^\omega \mathcal{D}C \, \delta(\partial_\mu A_\mu^\omega - C) \, \det\left[\frac{\partial}{\partial\omega} \partial^\mu A_\mu^\omega\right]$$

$$\times \exp\left[i S(A_\mu^\omega) - \frac{i}{2\alpha} \int C(x)^2 dx \right]$$

This expression is <u>local</u> in ω-parameter space.

$\det[\frac{\partial}{\partial\omega} \partial^\mu A_\mu^\omega]$ depends on A_μ^ω

$\rightarrow$ (entire integrand is in general <u>no more degenerate</u> with respect to gauge copies,

- $\det[\frac{\partial}{\partial\omega} \partial^\mu A_\mu^\omega]$ <u>without</u> absolute sign.

- $\langle +\infty | -\infty \rangle$ is rewritten as

$$\langle +\infty | -\infty \rangle = \frac{1}{\tilde{N}} \int \mathcal{D}A_\mu^\omega \mathcal{D}B \mathcal{D}\bar{c} \mathcal{D}c \, \exp\left[i S(A_\mu^\omega) + i \int \mathcal{L}_g dx \right],$$

$$\mathcal{L}_g = -\partial_\mu B^a A_\mu^{a\omega} + i \, \partial_\mu \bar{c}^a (\partial_\mu - g f^{abc} A_\mu^{\omega b}) c^c + \frac{\alpha}{2} B^a B^a,$$

$(\bar{c}^a)^\dagger = \bar{c}^a$, $(c^a)^\dagger = c^a$; hermitian ghosts.

$\mathcal{L}_{eff} = \mathcal{L}_{YM} + \mathcal{L}_g$ is invariant under BRST

$$\begin{cases} \delta_\theta \underset{\sim}{A}_\mu^{a\omega} = i\theta\,[\partial_\mu C^a - g f^{abc} \underset{\sim}{A}_\mu^{b\omega} C^c] \\ \delta_\theta C^a = i\theta\,(\frac{g}{2}) f^{abc} c^b c^c \\ \delta_\theta \bar{C}^a = \theta B^a \\ \delta_\theta B^a = 0 \end{cases}$$

- path integral measure is also BRST invariant
- Normalization factor $\tilde{N}$ is independent of the parameter α

To make our $\langle +\infty | -\infty \rangle$ agree with

$$\langle +\infty | -\infty \rangle = \int \frac{\mathcal{D}\underset{\sim}{A}_\mu^\omega}{\text{gauge volume}(\omega)} \exp\left[i S(\underset{\sim}{A}_\mu^\omega)\right]$$

the normalization constant needs to be defined as

$$N = \int \mathcal{D}\omega\, \mathcal{D}C\, \delta(\partial_\mu \underset{\sim}{A}_\mu^\omega - C)\, \det\left[\frac{\partial}{\partial\omega} \partial^\mu \underset{\sim}{A}_\mu^\omega\right] \exp\left[-\frac{i}{2\alpha}\int C(x)^2 dx\right]$$

$$= \int \mathcal{D}\omega\, \det\left[\frac{\partial}{\partial\omega} \partial^\mu \underset{\sim}{A}_\mu^\omega\right] \exp\left[-\frac{i}{2\alpha}\int (\partial_\mu \underset{\sim}{A}_\mu^\omega)^2 dx\right]$$

$$= \int \mathcal{D}\tau\, \exp\left[-\frac{i}{2\alpha}\int \tau(x)^2 dx\right] \;; \quad \tau(x) \equiv \partial^\mu \underset{\sim}{A}_\mu^\omega(x)$$

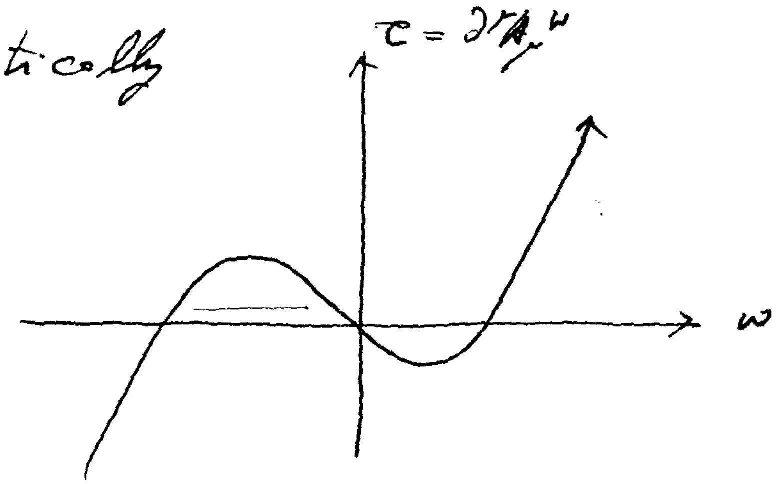
It is crucial to establish that

$$N = \int \mathcal{D}\tau \, \exp\left[-\frac{i}{2\alpha}\int \tau(x)^2 dx\right] \quad , \quad \tau(x) \equiv \partial^\mu A_\mu^\omega(x),$$

is <u>independent</u> of A_μ, i.e.,

"Range of the function $\tau(x)$ is the total τ-field space, no matter what gauge field A_μ takes place". (P. van Baal)

Schematically



Evaluation of the factor N is the <u>only place</u> where we encounter Gribov copies in this formulation.

If N depends on A_μ, it should be taken inside the path integral $\Rightarrow$ complications

3. Gauge Independence and Gribov problem

gauge independence of

$$\langle +\infty | -\infty \rangle = \frac{1}{\tilde{N}} \int \mathcal{D}A_\mu^a \mathcal{D}B \mathcal{D}\bar{c} \mathcal{D}c \; \exp\left[i \int \mathcal{L}_{eff} \, dx \right]$$

$$\mathcal{L}_{eff} \equiv \mathcal{L}_{YM} - \partial_\mu B^a A_\mu^{a\mu} + i\, \partial_\mu \bar{c}^a (D_\mu c)^a + \frac{\alpha}{2} B^a B^a$$

Schwinger's action principle:

$$\delta_\alpha \langle +\infty | -\infty \rangle = i\delta\alpha \int d^4x \langle +\infty | B^a(x)^2 | -\infty \rangle$$

$$= i\delta\alpha \int d^4x \langle +\infty | \{ Q_{BRST}, \bar{c}^a(x) B^a(x) \}_+ | -\infty \rangle$$

$$= 0$$

if $Q_{BRST} | -\infty \rangle = \langle +\infty | Q_{BRST} = 0$.

In the path integral analysis, one may consider

$$Z(\Lambda) = \frac{1}{N} \int \mathcal{D}A_\mu \mathcal{D}B \det[\partial^\mu D_\mu]$$

$$\times \exp\left[i \int \left(-\frac{1}{4} F^{\mu\nu} F_{\mu\nu} - B^a (\partial^\mu A_\mu^a - \Lambda^a) \right) d^4x \right]$$

If one can bring the configuration

$$\partial^\mu A_\mu^a(x) = \Lambda^a(x)$$

for "arbitrary" $\Lambda^a(x)$ to

$$\partial^\mu A_\mu^a(x) = 0$$

by a suitable gauge transformation, or equivalently, if one can find a solution $\delta\omega^a(x)$,

$$\underline{(\partial^\mu D_\mu)^{ab}}\,\delta\omega^b(x) \equiv \partial^\mu\left(\partial_\mu\delta^{ab} - g f^{acb} A_\mu^c(x)\right)\delta\omega^b(x) = \delta\Lambda^a(x)$$

for <u>arbitrary</u> $\delta\Lambda^a(x)$, one can establish

$Z(\Lambda)$ is independent of $\Lambda(x)$. (Fradkin-Tyutin

$\Rightarrow$

$$\frac{\delta^2}{\delta\Lambda^a(x)\,\delta\Lambda^a(x)} Z(\Lambda) = \langle B^a(x)\, B^a(x) \rangle = 0$$

in agreement with formal operator analysis above.

M. Schader

Resolution of the Gribov Problem – or How to Live With the Ambiguity

K. Fujikawa('79,'83), P. Hirschfeld('79), B. Sharpe('84), R. Friedberg, T.D. Lee, Y. Pang, H.C. Ren('95), L. Baulieu, M. S., A. Rozenberg('96,'98), and many others

(talk at BNL on May 26, 1999)

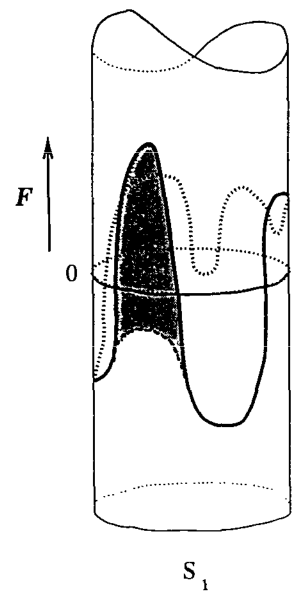
SETTING: SU(n) Gauge theory on compact Euclidean space-time without boundary
PROBLEM: V.N. Gribov & I.M. Singer ('78):

$$\mathcal{E}_{\mathcal{F}}[A] = \{U : U(x) \in SU(n), \mathcal{F}(A^U) = 0\}$$

is not simply connected for all local $\mathcal{F}(A)$.

PROPOSAL: Compute a top# of $\mathcal{E}_{\mathcal{F}}[A]$:
1) top# constant on *connected* set of orbits
2) top# constant on *connected* set of $\mathcal{F}$'s
IMPLEMENTATION: top# $\Leftrightarrow$ BRST of TQFT
3) "Covariant" local effective action
4) Global Gribov Problem (GGP): top# $= 0$
5) Generalizes to $SU(n)^{sites}$-invariance of LGT
COMMENT: 1)+2) are related. 3) requires a "covariant" and local $\mathcal{F}(A)$ that preserves *all* isometries of the compact space-time, such as $\mathcal{F}(A) = \partial \cdot A$. The resolution of 4) and the generalization 5) are subjects of this talk.

TRIVIALIZATION: BRST, FP-Integral and a top#(S_1)



$$\mathcal{F} : g \in S_1 \to \mathcal{F}(g) \in R$$

$$\chi(S_1) = \int_{S_1} dg\, \delta(\mathcal{F}(g))\, \mathcal{F}'(g)$$

$$\Updownarrow$$

$$\text{top\#} = \sum_{\mathcal{F}(g_i)=0} \text{sign}[\mathcal{F}'(g_i)] = 0!$$

$$\Updownarrow$$

$$\text{FP} = \int_{-\infty}^{\infty} db \int dc d\bar{c} \int_{S_1} dg \exp[s\bar{c}\mathcal{F}(g)]$$

with the nilpotent BRST symmetry,

$$sg = c,\ sc = 0\ ;\ s\bar{c} = ib,\ sb = 0$$

COMMENT: The model TQFT "action" is s-exact:

$$s\bar{c}\mathcal{F}(g) = ib\mathcal{F}(g) - \bar{c}\mathcal{F}'(g)c$$

$\Rightarrow$ the integral is a top#(S_1) that does not depend on $\mathcal{F}$. By deforming to $\mathcal{F} = \mathcal{V}' \Rightarrow \chi(S_1)$ is Morse's definition of the Euler# of S_1 – which vanishes!$\Rightarrow$ GGP

MORAL: The FP-Integral is a constant on a *connected* set of orbits that may vanish: $\chi(\mathcal{G} = SU(n) \times \mathcal{G}_p) = 0$
STRATEGY: avoid GGP by either
1)Equivariant cohom. for $\chi(\mathcal{M} = \mathcal{G}/\mathcal{H}) \neq 0$
2)Use BRST to compute a top$\# \neq 0$ of $\mathcal{G}$!

RESULTS for $SU(2)$ gauge group:
on T_4: $\chi(\mathcal{E}_{\mathcal{F}}/SU(2))$ =odd
on S_4, $n = 0$ sector: $\chi(\mathcal{E}_{\partial \cdot A}/SU(2)) = 1$
on S_4, $n = 1$ sector: $\chi(\mathcal{E}_{\partial \cdot A}/SU(2))$ =even,
etc....
"Cov." & BRST-inv., $SU(2)$-LGT in MAG:
$\chi(\mathcal{G} = SU(2)^N) = \chi(SU(2))^N = 0 \Rightarrow$GGP
BUT: $\chi(\mathcal{M} = SU(2)^N/U(1)^N \sim S_2^N) = 2^N$!
Use $U(1)^N$ invariant Morse potential:

$$V : \mathcal{M} \to R$$

$$V_{\{U\}}[g] = V[U^g] \text{ with } V[U] = \sum_{\text{links}} \left| \mathrm{Tr}\tau_+ U_{ij} \right|^2$$

to construct the TQFT with an equivariant BRST-symmetry and partition function $\chi(\mathcal{M})$.

THE TQFT on $\mathcal{M} = SU(2)^N/U(1)^N$
equivariant BRST algebra of TQFT:

$$
\begin{aligned}
sU_{ij} &= 0 \\
sg_i &= -g_i(c_i + \omega_i) \ , \ \omega_i \in \text{Cartan} \\
sc_i &= c_i^2 + [\omega_i, c_i] + \phi_i \ , \phi_i \in \text{Cartan} \\
s\omega_i &= -\phi_i \\
s\phi_i &= 0
\end{aligned}
$$

and multiplier doublets to enforce:

$$
\mathcal{F}_i[U^g] = \mathrm{Tr}\tau_0 c_i = \mathrm{Tr}\tau_0 \bar{c}_i = 0
$$

with $\mathcal{F}_i[U] = \delta V[U^g]/\delta g_i|_{g=1}$

$$
\begin{aligned}
s\bar{c}_i &= [\omega_i, \bar{c}_i] + b_i \ , & sb_i &= [\omega_i, b_i] - [\phi_i, \bar{c}_i] \\
s\bar{\sigma}_i &= \sigma_i \ , & s\sigma_i &= 0 \\
s\bar{\gamma}_i &= \gamma_i \ , & s\gamma_i &= 0
\end{aligned}
$$

with $\sigma, \bar{\sigma}, \gamma, \bar{\gamma}$ in Cartan subalgebra.
BRST-algebra is NILPOTENT: $s^2 = 0$
BRST-exact action: $S_{TQFT}[U^g, c, \bar{c}, \ldots] = sW$

$$
W = \sum_i \mathrm{Tr}\left[\bar{c}_i(\mathcal{F}_i[U^g] + \frac{\alpha}{2} b_i) + \beta \bar{c}_i^2 c_i + \bar{\gamma}_i \bar{c}_i + \bar{\sigma}_i c_i\right]
$$

S_{TQFT} does not depend on ω.

$\Rightarrow U(1)^N$-invariant gauge fixing action:

$$\sum_{sites\ i} \mathrm{Tr}\left[-\frac{1}{2\alpha}\mathcal{F}_i[U]\mathcal{F}_i[U] - \bar{c}_i M_i[U,c] - \rho_i \tau_0[\bar{c},c]\right] + \frac{1}{4\alpha}\rho_i^2$$

ρ_i is an auxiliary scalar site field that could be eliminated in favor of a quartic ghost interaction. The action is bilinear in the complex ghost fields

$$c_i = C_i\tau_+ - C_i^*\tau_- \ , \quad \bar{c}_i = \bar{C}_i\tau_+ + \bar{C}_i^*\tau_-$$

which, when integrated, give a sparse determinant that depends on the link configuration $\{U_{ij}\}$ *and* $\{\rho_i\}$. In a numerical simulation its gaussian average over $\{\rho_i\}$ determines the measure for the link variables.

$$\mathcal{F}_i[U] = \sum_{j\sim i} U_{ij}\tau_+(\mathrm{Tr}U_{ij}^\dagger\tau_-) + U_{ij}\tau_-(\mathrm{Tr}U_{ij}^\dagger\tau_+) - h.c.$$

gives the variation of $V[U]$ under infinitesimal gauge transformations.

THE EQUIVARIANT COHOMOLOGY:
The action is (on-shell) BRST invariant:

$$\begin{aligned} sU_{ij} &= c_i U_{ij} - U_{ij} c_j & s\bar{c}_i &= -\mathcal{F}_i[U]/\alpha \\ sc_i &= 0 & s\rho_i &= 2\mathrm{Tr}\tau_0[c_i, \mathcal{F}_i[U]] \end{aligned}$$

$$\text{with} \quad sF_i[U] = M_i[U,c]$$

defining $M_i[U,c]$.
HOWEVER: on-shell (using EM's of $\bar{c}, \rho$)

$$s^2 = \text{infinitesimal}\mathrm{U}(1)^{\mathrm{N}}, \text{parameters } c_i^2$$

and is nilpotent on $U(1)^N$-*invariant* functionals

$$\mathcal{B} := \{A[U,c] = A[U^h, c^h],\ \forall\ h \in U(1)^N\}$$

that do not depend on $\bar{c}, \rho$ only. On $\mathcal{B}, s$ thus defines an *equivariant* cohomology

$$\Sigma := \{O \in \mathcal{B} : sO = 0, O \neq sE,\ \forall E \in \mathcal{B}\}$$

Observables are in the equivariant cohomology and have ghost#=0.

ONE PROVES (explicitly):
1) (Equivariant) U(1)-LGT is normalizable
2) Expectation values of observables in the U(1)-LGT are the same as in the SU(2)-LGT
3) $<: \rho_i :> \neq 0$ in $D \leq 4$ dimensions scales like a physical mass^2 in the *critical* gauge $\alpha = g^2(11 - n_f) + O(g^4)$
4) Residual $U(1)^N$-invariance can be "gauge fixed" to a discrete Z_2^N with a TQFT whose partition function is the number of *connected* components $b_0(U(1)^N) = 1 \neq 0$
5) Equivariant construction of equivalent abelian LGT for $SU(n)$-LGT relies on $SU(n+1)/U(n) \sim CP_n$ with $\chi(CP_n) = n + 1 \neq 0$
CONCLUSION: It appears that this construction is a viable field theoretic alternative to Dirac's quantization. It relies on a nilpotent symmetry whose global breaking can be analyzed as in other TQFT's (and SUSY).
FOR DETAILS and APPLICATIONS SEE:

1)M.S.,Phys.Rev.D59:014508;*ibid*D58:025016
2)A.Rozenberg,M.S.,Phys.Rev.D57:3670
3)L.Baulieu,A.R.,M.S.,Phys.Rev.D54:7825
4)L.Baulieu,M.S.,Int.J.Mod.Phys.A13:985

Dynamics and Topology in a Gauge-Invariant Formulation of QCD.

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Summary

This investigation addresses the properties of both, the gauge-invariant gauge field, and the "resolvent field" $\overline{\mathcal{A}_i^\gamma}(\mathbf{r})$, one of its essential constituent elements. Both of these operator-valued quantities were constructed in earlier work.[2] The resolvent field is a functional of the gauge field, and it has a pivotal role in our work. It first appears in the operator that implements the non-Abelian Gauss's law, where it is folded into $\Pi_i^\gamma(\mathbf{r})$, the canonical momentum for the gauge field and the negative chromoelectric field. In addition, the resolvent field has an important role in establishing gauge-invariant gauge (gluon) and spinor (quark) fields. And it also is an important component of the unitary operator that enables us to carry out a similarity transformation to a representation in which the low-energy dynamics of QCD can largely be described by a nonlocal interaction — the QCD analog of the Coulomb interaction in QED.[3]

We have obtained a nonlinear integral equation that determines the resolvent gauge field for the SU(2) form of QCD,[2] in the form $\overline{\mathcal{A}_i^\gamma}(\mathbf{r}) = \overline{\mathcal{A}_i^\gamma}(\mathbf{r})_{\mathcal{X}} + \overline{\mathcal{A}_i^\gamma}(\mathbf{r})_{\overline{\mathcal{Y}}}$, where $\overline{\mathcal{A}_i^\gamma}(\mathbf{r})_{\mathcal{X}}$ and $\overline{\mathcal{A}_i^\gamma}(\mathbf{r})_{\mathcal{Y}}$ are displayed on one of the accompanying transparencies. When we treat the resolvent field (as well as all other fields) in this equation as ordinary functions of spatial variables, and make an appropriate *ansatz* delimiting their forms, we obtain a nonlinear differential equation that associates gauge fields with topological sectors in which the corresponding gauge-invariant gauge fields live. The topological sectors are characterized by winding numbers $2(N-n)+1$, where the integer indices N and n correspond to the $r\rightarrow\infty$ and $r\rightarrow 0$ limits, respectively, of the magnitude $\overline{\mathcal{N}} = g\sqrt{\frac{\partial_j}{\partial^2}\overline{\mathcal{A}_j^\gamma}(\mathbf{r})\frac{\partial_j}{\partial^2}\overline{\mathcal{A}_j^\gamma}(\mathbf{r})}$. These limiting values of $\overline{\mathcal{N}}$ are determined by the nonlinear differential equation referred to above. We observe that the dynamics of QCD — in particular the requirement that it respect the non-Abelian Gauss's law — mandates that the $r\rightarrow\infty$ and $r\rightarrow 0$ limits of $\overline{\mathcal{N}}$ be integer multiples of π. The dynamical equations, therefore, have an important role in selecting topological sectors defined by winding numbers that depend on these $r\rightarrow\infty$ and $r\rightarrow 0$ limits of $\overline{\mathcal{N}}$.

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The program of this work:

I) Constructing a set of state vectors for temporal gauge ($A_0 = 0$) QCD that obey the non-Abelian Gauss's Law.

II) Constructing gauge-invariant operator-valued gauge (gluon) and spinor (quark) fields.

III) Transforming the QCD Hamiltonian to a representation in which the interactions between the quarks and the 'pure gauge' components of the gauge field are replaced by nonlocal interactions between quarks — a non-Abelian analog to the Coulomb interaction in QED.

IV) Investigating the gauge-invariant gauge field — its topology and its multiple recurrences in different topologically distinct sectors.

SOLVING THE 'PURE GLUE' GAUSS'S LAW: We first construct a state $\Psi\,|\phi\rangle$ for which $\{\,\partial_i\Pi_i^a(\mathbf{r})+J_0^a(\mathbf{r})\,\}\Psi\,|\phi\rangle=0$, where $J_0^a(\mathbf{r})$ is the gluon color charge density only. We choose a state $|\phi\rangle$ annihilated by $\partial_i\Pi_i^a(\mathbf{r})$ (they are easy to construct) and seek a Ψ for which

$$\{\,\partial_i\Pi_i^a(\mathbf{r})+J_0^a(\mathbf{r})\,\}\,\Psi\,|\phi\rangle=\Psi\,\partial_i\Pi_i^a(\mathbf{r})\,|\phi\rangle\,,$$

or, equivalently,

$$[\,\partial_i\Pi_i^a(\mathbf{r}),\,\Psi\,]=-J_0^a(\mathbf{r})\,\Psi\,+\,B_Q^a(\mathbf{r}),$$

where $B_Q^a(\mathbf{r})$ is an operator that has $\partial_i\Pi_i^a(\mathbf{r})$ on its extreme right. THIS IS A KIND OF AN OPERATOR DIFFERENTIAL EQUATION. The commutator $[\,\partial_i\Pi_i^a(\mathbf{r}),\,\Psi\,]$ is, essentially, an operator derivative of Ψ.

The solution of this equation is:

$$\Psi=||\exp\left(i\int d\mathbf{r}\overline{\mathcal{A}}_i^\gamma(\mathbf{r})\Pi_i^\gamma(\mathbf{r})\right)||,$$

where the double-bar bracket denotes a normal ordering in which all the gauge fields are placed to the left of all canonical momenta, so that when the exponential is expanded, all powers of $\overline{\mathcal{A}_i^\gamma}(\mathbf{r})$ appear to the left of any of the $\Pi_i^\gamma(\mathbf{r})$.

$\overline{\mathcal{A}_i^\gamma}(\mathbf{r})$, is the <u>resolvent field</u>.

$\mathcal{X}^\alpha(\mathbf{r}) = \frac{\partial_j}{\partial^2} A_j^\alpha(\mathbf{r})\,,$ and $\overline{\mathcal{Y}^\alpha}(\mathbf{r}) = \frac{\partial_j}{\partial^2}\overline{\mathcal{A}_j^\alpha}(\mathbf{r})$ appear in the equation for $\overline{\mathcal{A}_i^\gamma}(\mathbf{r})$

<u>Gauge-invariant operator-valued fields:</u>
Attaching quarks to gluons — the basic idea:

$$\hat{\mathcal{G}}^a(\mathbf{r}) = \partial_i \Pi_i^a(\mathbf{r}) + g f^{abc} A_i^b(\mathbf{r}) \Pi_i^c(\mathbf{r}) + j_0^a(\mathbf{r})$$

and $\mathcal{G}^a(\mathbf{r}) = \partial_i \Pi_i^a(\mathbf{r}) + g f^{abc} A_i^b(\mathbf{r}) \Pi_i^c(\mathbf{r})$

are unitarily equivalent.

$$\hat{\mathcal{G}}^a(\mathbf{r}) = \mathcal{U}_{\mathcal{C}}\, \mathcal{G}^a(\mathbf{r})\, \mathcal{U}_{\mathcal{C}}^{-1}\,,$$

where $\mathcal{U}_{\mathcal{C}} = e^{\mathcal{C}_0} e^{\bar{\mathcal{C}}}$ and $\mathcal{C}_0$ and $\bar{\mathcal{C}}$ are given by $\mathcal{C}_0 = i \int d\mathbf{r}\, \mathcal{X}^\alpha(\mathbf{r})\, j_0^\alpha(\mathbf{r})$ and $\bar{\mathcal{C}} = i \int d\mathbf{r}\, \overline{\mathcal{Y}^\alpha}(\mathbf{r})\, j_0^\alpha(\mathbf{r})$

We have two options for gauge-transforming operators: in the $\mathcal{C}$ representation,

$$\mathcal{O}(\mathbf{r}) \rightarrow \mathcal{O}'(\mathbf{r}) = \exp\left(-\frac{i}{g}\int \hat{\mathcal{G}}^a(\mathbf{r}')\,\omega^a(\mathbf{r}')\,d\mathbf{r}'\right) \times \mathcal{O}(\mathbf{r})\,\exp\left(\frac{i}{g}\int \hat{\mathcal{G}}^a(\mathbf{r}')\,\omega^a(\mathbf{r}')\,d\mathbf{r}'\right),$$

where $\mathcal{O}(\mathbf{r})$ and $\mathcal{O}'(\mathbf{r})$ are in the "common" or $\mathcal{C}$-representation. Alternatively, we can transform to the $\mathcal{N}$ representation, in which case

$$\mathcal{O}_{\mathcal{N}}(\mathbf{r}) = \mathcal{U}_{\mathcal{C}}^{-1}\mathcal{O}(\mathbf{r})\,\mathcal{U}_{\mathcal{C}},$$

and the gauge transformation $\mathcal{O}_{\mathcal{N}}(\mathbf{r}) \rightarrow \mathcal{O}'_{\mathcal{N}}(\mathbf{r})$ is expressed as

$$\mathcal{O}'_{\mathcal{N}}(\mathbf{r}) = \exp\left(-\frac{i}{g}\int \mathcal{G}^a(\mathbf{r}')\,\omega^a(\mathbf{r}')\,d\mathbf{r}'\right) \times \mathcal{O}_{\mathcal{N}}(\mathbf{r})\,\exp\left(\frac{i}{g}\int \mathcal{G}^a(\mathbf{r}')\,\omega^a(\mathbf{r}')\,d\mathbf{r}'\right).$$

It is easy to see that the spinor field $\psi(\mathbf{r})$ is gauge-invariant in the $\mathcal{N}$ representation, because $\psi(\mathbf{r})$ and $\mathcal{G}^a(\mathbf{r}')$ trivially commute. To produce $\psi_{\rm GI}(\mathbf{r})$, this gauge-invariant spinor is transformed to the $\mathcal{C}$ representation:

$$\psi_{\rm GI}(\mathbf{r}) = \mathcal{U}_{\mathcal{C}}\,\psi(\mathbf{r})\,\mathcal{U}_{\mathcal{C}}^{-1} \quad \text{and} \quad \psi_{\rm GI}^{\dagger}(\mathbf{r}) = \mathcal{U}_{\mathcal{C}}\,\psi^{\dagger}(\mathbf{r})\,\mathcal{U}_{\mathcal{C}}^{-1}\,;$$

with the Baker-Hausdorff-Campbell theorem,

$$\psi_{\rm GI}(\mathbf{r}) = V_{\mathcal{C}}(\mathbf{r})\,\psi(\mathbf{r}) \quad \text{and} \quad \psi_{\rm GI}^{\dagger}(\mathbf{r}) = \psi^{\dagger}(\mathbf{r})\,V_{\mathcal{C}}^{-1}(\mathbf{r}).$$

Similarly,, the gauge-invariant gauge field is

$$\begin{aligned} A_{{\rm GI}\,i}^{b}(\mathbf{r})\,\frac{\lambda^b}{2} = \quad & V_{\mathcal{C}}(\mathbf{r})\,[\,A_i^b(\mathbf{r})\,\frac{\lambda^b}{2}]\,V_{\mathcal{C}}^{-1}(\mathbf{r}) + \\ & \frac{i}{g}\,V_{\mathcal{C}}(\mathbf{r})\,\partial_i V_{\mathcal{C}}^{-1}(\mathbf{r})\,, \end{aligned}$$

or, equivalently,

$$A_{{\rm GI}\,i}^{b}(\mathbf{r}) = A_{T\,i}^{\,b}(\mathbf{r}) + [\delta_{ij} - \frac{\partial_i\partial_j}{\partial^2}]\,\overline{\mathcal{A}_i^b}(\mathbf{r})\,.$$

<u>Two-color — SU(2) — QCD :</u>

$$\overline{\mathcal{A}_i^\gamma}(\mathbf{r}) = \overline{\mathcal{A}_i^\gamma}(\mathbf{r})_{\mathcal{X}} + \overline{\mathcal{A}_i^\gamma}(\mathbf{r})_{\overline{\mathcal{Y}}}\,,$$

where

$$\overline{\mathcal{A}_i^\gamma}(\mathbf{r})_{\mathcal{X}} = g\epsilon^{\alpha\beta\gamma}\quad \mathcal{X}^\alpha(\mathbf{r})\,A_i^\beta(\mathbf{r})\,\frac{\sin(\mathcal{N})}{\mathcal{N}} -$$

$$g\epsilon^{\alpha\beta\gamma}\,\mathcal{X}^{\alpha}(\mathbf{r})\,\partial_i\mathcal{X}^{\beta}(\mathbf{r})\,\frac{1-\cos(\mathcal{N})}{\mathcal{N}^2}$$
$$-g^2\epsilon^{\alpha\beta b}\epsilon^{b\mu\gamma}\,\mathcal{X}^{\mu}(\mathbf{r})\,\mathcal{X}^{\alpha}(\mathbf{r})\,A_i^{\beta}(\mathbf{r})\,\frac{1-\cos(\mathcal{N})}{\mathcal{N}^2}$$
$$+g^2\epsilon^{\alpha\beta b}\epsilon^{b\mu\gamma}\,\mathcal{X}^{\mu}(\mathbf{r})\,\mathcal{X}^{\alpha}(\mathbf{r})\quad\partial_i\mathcal{X}^{\beta}(\mathbf{r})\,[\frac{1}{\mathcal{N}^2}-\frac{\sin(\mathcal{N})}{\mathcal{N}^3}]$$

and

$$\begin{aligned}\overline{\mathcal{A}_i^{\gamma}}(\mathbf{r})_{\overline{\mathcal{Y}}} &= g\epsilon^{\alpha\beta\gamma}\,\overline{\mathcal{Y}^{\alpha}}(\mathbf{r})\,A_{\mathrm{GI}\,i}^{\beta}(\mathbf{r})\,\frac{\sin(\overline{\mathcal{N}})}{\overline{\mathcal{N}}} \\ &\quad+g\epsilon^{\alpha\beta\gamma}\,\overline{\mathcal{Y}^{\alpha}}(\mathbf{r})\,\partial_i\overline{\mathcal{Y}^{\beta}}(\mathbf{r})\,\frac{1-\cos(\overline{\mathcal{N}})}{\overline{\mathcal{N}}^2} \\ &+g^2\epsilon^{\alpha\beta b}\,\epsilon^{b\mu\gamma}\,\overline{\mathcal{Y}^{\mu}}(\mathbf{r})\,\overline{\mathcal{Y}^{\alpha}}(\mathbf{r})\,A_{\mathrm{GI}\,i}^{\beta}(\mathbf{r})\,\frac{1-\cos(\overline{\mathcal{N}})}{\overline{\mathcal{N}}^2} \\ &+g^2\epsilon^{\alpha\beta b}\epsilon^{b\mu\gamma}\,\overline{\mathcal{Y}^{\mu}}(\mathbf{r})\,\overline{\mathcal{Y}^{\alpha}}(\mathbf{r})\,\partial_i\overline{\mathcal{Y}^{\beta}}(\mathbf{r})\,[\frac{1}{\overline{\mathcal{N}}^2}-\frac{\sin(\overline{\mathcal{N}})}{\overline{\mathcal{N}}^3}]\,.\end{aligned}$$

where

$$\mathcal{N}=\left[g^2\,\mathcal{X}^{\delta}(\mathbf{r})\,\mathcal{X}^{\delta}(\mathbf{r})\right]^{\frac{1}{2}}\text{ and }\overline{\mathcal{N}}=\left[g^2\,\overline{\mathcal{Y}^{\delta}}(\mathbf{r})\,\overline{\mathcal{Y}^{\delta}}(\mathbf{r})\right]^{\frac{1}{2}}.$$

From an *ansatz*, we find that we can express

$$\overline{\mathcal{A}_i^{\gamma}}(\mathbf{r})=\left(\delta_{i\,\gamma}-\frac{r_i\,r_\gamma}{r^2}\right)\left(\frac{\overline{\mathcal{N}}}{gr}+\varphi_A\right)+\epsilon_{i\gamma n}\frac{r_n}{r}\,\varphi_C\,.$$

We can set $gr\Phi=\overline{\mathcal{N}}$ and $gr\mathcal{S}=\mathcal{N}$ and use this form of the resolvent field to transform the nonlinear integral equation into the nonlinear differential equation

$$\begin{aligned}\frac{d^2\overline{\mathcal{N}}}{du^2} &+ \frac{d\overline{\mathcal{N}}}{du}+2\left[\mathcal{N}\cos(\overline{\mathcal{N}}+\mathcal{N})-\sin(\overline{\mathcal{N}}+\mathcal{N})\right] \\ &+ 2gr_0\exp(u)\left\{\mathcal{T}_A\left[\cos(\overline{\mathcal{N}}+\mathcal{N})-1\right]\right. \\ &\left. -\mathcal{T}_C\sin(\overline{\mathcal{N}}+\mathcal{N})\right\}=0,\end{aligned}$$

where $u=\ln(r/r_0)$, and $\mathcal{T}_A$ and $\mathcal{T}_A$ are determined by the gauge field A_i^γ. With the ansatz we have made,

$$V_C(\mathbf{r})=\exp\left(-i\hat{r}_n\tau_n\frac{\overline{\mathcal{N}}}{2}\right),$$

so that we can represent $\left[A^\gamma_{\mathrm{GI}\,i}(\mathbf{r})\right]_{A_i^\gamma=0}$ as

$$\mathsf{A}_i(\mathbf{r})=\tau^\gamma\left[A^\gamma_{\mathrm{GI}\,i}(\mathbf{r})\right]_{A_i^\gamma}=V_C^{-1}(\mathbf{r})\partial_iV_C(\mathbf{r})$$

and define the winding number

$$Q=-i(24\pi^2)^{-1}\epsilon_{ijk}\int d\mathbf{r}\,\mathrm{Tr}[\mathsf{A}_i(\mathbf{r})\mathsf{A}_j(\mathbf{r})\mathsf{A}_k(\mathbf{r})].$$

This leads to

$$Q=2\,(N-n)+1\,.$$

PHYSICAL CHARGES IN GAUGE THEORIES

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Summary: In QCD colour can only be defined on gauge invariant states. Hence to go beyond naive QCD partonic phenomenology requires gauge invariant descriptions of quarks and gluons. Obvious applications include the construction of constituent quarks; the onset of hadronisation and QCD description of the glue which makes up the pomeron. In this talk the highlights of a programme to systematically construct and apply such descriptions of charged states will be presented. The general method will be explained and then tested in various contexts. In particular we will see that in QED the on-shell Green's functions and S-matrix elements for such physical charged states are free of infra-red divergences at all orders. This result will be shown to be a natural consequence of the emergence of a particle structure at asymptotic times in our construction. The ultra-violet behaviour of these fields is studied and is shown to have many attractive properties. In QCD it is demonstrated that this construction captures the dominant gluonic fields around static quarks and that a constituent structure emerges at short distances.

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Moral: We need to construct operators that create gauge invariant charges.

How?

> Let $\mathcal{G}$ be the gauge group and $U \in \mathcal{G}$
>
> $$\begin{aligned} \psi^U &= U^{-1}\psi \\ A^U &= U^{-1}AU + \frac{1}{g}U^{-1}\partial U \end{aligned}$$

We need $h \equiv h[A] \in \mathcal{G}$ such that

$$h^U \equiv h[A^U] = U^{-1}h$$

and use this to define

$$\begin{aligned} \psi_{\text{ph}} &= h^{-1}\psi \\ A_{\text{ph}} &= A^h \end{aligned}$$

$\boxed{\psi_{\text{ph}} \text{ and } A_{\text{ph}} \text{ are gauge invariant}}$

h^{-1} is called a *dressing*

Physically relevant variables

• Naive requirement [Dirac]

$$\boxed{\text{Right electric field}}$$

Example [Dirac]:

$$\psi_D(x) \equiv \exp\left(-ie\frac{\partial_i A_i}{\nabla^2}\right)\psi(x)$$

we obtain

$$E^i(x_0,\vec{y})\psi_D(x)|0\rangle = -\frac{e}{4\pi}\frac{x_i - y_i}{|\vec{x}-\vec{y}|^3}\psi_D(x)|0\rangle$$

i.e., the electric field of a static charge

• Kinematical requirement [e.g., Buchholz]
Recall that physical state space breaks into distinct superselection sectors labelled by the $\vec{E}$ flux distribution at spatial infinity. In the asymptotic regime the velocity of the charges is also superselected.

$$\vec{v} \leftrightarrow \vec{E} \text{ flux distribution at } \infty$$

Demand dressing eq.: [redacted]
- Analogy with HQET $\Rightarrow$ sharp momenta
- From asymptotic dynamics:

$u \cdot A^h = 0 \Rightarrow H_I \to 0$

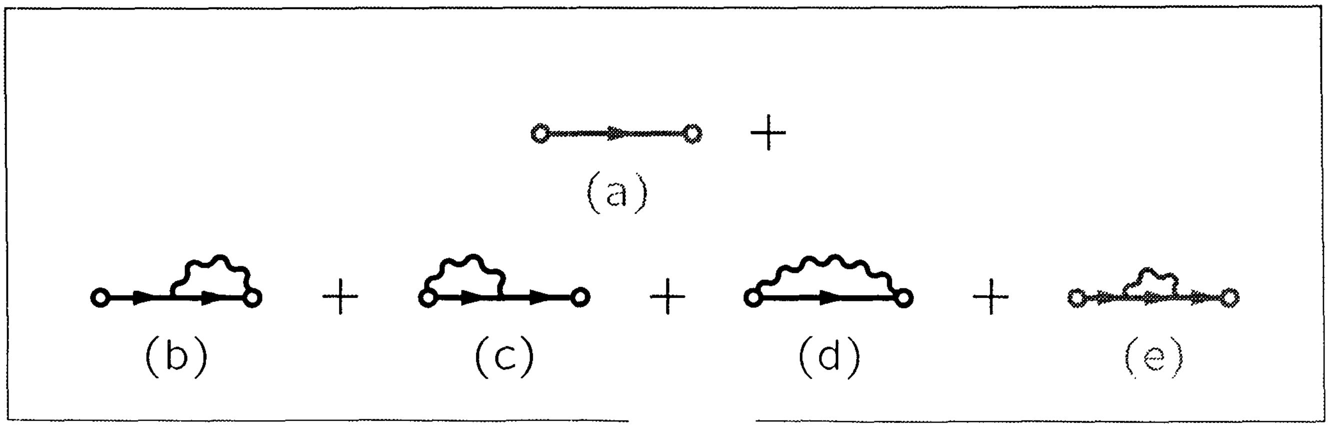
IR Structure

Usual on-shell Green's functions and S-matrix elements have IR divergences.

The two dressing structures remove these IR divergences (at all orders):
χ (minimal) removes soft and K (additional) cancels phase. [QED, scalar QED] for both (on-shell) Green functions and S-matrix elements of any scattering process.

Note: Have to go on-shell at appropriate point, e.g., static dressing $\Leftrightarrow p = (m, 0, 0, 0)$.

We also have explicit full calculations at one loop order in both QED and scalar QED.



UV Structure

•Propagator: multiplicative wave function renormalisation possible. On-shell & $\overline{\text{MS}}$ schemes

- Scalar QED: $\phi_v \to \sqrt{Z_2}\phi_v$. Where $\sqrt{Z_2}$ becomes a function (which is IR finite even in the on-shell scheme) of the velocity

- Fermionic QED: IR finite matrix multiplication. $\psi_v \to \sqrt{\mathbf{Z}_v}\psi_v$. This can happen because the renormalization constant depends on two vectors, η and v.

•Renormalisation of ψ_v as a composite operator well behaved. Does not mix with other operators.

•Vertex renormalisation. Additional renormalization constant $Z(v_{\text{rel}})$ linked to the anomalous dimension of the Isgur-Wise function

•Ward identity for vertex holds,

$$\boxed{Z(v_{\text{rel}} = 0) = 1}$$

Charge universality.

•Usual physical predictions for IR safe observables $(g-2)$ hold

•Charge radius?

Interquark Potential

- use minimally dressed quark/ antiquark states,

$$\bar{\psi}(y)h(y)h^{-1}(y')\psi(y')|0\rangle.$$

- Take expectation value of Hamiltonian,

$$H = \frac{1}{2}\int (E_i^a E_i^a + B_i^a B_i^a) d^3x$$

- Equal-time commutator:

$$[E_i^a(x), A_j^b(y)]_{\text{et}} = i\delta^{ab}\delta_{ij}\delta(\vec{x}-\vec{y})$$

- At order g^2 obtain Coulomb potential:

$$V^{g^2}(r) = -\frac{g^2 N C_F}{4\pi r}$$

where r is separation. This is QED with coloured icing.

What about QCD with non-abelian ingredients?

- At order g^4 need dressing to order g^3
- Find potential to g^4

$$V^{g^4}(r) = -\frac{g^4}{(4\pi)^2}\frac{NC_F}{2\pi r}\underbrace{4C_A}_{\substack{\text{Should be} \\ -\beta_1 = (4-1/3)C_A}} \log(\mu r)$$

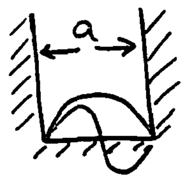
Dressing generates the dominant antiscreening contribution to the β function. It comes from longitudinal glue. The screening contribution comes from gauge invariant glue (phase dressing?)

- Renormalization of metric on configuration space.

G. Alexanian and E. F. Moreno

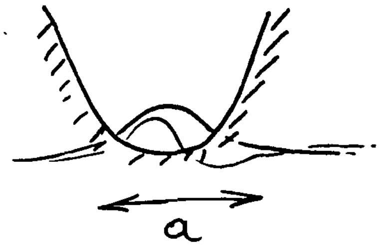
<u>Motivation</u>: Look at the Q.M. problem:

a) Particle in the box of finite size a
Particle on a circle, $r = a$
Gap $= \Delta E \sim 1/a^2$



b) H.O. $U = x^2/a^2$

Gap $= \Delta E \sim 1/a \sim k$



Q: Can this concept be applied to Gauge Fields?
$\Rightarrow$ Look at the geometric properties of configuration space. (Feynman, 81)
(Atiyah, Singer, Babelon & Viallat, Orland) <u>2+1 YM</u>

Problems: Conf. spase ∞ dim.
Has "spikes" and "valleys"
May have "final" volume in 2+1 ... (Karabali, Kim, Nair)
but not clear in 3+1 ....

- However, If Effective size of the preferred field configuration is finite ("a"),
Gap $\sim \frac{1}{\text{size}}$;

? <u>Question</u>: How does this distance (size) change under the Renormalization ?

Distance in the configuration space:

$$\rho(A_1, A_2) = \inf_g \left| \int \mathrm{tr}(A_1^g - A_2) \right.$$

<u>Program:</u>

- Consider configuration space as a metric space $ds^2 = g_{ij}\, dx^i dx^j \to \int G^{ij}_{ab}\, \delta A^a_i\, \delta A^b_j$

- Identify G^{ij}_{ab} by looking at kinetic energy part of Hamiltonian as Laplacian on configuration space:
$$\Delta = \frac{1}{\sqrt{g}}\, \partial_i \sqrt{g}\, g^{ij} \partial_j \sim g^{ij} \partial_i \partial_j \to \int G^{ij}_{ab} \frac{\delta^2}{\delta A^a_i\, \delta A^b_j}$$

- Renormalize H by integrating out high-frequency modes.

- Use this renormalized H to read off new metric and determine effective distance. $\Longrightarrow$

Need to develop method for renormalizing H.

- Renormalization of H (2-loop $\lambda\varphi^4$).
- 1-loop QCD H
- 1-loop QED H
- Geometric interpretation of asymptotic free…

Renormalization of Hamiltonian.

$\underbrace{0 < \mu < \Lambda}$ Start with $H(\Lambda) \xrightarrow[\mu < k < \Lambda]{\text{"integrate out"}} H(\mu)$

$H(\mu)$ – describes the same physics for all the modes $k < \mu$.

- Split H as: $H = \underbrace{H_1}_{k<\mu} + \underbrace{H_2}_{\substack{\text{free part}\\ \mu<k<\Lambda}} + \underbrace{V_{12}}_{\text{mixing terms}}$

- Introduce unitary transformation $U(\Lambda,\mu)$ which:
  a) Decouples "high" & "low" modes
  b) Partially Diagonalizes "high" part.

(Frohlich, Wegner, Nair & Minic, Glazek & Wilson)

- Define
$$\boxed{H_{eff} = \langle 0|_{high} \, U^{-1} H U \, |0\rangle_{high}}$$

Determine U in perturbation theory in λ and μ/Λ

$$U = U_0 U_1 \dots U_n \dots, \quad U_n = e^{i\Omega_n} \quad \underline{\underline{\Omega_n^+ = \Omega_n}}$$

$$e^{-i\Omega_0}(H)e^{i\Omega_0} = H_1 + H_2 + V_{12} + i[H_1,\Omega_0] + i[H_2,\Omega_0] + \dots$$

$\Rightarrow \; i[H_2,\Omega_0] + V_{12} = 0 \;\Rightarrow\; \Omega_0 = \mathcal{O}(\lambda, \mu/\Lambda)$

$\Rightarrow$ new term of order λ: $[H_1,\Omega_0] \Rightarrow$ higher in μ/Λ!

use next Ω_1 to remove it: $e^{-i\Omega_1}e^{-i\Omega_0} H e^{i\Omega_0} e^{-i\Omega_1} =$

$= \dots i[H_1,\Omega_0] + \dots i[H_2,\Omega_1] \Rightarrow [H_1\Omega_0] + [H_2\Omega_1] = 0$

Generally $\boxed{\Omega_{n+1} \sim \mu/\Lambda \; \Omega_n}$

Results.

I. 2-loop renormalization of $\lambda\varphi^4$. (correct β)

II. 1-loop QCD (no quarks) $\rightarrow$ (Correct β)

III. 1-loop QED (non-renormalization of fermion terms) but! still correct β-function!

IV. Geometric interpretation of asymptotic freedom.

Conjecture: asymptotically free theory $\rightarrow$ $\rightarrow$ eff. distance decreases as high-frequency modes are being integrated out. (opposite for QED)

$$\underset{QCD}{G_{(ia)(jb)}(\mu)} = \left(1 + e_R^2 \frac{11N}{3(4\pi)^2} \ln\left(\frac{\mu}{m_R}\right)^2\right) \overset{0}{G}_{(ia)(jb)} + \ldots$$

$$\underset{QED}{G_{ij}(\mu)} = \left(1 - e_R^2 \frac{4}{3} \frac{1}{(4\pi)^2} \ln\left(\frac{\mu}{m_R}\right)^2\right) G^0_{ij} + \ldots$$

Properties: $E^2 \rightarrow Z_{E^2} E^2$; $B^2 \rightarrow Z_{B^2} B^2$ $\quad Z_B = Z_3 = \frac{1}{Z_E}$

I. Conjugated variables pick up opposite (inverse) R. coefficients.

II. Coefficients in G^{ab}_{ij} renormalization come from gauge-inv. combinations of Z-factors (β functions)

Example: QCD

$$A_0 = 0\,; \qquad [\,A^a_i(x), E^{bj}(y)\,] = i\,\delta^{ab}\delta_i^j(x-y).$$

Gauss Law: $G^a(x)\,\Psi[A] = -i\,D[A]_i^{ab}\dfrac{\delta}{\delta A^b_i(x)}\Psi[A] = 0$

New Coordinates: $(\mathcal{A}, g)$ such as:

$$A_i(x) = g^{-1}\mathcal{A}_i\, g + i g^{-1}\partial_i g(x) \qquad g \in SU(N)$$

$$\mathcal{A}_i : \quad \nabla_i \mathcal{A}_i = 0$$

In this coordinates: Gauss Law: $\dfrac{\delta\Psi}{(i\delta g g^{-1})^b} = 0 \rightarrow$

$\rightarrow \Psi[A]$ independent of g;

Perturbative analysis $\rightarrow$ no Gribov problem.

Kinetic Energy $\langle \Psi_1 | T | \Psi_2 \rangle = \dfrac{1}{\mathrm{Vol}\,G}\displaystyle\int [DA]\, \tfrac{1}{2}\int \frac{\delta\Psi_1^*}{\delta A^a_i}\frac{\delta\Psi_2}{\delta A^a_i}$

$$T = \tfrac{1}{2}\int d^3x\, d^3y\; G^{(ia)(jb)}_{\mathcal{A}/G}(x,y)\; \mathcal{E}^{ai}(x)\, \mathcal{E}^{bj}(y)$$

$$G^{(ia)(jb)}_{\mathcal{A}/G}(x,y) = \delta^{ij}\delta^{ab}\delta(x-y) - e^2 f^{acd} f^{bce}\, \mathcal{A}^d_i(x)\left(\frac{1}{\partial^2}\right)_{xy}\mathcal{A}^e_j(y)$$
$$+\; \mathcal{O}(e^3)$$

$$[\mathcal{A}^a_i(x), \mathcal{E}^b_j(y)] = i\,\delta^{ab} P_{ij}(x,y)$$

$$H = \tfrac{1}{2} Z_E E^2 + \tfrac{1}{2} Z_B B^2 + e Z_1 f^{abc}\partial_i A^a_j A^b_i A^c_j +$$
$$+ \tfrac{1}{2} Z_{AAEE}\, e^2 f^{adc} f^{bde} A^c_i(x) A^e_j(y)\, G(x,y)\, \mathcal{E}^a_i(x)\, \mathcal{E}^b_j(y) +$$
$$+ \frac{e^2}{4} Z_4 f^{abc} f^{dec} A^a_i A^b_j A^d_i A^e_j + \ldots$$

$Z = 1 + f_1\lambda + f_2\lambda^2 + \ldots$

Solitons (monopoles and dyons) in the Einstein-Yang-Mills theory in AdS_4

by Jeff Bjoraker & Yutaka Hosotani
(Univ. of Minnesota)

* In the pure Einstein-Yang-Mills theory everywhere regular monopole/dyon solutions exist in the anti-de Sitter space.

* There exist solutions with no node. They are stable against linear fluctuations.

* Dyon solutions are parametrized by 2 parameters

3. Spherically sym. solutions

$$ds^2 = -\frac{H}{p^2}\,dt^2 + \frac{dr^2}{H} + r^2(d\theta^2 + \sin^2\theta\, d\phi^2)$$

$$A^{(0)} = \frac{1}{2e}\Big\{ A_0\, \hat{x}_j \tau_j\, dt + A_1\, \hat{x}_j \hat{x}_k \tau_j\, dx_k$$

$$+ \frac{\phi_1}{r}(\delta_{jk} - \hat{x}_j \hat{x}_k)\tau_j\, dx_k + \frac{\phi_2 - 1}{r}\epsilon_{jkl}\, \hat{x}_j \tau_l\, dx_k\Big\}$$

$$\downarrow \quad A = S A^{(0)} S^{-1} - \frac{i}{e}\, dS \cdot S^{-1}$$

singular gauge

$$A = \frac{1}{2e}\Big\{ u\,\tau_3\, dt + v\,\tau_3\, dr$$

$$+ (w\tau_1 + \tilde{w}\tau_2)\,d\theta + (\cot\theta\,\tau_3 + w\tau_2 - \tilde{w}\tau_1)\sin\theta\, d\phi\Big\}$$

At $r = 0$ $\quad u = v = 0$

$$w^2 + \tilde{w}^2 = 1$$

Choose $\nu = 0$ & $\tilde{w} = 0$

YM eq.

$$(r^2 p u')' = \frac{2p}{H} u w^2$$

$$\left(\frac{H}{p} w'\right)' = \frac{w(w^2-1)}{r^2 p} - \frac{p}{H} u^2 w$$

E eq.

$$H(r) \equiv 1 - \frac{2m(r)}{r} - \frac{\Lambda}{3} r^2$$

$$\begin{cases} m' = \dfrac{4\pi G}{e^2} (A + B) \\ p' = -\dfrac{8\pi G}{e^2} \dfrac{p}{rH} B \end{cases}$$

$$A = \frac{1}{2} r^2 p^2 u'^2 + \frac{(w^2-1)^2}{2r^2}$$

$$B = \frac{p^2}{H} u^2 w^2 + H w'^2$$

6. Solutions in $\Lambda < 0$ (AdS)

$$H = 1 - \frac{2m(r)}{r} + \frac{|\Lambda|}{3} r^2$$

> 0 for solitons

no horizon , $u_\infty \neq 0$ is allowed
w_∞

[1] Purely magnetic sols.

* Solutions with no node.

— only in AdS —

$w(r) > 0$

$1 \longrightarrow w_\infty$, $Q_M \neq 0$

(meta) Stable !

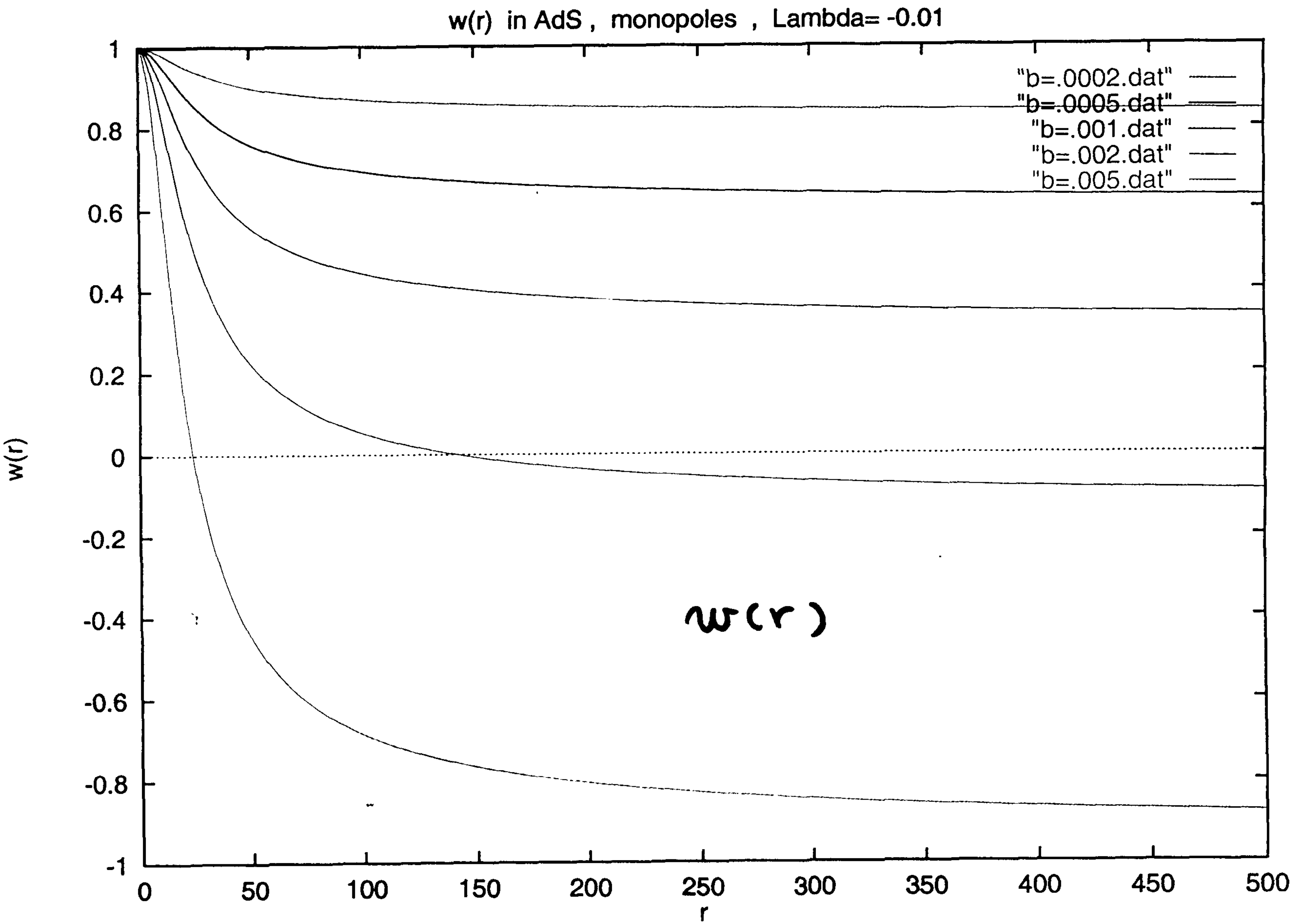
* Solutions with nodes.

...

* Continuous family of solutions.

[Black hole sol. E. Winstanley (1998)]

6.



w(r) in AdS , monopoles , Lambda= -0.01
"b=.0002.dat"
"b=.0005.dat"
"b=.001.dat"
"b=.002.dat"
"b=.005.dat"
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
w(r)
0
50
100
150
200
250
300
350
400
450
500
r
w(r)

71

[2] Dyons

$u(r)\ ,\ w(r) \neq 0$ — only in AdS —

$u'(r) > 0 \qquad u(0) = 0 \quad \rightarrow \quad u_\infty + \dfrac{u_1}{r} + \cdots$

* 2 parameters family of solutions.

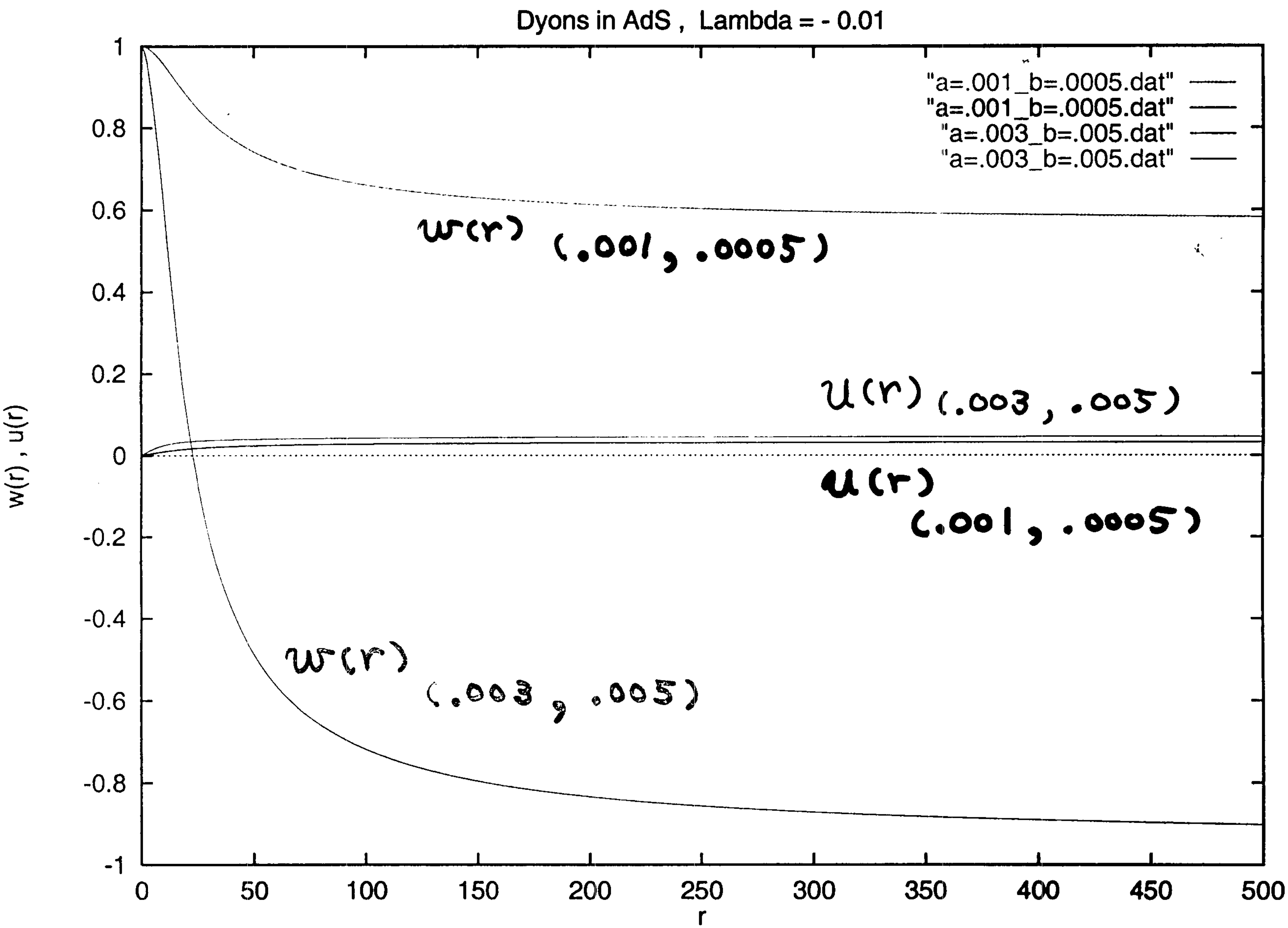
* Solutions with no node.

maybe meta- stable.

* Solutions with nodes.

$$Q_M\ ,\ Q_E \neq 0$$

6-5



$Q_E \sim -0.46 \frac{4\pi}{e}$, $Q_M = 0.66 \frac{4\pi}{e}$ $(-0.36, 0.19)$

7.

Solitons

No purely electric sol.

$w(r) \neq 0$

	$\Lambda = 0$	$\Lambda > 0$	$\Lambda < 0$	
	magnetic	magnetic	magnetic	magnetic & electric
Q_M	0	$\neq 0$	$\neq 0$	$\neq 0$
Q_E	0	0	0	$\neq 0$
nodes of $w(r)$	≥ 1	≥ 1	≥ 0	≥ 0
	Lumps	monopoles	monopoles	dyons
	Bartnik - McKinnon	Volkov et al.	Bjoraker - Hosotani	
	unstable	unstable	meta-stable / unstable	

$\rho(r)$: $\quad d\rho = \frac{P(r)}{H(r)}\, dr$

$$\beta = \frac{r^2 p}{w}\, \delta\nu$$

$$\left\{ -\frac{d^2}{d\rho^2} + U_\beta(\rho) \right\} \beta(t,\rho) = -\ddot{\beta}(t,\rho) = \omega^2 \beta(t,\rho)$$

$$U_\beta = \frac{H}{r^2 p^2}(1 + w^2) + \frac{2}{w^2}\left(\frac{dw}{d\rho}\right)^2$$

$\Lambda = 0$ or $\Lambda < 0$: $\quad U_\beta > 0 \quad \Rightarrow \quad \omega^2 > 0$

δw

$$\left\{ -\frac{d^2}{d\rho^2} + U_w(\rho) \right\} \delta w = -\delta \ddot{w} = \omega^2 \delta w$$

$$U_w = \frac{H}{r^2 p^2}(3w^2 - 1) + \frac{16\pi G}{e^2} \frac{d}{d\rho}\left(\frac{H w'^2}{r p}\right)$$

* $|w_\infty| > \frac{1}{\sqrt{3}}$ for the stability

* If $w(r_j) = 0$ (node),

negative potential.

Numerical study of the gluon propagator in Landau and Coulomb gauge

Attilio Cucchieri

UNIVERSITÄT Bielefeld

Abstract

We study numerically the infrared behavior and the influence of Gribov copies for the gluon propagator in pure SU(2) lattice gauge theory. We consider the momentum-space gluon propagator $D(k)$ in the lattice Landau gauge, and the (equal-time) time-time $D_{00}(\vec{k})$ and space-space $D_{ii}(\vec{k})$ components of the gluon propagator in the lattice Coulomb gauge. We also address the problem of discretization errors introduced by the lattice regularization, and their effects on the ultraviolet behavior of the gluon propagator. This issue is important in the numerical evaluation of the QCD running coupling constant using a recently proposed nonperturbative definition.

Infinite-Volume Limit

Let us note that the zero-momentum gluon propagator $D(0)$ can be written as

$$D(0) \;=\; \frac{V}{3\,d} \sum_{\mu,b} \langle \left(\phi^b_\mu \right)^2 \rangle$$

where

$$\phi^b_\mu \equiv \; V^{-1} \sum_x \; A^b_\mu(x)$$

is the zero-momentum component of the gluon field $A^b_\mu(x)$.

A nonzero value for the constants ϕ^b_μ is (in Landau gauge) a lattice artifact related to the use of periodic boundary conditions and to the finiteness of the volume.

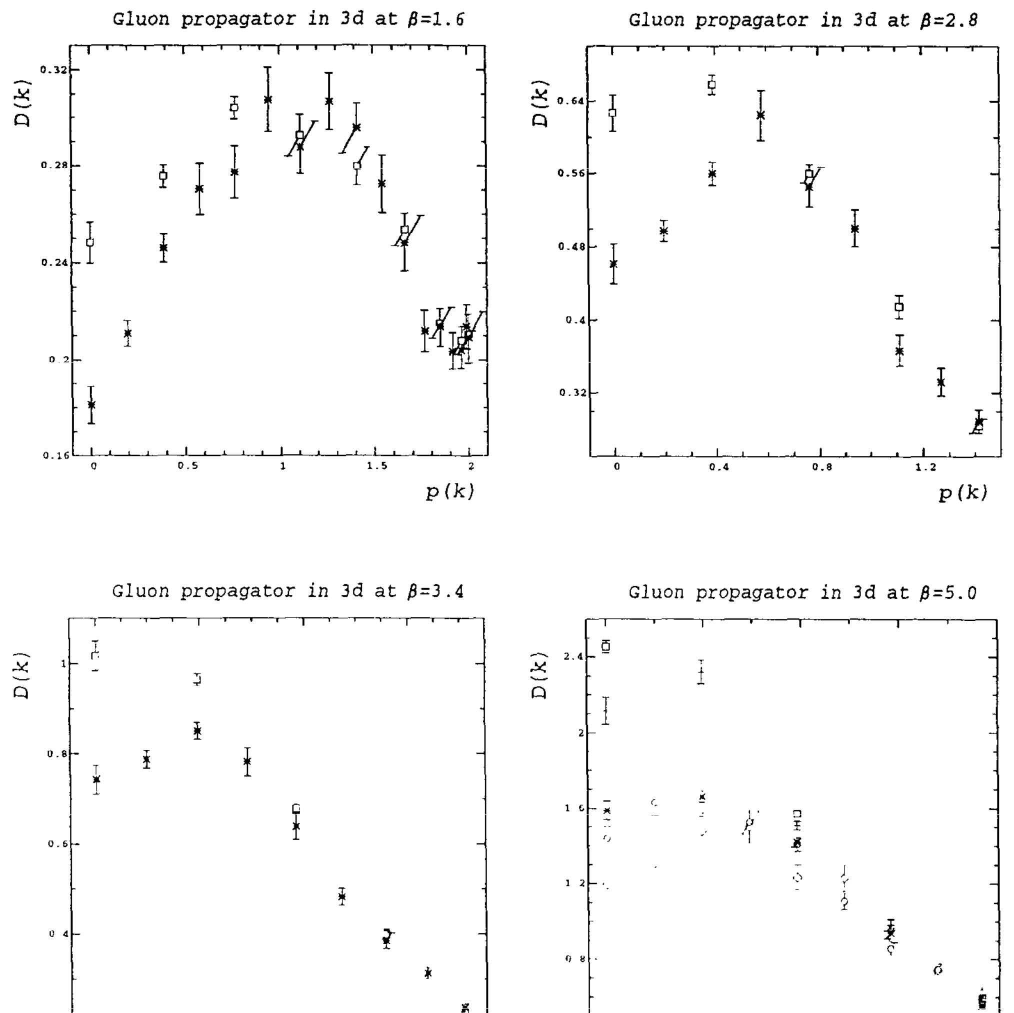
Vanishing of $D(0)$

Zwanziger proved that, for lattice gauge configurations $\{U\} \in \Omega$ and in the infinite-volume limit, $D(k)$ is less singular than $p^{2-d}(k)$ in the infrared limit. Moreover,

$$D(k = 0, H) \to 0 \qquad \text{as} \qquad V \to \infty$$

for almost every H, where $H^b_\mu \cos{(k \cdot x)}$ is a "spatially modulated magnetic field" coupled to $A^b_\mu(x)$.

Infrared Behavior in $3d$



Data for the gluon propagator (in Landau gauge) at: $\beta = 1.6$ and $V = 16^3$ ($\square$), 32^3 ($*$); $\beta = 2.8$ and $V = 16^3$ ($\square$), 32^3 ($*$); $\beta = 3.4$ and $V = 16^3$ ($\square$), 32^3 ($*$); $\beta = 5.0$ and $V = 16^3$ ($\square$), 16^2x32 ($+$), 32^3 ($*$), 32^2x64 ( ), 64^3 ( ). A decreasing propagator as $p(k) \to 0$ can clearly be seen for the larger lattice sizes.

Theoretical Predictions: Coulomb gauge

The equal-time space-space gluon propagator (in momentum space)

$D(0) \equiv \frac{1}{9} \sum_{i,b} D^{bb}_{ii}(0)$

$D(\vec{k}) \equiv \frac{1}{6} \sum_{i,b} D^{bb}_{ii}(\vec{k})$

$D^{bb}_{\mu\mu}(\vec{k}) \equiv \frac{1}{N_x N_y N_z} \langle\, \widetilde{A}^b_\mu(\vec{k})\, \widetilde{A}^b_\mu(-\vec{k})\, \rangle$

should be suppressed in the infrared limit due to the proximity of the first Gribov horizon in the infrared directions.

The quantity

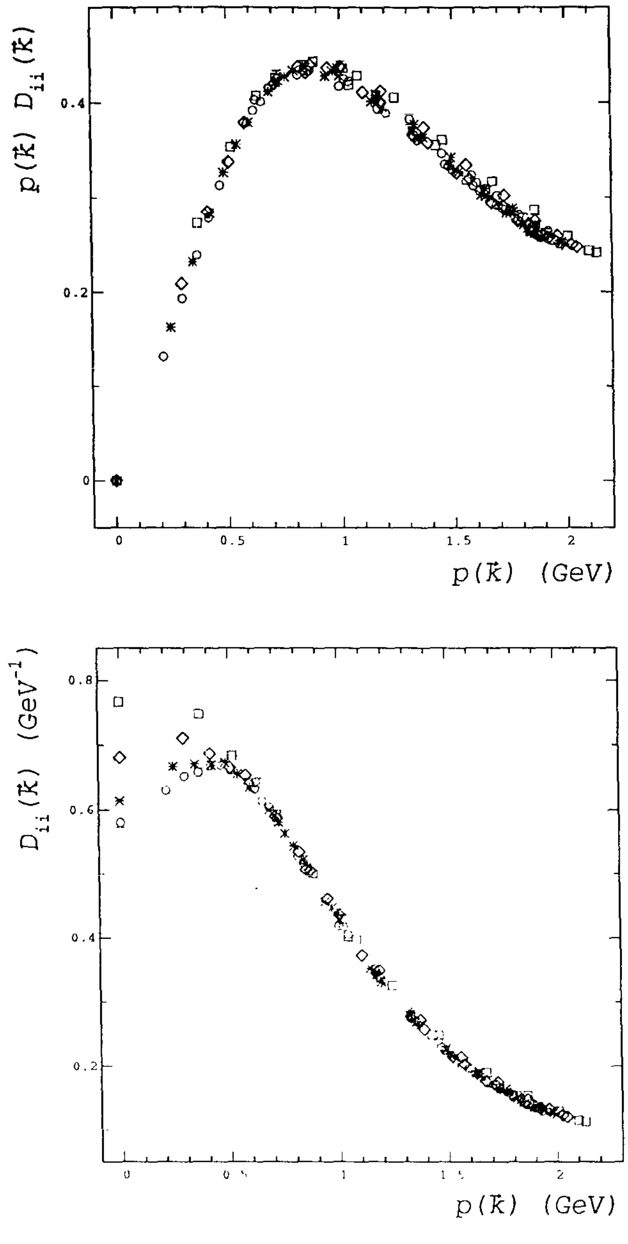
$$g^2(\vec{k}) \equiv \vec{k}^2\, V(\vec{k}) \equiv g_0^2\, \vec{k}^2\, \mathcal{D}_{00}(\vec{k}) \equiv g_0^2\, \vec{k}^2\, \frac{1}{3} \sum_b D^{bb}_{00}(\vec{k})$$

is a renormalization-group-invariant quantity since (in Coulomb gauge) $Z_g\, Z_{A_0} = 1$. We can evaluate $g^2(\vec{k})$ on the lattice and check its asymptotic behavior

$$g^2_{UV}(\vec{k}) \approx \frac{12/11}{b_0 \log \left[p^2(\vec{k}) / \Lambda_C^2 \right]}$$

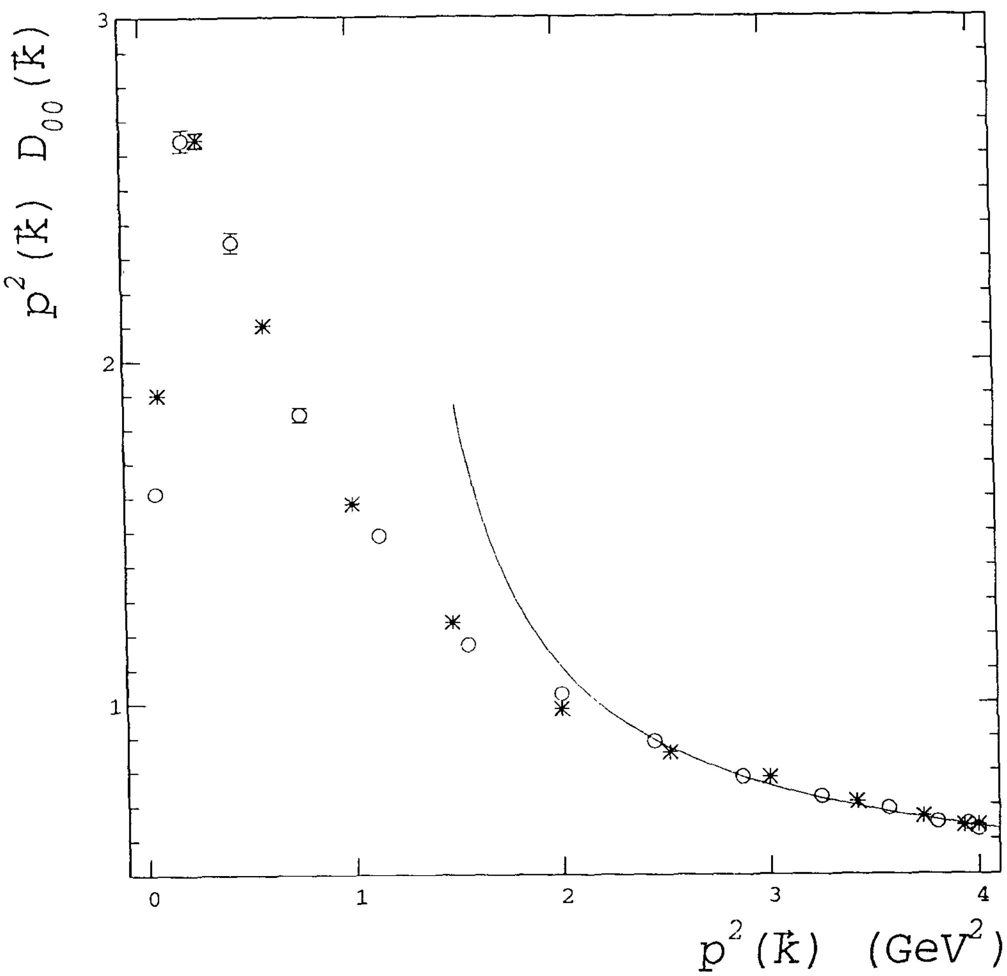
in the limit of large momenta $p(\vec{k})$, with $\Lambda_C = 1.8968\, \Lambda_{\overline{MS}}$.

Coulomb Gauge at $\beta = 2.2$



Data for the gluon propagators $p(\vec{k})D_{ii}(\vec{k})$ and $D_{ii}(\vec{k})$ for $V = 16^4$ ($\square$), $V = 20^4$ ($\diamond$), $V = 24^4$ ($*$) and $V = 28^4$ ($\circ$). The propagator $D_{ii}(\vec{k})$ clearly decreases as $p(k) \to 0$ for the larger lattice sizes.

The Running Coupling Constant



The quantity $p^2(\vec{k})D_{00}(\vec{k})$ at $\beta = 2.2$ for $V = 24^4$ ($*$), and $V = 28^4$ ($\bigcirc$), compared to the two-loop running coupling constant $c\, g_I^2(\vec{k})$. We use $c = 0.3068$ and $\Lambda_C = 0.9596$. In physical units and in the $\overline{MS}$-scheme we obtain $\Lambda_{\overline{MS}} \approx 475$ MeV.

On Field/String Theory Approach to Theta Dependence in Large N Yang-Mills Theory

Gregory Gabadadze
(NYU)

Summary:

- A composite field is identified which saturates the expression for the theta dependent vacuum energy in large N Yang-Mills theory.
- Large N corrections are calculated to the leading order expression of the vacuum energy.
- Decays and stability of the false vacua are studied for various realisations of initial conditions of the theory. The vacuum structure appears to be different depending on whether N is infinite, or alternatively, large but finite.
- The reconsiliation of the string and field theory approaches is based on the fact that the gauge theory instantons carry zerobrane charge in the corresponding D-brane construction of Yang-Mills theory.

On Field/String Theory Approach to Theta Dependence in Large N Yang-Mills Theory

Gregory Gabadadze
(NYU)

- Pure YM theory with θ:

$$\mathcal{L} = -\frac{1}{2g^2} \mathrm{Tr}\, F_{\mu\nu}^2 + \frac{\theta}{16\pi^2} \varepsilon^{\mu\nu\alpha\beta} \mathrm{Tr}\, F_{\mu\nu} F_{\alpha\beta}$$

- Vacuum energy (normalised as $\mathcal{E}_0(\theta=0)=0$):

$$(1): \quad \mathcal{E}_0(\theta) = C \min_k (\theta + 2\pi k)^2 + \mathcal{O}\left(\frac{1}{N}\right)$$

C is independent of N

(Conjectured: Witten 80)
(Derived from string theory: Witten 98)

- Puzzle:

Naively $\mathcal{E}_{YM} \sim N^2$ (since there are N^2 gluons)
However in (1) $\mathcal{E} \sim O(1)$???

Possible explanation:

A massless singlet state saturates (1)

However, there are no massless PHYSICAL states in YM (all glueballs are massive).

Solution to the puzzle:

Antisymmetric tensor field $C_{\mu\nu\alpha}$ (Lüscher)

$$F\tilde{F} = \frac{1}{4!}\,\varepsilon^{\mu\nu\alpha\beta}\underbrace{\left(\partial_\mu C_{\nu\alpha\beta} - \partial_\nu C_{\mu\alpha\beta} - \partial_\alpha C_{\nu\mu\beta} - \partial_\beta C_{\nu\alpha\mu}\right)}_{H_{\mu\nu\alpha\beta}\ -\ \text{field strength of } C}$$

- $H_{\mu\nu\alpha\beta} \Rightarrow$ four-form field in $(3+1)$ dim $\Rightarrow$ no dynamics; produces the background energy (Aurilia, Nikoli Townsend)
- Coulomb propagator for $C_{\mu\nu\alpha}$

<u>Constant</u> solution

$$H_{\mu\nu\alpha\beta} = -\left(\theta + 2\pi k\right) \chi\, \varepsilon_{\mu\nu\alpha\beta} \qquad \chi - \text{top. susceptibility}$$

Energy for H:

$$\mathcal{E}_k(\theta) = \frac{1}{2}\chi\left(\theta + 2\pi k\right)^2 + \mathcal{O}\left(\frac{1}{N}\right)$$

Order parameter for these vacua:

$$\langle F\tilde{F}\rangle_k = \chi(\theta+2\pi k)$$

True vacuum energy:

$$\mathcal{E}_0(\theta) = \frac{1}{2}\chi \min_k (\theta+2\pi k)^2 + \mathcal{O}(1/N)$$

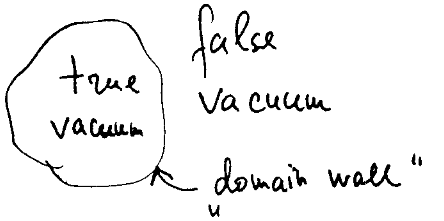
Large N corrections:

$$\mathcal{E}_0^{(2)}(\theta) = \frac{1}{2}\chi \min_k (\theta+2\pi k)^2 + \frac{\text{const.}}{N^2}\chi \min_k (\theta+2\pi k)^4 + \mathcal{O}\left(\frac{1}{N^3}\right)$$

There are false vacua in the theory.

Dynamics of the false vacua

(a) False vacuum with $k \sim 1$



$$\Delta\mathcal{E}_{volume} = 2\pi\chi(\theta+\pi) \sim O(1)$$

$$\Delta\mathcal{E}_{surface} = -T_D \sim O(N)$$

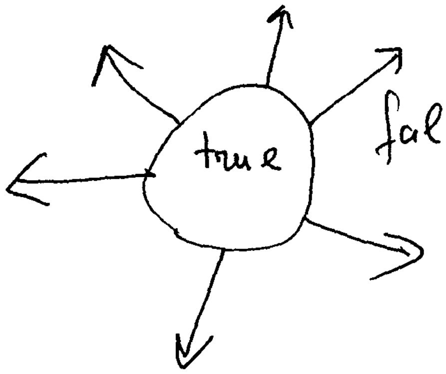
domain wall tension

$\Rightarrow$ in the limit $N \to \infty$ the false vacuum is stable:

$$\Gamma \sim \exp(-aN^4) \qquad a>0 \qquad \text{(Shifman 98)}$$

false vacuum decay width

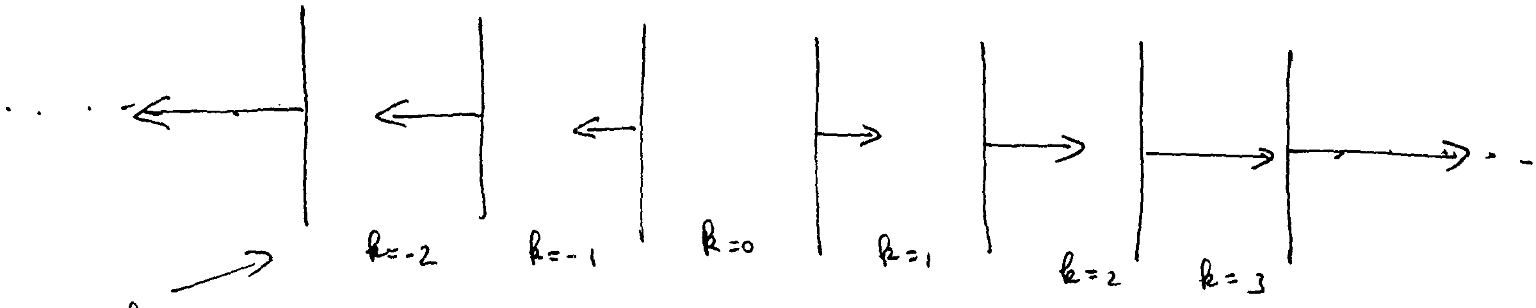
(b) False vacuum with $k \sim N$



$\Delta \mathcal{E}_{volume} \sim \chi N^2$

$\Rightarrow$ false vacuum decay.

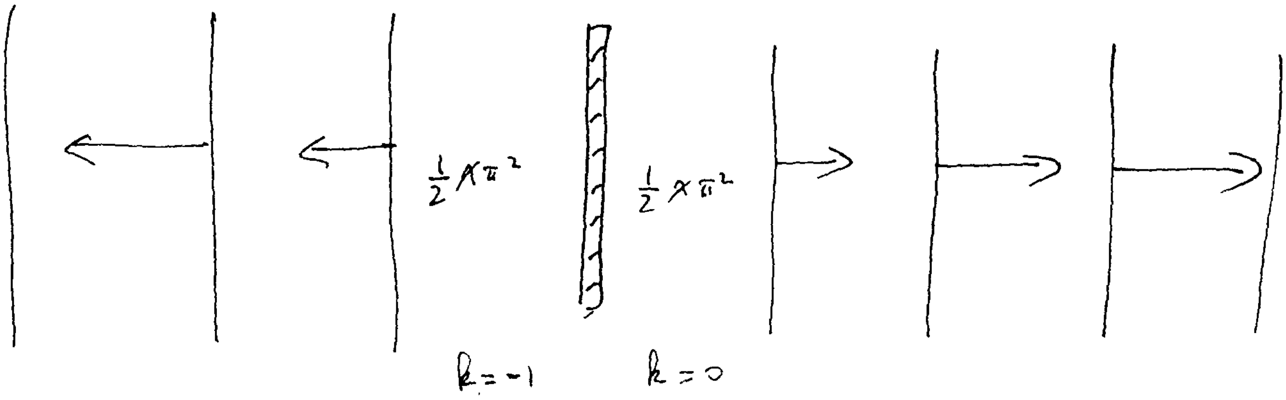
(c) All the vacua are simultaneously realised:



- For big but finite N all the walls seem to infinite
- For $N=\infty$ they are stable.

(d) $\underline{\underline{\theta=\pi}}$. CP is spontaneously broken $\Rightarrow$

$\Rightarrow$ $\mathbb{Z}_2$ domain wall



String theory vs. field theory results.

(Maldacena 97)

Type IIA construction : $R^4 \times S^1 \times R^5$ (Witten 98)

N # of D4 branes wrapped on $R^4 \times S^1$; antiperiodic boundary condition for fermions $\Rightarrow$ pure $SU(N)\times U(1)$ on the worldvolume.

In the dual picture SUGRA $R^4 \times (\text{Disc}) \times S^4$

In string theory $\mathcal{E}_k(\theta)$ is due to RR one-form B_μ $(\mu=1,\dots 5)$

B_μ couples to zerobranes in IIA. On the other hand

$$S_{WZW} = \int_{R^4\times S^1} \frac{B \wedge T_2\, F\wedge F}{16\pi^2}$$

& B couples to instanton charge density $\Rightarrow$ instantons in this construction carry zerobrane charge.

$\Rightarrow$ vacuum energy in YM is related to quantum fluctuations of topological charge. (as we found it before)

Conclusions:

- C_{pure} – no dynamics, just background energy;
- Studied decays of the false vacua for different initial conditions;
- String/Field theory reconsiliation: instantons carry zerobrane charge.

MAGNETIC SPATIAL GEOMETRY and the WU-YANG AMBIGUITY

by · R. Khuri
Baruch College, CUNY

- spatial geometry of magnetic field in D=3, SU(2) YM · $A^a_i, B^{bj} \Longleftrightarrow G_{ij}, K_{ij}{}^k$ (torsion)

- $B(A) \Longleftrightarrow R_{ij}(\Gamma) = -2G_{ij}$ Einstein geometry
  WY ambiguity — multiple solutions for $\Gamma = \mathring{\Gamma} + K$ ($\mathring{\Gamma}$ = Christoffel)

- construction of several examples of new 1-parameter, 3-parameter and infinite parameter families of WY ambiguities

- open questions:
  systematics of ambiguities (geometry, symmetry)
  significance of ambiguities (observables)

p①

<u>Wu-Yang ambiguity</u>: 2 or more gauge-inequivalent non-abelian potentials $A^a_i(x)$ with same field strength $F^a_{ij}(x)$

3 spatial dimensions, $SU(2)$, magnetic field

$$\boxed{B^{ai} = \epsilon^{ijk}\left[\partial_j A^a_k + \tfrac{1}{2}\epsilon^{abc} A^b_j A^c_k\right]} \quad (1)$$

Multiple solutions for A^b_j given B^{ai}

Hamiltonian form of gauge field dynamics in 3+1 dimensions. $A^a_i \longrightarrow B^{ai}$ field variable

Let $B^a{}_i(x) = (B^{ai}(x))^{-1}$, $\det B = \det B^{ai} \neq 0$
gauge invariant

Define metric
$$G_{ij} = \frac{B^a{}_i B^a{}_j}{\det B}$$
gauge invariant, symmetric tensor (under diffeomorphisms

(attempt to study action of electric field on gauge-invariant state functionals $\Psi[G]$)

p(2) $b^a_i = |\det B|^{1/2} B^a_i$ - frame for G_{ij}

Let $D_i b^a_j = \Gamma^k_{ij} b^a_k$

- Γ^k_{ij} transforms as a connection under diffeomorphisms
- well-defined in terms of A^a_i, B^{ai} for $\det B \neq 0$

$$\partial_i G_{jk} - \Gamma^l_{ij} G_{lk} - \Gamma^l_{ik} G_{jl} = 0$$

$\Longrightarrow$ Γ is a metric-compatible connection for G

$$\Gamma^k_{ij} = \underbrace{\overset{\circ}{\Gamma}{}^k_{ij}(G)}_{\text{Christoffel connection}} - \underbrace{K_{ij}{}^k}_{\text{Contortion tensor}} \quad (K_{ijk} = -K_{ikj})$$

$$R^l{}_{kij} = \delta^l_j G_{ik} - \delta^l_i G_{jk}$$

$\Longleftrightarrow$ $\boxed{R_{ij}(\Gamma) = -2G_{ij}}$ (2) (3 dimensions $R_{kj} = \delta^i_l R^l{}_{kij}$)

$K_{kj}{}^k = 0 \iff D_i B^{ai} = 0$
(Bianchi Identity)

Einstein Geometry

start with $A, B \longrightarrow G, \Gamma$ (or K)

p③ Inverse map - Start with G_{ij}, Γ^k_{ij}

frame b^a_i, $\det b > 0$

$$\partial_i b^a_j - \Gamma^k_{ij} b^a_k + \epsilon^{abc} A^b_i b^c_j = 0$$

A^a_i spin connection

$$A^a_i = -\frac{1}{2}\epsilon^{abc} b^{bj}(\partial_i b^c_j - \Gamma^k_{ij} b^c_k)$$

$$B^{ai}(x) = |\det G_{jk}|^{1/2} b^{ai}$$

$(G, \Gamma \longrightarrow A, B)$

$$\boxed{B(A) \text{ in eq. (1)} \Longleftrightarrow R_{ij}(\Gamma) = -2G_{ij}} \quad (2)$$

WY ambiguity $\Longleftrightarrow$ multiple solutions for Γ (or K) for given G

representation $K^i_{\;jk} = \epsilon_{jkn} S^{ni} \frac{1}{|\det G|^{1/2}}$, S^{ni} symmetric 6 components

$$\boxed{\frac{\epsilon^{jkl}}{|\det G|^{1/2}} \mathring{\nabla}_k S_{li} - (S^j_k S_i^{\;k} - S^k_k S^j_i) = \mathring{R}^j_i + 2\delta^j_i} \quad (3$$

$\mathring{\nabla}_k$, $\mathring{R}_{ij}$ covariant derivative & Ricci tensor for $\mathring{\Gamma}^i_{jk}$

$$A^a_i = -\frac{1}{2}\epsilon^{abc} b^{bj} \mathring{\nabla}_i b^c_j - b^{ak} S_{ki}$$

p(4)

Examples:

I) $G_{ij}(x) = \delta_{ij}$, $B^{ai} = \delta^{ai}$, $A^a_i = -\delta_{ai}$

$$A^a_i = \begin{pmatrix} \beta \pm\sqrt{\beta^2-1}\cdot\cos(z/\beta) & \pm\sqrt{\beta^2-1}\,\sin(z/\beta) & 0 \\ \pm\sqrt{\beta^2-1}\,\sin(z/\beta) & \beta\mp\sqrt{\beta^2-1}\,\cos(z/\beta) & 0 \\ 0 & 0 & 1/\beta \end{pmatrix}$$

1-parameter family (β) - extend to a 4-parameter family - arbitrary phase $z \longrightarrow z - z_0$
direction $(0,0,1) \longrightarrow \hat{k}$

($B^{ai} = \delta^{ai}$ invariant under rotations & translations of configuration space $\mathbb{R}^3$)

Transform under diffeomorphism to finite energy configuration

II) 3-dimensional hyperboloid $ds^2 = \frac{1}{c^2 z^2}(dx^2+dy^2+dz^2)$

$$R_{ij} = -2c^2 G_{ij}$$

$c^2 = 1$, $S_{ij}(z,x) = \delta_{i1}\delta_{j1}\,\frac{1}{z}h(x) \longleftarrow$ arbitrary function

(rotate by θ in (x,y)-plane $S_{ij} = \frac{1}{z}h(x\cos\theta + y\sin\theta)V_i V_j$
$V_i = (\cos\theta, \sin\theta, 0)$

$B^{ai} = \frac{\delta^{ai}}{z^2}$, $A^1_1 = -h(x)$, $A^1_2 = -A^2_1 = 1/z$

<u>Infinite parameter family</u> - also true for $c^2 \neq 1$

(transform to frame in which B regular)

III) 2+1 product metric

$$ds^2 = F(x,y)(dx^2+dy^2)+dz^2$$

$$\nabla^2 \ln F = c^2 F + \frac{m^2}{F}, \quad c^2<1, \; m>0$$

$$S_{ij} = \begin{pmatrix} \beta F \pm m\cos(z/\beta) & \pm m \sin(z/\beta) & 0 \\ \pm m\sin(z/\beta) & \beta F \mp m\cos(z/\beta) & 0 \\ 0 & 0 & 1/\beta \end{pmatrix} \quad \beta = \sqrt{1-c^2}$$

$m \neq 0$, 1 parameter family

$$ds^2 = \frac{1}{c^2 y^2}(dx^2+dy^2)+dz^2 \Longrightarrow \text{arbitrary function in finite-parameter family}$$

- Continuous WY ambiguities for nondegenerate SU(2) mag. field in D=3 using spatial geometry

  Systematics of ambiguity: what properties of B determine degree of ambiguity?

  - symmetry seems necessary, but not sufficient to explain, say, arbitrary function
  - topology seems not to play a role
    ambiguity in any compact subspace of $\mathbb{R}^3$
    gauge transf. can change topological class

are ambiguities related to any physical observable?
hard to see in absence of topological picture

Gauge Invariant Variables in Gauge Theories
at BNL, May 25-28, 1999

Reduction of Quantum Systems on Riemannian Manifolds with Symmetry and Application to Molecular Mechanics

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Abstract

Let $\mathcal{A}$ and $\mathcal{G}$ denote the configuration space of a gauge field and the group of gauge transformations, respectively. When one tries to define the moduli space $\mathcal{M} = \mathcal{A}/\mathcal{G}$, a singularity arises at a configuration which admits a non-trivial symmetry group. To quantize the moduli space $\mathcal{M}$ we should impose some boundary condition on wave functions to make them smooth and to make the Hamiltonian self-adjoint.

Molecular quantum mechanics provides a finite-dimensional analog of the quantum gauge theory. We make a general formulation to quantize the quotient space $Q = M/G$, which is a reduction of the Riemannian manifold M by the action of the compact Lie group G. A stratified bundle and a stratified connection are introduced as geometric concepts to describe quantization on Q. The space of equivariant functions on M provides a Hilbert space of the reduced quantum system and the reduced Laplacian provides a self-adjoint operator on the Hilbert space. We also apply the formulation to tri-atomic molecule to give an concrete example.

Notations

$\mathcal{A} = \{A(x)\}$: configuration space of gauge field

$\mathcal{G} = \{g(x)\}$: gauge transformation group :

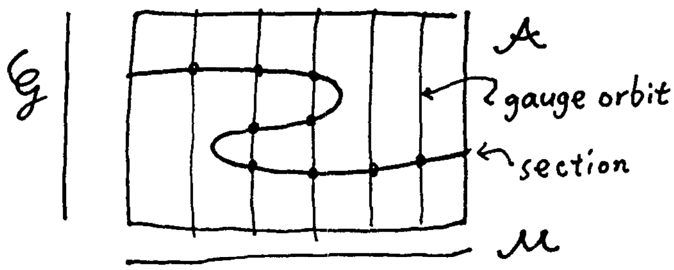
$$A \mapsto A^g = g^{-1}Ag + g^{-1}dg$$

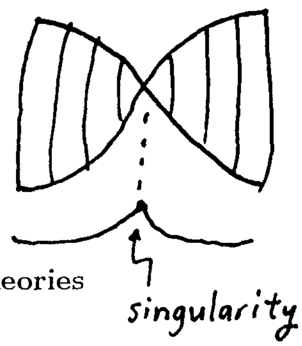
$\mathcal{M} = \mathcal{A}/\mathcal{G}$: moduli space

Difficulty in quantization of systems with gauge invariance

(i) No global section of the bundle $\pi : \mathcal{A} \to \mathcal{M}$.
Global gauge fixing is impossible.

(ii) The quotient space $\mathcal{M} = \mathcal{A}/\mathcal{G}$ is not a smooth manifold but an orbifold.
A : reducible connection $\Longleftrightarrow \exists g \neq e,\ A^g = A$,
which gives rise to singularity in the moduli space.





Molecular mechanics as a toy model of gauge theories

Falling cat phenomenon :

$$\text{net rotation with} \begin{cases} \text{zero angular momentum} \\ \text{zero torque} \end{cases}$$

$\Downarrow$ Analog in molecular mechanics

Non-separability of rotation and vibration :

(motion of molecule) = (translation) + (rotation) + (vibration)

$$\underset{\text{separable}}{\uparrow} \qquad \underset{\text{nonseparable}}{\uparrow}$$

$\uparrow$

centrifugal force, Coriolis force
change of moment of inertia

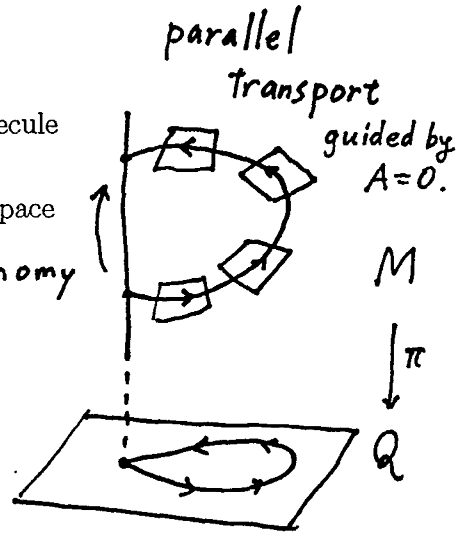
$\Downarrow$ useful description

Gauge theory or differential geometry of connections :

The net rotation of the cat with zero angular momentum is described as a holonomy associated with parallel transport.

Gauge theory for molecules

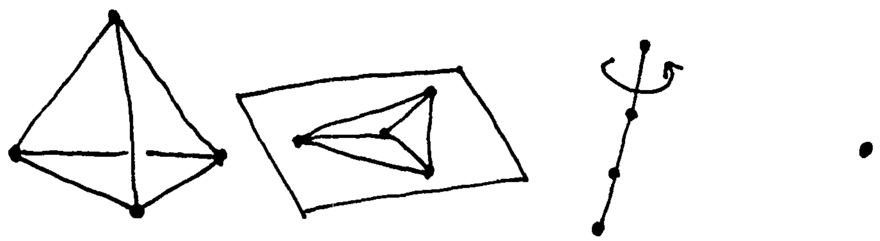
$M = \boldsymbol{R}^{3N}$: configuration space of the N-atomic molecule
$G = SO(3)$
$Q = M/G$: 'shape' space, or reduced configuration space
$\pi : M \to Q$: fiber bundle
I : inertia tensor
$A = I^{-1}(\sum_\alpha m_\alpha \boldsymbol{r}_\alpha \times d\boldsymbol{r}_\alpha)$: connection form on M



$\left\{ \begin{array}{l} \text{classical mechanics: Guichardet (1984)} \\ \text{quantum mechanics: Tachibana and Iwai (1986)} \end{array} \right.$

Difficulty

existence of "reducible" or degenerated configurations



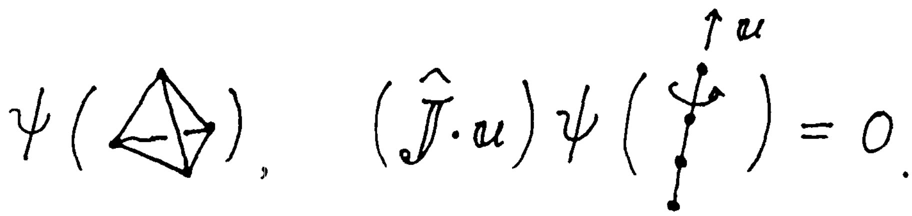
$\underbrace{\text{generic} \qquad\qquad \text{coplanar}}_{\text{irreducible}}$ $\qquad$ $\underbrace{\text{collinear} \qquad\qquad \text{collision}}_{\text{reducible: } \exists g,\ gx = g,\ g \neq e}$

$\downarrow$

orbifold singularity in $Q = M/G$

Solution

vector-valued wave function : $\psi : M \to C^{2\ell+1}; x \mapsto \psi(x)$
equivariance : $\psi(gx) = \rho_\ell(g)\psi(x),\ g \in SO(3)$
boundary condition : $\rho_*(\xi)\psi = 0$ for $\xi \in so(3)$ such that $\theta_x(\xi) = 0$



Formulation

(M, g_M) : Riemannian manifold

G : compact Lie group acting on M by isometry

$G_x = \{g \in G \,|\, gx = x\}$: isotropy group at x

$\mathbf{g}$: Lie algebra of G

$\mathbf{g}_x$: Lie algebra of G_x

$\mathcal{O}_x = \{gx \,|\, g \in G\} \cong G/G_x$: G-orbit through $x \in M$

$\pi : M \to Q = M/G$: stratified bundle

$V_x = \text{Ker}\ \pi_* = T_x\mathcal{O}_x \cong \mathbf{g}/\mathbf{g}_x \subset T_xM$: vertical subspace

$H_x = (V_x)^\perp$: horizontal subspace

$\theta_x : \mathbf{g} \to T_xM$: infinitesimal transformations

$\omega_x : T_xM \to \mathbf{g}/\mathbf{g}_x$: stratified connection form

$(\mathcal{H}^\chi, \rho^\chi)$: irreducible unitary representation of G

$\psi : M \to \mathcal{H}^\chi$: equivariant function; $\psi(gx) = \rho^\chi(g)\psi(x)$ for $g \in G$ and $x \in M$

$D\psi = d\psi - \rho^\chi_*(\omega)\psi$: covariant derivative

$I_x : \mathbf{g} \times \mathbf{g} \to \boldsymbol{R}$; $I_x(\xi, \eta) = g_M(\theta_x(\xi), \theta_x(\eta))$: inertia tensor field

$\tilde{I}_x : \mathbf{g}/\mathbf{g}_x \otimes \mathbf{g}/\mathbf{g}_x \to \boldsymbol{R}$: non-degenerated reduced inertia tensor

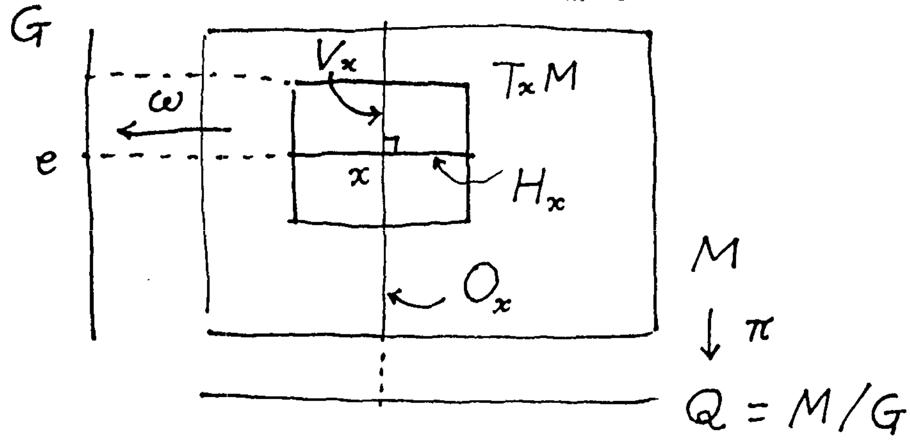
$(\tilde{I}_x)^{-1} :\in \mathbf{g}/\mathbf{g}_x \otimes \mathbf{g}/\mathbf{g}_x$: inverse of the reduced inertia tensor

$\Delta^\chi = D^\dagger D + \Lambda^\chi$: reduced Laplacian

$D^\dagger D$: horizontal component of the Laplacian or vibrational energy operator

$\Lambda^\chi = (\rho^\chi_* \otimes \rho^\chi_*) \circ (\tilde{I}_x)^{-1}$: vertical component of the Laplacian

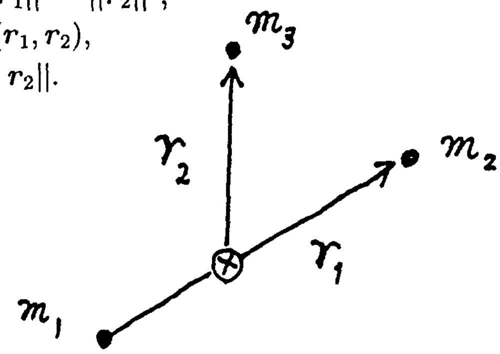
or rotational energy operator



Tri-atomic molecule

coordinate systems (q_1, q_2, q_3) and (ρ, ϕ, χ) in $Q = M/SO(3)$:

$$\begin{cases} q_1 = \rho^2 \cos\chi \cos\phi := \|r_1\|^2 - \|r_2\|^2, \\ q_2 = \rho^2 \cos\chi \sin\phi := 2(r_1, r_2), \\ q_3 = \rho^2 \sin\chi := 2\|r_1 \times r_2\|. \end{cases}$$



vector-valued wave function:

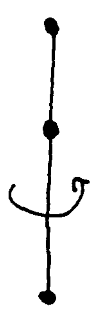
$$\psi(\rho, \chi, \phi) = (\psi_\ell, \psi_{\ell-1}, \cdots, \psi_{-\ell+1}, \psi_{-\ell})$$

the reduced Laplacian :

$$\begin{aligned} & -\Delta^\chi \psi(\rho, \chi, \phi) \\ = \ & -D^\dagger D\psi - \Lambda^\chi \\ = \ & \underbrace{\left\{ \frac{\partial^2}{\partial\rho^2} + \frac{5}{\rho}\frac{\partial}{\partial\rho} + \frac{4}{\rho^2}\left(\frac{\partial^2}{\partial\chi^2} + 2\cot 2\chi \frac{\partial}{\partial\chi} \right) + \frac{4}{\rho^2\cos^2\chi}\left(\frac{\partial}{\partial\phi} + \overbrace{\frac{1}{2}\sin\chi \widehat{J}_1}^{\text{gauge field}} \right)^2 \right\}\psi}_{\text{vibrational energy}} \\ & \underbrace{+\frac{1}{\rho^2}\left\{ (\widehat{J}_1)^2 + \frac{1}{\cos^2(\chi/2)}(\widehat{J}_2)^2 + \frac{1}{\sin^2(\chi/2)}(\widehat{J}_3)^2 \right\}\psi,}_{\text{rotational energy}} \end{aligned}$$

boundary condition:

$$\widehat{J}_3 \psi = 0 \quad \text{for} \quad \chi = 0,$$
$$\widehat{J}_1 \psi = \widehat{J}_2 \psi = \widehat{J}_3 \psi = 0 \quad \text{for} \quad \rho = 0$$



What we have done is

quantization of the reduced configuration space $Q = M/G$ with

- generalization of the concept of connection from principal fiber bundles to stratified bundles
- definition of the reduced Hilbert space of equivariant wave functions
- definition of the reduced Laplacian using the stratified connection
- determination of the boundary condition imposed on the equivariant function at the singularities to make the Laplacian self-adjoint

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New monopole solutions to the SU(2) Einstein Yang-Mills equations in asymptotically anti-de Sitter space

by Jefferson J. Bjoraker
and
Yutaka Hosotani

**School of Physics and Astronomy
University of Minnesota, May 4, 1999**

Overview

New Monopole solutions to the Einstein Yang-Mills (EYM) equations.

- General formalism and the EYM equations
- Monopole solutions in asymptotically Minkowski space and in asymptotically de Sitter space.
- New monopole solutions in asymptotically anti de sitter space
- Stability of the new monopole solutions.

Yang-Mills Equations

No Regular Static solutions -> Repulsive in nature.

Einstein equations

No Regular Static solutions -> Attractive in nature.

If one couples the attractive gravitational fields and the repulsive Yang-Mills fields together, are there static solutions?

YES-- This was done by Bartnik and Mckinnon (BK) in 1988 for purely magnetic Yang-Mills fields in Minkowski space.

Black hole solutions were found soon after, as were solutions in asymptotically de Sitter space.

Ansatz for the SU(2) Yang-Mills fields.

Most general Ansatz found by Witten (1977):

$$A_j^a = \frac{(\phi_2 - 1)}{r^2}\epsilon_{jak}x_k + \frac{\phi_1}{r^3}(\delta_{ja}r^2 - x_j x_a) + A_1 \frac{x_j x_a}{r^2}$$

$$A_0^a = A_0 \frac{x^a}{r} \qquad (1)$$

where A_0, A_1, ϕ_1 and ϕ_2 are functions that depend only on r and t.

Gauge invariance allows us to gauge rotate A and remain general.

Gauge rotations:

- Line up A_r with the z axis.
- Rotate about τ_3 (i.e. $S = e^{i\frac{\phi}{2}\tau_3}$).
- Rotate about $S = \exp[if(r)\tau_3/2]$, where $f(r)$ is some arbitrary function of r

The result is:

$$A = \frac{1}{2e}\left[u\tau_3 dt + (w\tau_1 + \tilde{w}\tau_2)d\theta + (\cot\theta\tau_3 + w\tau_2 - \tilde{w}\tau_1)\sin\theta d\phi\right].$$

3

Yang-Mills equations in curved space

Using the static metric

$$ds^2 = -f(r)dt^2 + h(r)dr^2 + r^2 d\theta + r^2 \sin^2\theta d\phi.$$

The Yang-Mills (YM) equations are,

$$\left(\sqrt{\frac{1}{fh}} r^2 u'\right)' - 2\sqrt{\frac{h}{f}}(uw^2 + u\tilde{w}^2) = 0.$$

$$\left(\sqrt{\frac{f}{h}} w'\right)' + \sqrt{\frac{h}{f}} u^2 w + \sqrt{fh}\frac{w(1 - w^2 - \tilde{w}^2)}{r^2} = 0.$$

$$\left(\sqrt{\frac{f}{h}} \tilde{w}'\right)' + \sqrt{\frac{h}{f}} u^2 \tilde{w} + \sqrt{fh}\frac{\tilde{w}(1 - w^2 - \tilde{w}^2)}{r^2} = 0.$$

$$-\tilde{w}w' + \tilde{w}'w = 0.$$

The last equation yields, $\tilde{w} = Cw$ where C is some constant. A constant rotation $S = \exp(i\tau_3 C/2)$, allows us to set $\tilde{w} = 0$.

Energy momentum Tensor

The energy momentum tensor is:

$$T_{ab} = \frac{1}{4\pi}\left(F_{ac}^{(i)}F_{b}^{c(i)} - \frac{1}{4}\eta_{ab}F_{de}^{(i)}F^{de(i)}\right).$$

The components of T_{ab} are:

$$T_{00} = \frac{1}{4\pi e^2}\left(\frac{1}{2}f^{-1}h^{-1}(u')^2 + f^{-1}\frac{u^2w^2}{r^2} + h^{-1}\frac{(w')^2}{r^2} + \frac{1}{2}\frac{(1-w^2)^2}{r^4}\right),$$

$$T_{11} = \frac{1}{4\pi e^2}\left(-\frac{1}{2}f^{-1}h^{-1}(u')^2 + f^{-1}\frac{u^2w^2}{r^2} + h^{-1}\frac{(w')^2}{r^2} - \frac{1}{2}\frac{(1-w^2)^2}{r^4}\right),$$

and,

$$T_{22} = T_{33} = \frac{1}{4\pi e^2}\left(\frac{(1-w^2)^2}{r^4} + f^{-1}h^{-1}(u')^2\right).$$

(SOME TOPICS ON THE)

• GRAVITY ⟺ LARGE N Y-M CORRESPONDENCE

A. JEVICKI

BNL MAY 1999

- PROGRESS IN STRING THEORY DURING THE LAST 3-4 YEARS (2nd SUPERSTRING REVOLUTION ?)

- DUALITY BETWEEN VARIOUS ALL POSSIBLE STRING THEORIES

- OPEN STRING ⟺ CLOSED STRING DUALITY

- IN THE ZERO SLOPE ($\alpha' \to 0$) LIMIT

GRAVITY ⟺ SUPER Y-M

MALDACENA CONJECTURE, 1998

700 References

CONTENT

- BASIC IDEAS BEHIND THE GR - YM DUALITY

- EXTRA DIMENSION : QUANTUM MECHANICAL (TOY) MODEL $d=1$

- $N=4$ SYM ($d=4$): RELEVANCE OF CONFORMAL SYMMETRY

- CLASSICAL GRAVITY $\Leftrightarrow$ QUANTUM Y-M (LARGE N)

  METHAMORPHOSIS OF CONFORMAL TRANSFORMATIONS

  T. YONEYA, Y. KAZAMA, A.J.
  hep-th / 9810146
  9808039

- APPLICATIONS : "GLUEBALL" MASSES

- $N=4$ 4d SYANG-MILLS THEORY | 4d X_μ

- THE CLEAREST CASE IN THE LIST OF MALDACENA

⇒ IN THE STRONG COUPLING LIMIT IT IS REPRESENTED BY II_B SUGRA ON

$$AdS_5 \times S_5$$

$$(ds)^2 = \alpha' \Big\{ \underbrace{\frac{U^2}{R^2}\, dx_{(4)}^2 + \frac{R^2}{U^2}\, dU^2}_{AdS_5} + \underbrace{R^2 d\Omega_5^2}_{S_5} \Big\}$$

The radius of AdS_5 AND S_5 is given by

$$R^2 = (\underbrace{2 g_{YM}^2 N}_{\text{'t Hooft}})^{1/2}$$

Extra radial dimension U

Extra spherical dimension $\vec{n}^2 = 1 : S_5$

SYMMETRIES (of AdS_5 S_5 BAGGROUND)

S_5 : J^{ab}

$a, b = 1, 2, 3$ 6

$SO(6)$ ROTATIONS

AdS_5 : $SO(2,4)$

$$(ds)^2 = - dy_{-1}^2 - dy_0^2 + dy_1^2 + \cdots dy_4^2$$

$$y_\alpha y^\alpha = -1$$

$$J^{\alpha\beta}_{Ads} = y^\alpha \partial^\beta - y^\beta \partial^\alpha$$

EQUIVALENT TO THE CONFORMAL GROUP IN 4 d

- CONFORMAL TRANSFORMATIONS (IN YM)

$$\boxed{\delta_K x_\mu = 2\varepsilon\cdot x\, x_\mu - \varepsilon_\mu x^2}$$. SPECIAL CONF. TR.

$$\delta\varphi = -2\varepsilon\cdot x\ \underline{\varphi(x)} + \delta x^\mu \partial_\mu \underline{\varphi(x)}$$.

SCALAR FIELD

ALL the Higgs (AND YM) FIELDS TRANSFORM LINEARLY UNDER SPECIAL CONF. TR.

$$\delta A_\mu(x) = -2\varepsilon\cdot x\, \underline{A_\mu} + 2x_\mu\, \varepsilon\cdot \underline{A} - 2\varepsilon_\mu\, x\cdot \underline{A}$$
$$+ \delta x^\nu \partial_\nu \underline{A_\mu(x)}$$

$$\delta \Phi^a(x) = -2\varepsilon\cdot x\, \Phi^a + \delta x^\nu \partial_\nu \Phi^a \qquad a = 4\ldots 9$$

- TRANSFORM OF A DOESNOT DEPEND ON Φ^a (Higgs)
AND NO DEP. ON $U \sim \sqrt{\vec{\Phi}^2}$?
appears in the transform of A_μ

BACKGROUND GAUGE COND.

$$\chi = \underset{\sim}{\partial_\mu A_\mu} - i \, [\underset{\sim}{B^a ,} \, Y_a]$$

NOT INVARIANT UNDER SPEC. CONF. TR.

$$\delta \chi = \delta x^\mu \partial_\mu \chi - 2 \varepsilon \cdot x \, \chi$$ almost a scalar

$$+ \boxed{4 \, \varepsilon^\mu \cdot A_\mu}$$ from DERIVATIVE

CONSEQUENTLY $\chi = 0$ INDUCES A VIOLATION OF CONF. SYMMETRY OR CONF. TR. INDUCES A CHANGE OF GAUGE

THE EXTRA TERM CAN BE COMENSATED BY A GAUGE TRANSFORMATION.

GAUGE PARAMETER

$$\Lambda = \Delta_B^{-1} \left(4 \varepsilon \cdot A(x) \right)$$

BACK. LAPLACIAN

COMMENTS AND SUMMARY:

We have exibited the origin of the AdS BULK NONLINEAR TERM IN THE ISOMETRY

$$\delta_{ADS} X_\mu = \delta x_\mu + \varepsilon_\mu \cancel{g}^2 \frac{R^4}{U^2}$$

From YANG-MILLS THEORY

$$\delta_{YM} X_\alpha = \left(\delta x_\mu + \varepsilon_\mu \frac{g_{YM}^2 N}{r^2} \right) \partial^\mu X_\alpha$$

- 1-LOOP Effect
- NONRENORMALIZATION
- EQUIVALENT TO EXIBITING THE AdS metric.

Gauge invariant monopoles for free

Pierre van Baal, Leiden Univ.

Summary of talk at Riken-BNL workshop on "Gauge invariant variables in gauge theories." (May 25-29, 1999)

Instanton solutions at finite temperature have a hidden variable, so far not fully exploited. At spatial infinity the Polyakov loop is constant, but may be non-trivial, $P_\infty \equiv \lim_{|\vec{x}|\to\infty} P\exp(\int_0^T A_0(\vec{x},t)dt) \notin Z_N$

In this case the instanton has for $SU(N)$, N constituent BPS mononopoles, which are regular and gauge invariant (for $P_\infty \in Z_N$, $N-1$ are massless and do not show-up in the action density profile).

At the spatial centers of each constituent the Polyakov loop is a center element, different for each of the constituents. Proven upto now for $SU(2)$, where it is the non-trivial center element constituent that supports the fermion zero-mode

We also show how this is related to "Taubes-winding" of monopole loops which is responsible for the topological charge.

We speculate on making a model that combines monopoles for confinement and instantons for chiral symmetry breaking.

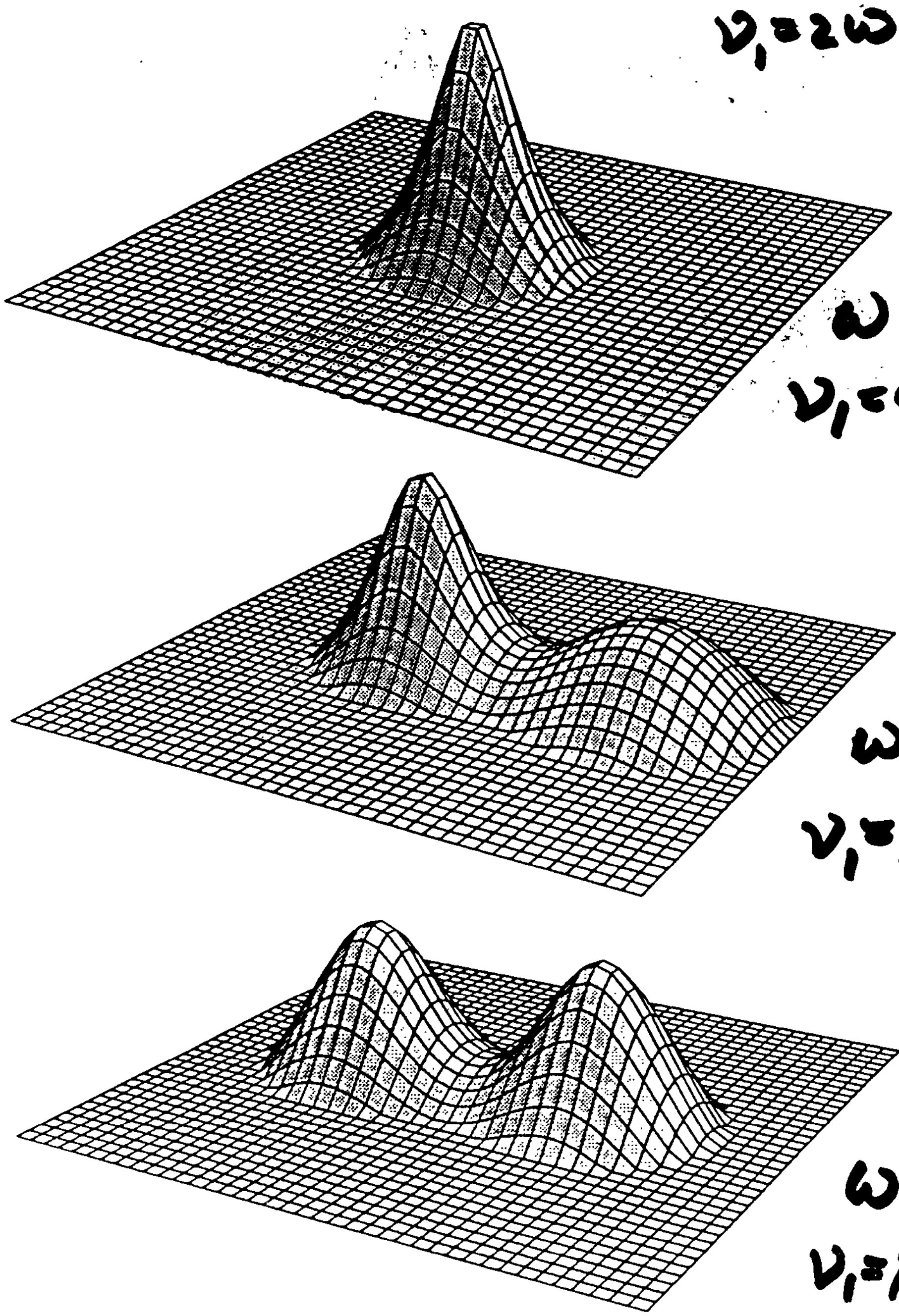
6.

$T = \rho = 1$

$P_\infty = \exp(2\pi i \vec{\omega}\cdot\vec{\tau})$

$\omega \equiv |\vec{\omega}|$

$\nu_1 = 2\omega,\ \nu_2 = 1 - 2\omega = 2\bar{\omega}$



$\omega = 0$

$\nu_1 = 0,\ \nu_2 = 1$

$\omega = 1/8$

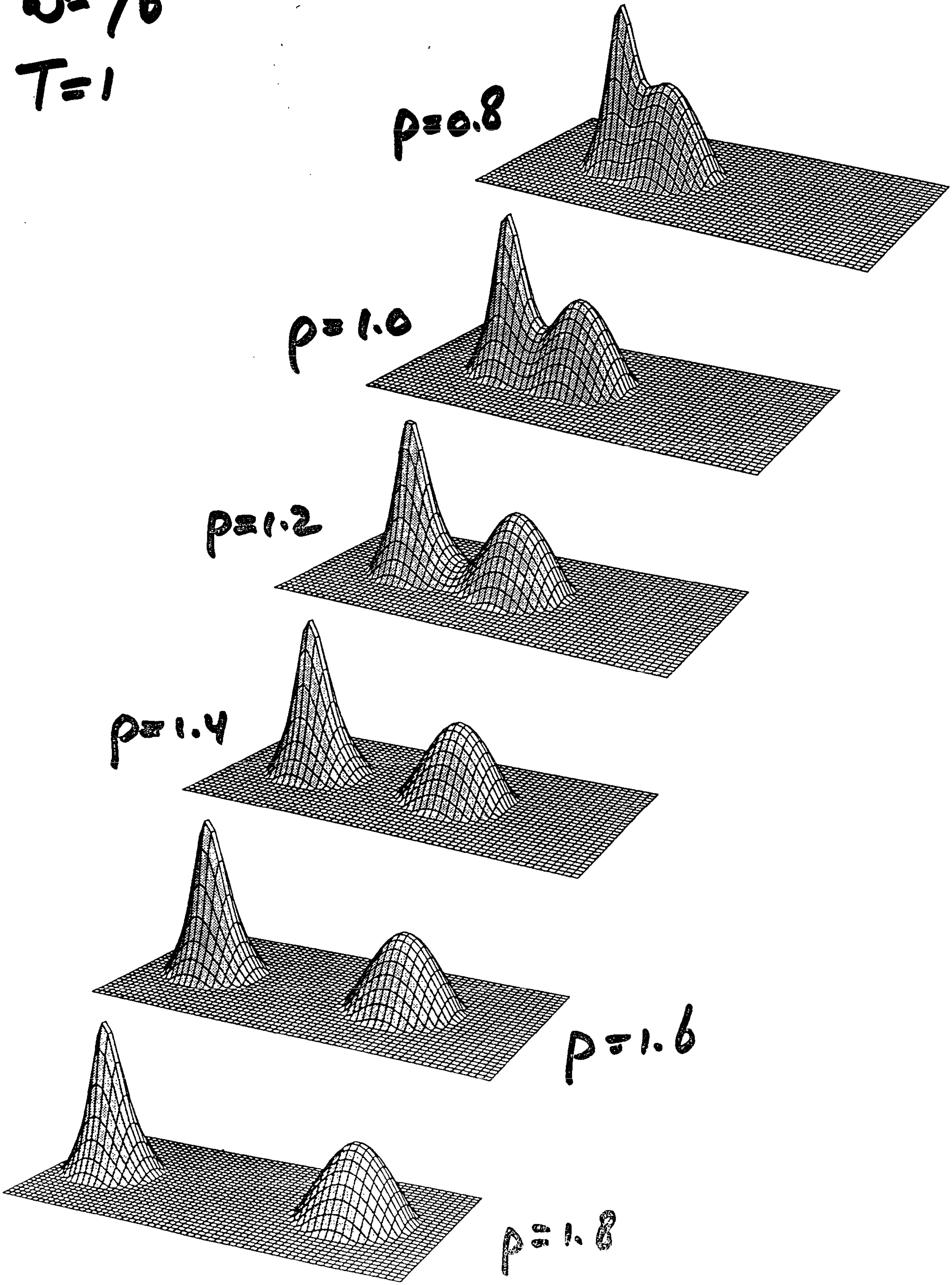
$\nu_1 = 1/4,\ \nu_2 = 3/4$

$\omega = 1/4$

$\nu_1 = 1/2,\ \nu_2 = 1/2$

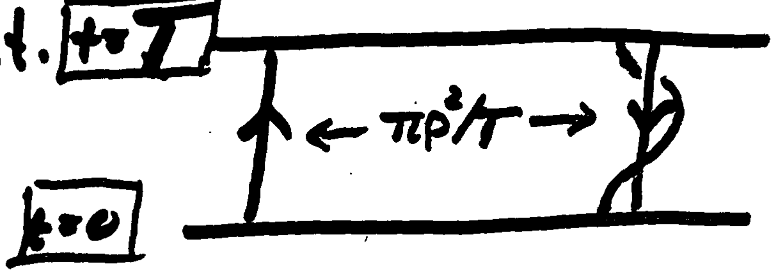
Figure 1: Profiles for calorons at $\omega = 0, 0.125, 0.25$ (from top to bottom) with $\rho = 1$. The axis connecting the lumps, separated by a distance π (for $\omega \neq 0$), corresponds to the direction of $\hat{\omega}$. The other direction indicates the distance to this axis, making use of the axial symmetry of the solutions. Vertically is plotted the action density, at the time of its maximal value, on equal logarithmic scales for the three profiles. The profiles were cut off at an action density below $1/e$. The mass ratio of the two lumps is approximately $\omega/\bar{\omega}$, i.e. zero (no second lump), a third and one (equal masses), for the respective values of ω.

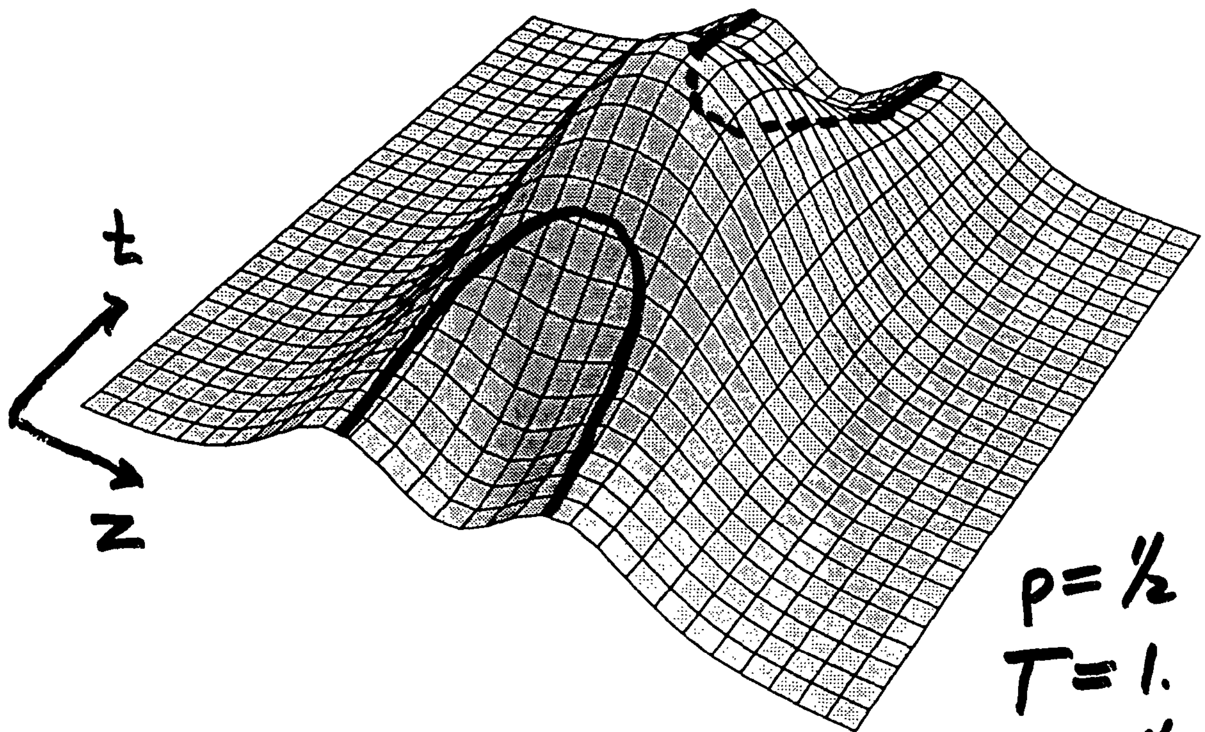
$\omega = 1/8$
$T = 1$



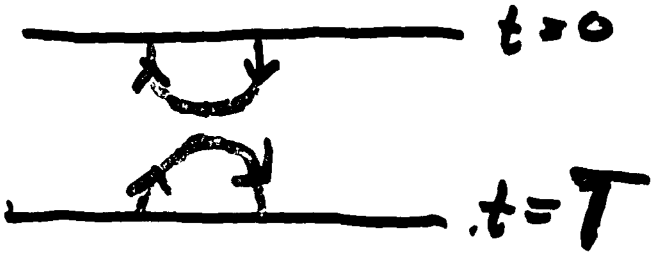
$\omega = 1/8$; cutoff $1/e^2$

ρ large, or T small – solution becomes static – monopole traject. are straight.





$\rho = \frac{1}{2}$
$T = 1.$
$\omega = \frac{1}{4}$



ρ^2/T <u>not</u> large
monopoles attract
(and annihilate)

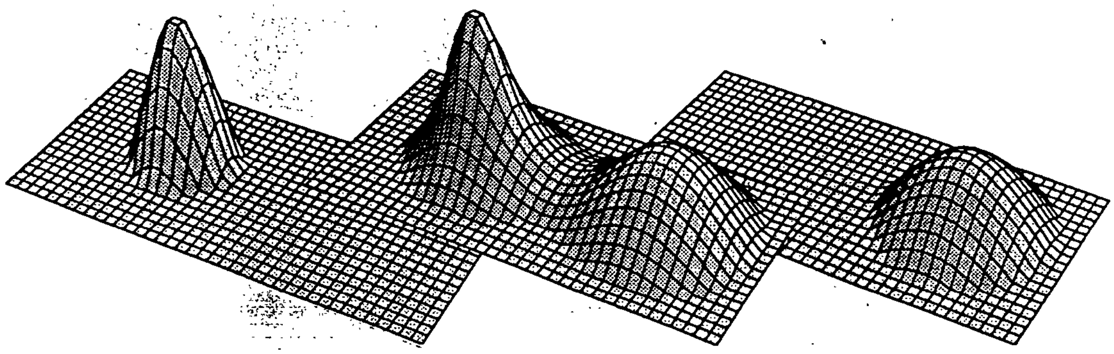


Figure 1: For the two figures on the sides we plot on the same scale the logarithm of the zero-mode densities (cutoff below $1/e^5$) for $\omega = 1/8$ (left Ψ^- / right Ψ^+) and $\omega = 3/8$ (right Ψ^- / left Ψ^+), with $\beta = 1$ and $\rho = 1.2$. In the middle figure we show for the same parameters (both choices of ω give the same action density) the logarithm of the action density (cutoff below $1/2e^2$).

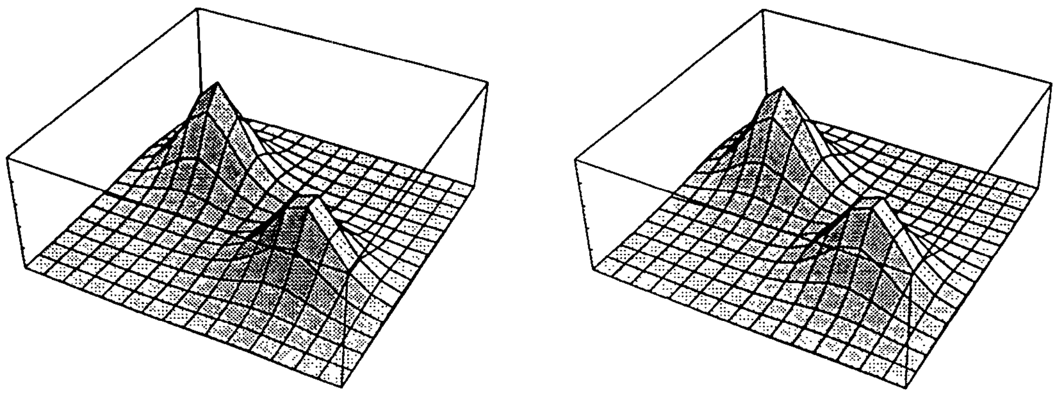
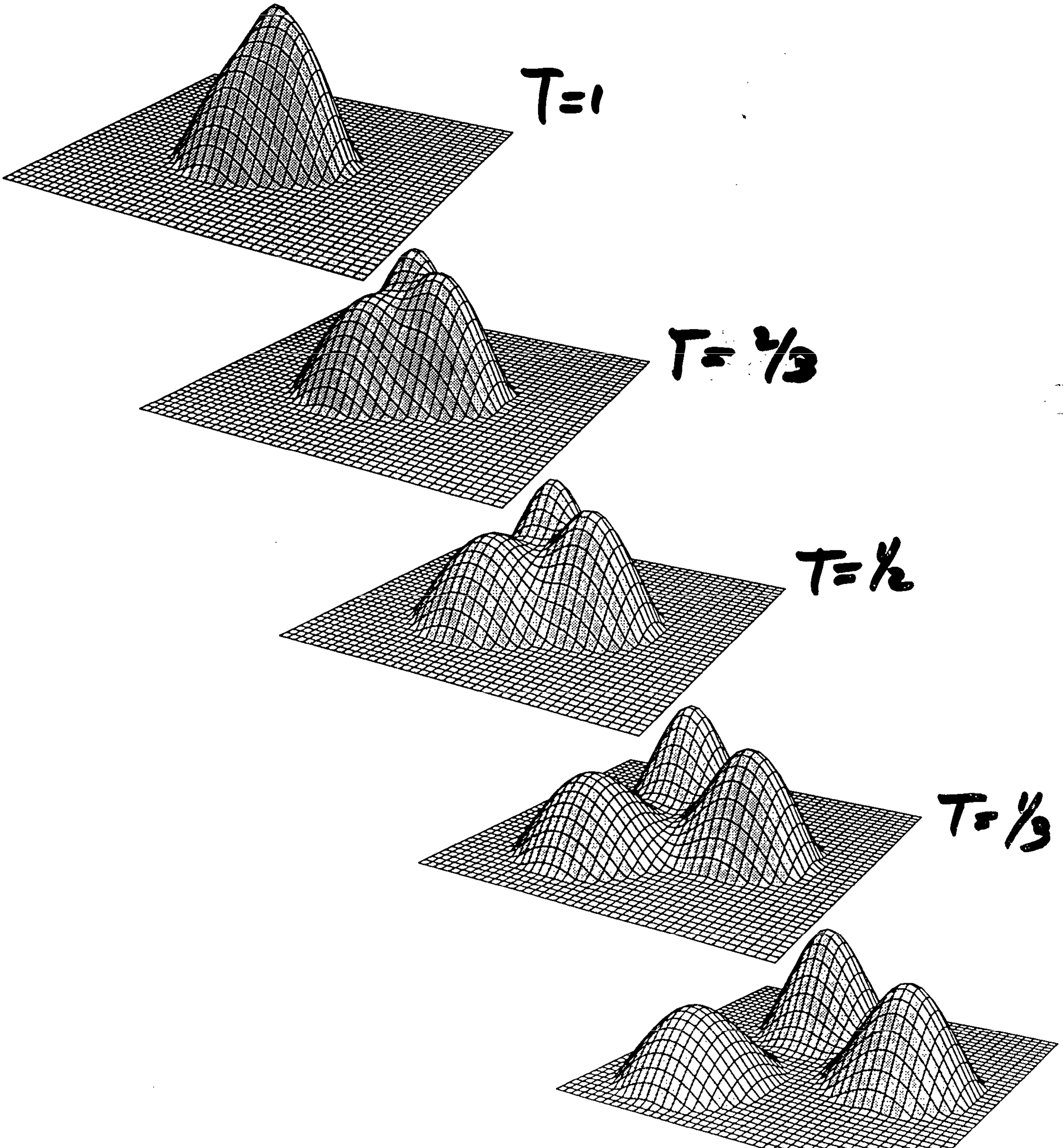


Figure 3: Zero-mode density profiles for the two zero-modes of the lattice caloron (left) on a 4×16^3 lattice for $\vec{k} = (1,1,1)$, created with improved cooling ($\varepsilon = 0$). The profiles fit well to the two zero-modes for the infinite volume analytic caloron solution (shown on the right at $y = t = 0$) with $\omega = \frac{1}{4}$ and constituents at $\vec{y}_1 = (2.50, 0.12, 0.95)$ and $\vec{y}_2 = (1.38, -0.24, 2.67)$, in units where $\beta = l_t = 1$ (or $a = \frac{1}{4}$) and the left most lattice point corresponding to $x = z = 0$. The plots give the added densities of the two zero-modes.

15.

SU(3)



Dynamics of confinement from 3 d gauge theories

Owe Philipsen CERN

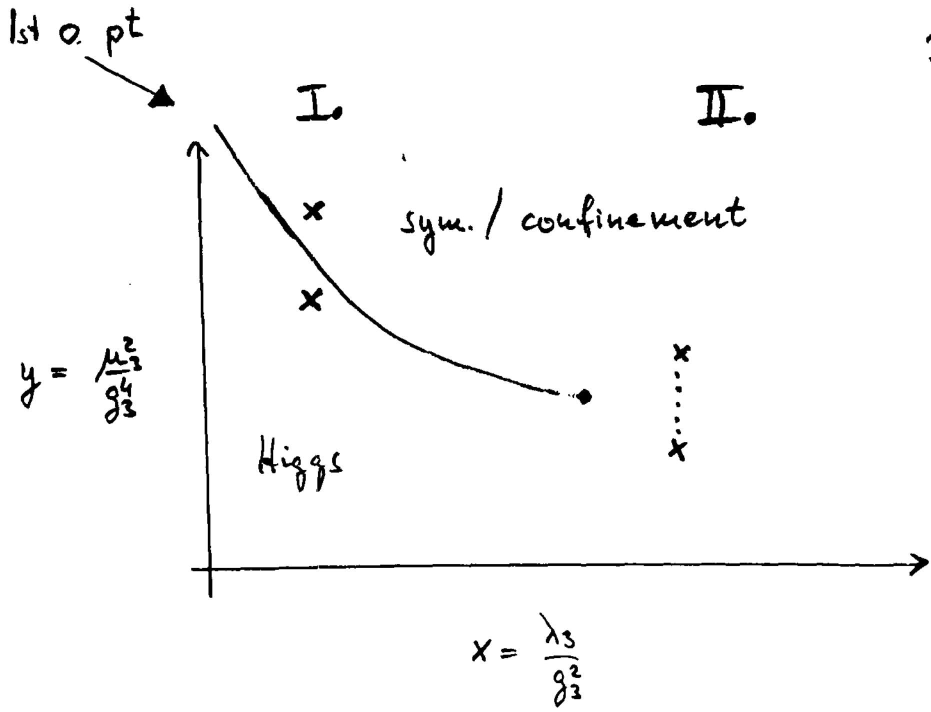
- 3 d gauge theories exhibit all qualitative properties of QCD: linear confinement, screening ...
- they are much more easily accessible by lattice simulations ⟹ detailed accurate results
- they describe thermodynamics of 4 d finite T QFT

This talk reviews non-perturbative results for

- the mass spectrum of SU(2) pure gauge theory and SU(2) Higgs model (fundamental + adjoint)
- the static potential of fund. and adjoint sources: string tension, screening at large distances
- the gauge field propagator and its possible connection with the physical properties

SU(2) Higgs in 3d.

phase diagram:



Buchmüller, O.P.

Kajantie, Laine, Rummukainen, Shaposhni

Karsch, Patkos, Neuhaus,

Gürtler, Ilgenfritz, Schi

measurements of mass spectrum: O. P., Teper, Witt

I. $\frac{\lambda_3}{g_3^2} = 0.0239$ ($M_H = 35\,GeV$)

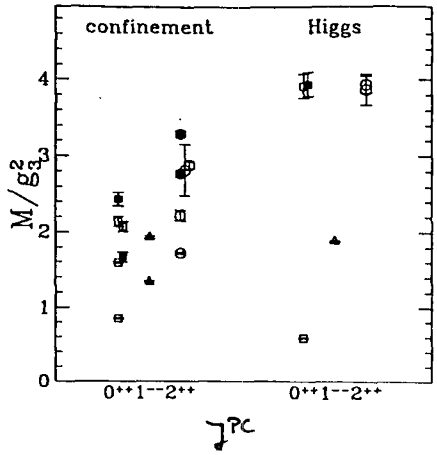
II. $\frac{\lambda_3}{g_3^2} = 0.274$ ($M_H = 120\,GeV$)

perturbative Higgs and conf. regime analytically connected, pure gauge theory $x, y \longrightarrow \infty$

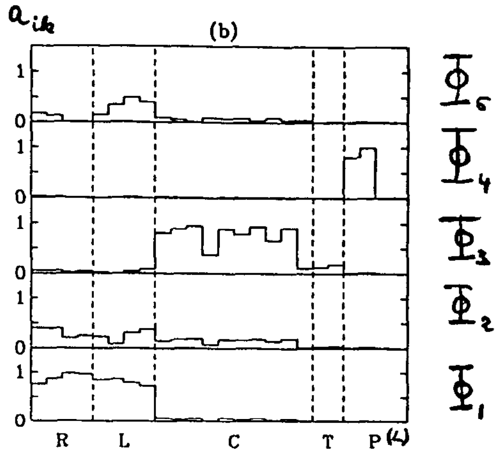
I. mass spectrum at small scalar coupling

?. P., Teper, Wittig

Spectrum



mixing in 0^{++}-channel



■ purely gluonic states

□ scalar and mixed states

$$\Phi_i = \sum_k a_{ik}\,\phi_k \quad , \quad \phi_k \in \{R, L, C, P$$

operator types:

$$R \sim \mathrm{Tr}\,(\phi^+\phi)$$
$$L \sim \mathrm{Tr}\,(\phi^+ D_i \phi)$$
$$C \sim \mathrm{Tr}\,(F_{ij}^2)$$
$$P^{(L)} \sim \mathrm{Tr}\,P \exp\left\{ ig \int_0^L A(x)\,dx \right\}$$

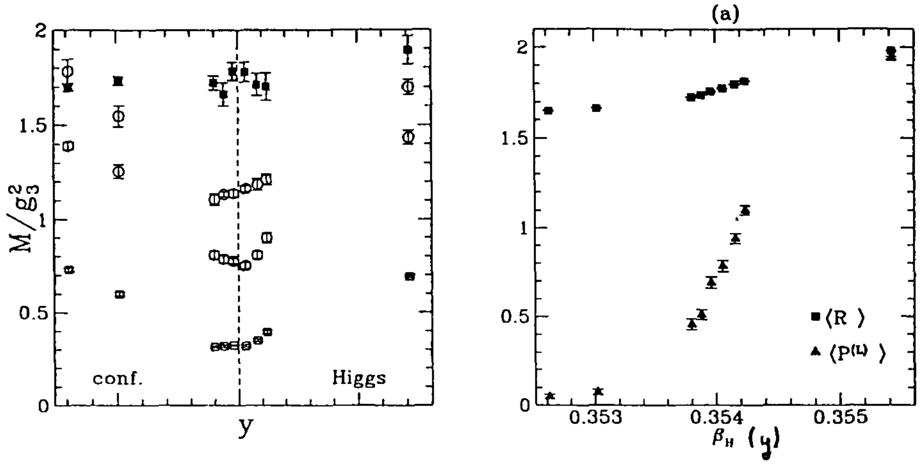
Higgs region: perturbative

symmetric/confinement region:

- purely gluonic states ⟺ glueball spectrum in pure SU(2)
- additional "mesonic" states of bound scalars
- little mixing between glueballs and mesons
- string tension ~ 9[illegible] % of pure SU(2)

II. mass spectrum at large scalar coupling

0^{++} masses vev of R and $P^{(L)}$



no phase transition between

Higgs "side" :
- Higgs characteristics disappear
- dense spectrum as on conf. side

confinement "side" :
- glueball spectrum insensitive to variation of scalar parameters by $O(10)$

But: Different dynamics !

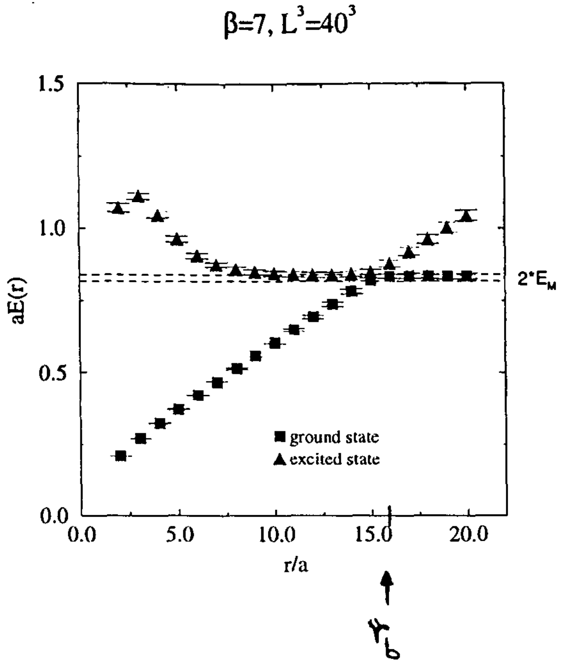
Higgs: $\langle P^{(L)} \rangle \neq 0$ no "gauge" confinement

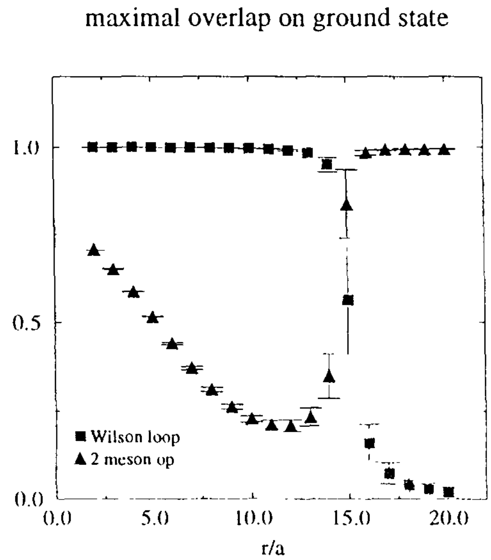
conf: $\langle P^{(L)} \rangle \longrightarrow 0$ linear confinement for intermediate r

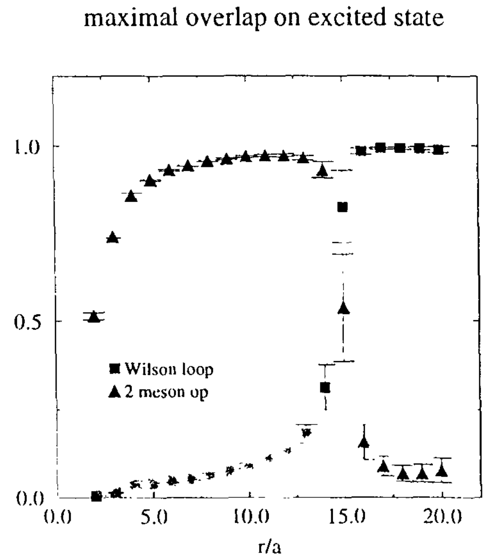
The static potential:

ground state + first excited state

O. P., Wittig







continuum extrapolation:

I. fund rep. with matter

$r_b g_3^2 \simeq 8.5$

$r_b m_{glueball} \simeq 13.6$

$m_{meson}/m_{glueball} = 0.52(2)$

II. adjoint rep. pure SU(2)

$r_b g_3^2 \simeq 6.4$

$r_b m_{glueball} \simeq 10.3$

interpretation of propagator mass : constituents ?

Higgs phase : free H ⊙ , W ◎ point particles

confinement : bound states ...

○ gauge - inv. ops ● ϕ ○ D_i field ops

➡ in Higgs phase they project on the same object

➡ in conf. phase they project on different objects

e. g. $\tilde{A}_i^a = \mathrm{Tr}\,(\tau^a \phi^+ D_i \phi)$ $m \approx 2m_\phi + m_A$

$C = \mathrm{Tr}\,(\vec{F}_{ij}^2)$ $m \approx 4 m_A$

$G_F = \langle F_{ij}^a(x)\, W^{ab}(x,y)\, F_{kl}^b(y) \rangle$ $m \approx 2 m_A$ (after suitable normalization)

W^{ab} : adjoint Wilson line

take $m_A \approx 0.4\, g_3^2$ Karsch et al

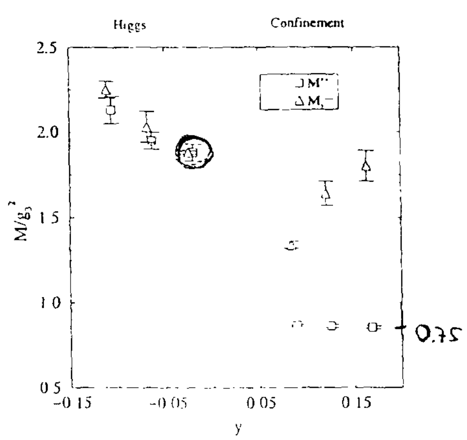


FIG. 1. The lowest physical 1^{--} state (the W-boson in the Higgs phase) and the mass M^{W} extracted from $G_F(x,y)$.

	constituent pic.	lattice
m_C	1.6	1.60(4)
m_{G_F}	0.8	0.75(?)
$m_{\tilde{A}}$	1.23	1.27(6)

⊙ : at this point divergence subtracted and normalized to W propagator

Lattice Gauge Fixing, Elitzur's Theorem and Abelian Projection

Mike Ogilvie
Washington University
St. Louis, MO USA

1. Consequences of Elitzur's Theorem
2. Lattice Gauge Fixing
3. Abelian Projection and Abelian Dominance
4. Lessons

Analytic Control of Lattice Gauge Fixing

It is very convenient to introduce an auxiliary gauge-fixing field $\phi(x)$, which takes values in G. This fields are applied to an unfixed field configuration as

$$g_\mu(x) \rightarrow \tilde{g}_\mu(x) = \phi(x) g_\mu(x) \phi^+(x+\hat{\mu})$$

so that $\tilde{g}_\mu(x)$ is used wherever the gauge-fixed field is required. The gauge fixing function depends on $\tilde{g}$, which is to say both g and ϕ. The expectation value of an observable O is given by

$$\langle O \rangle = \frac{1}{Z_g} \int [dg]\, e^{S_g[g]} \frac{1}{Z_{gf}[g]} \int [d\phi] e^{S_{gf}[\tilde{g}]}\, O$$

where

$$S_{gf} = \frac{\lambda}{2N} \sum_{x,\mu} Tr\left[\tilde{g}_\mu(x) + \tilde{g}_\mu(x)^+\right]$$

and$Z_{gf}[\tilde{g}]$ is defined as

$$Z_{gf}[\tilde{g}] = \int [d\phi]\, e^{S_{gf}[\tilde{g}]}$$

Z_{gf} is needed for gauge invariance

In the limit $\lambda \rightarrow \infty$, the procedural implementation of this formula is equivalent to the commonly used algorithm for lattice gauge fixing, modulo Gribov copies.

- Formally, the field ϕ is just a quenched, adjoint representation scalar field with two independent symmetry groups, $G_L \otimes G_R$.
- The generating function $Z_{gf}[g]$ is somewhat analogous to the Fadeev-Popov determinant.
- This lattice formalism sidesteps the problem of Gribov copies. By construction,gauge invariant observables are evaluated by integrating over all configurations. Gauge-variant quantities receive weighted contributions from Gribov copies.

Projection without Gauge Fixing

Projection to an abelian subgroup for lattice field configuration is carried out by maximizing the overlap of each link with an element in a subgroup H in the trace norm. For analytical purposes, we generalize the projection process. The weight function for this projection is:

$$S_{proj}[g,h] = \sum_l \left[\frac{p}{2N} Tr(g_l^+ h_l + h_l^+ g_l) \right]$$

Expectation values are given by

$$\langle O \rangle = \frac{1}{Z_g} \int [dg]\, e^{S_g[g]} \frac{1}{Z_{proj}[g]} \int [dh] e^{S_{proj}[g,h]}\, O$$

where

$$Z_{proj}[g] = \int [dh]\, e^{S_{proj}[g,h]}$$

- Reduces to usual case if O depends only on the g fields. $Z_{proj}[g]$ ensures that the h fields behave as quenched variables, and have no effect on the distribution of g fields.

- Computationally, this can be implemented as a Monte Carlo simulation inside a Monte Carlo simulation.

- As $p \to \infty$, the usual projection algorithm is reproduced
- Alternative (Faber, Greensite and Olejnik, 1998) for $SU(2) \to Z(2)$

$$z_l = sign(Tr_F U_l) = \sum_\alpha c_\alpha \chi^\alpha(U_l)$$

Consider a Wilson loop W in a representation β of H with no self-intersections:

$$\widetilde{\chi}^{\beta}(h_1..h_n) = \widetilde{D}^{\beta}_{j_1 j_2}(h_1)\widetilde{D}^{\beta}_{j_2 j_3}(h_2)..\widetilde{D}^{\beta}_{j_n j_1}(h_n)$$

1. Expand the weight function in characters of the group G:

$$\exp\Big[\frac{p}{2N} Tr(g^+ h + h^+ g)\Big] = \sum_{\alpha} d_\alpha c_\alpha(p)\chi_\alpha(h^+ g)$$

where

$$c_\alpha(p) = \frac{1}{d_\alpha}\int_G (dg)\,\chi_\alpha(g^+)\exp\Big[\frac{p}{2N} Tr(g+g^+)\Big]$$

$Z_{proj}[g]$ is given by

$$Z_{proj}[g] = \prod_l \widetilde{c}_0(p, g_l) \approx \prod_l c_0(p)$$

where $\widetilde{c}_0$ is given by

$$\widetilde{c}_0(p, g_l) = \int_H (dh_l)\,\exp\Big[\frac{p}{2N} Tr(g_l h_l + h_l^+ g^+)\Big]$$

Note that

$$e^{pM_1} \leq \widetilde{c}_0(g_m, p) \leq e^{pM_2}$$

2. Integrate over all possible gauge transformations: Consider two adjacent links on the curve g_m and g_{m+1}. We integrate over the variable ϕ_m on the common site.

$$\int_G (d\phi_m) D^{\alpha_m}_{l_m k_m}(g_m \phi_m) D^{\alpha_{m+1}}_{l_{m+1} k_{m+1}}(\phi_m^+ g_{m+1})$$
$$= \frac{1}{d_{\alpha_m}} \delta_{\alpha_m \alpha_{m+1}} \delta_{k_m l_{m+1}} D^{\alpha_m}_{l_m k_{m+1}}(g_m g_{m+1})$$

3. ***Key Result***:

$$\left\langle \tilde{\chi}^{\beta}(h_1..h_n) \right\rangle \approx \sum_{\alpha} \left(\frac{c_{\alpha}(p)}{c_0(p)} \right)^n \int_H (dh) \tilde{\chi}^{\beta}(h) \chi^{\alpha}(h^+) \langle \chi^{\alpha}(g_1..g_n) \rangle$$

Physics content: the Wilson loop as measured in the β representation of H is given as a sum of Wilson loops in the irreducible representations of G, each weighted by the number of times β appears in α and by a p-dependent perimeter renormalization factor which goes to 1 as $p \to \infty$.

4. *Strict Bounds*

Using the bounds on $\tilde{c}_0$ we can obtain the upper bound

$$\left\langle \tilde{\chi}^{\beta}(h_1..h_n) \right\rangle \leq \sum_{\alpha} (c_{\alpha}(p) e^{-pM_1})^n \int_H (dh) \tilde{\chi}^{\beta}(h) \chi^{\alpha}(h^+) \langle \chi^{\alpha}(g_1..g_n) \rangle$$

with a corresponding lower bound when M_1 is replaced by M_2.

5. ***Key Result***:

$$\tilde{\sigma}_{\beta} = \min_{\alpha} \sigma_{\alpha}$$

independent of p, where the minimum is taken over all representations α that have a non-zero contribution. This result relies only on gauge invariance, does not depend strongly on the projection subgroup, and is independent of dimensionality. Center symmetry is important.

Example $SU(2)$ projected to $U(1)$

$$\tilde{\sigma}_{1/2} = \min_{j=1/2,3/2,..} \sigma_j$$

and

$$\tilde{\sigma}_{3/2} = \min_{j=3/2,5/2,..} \sigma_j$$

but $\tilde{\sigma}_1 = 0$ because $\sigma_1 = 0$.

Lessons

- Elitzur's theorem tells us that lattice operators defined via gauge fixing can always be reduced to explicitly gauge invariant operators.
- Lattice gauge fixing is really not very much like continuum gauge fixing. Lattice Landau gauge does not reduce to continuum Landau gauge in the naive continuum limit. Lattice gauge fixing assumes that integration over all field configurations is the correct prescription for defining the functional integral.
- Landau gauge propagators can be viewed as propagators of composite objects or as complicated, extended objects.
- Coulomb gauge fixing has much in common with other lattice techniques for constructing improved operators, such as smeared operators.
- String tension measurements on projected gauge theories with or without gauge fixing give the same answers as the underlying gauge theory. This is independent of dimension, and unrelated to any scheme for confinement. The choice of subgroup is not important, as long as it contains the center of the group. Such measurements give no information on confinement mechanisms.

Smeared Gauge Fixing

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Abstract

We present a new method of gauge fixing to standard lattice Landau gauge, Max Re Tr $\sum_{\mu,x} U_{\mu,x}$, in which the link configuration is recursively smeared; these smeared links are then gauge fixed either by by standard extremization for SU(2), or by constructing a "gauge tree" which diagonalizes the links for SU(3). The resulting gauge transformation is simultanteously applied to the original links. Following this preconditioning, the links are gauge fixed again as usual.

We find that this method is *generally* free of Gribov copies and for physical parameters, generally results in the gauge fixed configuration with the globally maximal trace.

The method is hindered by the stability of instantons while smearing, and it is hoped that an alternative smearing method will help.

1 Introduction

There are two deficiencies of gauge fixing on the lattice:

1. *The Gribov problem*: that gauge fixing leads to multiple solutions of the gauge fixing condition.
2. *The Smoothness condition*: if one alters the gauge condition so that it is free of Gribov copies (like axial gauge) it is generally not smooth, and difficult to compare to perturbation theory which usually uses Landau gauge.

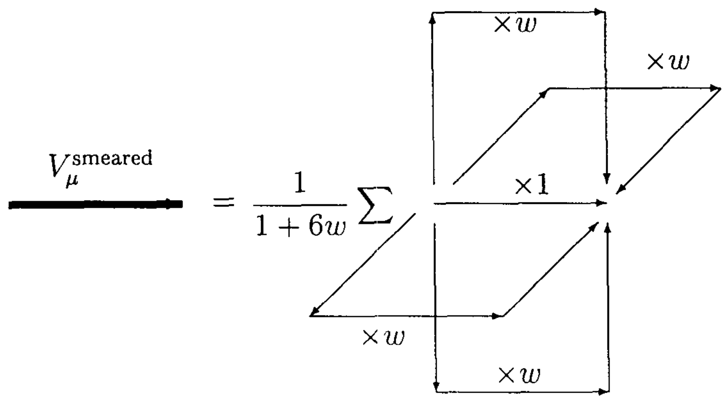
Below we present a method for addressing these two issues which is both simple and fast. It has so far been free of Gribov copies, and usually produces the smoothest (Max Re Tr $\sum_{x,\mu} U_\mu$) configuration.

2 Iterative smearing

By smearing we define new links V_μ from the old ones U_μ by the definition

$$V_\mu^{\text{smeared}}(x) = \frac{1}{1+6w}\Big\{U_\mu(x) + w \quad \textstyle\sum_{\nu\neq\mu}[U_\nu^\dagger(x-\hat\nu)U_\mu(x-\hat\nu)U_\nu(x-\hat\nu+\hat\mu) + U_\nu(x)U_\mu(x+\hat\nu)U_\nu^\dagger(x+\hat\mu)]\Big\} \tag{1}$$

where w is the weight for the staples, normalized so that $V_\mu^{\text{smeared}}(x) = \mathbf{1}$ when all U_μ's on the r.h.s of eq. (??) are $\mathbf{1}$. Pictorially:



- After smearing each link is reunitarized.

3 Smearing as gauge covariant cooling

As we smear the configuration is transported toward the trivial gauge orbit ($F_{\mu\nu} = 0$), ie. *it is cooled*. Furthermore this cooling is *gauge covariant:*

If starting configurations U^A_μ and U^B_μ are related by a gauge transformation G^{AB}, then the corresponding smeared configurations V^A_μ and V^B_μ are related by the same gauge transformation G^{AB}.

We can say that smearing (with $w < w_c$) transports the entire fiber toward the trivial orbit, preserving the vertical structure, as depicted in Figure ??.

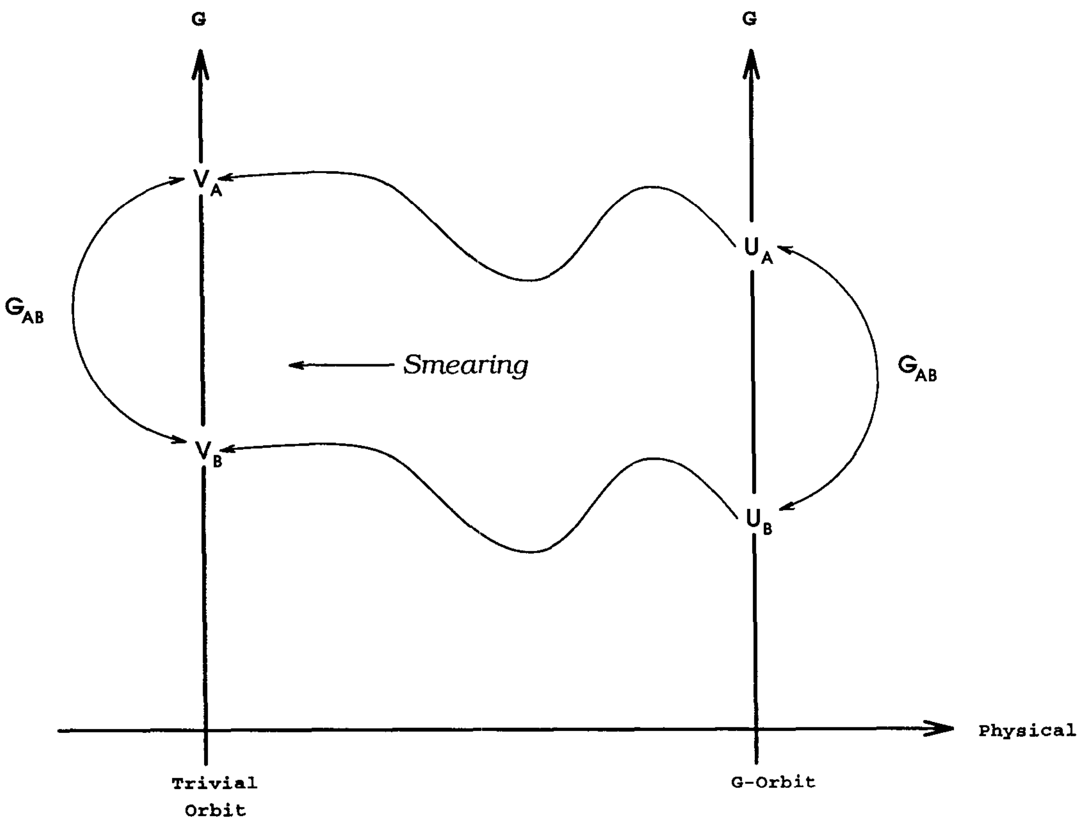


Figure 2: Gauge structure of the smearing process: Physical degrees of freedom ($\sim 1/\beta$) change horizontally and gauge degrees of freedom are represented as vertical.

On the trivial orbit, the *unique minimal Landau gauge* fixed configuration is easy to find:

$$V_\mu^{\text{Landau gauge}} = (D_\mu)^{1/n_\mu} \tag{3}$$

where there are n_μ links in the μ—direction.

We compute the gauge transformation G_{VD} that rotates the smeared links V_μ into D_μ^{1/n_μ}, and apply this gauge transformation to the unsmeared links U_μ, which rotates the original links to a unique point D'_μ. Because the gauge fixed surface is nonlinear†, the resulting D' does not satisfy the Landau gauge condition. However the point D' along the orbit is unique. Thus, every starting configuration is taken to D' by the gauge transformation which takes its smeared version to D. From D' we gauge fix as usual by extremizing $I(U_\mu, G)$; this results in a unique Landau gauge fixed configuration.

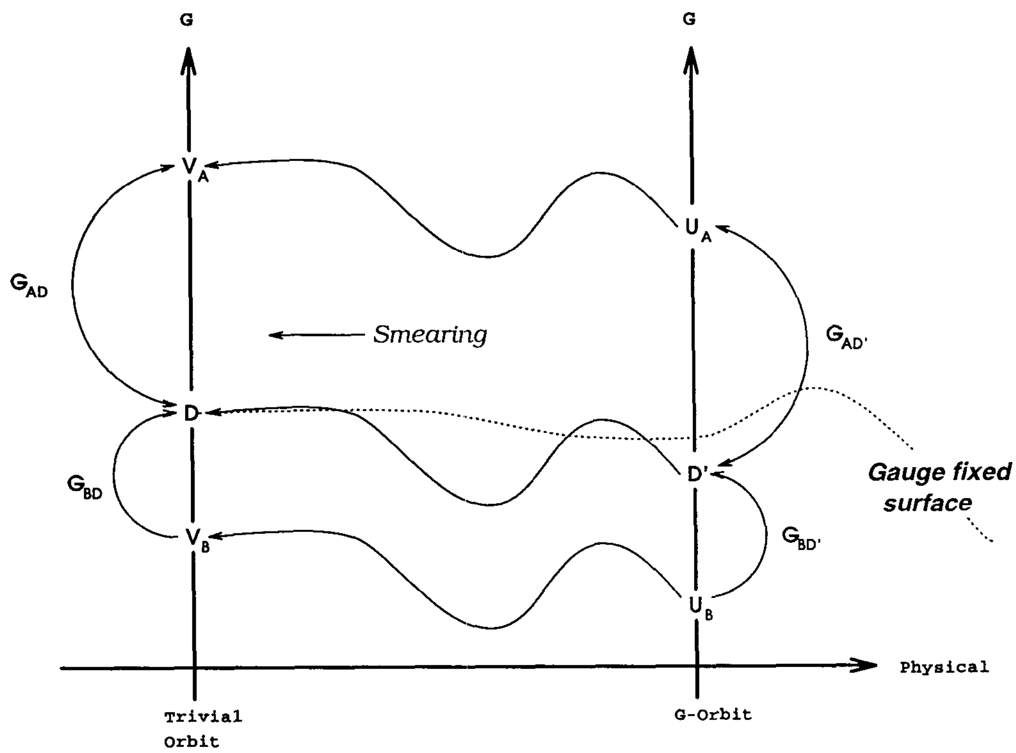


Figure 4: Gauge fix V_μ to D_μ and use the same G to fix U_μ to D'_μ.

†For U(1) gauge fields the resulting configuration D'_μ lies on the gauge fixed surface and satisfies the Landau condition, because U(1) gauge transformations are linear.

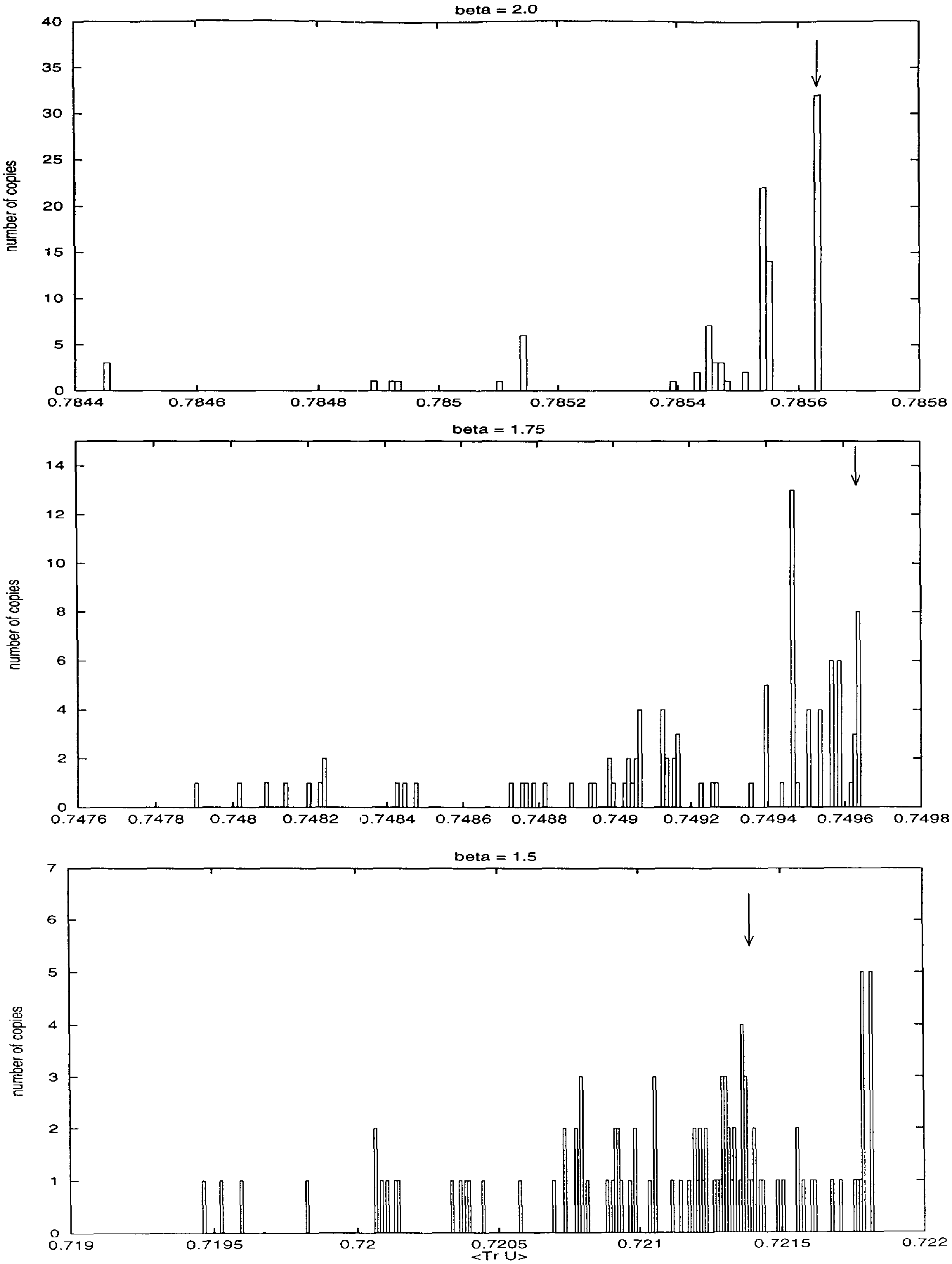


Figure 7: Standard Gribov copies and the <u>*unique copy*</u> ↓ from smearing.

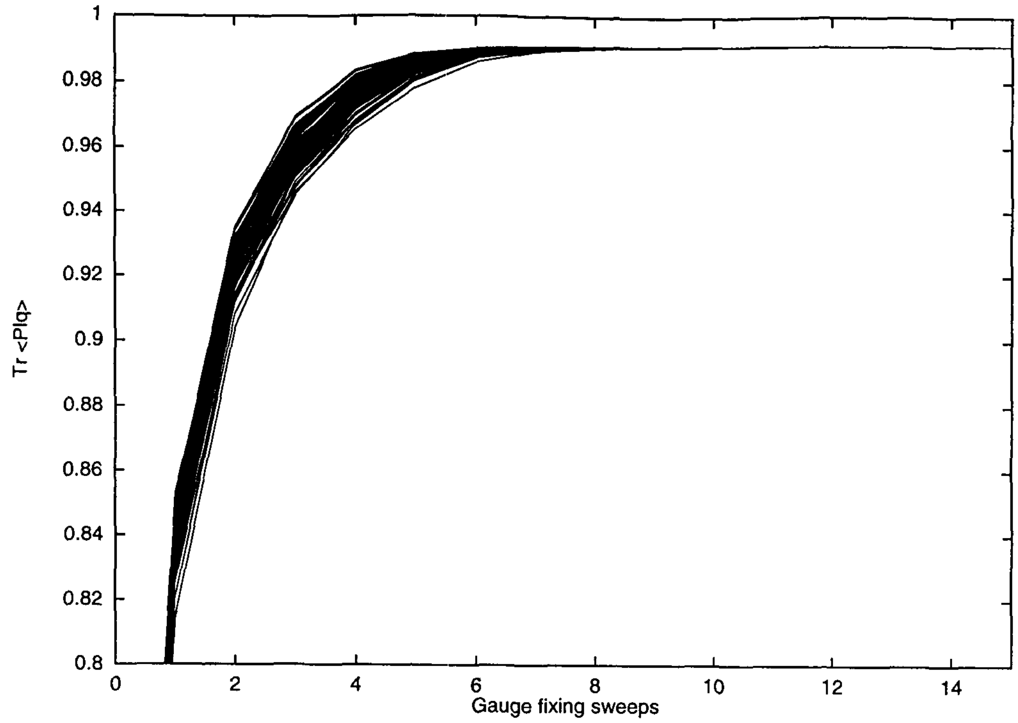


Figure 8: The gauge fixing history of the smeared links showing that $I(V_\mu, G)$ is very smooth.

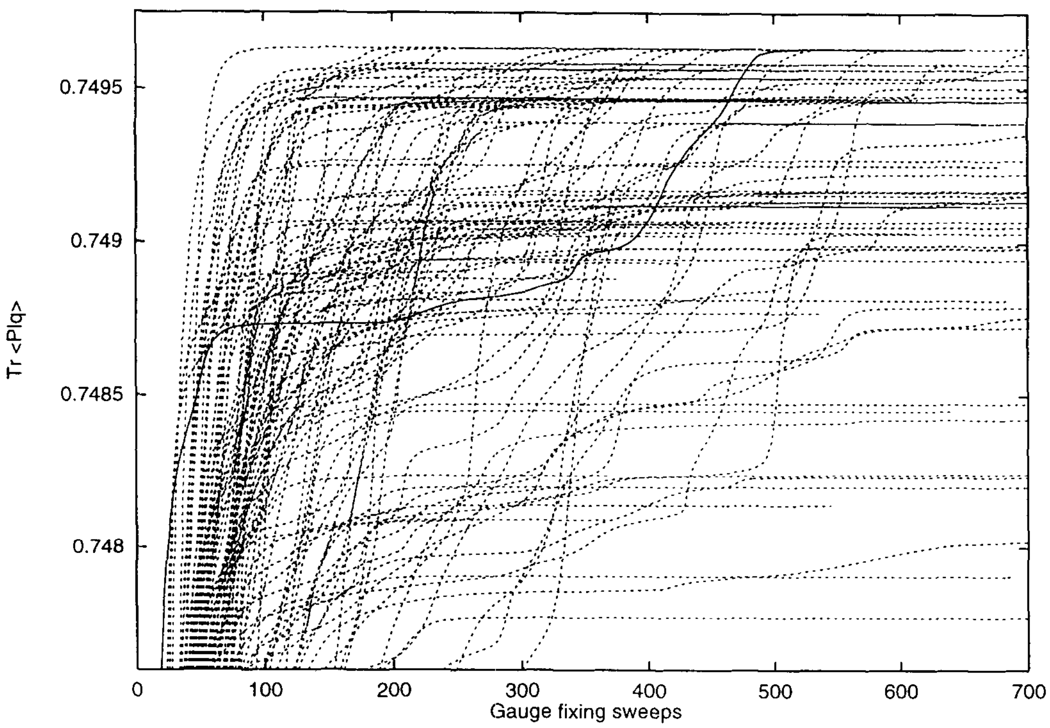


Figure 9: Gauge fixing histories: standard Gribov copies $(\cdots)$ and the unique copy $(—)$ from smeared gauge fixing.

$\star$ We end by summarizing our algorithm in its simplest form:

- Recursively smear a configuration U_μ into V_μ. We have found that smearing coefficient $w \sim 0.3$ works best, and have used as stopping condition that $\frac{1}{N}\mathrm{Tr}\ \langle\Box\rangle$ be within 10^{-5} of 1.
- Gauge fix V_μ by extremizing, $I(V,G) \equiv \sum_{x,\mu} \mathrm{Re}\ \mathrm{Tr}\ G^\dagger(x)V_\mu(x)G(x+\mu)$, and apply the same gauge transformation to U_μ.
- Finally, gauge fix U_μ as usual, by extremizing $I(U,G)$.

Lattice gauge fixing, Gribov copies and BRST symmetry

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Abstract

We discuss the problem of quantisation of Gauge Theories in presence of Gribov ambiguities.

In particular, through a very simple example, we argue that, in the framework of ghost formalism and BRST quantisation, Gribov copies are probably harmless.

Toy abelian model
(zero dimensional prototype of BRST symmetry with compact variables)

M. Testa Phys. Lett. B 429 (1998) 349

Take:

$$U = e^{iaA}$$

and define:

$$N \equiv \int_{-\pi/a}^{\pi/a} dA$$

The "gauge-fixed" version of the functional integral is:

$$N' = \int_{-\frac{\pi}{a}}^{\frac{\pi}{a}} dA \int_{-\infty}^{+\infty} d\lambda \int d\bar{c}\, dc\, e^{-\frac{\alpha}{2}\lambda^2} e^{\delta[\bar{c} f(A)]}$$

where δ denotes the (idempotent $\delta^2 = 0$) BRST transformation:

$$\delta A = c \qquad \delta c = 0$$
$$\delta \bar{c} = \lambda \qquad \delta \lambda = 0$$

N' suffers from the Neuberger disease. Going through the same steps as before, we conclude: $N'=0$

This can also be checked through an explicit calculation:

$$N' = \int_{-\frac{\pi}{a}}^{\frac{\pi}{a}} dA \int_{-\infty}^{+\infty} d\lambda \int d\bar{c}\, dc\, e^{-\frac{\alpha}{2}\lambda^2} e^{i\lambda f(A)} e^{-\bar{c} f'(A) c} =$$

$$= \int_{-\frac{\pi}{a}}^{\frac{\pi}{a}} dA \int_{-\infty}^{+\infty} d\lambda\, e^{-\frac{\alpha}{2}\lambda^2} e^{i\lambda f(A)} f'(A) =$$

$$= \sqrt{\frac{2\pi}{\alpha}} \int_{-\frac{\pi}{a}}^{\frac{\pi}{a}} df(A) e^{-\frac{f(A)^2}{2\alpha}} = 0$$

for periodic $f(A)$.

Why should $f(A)$ be periodic?

In order to satisfy BRST Identities
(crucial to show independence on α of gauge-invariant observables):

$$\langle \delta\Gamma \rangle = 0$$

In fact, if:

$$\Gamma \equiv \delta[\bar{c}F(A)] = i\lambda F(A) - \bar{c}F'(A)c$$

we have:

$$\langle\Gamma\rangle \equiv \int_{-\frac{\pi}{a}}^{\frac{\pi}{a}} dA \int_{-\infty}^{+\infty} d\lambda \int d\bar{c}dc e^{-\frac{\alpha}{2}\lambda^2} e^{\delta[\bar{c}f(A)]}\Gamma =$$

$$= \sqrt{\frac{2\pi}{\alpha}} \int_{-\frac{\pi}{a}}^{\frac{\pi}{a}} dA \frac{d}{dA}\left(F(A)e^{-\frac{f(A)^2}{2\alpha}}\right) = 0$$

for periodic "gauge fixing condition", $f(A)$, and $F(A)$.

In particular, for $\alpha = 0$:

$$N' = \lim_{\alpha\to 0} \int_{-\frac{\pi}{a}}^{\frac{\pi}{a}} dA \int_{-\infty}^{+\infty} d\lambda\, e^{-\frac{\alpha}{2}\lambda^2} e^{i\lambda f(A)} f'(A) =$$

$$= 2\pi \int_{-\frac{\pi}{a}}^{\frac{\pi}{a}} dA\, f'(A)\, \delta(f(A)) = 0$$

which displays the Gribov nature of the paradox (a periodic $f(A)$ has an even number of alternating zeroes).

Solution

$$\delta \Rightarrow \delta_P$$

$$\delta_P(x) \equiv \sum_{n=-\infty}^{+\infty} \delta(x - n\frac{2\pi}{a}) = \frac{a}{2\pi}\sum_{n=-\infty}^{+\infty} e^{inax} \equiv \frac{a}{2\pi}\sum_{n=-\infty}^{+\infty} e^{i\lambda_n x}$$

$$\lambda_n \equiv na$$

so that $N' \Rightarrow N''$ where:
(we extend the formulation with the inclusion of a λ^2 term)

$$N'' = a\sum_{n=-\infty}^{+\infty} \int_{-\frac{\pi}{a}}^{\frac{\pi}{a}} dA e^{-\frac{\alpha}{2}\lambda_n^2} e^{i\lambda_n f(A)} f'(A)$$

This formulation admits BRST symmetry, under:

$$\delta\bar{c} = i\lambda_n$$

and we have:

$$N'' = a\sum_{n=-\infty}^{+\infty} \int_{-\frac{\pi}{a}}^{\frac{\pi}{a}} dA \int d\bar{c}\, dc e^{-\frac{\alpha}{2}\lambda_n^2} e^{\delta[\bar{c} f(A)]}$$

Invariance under these transformations is enough for all purposes (gauge invariance)

We can now chose a "gauge fixing" $f(A)$ such that:

$$f(A+\frac{2\pi}{a}) = f(A)+\frac{2\pi}{a}$$

which evades Neuberger's argument ($f(A)$ has an odd number of zeroes), still leaving a periodic integrand.

The mechanism is general:

since $f(A)$ is not periodic, $\exp(it\lambda_n f(A))$ is only periodic for integer t's.

When $a \to 0$ we recover the continuum BRST formulation (Fourier series -->Fourier integral):

$$\lim_{a\to 0} N'' = \lim_{a\to 0} a \sum_{n=-\infty}^{+\infty} \int_{-\frac{\pi}{a}}^{\frac{\pi}{a}} dA e^{-\frac{\alpha}{2}\lambda_n^2} e^{i\lambda_n f(A)} f'(A) =$$

$$= \int_{-\frac{\pi}{a}}^{\frac{\pi}{a}} dA \int_{-\infty}^{+\infty} d\lambda \; e^{-\frac{\alpha}{2}\lambda^2} e^{i\lambda f(A)} f'(A)$$

Gauge-fixing approach to Lattice Chiral Gauge Theories

Maarten Golterman

collab: Wolfgang Bock
Ka-Chun Leung
Yigal Shamir

Introduction

LGT is finite
Need anomaly in cont. limit
$\Rightarrow$ either
fermions are anom. free $\to$ doublers
or
chiral symm. is explicitly broken

fermions are anomaly free:

$\{D, \gamma_5\} = 0 \quad \longrightarrow$ doublers, theory is vectorlike

$\{D, \gamma_5\} = D\gamma_5 D \quad \longrightarrow$ doubler-free construction may exist (Lüscher) (abelian ok)

Explicit chiral symm. breaking:
Fermions couple to longitudinal gauge field
– nonperturbative!

"Lattice artifact" (usually) destroys chiral spectrum.

This talk:
Control long. modes through gauge fixing

Other approach: "interpolation"
(Bodwin, 't Hooft, Hernandez/Sundrum/Boucaud,
Schierholz et al., Slavnov et al., ...)

Gauge fixing ("Rome approach")

Borelli, Maiani, Rossi, Sisto & Testa, revived by Shamir

$$S = S_{gauge} + S_{fermion} + S_{gaugefix} + S_{ghosts} + S_{counterterms}$$

S_{gauge}: plaquette

$S_{fermion}$: Wilson breaks gauge inv.

$S_{gaugefix}$: $\sim \frac{1}{2\xi}(\partial_\mu A_\mu)^2$ cov., renormalizable

S_{ghosts}: ?

$S_{counterterms}$: use BRST (?) to fix

perturbation theory → proper scaling behavior (like QCD)

counterterms: 1 dim 2 operator, (gauge-field mass)
0 dim 3
many dim 4
tune nonpert., pert. ?

Questions:

1.) What is $S_{gaugefix}$ ⟺ phase diagram ?

2) Fermion content ?

3) S_{ghost} ? ⟷ Gribov copies (nonabelian)

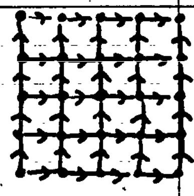
1) S_{gaugefix} :

want lattice pert. th. to work →

Don't take $S^{\text{naive}}_{\text{gaugefix}} = \frac{1}{2\xi g^2} \sum_x \left[\sum_\mu \left(\operatorname{Im} U_\mu(x) - \operatorname{Im} U_\mu(x-\mu) \right) \right]^2$

$U_\mu(x) = 1$ has many Gribov copies:

e.g. $g(x) = \begin{cases} -1 & x = x_0 \\ 1 & x \neq x_0 \end{cases}$



(there are more!)

There exists a gauge-fixing action

$$S_{\text{gaugefix}} = S^{\text{naive}}_{\text{gaugefix}} + \tilde{r}\, S_{\text{irrelevant}}$$

which has

- $S_{\text{gaugefix}}(U) \geq 0$ and $S_{\text{gaugefix}}(U) = 0 \Leftrightarrow U_\mu(x) = 1$
- $S_{\text{gaugefix}}(U) \sim \frac{1}{2\xi} (\partial_\mu A_\mu)^2$ + irrel. terms

Note : – no lattice BRST symmetry

$$- \frac{1}{2\xi} (\partial_\mu A_\mu)^2 \xrightarrow[A_\mu = \partial_\mu \lambda]{} \frac{1}{2\xi} (\Box \lambda)^2$$

Full action for $U(1)$:

$$S = S_{\text{gauge}} + S^{\text{Wilson}}_{\text{fermion}} + S_{\text{gaugefix}} + \sum_x \bar{c}(x) \Box c(x) + S_{\text{c.t.}}$$

$$S_{\text{c.t.}} = -\kappa \sum_{x,\mu} \left[U_\mu(x) + U^\dagger_\mu(x) \right] + \text{dim. 4}$$

Phase diagram (no fermions, small g)

$$V_{\text{classical}}(U) = \frac{1}{\text{vol.}}\, \tilde{r}\, S_{\text{irrel.}}(U) - \kappa \sum_\mu \left(U_\mu + U_\mu^\dagger\right) + \dots$$

$U_\mu = \exp(igA_\mu)$

$$= \tilde{r}\,\frac{g^4}{4\xi} \sum_\mu A_\mu^2 \sum_\nu A_\nu^4 + \dots + \kappa g^2 A_\mu^2 + \dots$$

$\kappa > 0$: min. at $A_\mu = 0$, $m_A^2 > 0$ broken phase

$\kappa = \kappa_c = 0$: min. at $A_\mu = 0$, $m_A^2 = 0$ critical point

$\kappa < 0$: min. at $A_\mu = \pm\left(\dfrac{-\xi\kappa}{3\tilde{r} g^2}\right)^{1/4}$ $\forall \mu$

new phase (FMD) with broken rot. symm. !

Take continuum limit at $\kappa \downarrow \kappa_c$:

recover massless gauge field.

(other counterterms: see MG & Shamir)

3) Nonabelian speculations

A. Ghosts → put in Fadeev-Popov det.

- with absolute value : no good
- without :

naive : Gribov copies cancel pairwise

→ correct weight for each orbit?

but Neuberger's theorem :

$$\int d\mu\, \mathcal{O}_{BRST\text{-}inv} \exp\left[-S_{BRST\text{-}inv}\right] = 0\ !$$

(however: see Testa, hep-lat/9803025)

B. No ghosts → Jona-Lasinio, Parrinello, Zwanziger

Insert

$$1 = \frac{\int \mathcal{D}g\, e^{-S_{gf}(A^g)}}{\int \mathcal{D}h\, e^{-S_{gf}(A^h)}} \quad \text{into} \quad \int \mathcal{D}A\ O_{inv}(A)\, e^{-S_{inv}(A)}$$

change variables:

$$= \underbrace{\int \mathcal{D}g}_{=1} \int \mathcal{D}A\ O_{inv}(A)\ \frac{e^{-S_{inv}(A)-S_{gf}(A)}}{\underbrace{\int \mathcal{D}h\, e^{-S_{gf}(A^h)}}_{\text{instead of } [\det FP(A)]^{-1}}}$$

- rigorously correct !
- positive measure !

But $S(A,\psi)$ is not invariant....

Gauge-Invariant Variables in Gauge Theories

May 25-28, 1999

Organizers: Peter Orland, Robert Pisarski and Pierre Van Baal

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RIKEN BNL Research Center

Workshop on Gauge-Invariant Variables in Gauge Theories
May 25–29, 1999
Physics Department - Room 2-160
Organizers:
Pierre van Baal, Peter Orland and Rob Pisarski

AGENDA

Tuesday 25 May

Morning

9:00		Registration - Rm. 2-160
9:45		Opening Remarks
10:00	I. Kogan	*Gauge Fields and AdS/CFT* (45 min.)
10:45		Coffee
11:00	V.P. Nair	$2+1$ *Yang-Mills Theory* (45 min.)
11:45		Lunch

Afternoon

13:30	W. Brown	*Instantons and the QCD Vacuum Wave Functional* (30 min.)
14:00		Coffee
14:30		*Panel Discussion - The Variational Principle in QCD* - W. Brown, I. Kogan, E. Moreno, P. Orland (chair) (1 hr.)
15:30		Rest for all jet-lagged participants

Wednesday 26 May

Morning

9:15	D. Zwanziger	*A Confinement Scenario in the Coulomb Gauge* (45 min.)
10:00	P. Orland	*Potential Topography on Orbit Space* (45 min.)
10:45		Coffee
11:00	K. Fujikawa	*The Gribov Problem and BRST Formalism* (45 min.)
11:45		Lunch

Afternoon

13:30	M. Schaden	*Resolution of the Gribov Problem* (30 min.)
14:00	K. Haller	*Dynamics and Topology in a Gauge-Invariant Formulation of QCD* (30 min.)
14:30		Coffee
15:00	E. Bagan	*Gauge-Invariant Charged Variables* (30 min.)
15:30		Panel Discussion - *What Do We Really Know About Orbit Space?* - J. Fuchs (chair), V.P. Nair, P. van Baal, D. Zwanziger (1 hr.)

Forthcoming RIKEN BNL Center Workshops

Title:	**OSCAR II: Predictions for RHIC**
Organizers:	Y. Pang/M. Gyulassy
Dates:	July 8–16, 1999

Title:	**Coulomb and Pion-Asymmetry Polarimetry and Hadronic Spin Dependence at RHIC Energies**
Organizer:	E. Leader
Date:	August 18, 1999

Title:	**RHIC Spin**
Organizer:	G. Bunce
Dates:	Oct. 6–8, 1999

Title:	**Event Generator for RHIC Spin Physics III**
Organizers:	N. Saito and A. Schaefer
Date:	March 6–20, 2000

Title:	**Prediction and Uncertainties for the RHIC Spin Physics Program**
Organizers:	W. Vogelsang and J. Qiu
Date:	March 6–31, 2000

Title:	**Equilibrium and Non-Equilibrium Aspects of Hot, Dense QCD**
Organizers:	H. de Vega, D. Boyanovsky, and D. Rischke
Dates:	July 17-30, 2000

For information please contact:

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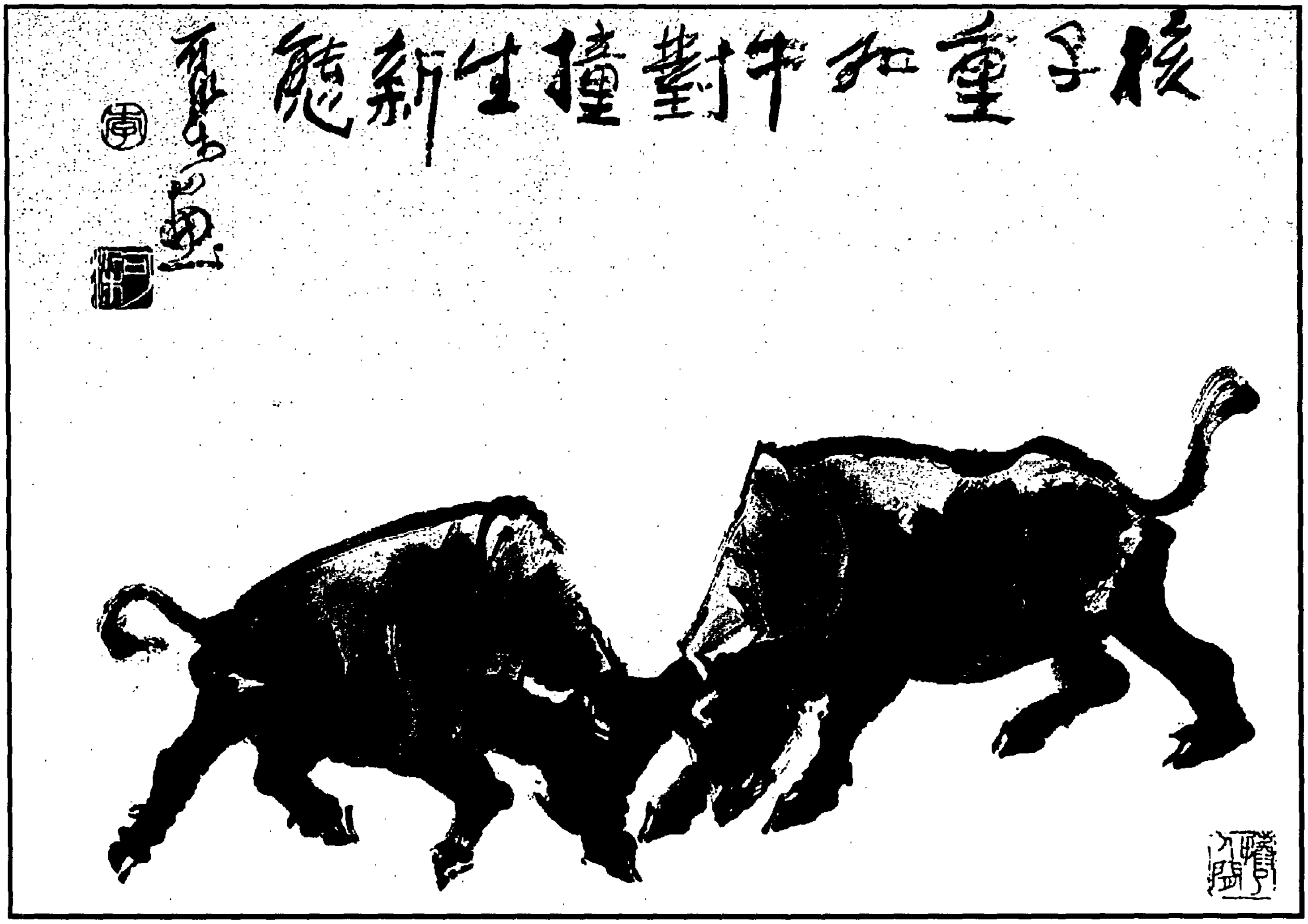
RIKEN BNL RESEARCH CENTER

GAUGE-INVARIANT VARIABLES IN GAUGE THEORIES

MAY 25–29, 1999



Li Keran

Nuclei as heavy as bulls
Through collision
Generate new states of matter.
T. D. Lee

Speakers:

G. Alexanian	E. Bagan	J. Bjoraker	W. Brown	A. Cucchieri
K. Fujikawa	G. Gabadadze	M. Golterman	K. Haller	J. Hetrick
Y. Hosotani	A. Jevicki	R. Khuri	I. Kogan	M. Ogilvie
P. Orland	O. Philipsen	V.P. Nair	R. Ray	M. Schaden
S. Tanimura	M. Testa	P. van Baal	D. Zwanziger	

Organizers: Pierre van Baal, Peter Orland and Rob Pisarski