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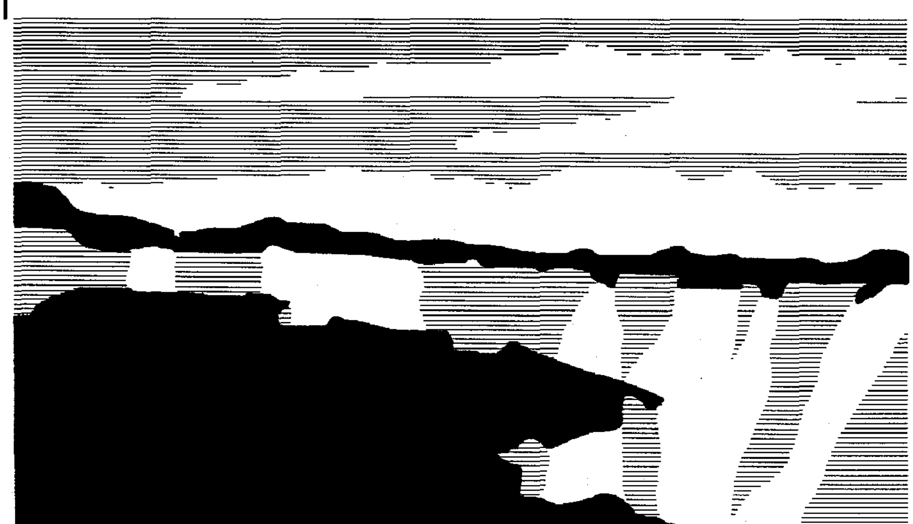

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Bounds on the Strength Distribution of Unidirectional Fiber Composites

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Abstract

Failure mechanisms under tensile loading of unidirectional fiber composites comprising of Weibull fibers embedded in a matrix are studied using Monte-Carlo simulations. Two fundamental mechanisms of failure are recognized — stress concentration driven failure and strength driven failure. It is shown that the cumulative distribution function for composite strength predicted by the stressconcentration -driven failure and strength-driven failure form apparent upper and lower bounds respectively and also that failure mechanism switches from one to the other as fiber strength variability changes.

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1 Introduction

Unidirectional (UD) fiber composite failure is a complex stochastic process. Primarily due to the randomness of fiber strengths, UD composite tensile strength is itself a random quantity and methods to determine its distribution are of considerable significance in assuring composite reliability.

Idealizations of composite structure and material properties are found to be inevitable before further analysis can be attempted. In this study, we assume linear elastic fibers arranged in a hexagonal array and embedded in an linear elastic non-debonding matrix so that material damage in our idealized composite is restricted to fiber failures alone. Although in a real fiber, flaws of random strengths are distributed along the fiber, we confine fiber failures to a plane perpendicular to the fiber direc-

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tion. This allows us to simulate composites consisting of a greater number fibers and a wider range of fiber properties than otherwise possible. Finally, we assume that fiber strengths (X) are Weibull distributed, an assumption experimentally well established ([Beyerlein and Phoenix (1996)]). Accordingly,

$$F(x) = \Pr\{X < x\} = 1 - e^{-(x/x_0)^\rho} \quad (1)$$

where x_0 is the scale parameter and ρ, the shape parameter. From the variance of this distribution,

$$\sigma_f^2 = {x_0}^2\{\Gamma(1+\frac{2}{\rho}) - \Gamma^2(1+\frac{1}{\rho})\} \quad (2)$$

one observes that smaller ρ corresponds to a higher fiber strength variance.

The in-plane failure of our idealized composite model takes place as follows. Consider a rhombus-shaped patch of s^2 hexagonally-arranged Weibull fibers loaded with stress per fiber x. If x is instantaneously applied to the composite and the progression of fiber breaks in "time increments" is monitored (dynamic effects are ignored) those fibers that have strengths smaller than x fail at time 1. The load dropped by these fibers is now redistributed among the intact fibers. This overload may cause more fiber failures (at time 2) which in turn overload yet another set of fibers beyond their strengths and fail them (at time 3) and so on. The smallest applied load per fiber x that will cause all the fibers in a particular specimen to fail is its tensile strength.

Exact solution of this process despite its Markov nature is impossible for composites of realistic sizes due to computational limitations. We therefore take the following approach: from Monte-Carlo simulations of the failure process, we obtain the dominant mechanisms of failure (§2) and model these mechanisms to get an estimate of the failure probabilities (§3).

2 Failure Simulation

2.1 Simulation Algorithm

Failure simulations are carried out on a rhombus shaped patch (Fig. 1) consisting of s^2 hexagonally arrayed Weibull fibers on which periodic boundary conditions are imposed. An increasing load is applied on the composite until all fibers in it fail by the process described in §1. A detailed description of the simulation procedure used in this work can be found in [Mahesh et al. (1999)].

An important component of the simulations is the manner in which the load dropped by a broken fiber is redistributed amongst other fibers. Two *load sharing* models are used – the Hedgepeth and Van Dyke load sharing model(HVLS) [Hedgepeth and Van Dyke (1967)] and

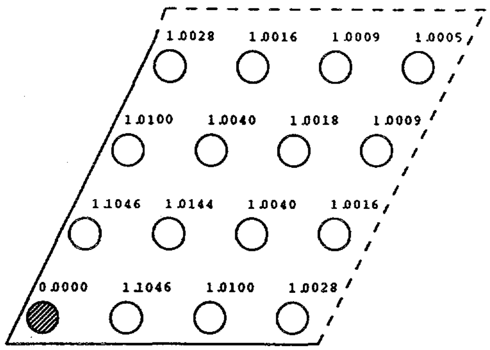

Figure 1: HVLS stress concentrations near a single break in a 30 × 30 periodic patch. The hatched fiber is broken. For this patch, the ELS stress concentration on the surviving fibers is $1 + \frac{1}{899} \approx 1.0011$.

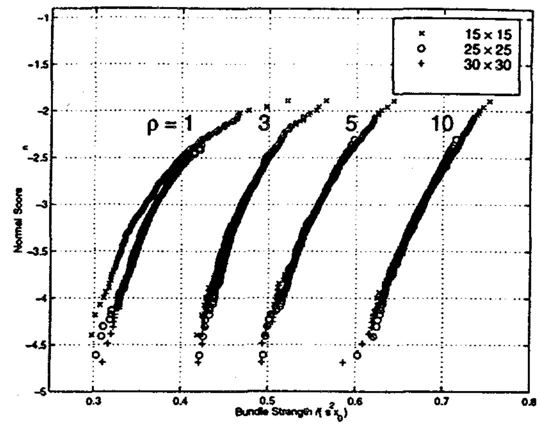

Figure 2: The weakest-link distribution $W(x) = 1 - (1 - F_c(x))^{s^2}$ on Normal probability paper.

the Equal Load Sharing (ELS) model [Daniels (1945)]. While the HVLS is a local load sharing model — a large part of the load dropped by a broken fiber is distributed amongst its nearest neighbors, the ELS model is global in its load sharing — broken fibers transfer their load equally amongst all the surviving fibers. Fig. 1 shows the stress concentrations on the fibers surrounding a broken fiber. While HVLS is considered a realistic model for load transfer in a composite with matrix, the ELS model is considered realistic in the case of a loose bundle of fibers (without matrix).

2.2 Simulation Results

Simulations were performed on rhombus shaped patches of a range of composite sizes s^2 and fiber Weibull moduli, ρ. Empirical composite strength distributions generated from these simulations are denoted by $F_c(x)$.

Figure 2 shows a plot of the weakest-link distribution function $W(x)$ derived from $F_c(x)$ on normal probability paper where,

$$W(x) = 1 - (1 - F_c(x))^{1/s^2} \qquad (3)$$

Note that in the probability range of simulation, $W(x)$ is independent of the composite size for $\rho \geq 3$. However, for $\rho < 3$, agreement between the weakest-link distributions ceases.

The reason behind the independence of $W(x)$ on s^2 for higher ρ and its dependence on s^2 for lower ρ is seen by examining the breaks in the composite just prior to catastrophic crack growth or the *critical cluster* (see Fig. 3). In the $\rho = 10$ and $\rho = 5$ cases the critical cluster is much smaller than the composite itself (comprising of two to five breaks) and is therefore not affected by the finite composite size (or boundary effects). However, in the $\rho = 1/2$ case, the critical cluster occupies a substantial portion of the composite. Therefore, if $W(x)$ is identified with the probability of formation of the critical cluster, it will be strongly influenced by the composite size. The size independence of $W(x)$ will be used in obtaining the upper bound on strength in §3. Note here that the dependence or independence of $W(x)$ on s^2 is a function of s^2 as well. $W(x)$ derived from $F_c(x)$ for a 5×5 with $\rho = 10$ fibers for example, is found not to coincide with the $W(x)$ plots shown in Fig. 2. Similarly, we expect that among large enough patches, the $\rho = 1$ strength distributions will also show a weakest link nature.

Fig. 4 compares HVLS and ELS composite strengths. As seen, the agreement between the composite strengths predicted by the two different load sharing models gets increasingly better as $\rho \downarrow 0$. In a qualitative manner, this can again be understood by observing that as ρ decreases, fiber failure becomes increasingly insensitive to x. For example, using (1), for a fiber with $\rho = 1/2$, $F(0.5) = 0.5069$ and $F(1) = 0.6321$. Contrast this with the case of a $\rho = 10$ fiber, with $F(0.5) = 0.000936$ and $F(1) = 0.6321$. Thus, in this case, the ELS assumption for stress redistribution agrees quite well with the simulated HVLS composite strength distribution.

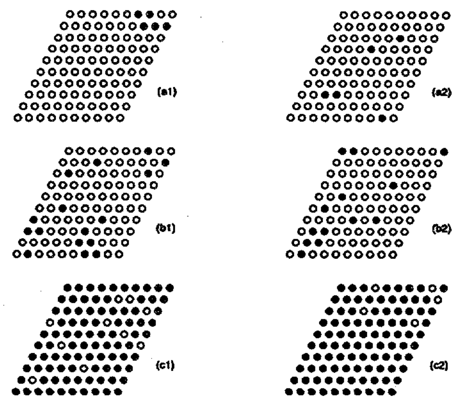

Figure 3: Failure patterns in a 10×10 composite patch. (a1) and (a2) are $\rho = 10$ lower and upper tail specimens respectively, (b1) and (b2) are $\rho = 5$ lower and upper tail specimen respectively, and (c1) and (c2) are $\rho = \frac{1}{2}$ lower and upper tails respectively. Open circles denote intact fibers and circles with a "×" in them denote broken fibers.

3 Strength Bounds

In this section, we will develop an expression that will always be an upper bound on the simulated distribution function, one that gets tighter as $\rho \uparrow \infty$. On the other hand, we will use an analytical solution to the ELS problem [Smith (1982)] as the lower bound on composite strength; this bound getting tighter as $\rho \downarrow 0$.

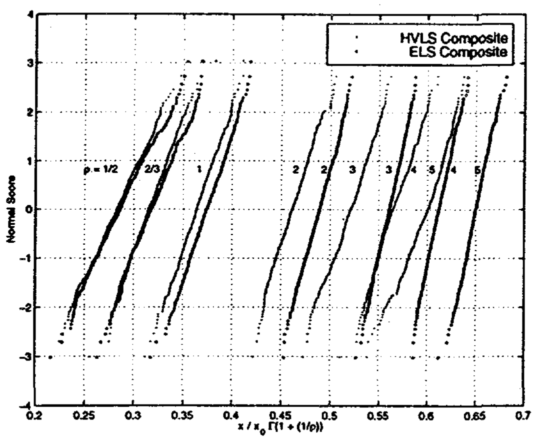

Figure 4: Comparison of HVLS and ELS strength distributions for $s^2 = 30 \times 30$ composite. Strengths are normalized with respect to mean fiber strength on Normal probability paper.

3.1 Upper Bound

The upper bound is arrived at by viewing composite failure as described below and computing its probability of occurrence. Composite failure occurs when at least one of the following s^2 events occur: one of the s^2 fibers fail, it drops part of its stress on its six nearest neighbors of which one fails, the pair of breaks thus formed fails another of its two most overloaded neighbors, the resulting triplet then fails one of its four most overloaded neighbors and so on until all the s^2 fibers in the composite are broken. If the probability of this event under applied load per fiber x is $W^u(x)$, the estimated probability of composite failure, $F_c^u(x)$ is (see (3))

$$F_c^u(x) = 1 - (1 - W^u(x))^{s^2} \quad (4)$$

$W^u(x)$ may thus be evaluated as:

$$\begin{aligned} W^u(x) &= F(x) \times \quad (5) \\ &\quad (1 - (1 - F(K_1 x))^{\eta N_1}) \times \\ &\quad (1 - (1 - F(K_2 x))^{\eta N_2}) \times \\ &\quad (1 - (1 - F(K_3 x))^{\eta N_3}) \times \cdots \end{aligned}$$

where $K_n = \sqrt{1 + \frac{2\sqrt{n}}{\pi^{3/2}}}$ is approximately the maximum stress concentration around a tight cluster of n fibers, $N_n \approx \sqrt{4n\pi}$ is the number of neighbors around a cluster of n breaks and η is a parameter to account for non-uniformity of stresses on the fibers surrounding the cluster. Physically, $\eta \in (0, 1]$ is approximately the fraction of neighbors of the cluster that are the most overloaded.

3.2 Lower Bound

As mentioned in §2, at smaller ρ, the failure process is dominated more by fiber strengths than by the stress concentrations. Composite strength calculated by assuming ELS stress concentrations is found to result in a tight lower bound on the simulated stress of a composite of low ρ.

The Smith corrected Daniel's formula [Smith (1982)] predicts very accurately that the strength of an ELS bundle comprising of s^2 fibers is normally

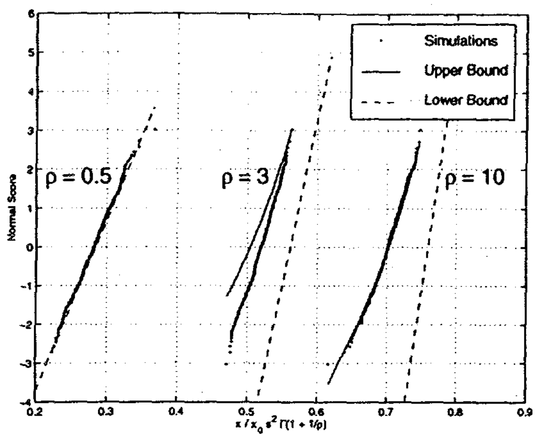

Figure 5: Upper and lower bounds on $s^2 = 30 \times 30$ composite strength on Normal probability paper. Note that the upper bound for $\rho = \frac{1}{2}$ lies well outside this plot.

distributed, $N(\mu^*, \sigma^*)$ where,

$$\mu^* = \sigma_0 s^2 (\rho e)^{-1/\rho}(1+0.996 s^{-4/3}(\frac{e^{2/\rho}}{\rho})^{1/3}) \quad (6)$$

and

$$\sigma^* = \sigma_0 \rho^{-1/\rho}\sqrt{s^2 e^{-1/\rho}(1-e^{-1/\rho})} \quad (7)$$

4 Conclusion

Fig. 5 compares the simulated strengths with the two bounds proposed in §3 for composite strength. Although, not all ρ cases are shown, it is found that the upper bound given by (4) – (6) fits the simulations quite tightly for $\rho \geq 5$. Also, the lower bound given by (6) and (7) is found to fit the simulations very well for $\rho \leq \frac{2}{3}$. It is further noted that composite failure process is a combination of the two mechanisms that yielded the upper and lower bounds.

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