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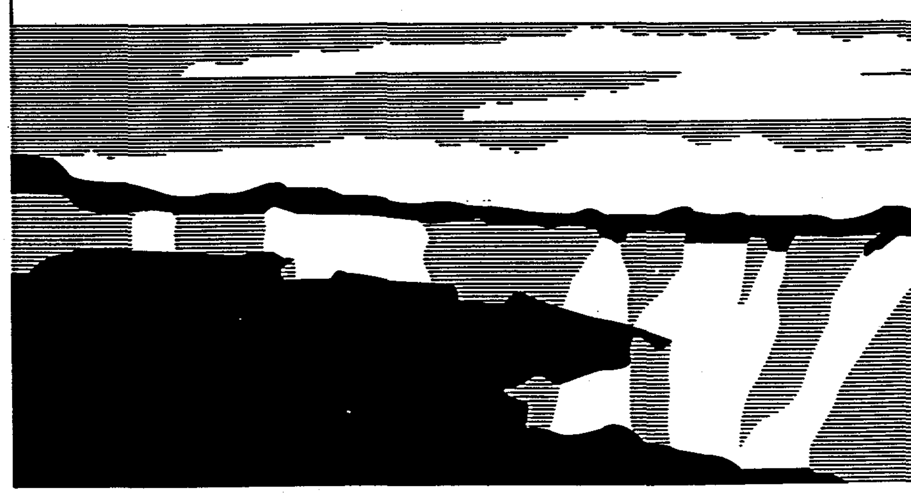

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BILINEAR SYSTEM CHARACTERISTICS FROM NONLINEAR TIME SERIES ANALYSIS

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ABSTRACT

Detection of changes in the resonant frequencies and mode shapes of a system is a fundamental problem in dynamics. This paper describes a time series method of detecting and quantifying changes in these parameters for a ten degree-of-freedom bilinear system excited by narrow band random noise. The method partitions the state space and computes mode frequencies and mode shapes for each region. Different regions of the space may exhibit different mode shapes, allowing diagnosis of stiffness changes at structural discontinuities. The method is useful for detecting changes in the properties of joints in mechanical systems or for detection of damage as the properties of a structure change during use.

NOMENCLATURE

p vector of delayed inputs and responses.

J local matrix relating measurements to states. **s = Jp**.

P matrix of past delay vectors.

F matrix of future delay vectors.

$d_{r,n}$ Euclidian distance between two vectors.

A, B, C, D Matrices describing the state space and state to measurements transformation.

k_{ij} stiffness connecting masses k and j.

c_{ij} damping coefficient between masses k and j.

k_{ref} reference stiffness, bilinear dynamic system.

NONLINEAR STRUCTURAL ELEMENTS.

During a typical vibration the test item is excited with a relatively complex waveform and a complex response obtained. The excitation and the response are composed of a wide range of frequencies. Global, linear estimation of the frequency response functions and mode shapes is used to obtain the modal characteristics of the structure. Modal characteristics are the best "average" structural properties describing the input-response relationship. For a linear system, the average properties are usually an excellent estimate of the true properties. For a nonlinear system, however, the average properties may be a poor description of the system characteristics. Nonlinear models represent changing system characteristics as functions of an input or response parameter. Nonlinear models usually require more parameters than linear models of the same order. More data is needed to accurately estimate this larger parameter set. Data requirements are quite demanding for some forms of nonlinear models.

A nonlinear model may describe response changes in the frequency domain (Bendat, 1990, Nikias, 1987, higher order spectra), changes in structural elements as functions of displacement or velocity (Masri, 1979, force state mapping), or trajectories in the state space (Farmer, 1988). Each discipline has evolved an approach to the analysis of nonlinear systems. Physicists often use the state trajectory approach, control theorists nonlinear time series models, and engineers, higher order spectral techniques or nonlinear time series modeling techniques .

NONLINEAR TIME SERIES ANALYSIS

Consider a dynamic system with force input f(t) and acceleration responses at k locations $a_k(t)$. Formulate the functional relationship:

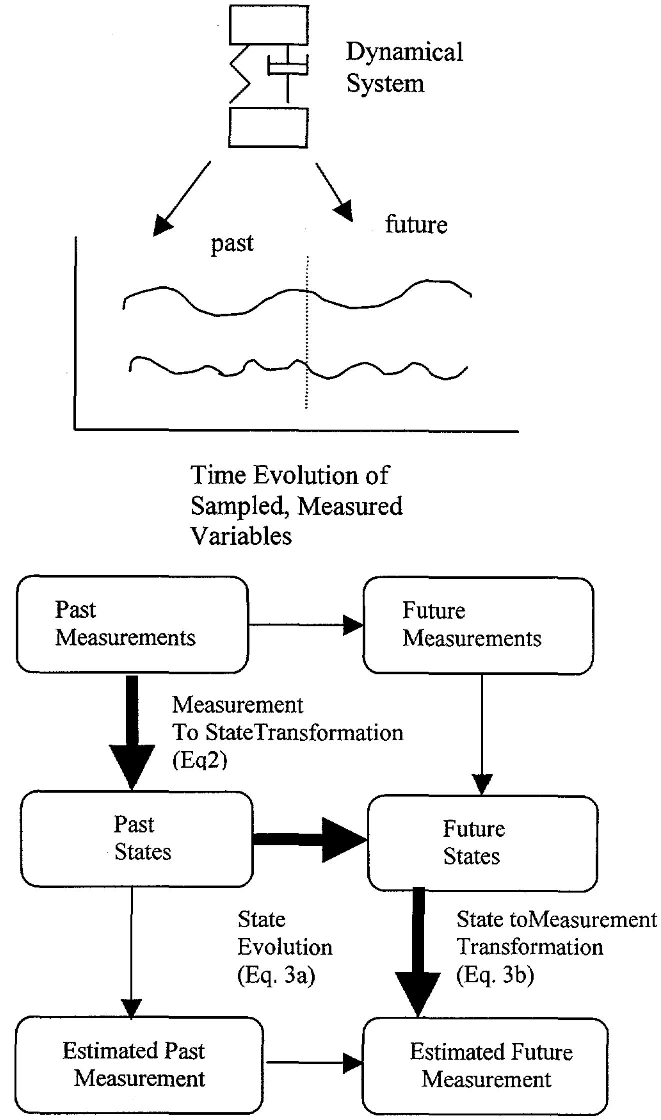
$$a(t) = f\begin{pmatrix} a(t-\tau), a(t-2\tau), \cdots, a(t-l\tau), \\ u(t), u(t-t), u(t-2t), \cdots, u(t-jt), \\ e(t-\tau), e(t-2\tau), \cdots, e(t-l\tau) \end{pmatrix} \quad (1)$$

Equation 1 relates the current acceleration response, at time t, often a vector quantity, to past acceleration responses and the input u(t). Error terms are modeled by e(t). If f is a linear functional relationship Equation 1 describes the standard ARMA (Autoregressive Moving Average) model.

The functional relationship of equation 1 is simply stated. In practice computation of the functional relationship may be quite complex. The number of lags must be selected judiciously, noise effects minimized, and the effective rank of the system estimated. Together these problems make accurate computation of the functional relationship a challenging problem. This problem is much more demanding when the functional form f is nonlinear, since an appropriate functional form must be chosen. Often we do not know the proper form a priori. Functional forms are often selected based on their ability to approximate relatively general relationships, for computational convenience, or for the potential insight they lend into system properties. Popular functional forms include polynomials (Billings, 1988), rational polynomials, radial basis functions (Casdagli, 1989) and neural networks (Lapedes, 1987).

CANONICAL VARIATE ANALYSIS

Canonical Variate Analysis (CVA) is an extension of the ARMA model technique. Originally developed by Hotelling (Hotelling,1936), this method has been improved by Larimore (Larimore,1983), whose current implementation of CVA gives accurate estimates of transfer functions and mode shapes for complex systems described by noisy signals CVA is described in detail in several references (Larimore,1983, Hunter,1997). Figure 1 schematically illustrates the method.



Canonical Variate Analysis
Figure 1

Figure 1 shows the three critical transformations involved in CVA, namely the Measurement to State Transformation, the evolution of past states to future states, and the future state to estimated measurement transformation, each outlined by a dark arrow. Equations 2 and 3 below implement the steps indicated in Figure 1.

$$\mathbf{s}(\mathbf{t}) = \mathbf{J}\begin{pmatrix} \mathbf{a}(t-\tau), \mathbf{a}(t-2\tau), \cdots, \mathbf{a}(t-l\tau), \\ u(t), u(t-t), u(t-2t), \cdots, u(t-jt) \end{pmatrix} \quad (2)$$

J selects the critical waveform components of the delay vector formed by a and u.

$$\mathbf{s}(\mathbf{t}+\tau)=\mathbf{A}\mathbf{s}(\mathbf{t})+\mathbf{B}\mathbf{u}(\mathbf{t})+\mathbf{e}(\mathbf{t}) \quad (3a)$$

$$a(t)=\mathbf{C}\mathbf{s}(\mathbf{t})+\mathbf{D}\mathbf{u}(\mathbf{t})+\mathbf{E}\mathbf{e}(\mathbf{t})+w(t) \quad (3b)$$

Equations 3a and 3b evolve the past states to future states and transform past (or future) states to measurements. The measurement to state transformation matrix is based on correlations between the past and future waveforms. A matrix formulation of the past and future waveforms is defined in Equations 4.

$$\mathbf{P}=\begin{bmatrix}\mathbf{a}(t_1-\tau),\cdots,\mathbf{a}(t_1-l\tau),\ u(t_1),\cdots,u(t_1-jt)\\ \mathbf{a}(t_2-\tau),\cdots,\mathbf{a}(t_2-l\tau),\ u(t_2),\cdots,u(t_2-jt)\\ \vdots \qquad\qquad \vdots \qquad\qquad \vdots\\ \mathbf{a}(t_k-\tau),\cdots,\mathbf{a}(t_k-l\tau),\ u(t_k),\cdots,u(t_k-jt)\end{bmatrix}$$

$$\mathbf{F}=\begin{bmatrix}\mathbf{a}(t_1), & \cdots, & \mathbf{a}(t_1+l\tau)\\ \mathbf{a}(t_2), & \cdots, & \mathbf{a}(t_2+l\tau)\\ \vdots & & \vdots\\ \mathbf{a}(t_k), & \cdots & ,\mathbf{a}(t_k+l\tau)\end{bmatrix} \quad (4)$$

Minimizing the error in the prediction of the future waveforms results in the formulation of Equation 5, which defines the measurement to state transformation **J** in terms of **P** and **F**.

$$\mathbf{J}=\mathbf{U}^{\mathrm{T}}\left[\mathbf{P}^{\mathrm{T}}\mathbf{P}\right]^{-\frac{1}{2}}$$

$$\mathbf{SVD}\left[\left[\mathbf{P}^{\mathrm{T}}\mathbf{P}\right]^{-\frac{1}{2}}\left[\mathbf{P}^{\mathrm{T}}\mathbf{F}\right]\left[\mathbf{F}^{\mathrm{T}}\mathbf{F}\right]^{-\frac{1}{2}}\right]=\mathbf{U}\mathbf{W}\mathbf{V}^{\mathrm{T}} \quad (5)$$

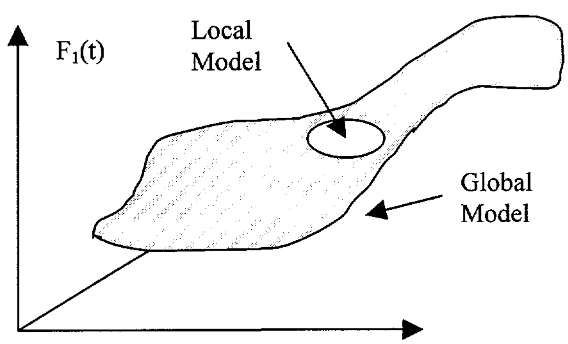
It is important to realize that the "s(t)" terms in equation 3 Derived from **s=Jp**, refer to selected critical waveforms (components) of the past and future rather than velocity and/or displacement. Matrix **J** reduces the dimension of **P** from the somewhat arbitrary value based on the number of lags l+j to the dimension of **s**, a number based on the number of functionally related orthogonal waveforms composing **P** and **F**. For a single degree of freedom,vibration system, for example, there are two states **s**, and the rank of **A** in equation 3a is two.

LOCAL CANONICAL VARIATE ANALYSIS

Local Canonical Variate Analysis combines the CVA approach defined in equations 2-5 and the idea of local modelling, which is the old idea of expanding a nonlinear system as a series of linear models. Each linear model is expanded about a different operating point. In local modeling equations 2-5 are applied to a series of local regions, as illustrated in figure 2. Each local region is defined by judiciously selecting a subset of the past and future waveforms from the global past and Future Matrices **P** and **F**. Judicious selection of the criteria for the local waveform is crucial. In general, the criteria for a local waveform is the Euclidian distance from a reference waveform and is defined by the equation:

$$d(r,n)=\left\{\sum_{i=1}^{c}\left(a_{ri}-a_{ni}\right)^2\right\} \quad (6)$$

The summation in Equation 6 is taken over the c components of the reference waveform a_r. The reference waveform is chosen as the past waveform for which a prediction of the future waveform is desired. From the complete set of measured past and future waveforms only those close (in the sense of equation 6) to the waveform for which prediction of the future is desired are used..

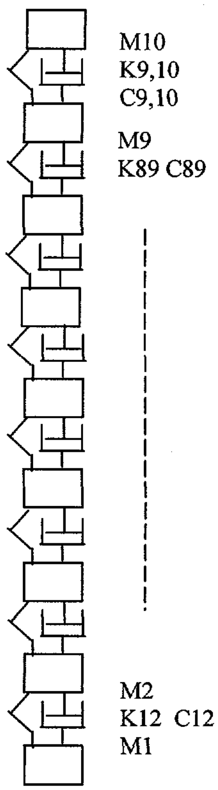


Local vs. Global Modelling
Figure 2

A different local model is used in each region of the waveform space. For a linear system, except for perturbations caused by noise, the local models are identical, since the space is defined by hyperplanes. For a nonlinear system, some systematic variation in the model form should be observed. To minimize the complexity of this functional variation careful selection of the distance criteria is critical. Preferential selection of waveforms at certain response locations (perhaps based on nonlinear effects) or, for a time varying systems, selection of waveforms which are close in time is necessary. The following example illustrates the application of the local linear CVA method. Local frequencies and local mode shapes are obtained for a nonlinear system.

A NONLINEAR SYSTEM WITH DIFFERENT STIFFNESSES IN TENSION AND COMPRESSION

Consider the system illustrated in Figure 3. This system is composed of ten masses interconnected by springs and dampers, forming a ten degree of freedom assembly. In the linear case each of the springs has a stiffness k=kr=(2*π*100). Nonlinearity is introduced by modifying k45, the stiffness interconnecting masses 4 and 5, to 0.250*kr when k_{45} is in tension and 4.0*kr when k_{45} is in compression. Differing stiffness between the cases of tension and. compression is observed in physical systems. The phenomena is caused by factors like loss of preload or changes in the properities of joints..



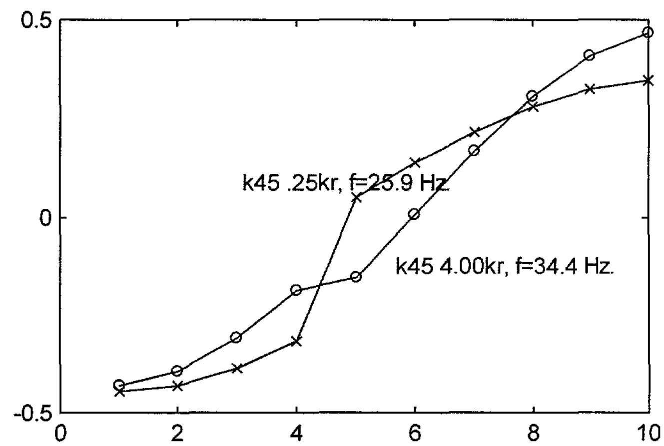
Ten Degree of Freedom Bilinear System
Figure 3

Numerical analysis of the system in Figure 3 is accomplished for three cases: k_{45}=0.250kr, k_{45}=1.00kr, and k_{45}=4.0kr. The results are summarized in Table 1 below. Eigenfrequencies are computed from the eigenvalues of the stiffness matrix. The first (rigid body) mode has been omitted from Table 1. Calculated frequencies range from a minimum of 26 Hz. to a maximum 302 Hz.

Mode	K45=0.250 kr	K45=1.00kr	K45=4.0kr
2	25.9392	31.2869	34.4308
3	57.5348	61.8034	64.1400
4	84.0750	90.7981	93.1504
5	104.2248	117.5571	126.0292
6	141.5366	141.4214	141.5420
7	145.9681	161.8034	169.4889
8	173.9412	178.2013	182.2436
9	185.4712	190.2113	192.4603
10	193.3781	197.5377	302.3715

Eigenfrequencies of The Bilinear System
Table 1

It is surprising that this rather large change in stiffness (8/1) in k_{45} results in only modest frequency changes for many modes. For example the frequency of mode 9 changes from 185 to 192 Hz. Mode 10, the highest mode, shows the largest frequency change. Computed Mode Shapes for the first mode is illustrated in Figure 4.



Mode Shapes for K45=0.250 kr
and k45=4.00 kr
Figure 4

A reasonable change in mode shape is noted for this relatively drastic change in stiffness. The lower frequency Mode shape in figure 4 corresponds to stable stiffness k_45 =0.250 kr, and the higher frequency mode shape corresponds to k_{45}=4.00 kr. When the stiffness changes dynamically from 0.25 kr to 4.00 kr the transient mode shapes may not match the two shapes in Figure 4. Still, the shapes in Figure 4 provide a baseline for something like the expected dynamic mode shapes.

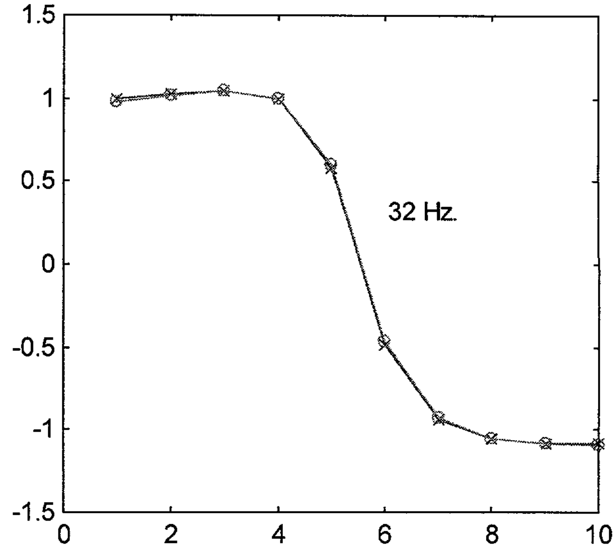
MODELLING THE BILINEAR SYSTEM USING TIME SERIES RESPONSES

The system in Figure 3 is simulated using a fourth order Runge-Kutta ordinary differential equation solver. Nine thousand data points are computed with at effective sample rate of 1000 samples/second. The force excitation, applied to mass 1 is Gaussian Noise with power concentrated between 10 Hz. and 200 Hz. This bandwidth was selected to emphasize excitation of the lower system modes. For a nonlinear system, band limited inputs are desirable to minimize the complexity of the system response.

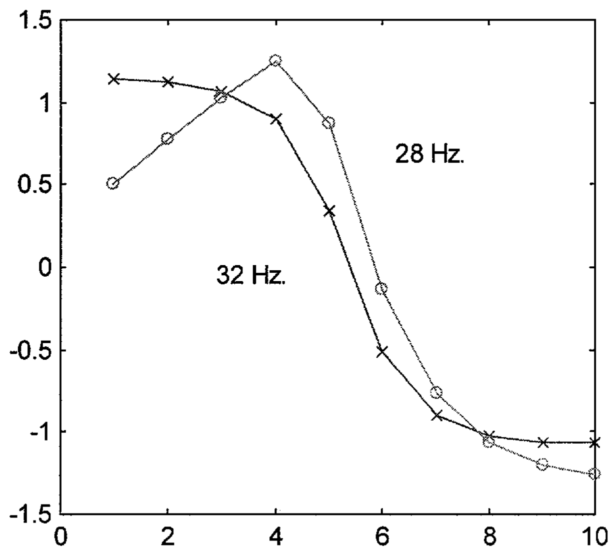
Since the major nonlinearity is a function of displacement the past and future are constructed from band limited double integrals of the 10 acceleration waveforms. Acceleration waveforms could also be used.. Integration must be done carefully, especially in a realistic case, where noise is a major contaminant of the time series response.

The model was constructed using 12 lagged displacement values at each of the ten response locations. Four thousand training points were used to build the model. The input is the driving force signal. The system response was predicted for 390 successive time values.. Eighteen states were detected in the time series responses. It is interesting to note that these eighteen states are detected in the nonlinear response when it is excited by narrow band driving force. The nonlinear nature of the system effectively couples the drive frequency to a myriad of response frequencies., including frequencies outside of the excitation range.

Bounding mode shapes are computed for two cases, large observed positive relative displacement beween masses 4 and 5, and large observed negative relative displacement between masses 4 and 5. For a reference calculation, and to partially validate the CVA model, bounding mode shapes were computed using a separate model for the linear case , where k45 is invariant with respect to displacement. Bounding mode shapes for the linear case are shown in Figure 6 for the second mode. The computed modal frequency of 32 Hz. This frequency compares favorably with the theoretical linear second modal frequency of 31.3 Hz. For the nonlinear case, the bounding mode shapes are illustrated in figure 7. Estimated frequencies range from a low of 28 Hz. To a high of 32 Hz.. The flexibility change between the mode extremes in Figure 7 is estimated using the second spacial derivative of the mode shapes to obtain the flexibility deviation between the two bounding mode shapes in Figure 7. Delta flexibility is plotted vs. mass location in Figure 8. The maximum flexibility change occurs in the vicinity of the link between masses 4 and 5.



Experimentallly Observed First Local Mode Shapes, k45 Linear
Figure 6



Estimated Mode Shape Extremes, Bilinear System, k45 Nonlinear
Figure 7

Figures 6,7, and 8 illustrate the experimental detection of a significant stiffness change for a somewhat contrived nonlinear system using local linear modeling techniques. The first mode is used to illustrate changes in mode shape. Higher modes also show changes in Frequency though the changes in mode shape are the most striking.

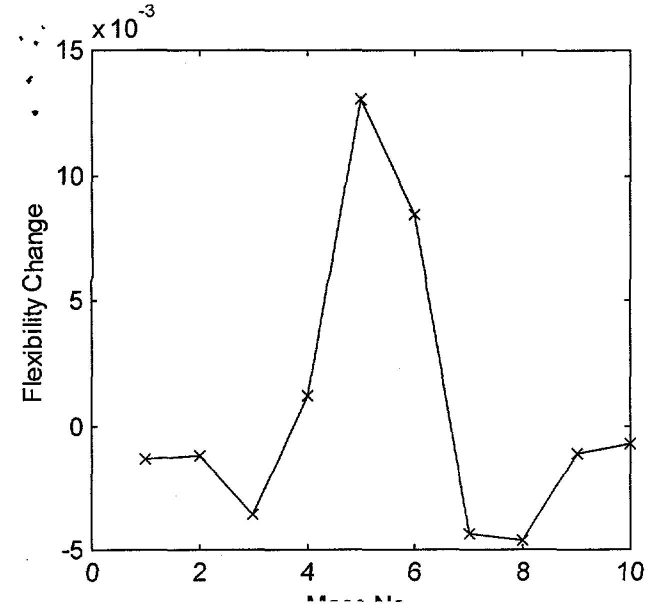
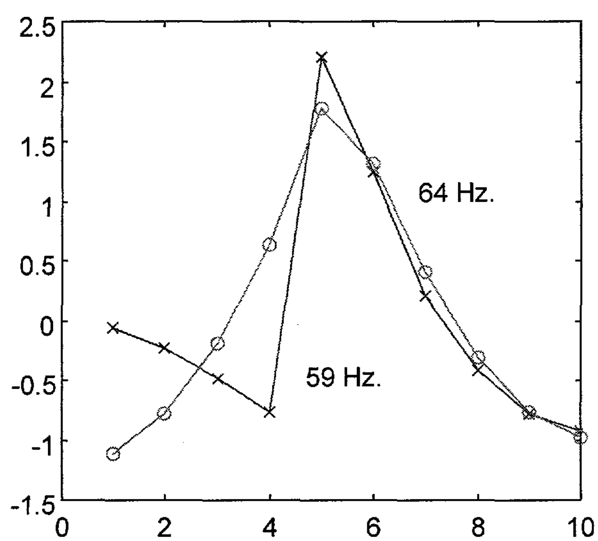


Figure 7 vs. Mass Location.
Figure 8

Figure 9 illustrates the extreme estimated shapes for the 3rd mode. Most modes show considerable differences in bounding mode shapes. Model mode shapes for the nonlinear case are invariably complex. Generally the real part of the mode shape dominates, so the real part is shown in Figures 5-8.



Estimated Mode Shape Extremes, Bilinear System
K45 Nonlinear, Mode 3
Figure 8

The bounding frequencies detected by the local model are 28 Hz. And 32 Hz. for the second mode and 59 Hz. and 64 Hz. for the third mode. An equivilant linear system, with the bounding stiffnesses, would have second modal frequencies of 25 Hz. and 34 Hz. and third modal bounding frequencies of 58 and 64 Hz. Estimated model frequencies are deemed reasonable. Model bounding mode shapes have a resembelance to the bounding analytical mode shapes. The model shapes do show strong differences, which appear related to the flexibility changes in the vicinity of k45.

Application to Physical Systems

Local Nonlinear CVA was applied to a wind turbine blade subjected to varying frequency sinusoidal vibration. Long term vibration damaged the blade, changing the frequency and shape of the first mode. The change in mode shape was readily detected when time was used as the local selection criteria. The damage location was determined using the flexibility change between the damaged and undamaged mode shapes.

Conclusions

Results for This System/Advantages

This paper illustrates a nonlinear time series analysis method, called Local linear canonical variate analysis, which fits a series of linear multidimensional functions to "local " regions of the response time series. A ten degree-of-freedom, analytically simulated nonlinear system served as an example to illustrate the technique. extract Local eigenvalues and eigenvectors of the state transtition matrix were estimated. Eigenvectors were multiplied by the state to measurement transition matrix to obtain local displacement eigenshapes. These were called local modes of the system, though a more appropriate name is local eigenshapes, since the computed shapes may not bear a direct relation to ordinary system modes. For this simple and noise free system eigenshape changes characteristic of different local system states were detected and the change in stiffness tracked to k45, the truly variable stiffness. A practical example, where a change in stiffness was detected in a real physical system, was also noted.

Nonlinear time series analysis methods are quite powerful and sensitive. For at least some simple cases, changes in eigenshapes characteristic of damage onset are detected. In many cases, the method seems almost too sensitive, especially when acceleration signals are used for analysis.

Method Limitations and Future Work

As Larimore (Larimore, 1983) demonstrated, the method of Canonical Variate Analysis is an effective, powerful means for estimating transfer functions for linear systems, even when these systems are characterized by small datasets. For nonlinear systems model optimization, function selection and noise reduction are very daunting problems. For special cases like those mentioned in this paper successful modeling using local linear functional methods has been achieved. Central features of the successful cases include appropriate density of measurment locations (so the true location of flexibility changes can be located), limited active dimensions in the estimated state space (a few to prehaps 20 dimensions) and relatively noise free datasets. General cases in which the dimensionality is high (perhaps a hundred), and the input excites strong, complex, nonlinear behavior are much more daunting. Even for the more tractable cases E general use of this technique requires either time consuming trial and error or automated model optimization techniques.. Optimal function selection and noise reduction are potentially computationally intensive tasks. Reasonable local eigenshapes for complex, noisy or poorly instrumented (in the sense of transducer density) systems have not been demonstarted.

For local linear models it is improtant to select a sensible criteria for the definition of ";local". Local may mean local in time, acceleration amplitude or amplitude of some other location specific parameter . In some cases this parameter may be selected based on a-priori knowledge of the system. Finding the best definition of local is a tricky task.

Our plans for future work include the systematic incorporation of better functional fitting methods into the measurement to state computation and more effective model optimization techniques.

Nonlinear Canonical Variate Analysis fills a particular niche in the suite of methods available for experimental characterization of nonlinear behavior. For very well defined systems first principles analysis, or nonlinear finite element analysis appear optimal. For experimental characterization of systems not quite so well characterized, the force state mapping technique of Masri looks very promising. Nonlinear CVA has a place in the analysis of systems which are too poorly characterized or instrumented to be amenable to force state mapping, and for which some sort of functional response surface is desired. For these sytems, higher order frequency response functions like the bispectrum and trispectrum also provide interesting and useful approaches.

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