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Using Local Born and Local Rytov Fourier Modeling and Migration methods for investigation of heterogeneous structures

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ABSTRACT

During the past few years, there has been interest in developing migration and forward modeling approaches that are both fast and reliable particularly in regions that have rapid spatial variations in structure. We have been investigating a suite of modeling and migration methods that are implemented in the wavenumber-space domains and operate on data in the frequency domain. The best known example of these methods is the split-step Fourier method (SSF). Two of the methods that we have developed are the extended local Born Fourier (ELBF) approach and the extended local Rytov Fourier (ELRF) approach. Both methods are based on solutions of the scalar (constant density) wave equation, are computationally fast and can reliably model effects of both deterministic and random structures. We have investigated their reliability for migrating both 2D synthetic data and real 2D field data [Huang et al, 1997; Huang et al., 1998]. We have found that the methods give images that are better than those that can be obtained using other methods like the SSF and Kirchhoff migration approaches [Fehler et al., 1998]. More recently, we have developed an approach for solving the acoustic (variable density) wave equation and have begun to investigate its applicability for modeling one-way wave propagation. The methods will be introduced and their ability to model seismic wave propagation and migrate seismic data will be investigated. We will also investigate their capability to model forward wave propagation through random media and to image zones of small scale heterogeneity such as those associated with zones of high permeability.

Key Words:

INTRODUCTION

With increased emphasis on finding petroleum in regions of complex structure, there is increased interest in finding seismic modeling and migration methods that are both fast and more reliable than the Kirchhoff approach. We have been investigating a suite of migration methods that are implemented in the wavenumber and space domains and operate on data in the frequency domain. The best known example of these methods is the split-step Fourier method (Stoffa et al. 1990). Two methods that we have developed, whose implementation procedure is similar to that of the SSF method are the extended local Born Fourier migration approach (Huang et al. 1997) and the extended local Rytov Fourier migration approach (Huang et al., 1998). Both of these new methods use approximations that are less restrictive than the conventional SSF approach and tests using numerical data for the SEG/EAGE salt model and the Marmousi model demonstrate that they give better images than those obtained using the SSF approach (Huang et al. 1997; Huang et al. 1998). In addition to the methods themselves, we have investigated implementation issues such as the use of the multiple reference velocity approach introduced by Kessinger et al. (1992) into the SSF approach to limit the velocity perturbation and thus improve the reliability of the images obtained (Huang et al. 1997).

To date, we have reported on tests of these methods using numerical data. We will now discuss application of these methods for both modeling and migration.

SPLIT-STEP FOURIER METHOD

The constant density scalar wave equation in the frequency domain is

$$\left[\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}+\frac{\omega^2}{v^2(\mathbf{x}_T,z)}\right]p(\mathbf{x}_T,z;\omega)=0$$

where $\mathbf{x}_T \equiv (x,y); p(\mathbf{x}_T,z;\omega)$ is the pressure in the frequency domain, $v(\mathbf{x}_T,z)$ is the velocity of the medium, and ω is the circular frequency. The spilt-step Fourier approach is a one-way wave propagation method that is based on the use of a small angle approximation to arrive at an algorithm for propagation across layers perpendicular to the main propagation direction using the following propagator

$$p(\mathbf{x}_T,z_{i+1};\omega)=p_0(\mathbf{x}_T,z_{i+1};\omega)e^{i\omega\left[\int_{z_i}^{z_{i+1}}\Delta s(\mathbf{x}_T,z)dz\right]}$$

where $p_0(\mathbf{x}_T,z_{i+1};\omega)$ is the wavefield at z_{i+1} obtained by propagation of the wavefield at z_i across the interval Δz where the interval is assumed to have homogeneous velocity v_0 and

$$p_0(\mathbf{x}_T,z_{i+1};\omega)=F^{-1}_{\mathbf{k}_T}\left\{e^{ik_{0z}\Delta z}F_{\mathbf{x}_T}\left\{p(\mathbf{x}_T,z_i,\omega)\right\}\right\}$$

where $k_{0z}=\sqrt{k_0^2-\mathbf{k}_T^2}$, $\mathbf{k}_T=(k_{ox},k_{0y},0)$, $k_0=\omega/v_0$ and $F_{\mathbf{x}_T}$, $F^{-1}_{\mathbf{k}_T}$ are 2D Fourier and inverse Fourier transforms over $\mathbf{x}_T$ and $\mathbf{k}_T$, respectively. The split-step Fourier propagator is applied to data in the frequency domain. Propagation is done in two steps: a free space propagation in the wavenumber domain across each depth interval using the background slowness for the interval s_0 followed by a correction for the heterogeneity described by Δs within the propagation interval, which is done in the space domain. The heterogeneity Δs is assumed to be small. The wavefield is transferred between the space and wavenumber domains using a Fast Fourier Transform. Since the propagator depends only on the local properties of the medium, one would not have to have access to the entire velocity structure of a model to propagate through a portion of the model. This gives the method a great computational advantage over some other wave-equation based methods. The reliability of the split-step Fourier method is discussed by Huang and Fehler (1998).

EXTENDED LOCAL BORN FOURIER PROPAGATOR

The extended local Born Fourier method is based on an application of the Born approximation within each layer in the model. Huang et al. (1997) showed that the propagation equation is

$$p(\mathbf{x}_T, z_{i+1};\omega) = p_0(\mathbf{x}_T, z_{i+1};\omega) + p_s(\mathbf{x}_T, z_{i+1};\omega)$$

where

$$p_s(\mathbf{x}_T, z_{i+1};\omega) = i\omega\Delta s(\mathbf{x}_T, z_i)\Delta z \bullet F^{-1}_{\mathbf{k}_T}\left\{\frac{k_0(z_i)}{k_z(z_i)} e^{ik_{0z}(z_i)\Delta z} F_{\mathbf{x}_T}\{p(\mathbf{x}_T, z_i;\omega)\}\right\}$$

Huang et al. (1997, 1998) discussed a computational approach for dealing with the possibility that $k_{oz}(z_i)$ is equal to or nearly equal to zero. Huang et al. (1997) show that the extended local Born Fourier propagator reduces to the split-step Fourier propagator for the case of small perturbation and small propagation angle.

EXTENDED LOCAL RYTOV FOURIER PROPAGATOR

The extended local Rytov Fourier method is based on an application of the Rytov approximation within each layer in the model. Sneider and Lomax (1996) present a good discussion of the relation between the Rytov and Born approximations. The method is discussed by Huang et al. (1998). The propagation equations are

$$p(\mathbf{x}_T, z_{i+1};\omega) = p_0(\mathbf{x}_T, z_{i+1};\omega) e^{i\omega\left[\int_{z_i}^{z_{i+1}} \Delta s(\mathbf{x}_T, z)dz\right]\chi(\mathbf{x}_T, z_{i+1};\omega)}$$

where

$$\chi(\mathbf{x}_T, z_{i+1};\omega) \equiv p_1(\mathbf{x}_T, z_{i+1};\omega)/p_0(\mathbf{x}_T, z_{i+1};\omega)$$

and

$$p_1(\mathbf{x}_T, z_{i+1};\omega) = F^{-1}_{\mathbf{k}_T}\left\{\frac{k_0(z_i)}{k_{oz}(z_i)} e^{ik_{0z}\Delta z} F_{\mathbf{x}_T}\{p(\mathbf{x}_T, z_i;\omega)\}\right\}$$

As discussed by Huang et al. (1998), the extended local Rytov Fourier approach is more stable than the extended local Born Fourier approach, handles amplitudes more correctly then either the SSF or ELBF approaches and gives better images when migrating numerical datasets. This approach can be shown to lead to the split-step Fourier method for small propagation angle (Huang et al., 1998).

EXTENDED LOCAL RYTOV FOURIER PROPAGATOR FOR THE VARIABLE DENSITY ACOUSTIC WAVE EQUATION

The variable-density acoustic wave equation is given by

$$\left[\frac{\partial}{\partial x}\frac{1}{\rho}\frac{\partial}{\partial x} + \frac{\partial}{\partial y}\frac{1}{\rho}\frac{\partial}{\partial y} + \frac{\partial}{\partial z}\frac{1}{\rho}\frac{\partial}{\partial z} + \frac{\omega^2}{\rho v^2(\mathbf{x}_T, z)}\right] p(\mathbf{x}_T, z;\omega) = 0$$

When density is variable, the extended local Rytov Fourier propagator is given by

$$p(\mathbf{x}_T, z_{i+1};\omega) = p_0(\mathbf{x}_T, z_{i+1};\omega) e^{i\omega\left[\int_{z_i}^{z_{i+1}} \Delta s(\mathbf{x}_T, z)dz\right]\chi(\mathbf{x}_T, z_{i+1};\omega)}$$

where

$$\chi(\mathbf{x}_T, z_{i+1};\omega) \equiv p_1(\mathbf{x}_T, z_{i+1};\omega)/p_0(\mathbf{x}_T, z_{i+1};\omega)$$

and

$$p_1(\mathbf{x}_T, z_{i+1};\omega) = F^{-1}_{\mathbf{k}_T}\left[\frac{k_0(z_i)}{k_z(z_i)} e^{ik_{0z}\Delta z} F_{\mathbf{x}_T}\{\varepsilon_a(\mathbf{x}_T, z_i) p(\mathbf{x}_T, z_i;\omega)\} + \frac{i}{k_o}\hat{\mathbf{k}}_\mathbf{o} \bullet F_{\mathbf{x}_T}\{\varepsilon_b(\mathbf{x}_T, z_i)\nabla p(\mathbf{x}_T, z_i;\omega)\}\right]$$

with

$$\hat{\mathbf{k}}_\mathbf{o} = \frac{1}{k_o}(\mathbf{k}_\mathbf{T}, k_z)$$

and

$$\varepsilon_b(\mathbf{x}_\mathbf{T}, z_i) = \int_{z_i}^{z_{i+1}} dz \frac{s_o}{2}\left[\frac{\rho_o(z)}{\rho(\mathbf{x}_\mathbf{T}, z)} - 1\right]$$

$$\varepsilon_a(\mathbf{x}_\mathbf{T}, z_i) = \varepsilon_b(\mathbf{x}_\mathbf{T}, z_i) + \int_{z_i}^{z_{i+1}} dz\left[s(\mathbf{x}_\mathbf{T}, z) - s_o(z)\right]$$

where s is slowness and ρ is density. Note that this formulation gives the correct phase correction for normal incidence propagation, e.g., that in the z direction.

MULTIPLE REFERENCE SLOWNESS APPROACH

Kessinger et al. (1992) introduced the concepts of using multiple reference slownesses into the SSF method. Since the SSF method is more reliable when the lateral slowness perturbation within a given extrapolation interval is small, the interval is broken into a number of sections, the average slowness in each section is used as a reference slowness, and the wavefield is extrapolated across that section using that slowness. Huang et al. (1997) show that the multiple reference slowness approach helps to stabilize the otherwise unstable ELBF method when migrating data in regions having large lateral slowness contrasts. Without the multiple reference slowness approach, the ELBF method cannot be applied to datasets such as the one for the SEG/EAGE salt model.

RANDOM MEDIA MODELS

We study forward wave propagation in models that are defined by an autocorrelation function of the velocity perturbation. We write the velocity as

$$V(\mathbf{x}) \equiv V_0 + \delta V(\mathbf{x}) = V_0\left(1+\xi(\mathbf{x})\right)$$

where we call $\xi(\mathbf{x})$ the fractional fluctuation of wave velocity. V_0 is chosen so that

$$V_0 = \langle V(\mathbf{x})\rangle \quad \text{and} \quad \langle \xi(\mathbf{x})\rangle = 0$$

We define the autocorrelation function (ACF) of the medium as

$$R(\mathbf{x}) \equiv \langle \xi(\mathbf{y})\xi(\mathbf{y}+\mathbf{x})\rangle$$

The magnitude of the fractional fluctuation is given by the mean square (MS) fractional fluctuation:

$$\varepsilon^2 \equiv R(0) = \langle \xi(\mathbf{x})^2\rangle$$

One form for the ACF is the Gaussian ACF

$$R(\mathbf{x}) = R(r) = \varepsilon^2 e^{-\frac{x^2}{a_x^2}} e^{-\frac{z^2}{a_z^2}}$$

where a_x and a_z are correlation distances.

Figure 1 shows an example of a Gaussian random media whose correlation distance is 5 km and fractional fluctuation is .05.

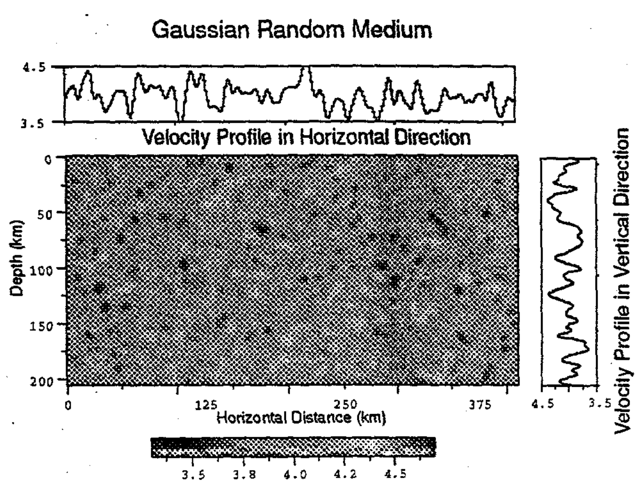

Figure 1. Gaussian random media model. Correlation length is 5 km in both directions. Fractional velocity fluctuation is .05. Cross-sections through the model in the vertical and horizontal directions are shown to the right and top of the velocity model, respectively.

COMPARISON WITH FINITE DIFFERENCE CALCULATIONS OF WAVE PROPAGATION IN RANDOM MEDIA

We use an extended local Rytov Fourier method for fast simulation of primary forward acoustic wave propagation in a 2D heterogeneous medium. The transverse derivatives in the acoustic wave equation are solved exactly. The method requires storage of wavefields only in a 1-D slice of a 2-D model and makes it possible to use a limited amount of computer memory to simulate wave propagation in a reasonably large 3-D space. It is computationally efficient due to the use of Fast Fourier Transform. It takes into account multiple forward scattering and neglects backward scattering. It calculates the exact phase for waves propagating along the main propagation direction no matter how large the lateral impedance variations are. We use the method to simulate primary forward acoustic wave propagation in heterogeneous media. The method can also be used to simulate primary reflected acoustic waves in a heterogeneous medium.

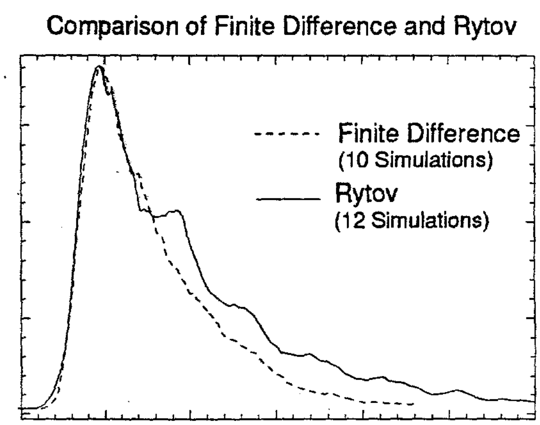

Figure 2. Comparison of finite difference and Extended local Rytov Fourier calculations of trace envelopes calculated wave propagation through a Gaussian random media. Trace envelopes are averaged from calculations through numerous realizations of random media models.

We compared the envelopes of seismograms calculated using the Rytov approach with those calculated using a 2D fourth order finite difference scheme. We average smoothed envelopes calculated for a number of random media having the same media characterization. The envelopes are not a perfect match; however, we used different media for the finite difference and Rytov calculations.

CONCLUSIONS

Dual domain methods like the extended local Born Fourier and Extended Local Rytov Fourier methods provide powerful tools for seismic modeling and wave propagation calculations. They can also be used to study forward wave propagation in heterogeneous media. With the ability to perform rapid calculations of seismic modeling in heterogeneous media and migrate data for such media, we have a powerful tool for studying seismic imaging capabilities in heterogeneous media.

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