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Implementations of the Superhistory Method*

by

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The superhistory method[1] is incorporated, in different implementations, into two versions of MONK.[2] Below we intercompare the efficiencies of these implementations via the Figure Of Merit ("FOM"), and compare the efficiencies of each with that of conventional Monte Carlo ("MC"). Finally, we suggest "preferred" versions of MC for eigenvalue calculations. Here, FOM $\approx 1/N\sigma^2$, where N is the number of histories, and σ^2 is the variance of a quantity of interest.

In the criticality-safety version MONK, fission is simulated as suggested in Ref. 1 ("Method-1"). Every absorption site is a potential fission site, with weight $W = \sigma_f/(k\times\sigma_a)$, where σ_f and σ_a are fission and absorption cross sections, and k is an estimate of the eigenvalue. If $W < 1$, W is taken as pf, the fission probability. A Method-1 fission produces, on average, ν offspring at each site. The reactor-physics MONK uses the standard MC fission treatment ("Method-0"), i.e. $\nu\times W$ is the average number of neutrons born in a fission, and pf = 1. For consistency, we take absorption sites as potential fission sites in both methods. For $\nu = 1$ and a single generation per supergeneration, conventional and superhistory methods coincide.

I. One-Region One-Group Slab Cell

This problem configuration is a reflected slab, 15.5 cm thick, divided into three equally thick edit-zones. In each, we estimate the absorption rate and its variance. In addition, we compute the eigenvalue and its variance, using collision estimators. Table 1 lists the problem parameters, variance, σ_k^2, in eigenvalues, and average, σ_{abs}^2, of absorption rate variances over edit zones with 500 absorptions per second per cell.

We see that Method-1 flux shapes are noisier than shapes generated by conventional MC, perhaps because:

1. in Method-1 the number of fission-neutron starters in a supergeneration may fluctuate substantially among supergeneration,[3] and
2. the number of fission sites is generally smaller in Method-1 than in conventional MC, so that less information resides in the fission source.

In the row-3 computation of Table 1, random selection of source-sites occurs every tenth generation, while in the row-1 MC it occurs after each generation. Apparently, the extra random-sampling produces the observed increase in variance, as predicted in Ref. 1.

One can, however, easily eliminate the random source-sampling in all methods discussed above. In conventional MC, for example, define $W(i) = \nu(i)\times\sigma_f(i)/(k\times\sigma_a(i))$. Let W_t be the sum of the W(i). Define $W(i)' = W(i)\times N_s/W_t$, where N_s is the desired number of starters per generation. Now, use W' to determine the mean number of offspring at site i. N_s will be the expected number of starters. In Section II, "conventional" and Method-0 MC have been modified as above. Next we turn to a slightly different model problem to test the robustness of our observations.

II. Three-Region One-Group Slab Cells

In problem sets 1-3, respectively, $\sigma_{f1,3} = 0.1/\nu$, $0.05/\nu$, and 0. Here $\sigma_{f1,3}$ is σ_f (in cm^{-1}) in regions one and three. In all cases the regions are equally thick, and the net cell-thickness is 15.5 cm. Each region is an edit zone. Results are displayed in Table 2 where computational methods are defined.

III. Conclusions

1. In Table 2, the differences between Method-0 and conventional MC are small, i.e. most deleterious effects of the random source-sampling have been removed.
2. Method-1 is noticeably less efficient than Method-0.
3. Method-0 is about as efficient as conventional MC. Bias reduction is accomplished, here, at no cost in efficiency and small cost in programming effort.

The above results suggest that Method-0 be used instead of Method-1 in superhistory computations, and that the superhistory method be used more routinely. Our proposed elimination of random sampling will alter the expected values of physical quantities, but seem unlikely to increase biases. Elimination of random source-sampling may also be worthwhile in conventional MC.

This preliminary study of simple test problems can't cover the range of MC problems encountered in practice. In particular none of our problem configurations has the features of the Eigenvalue of the World Problem.[4] It is hoped that it will be possible to extend this study to problems of this type.

References:

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Table 1. Homogeneous Slab, Reflecting Boundaries
$\sigma_t = 1$ in all cases

Method	σ_f	ν	Number of Edit Zones	Generations per Super	Number of Supers	σ^2_{abs}	σ^2_k
Conventional	0.1	1.	3	1	100	6.06	1.8e-5
Superhistory (Method-1)	0.1/ν	3.	3	10	10	9.28	1.7e-5
Superhistory (Method-0 and Method-1)	0.1	1.	3	10	10	3.92	1.7e-5

In the row-1 case with $\nu = 1$, 1 generation per supergeneration, superhistory method is equivalent to "conventional" MC. Row 2 displays results for same slab with ν raised to 3 and σ_f correspondingly lowered. With row-3 parameters, Method-1 and Method-0 are the same. All cases correspond to same just critical cell. Variances computed among 1000 replicas. Closely similar results were obtained for $\nu = 2.5$. Cross sections in cm^{-1}, 500 histories per generations, 500 absorptions per second per cell.

Table 2. Three-Region Slab Cells, Three Edit Zones
In all regions $\sigma_a = 0.1$ and $\sigma_t = 1$, while in Region 2, $\sigma_f = \sigma_a/\nu$.

Problem Set	Method	σ^2_{abs}		Eigenvalue		σ^2_k	
$\nu =$		3.0	2.5	3.0	2.5	3.0	2.5
1	Conventional	3.86	3.86	0.99993	0.99993	1.63e-5	1.63e-5
1	1	9.52	7.94	0.99984	0.99973	1.78e-5	1.63e-5
1	0	3.64	3.64	0.99993	0.99993	1.65e-5	1.63e-5
2	Conventional	2.71	2.71	0.77045	0.77045	1.04e-5	1.04e-5
2	1	4.99	4.09	0.77000	0.77030	0.37e-5	1.28e-5
2	0	2.46	2.46	0.77024	0.77024	1.04e-5	1.04e-5
3	Conventional	1.45	1.45	0.69191	0.69191	9.64e-6	9.64e-6
3	1	1.68	1.68	0.69201	0.69185	9.56e-6	9.27e-6
3	0	1.57	1.57	0.69200	0.69200	1.04e-5	1.04e-5

Runs replicated 500 times. Variances were computed in each region, then averaged over regions to give table entries. Problems in set 1 are physically the same as the problem of Table 1. Cross sections in cm^{-1}, all problems with 500 histories per generation, 500 absorptions per second per cell.

Terminology:

1. "Conventional": 1 generation per supergeneration, 130 supergenerations, 30 generations skipped in the edits.
2. Method-1: 10 generations per supergeneration, 13 supergenerations, and 3 supergenerations skipped.
3. Method-0: parameters as in 2.