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Title: Neutron Chain Length Distributions in Subcritical Systems

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Neutron Chain Length Distributions in Subcritical Systems

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Abstract

In this paper, we present the results of the chain-length distribution as a function of k in subcritical systems. These results were obtained from a point Monte Carlo code and a three-dimensional Monte Carlo code, MC++. Based on these results, we then attempt to explain why several of the common neutron noise techniques, such as the Rossi-α and Feynman's variance-to-mean techniques, are difficult to perform in highly subcritical systems using low-efficiency detectors.

1 Introduction

Numerous techniques for monitoring subcriticality in multiplying systems have been attempted over the past 50 years. One general class of these techniques is based on neutron noise theory (Williams, 1974) in which one attempts to observe correlated pairs of prompt neutrons belonging to the same fission chain. (A neutron chain is defined herein as all the neutrons spawned over time from a single initiating neutron.) In order to observe correlated pairs, it is, of course, necessary that the detector efficiency be high enough to allow for the detection of two or more neutrons from most of the chains that are expected to occur at a particular value of k_{eff}. Therefore, to better understand detector efficiency requirements, the chain-length distribution as a function of k_{eff} has been calculated using a point Monte Carlo code and a three-dimensional Monte Carlo code that simulate the random propagation of prompt fission chains in subcritical assemblies. Previous work in this area has been conducted by Mihalczo et al. (Mihalczo *et al.*, 1991) in which chain lengths in a BWR for k=0.9 were studied. In their work, however, they did not discuss the effect of detector efficiency on the ability to detect correlated pairs of counts. In this work, we have broadened the range of study to include k=0.3 to k=0.999. From these results, we then explain why some neutron noise measurements are difficult to perform in highly subcritical systems using low-efficiency detectors.

2 Theory

2.1 Point Model

In every multiplying system, source neutrons originating in or near the system initiate prompt fission chains. These source neutrons appear from spontaneous fission sources, (α,n) sources, start-up sources, (γ,n) sources, delayed neutron sources, etc. Once initiated, these prompt fission chains can die out very quickly or can propagate for relatively long periods of time in which millions of fission neutrons can be produced before the prompt fission chain eventually dies out. The propagation of these neutron chains is nearly identical to the process encountered in a much older

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problem that was previously analyzed in the mid-1800s dealing with the growth of families (Harris, 1989). In what Harris denotes as the Galton-Watson process, a family is followed through several generations with each generation having some associated probability distribution function for producing an integer number of male offspring. The number of generations passing before the surname is lost becomes the question of interest.

Without too many modifications, the neutron-chain process encountered in a multiplying system should exhibit mathematical traits similar to this historical problem. For example, it is assumed in both problems that the generations (whether they be neutrons or off-springs) act independently of any other generation (i.e., form a Markov chain) and that the number of neutrons or off-springs produced in one generation is independent of the existence of any other members. Because of these similarities, we assume that the solution of the Galton-Watson problem can be applied to the propagation of prompt fission chains as well. However, because of differences in the probability distribution required to produce 0, 1, 2, 3, etc. offspring, we have chosen to analyze the neutron chain length problem numerically rather than analytically.

To ascertain a quantitative measure of the neutron chain length distribution of prompt fission chains as a function of subcriticality, a simple Monte Carlo computer code was written at Los Alamos National Laboratory that simulates the evolution of prompt fission chains. In this Monte Carlo code, it is assumed that all fission neutrons have the same probability, P_f, of causing another fission. That is,

$$P_f = \frac{K}{\overline{\nu}_p}, \tag{1}$$

where K is the prompt multiplication factor [defined as $K=k_{eff}(1-\beta_{eff})$ where k_{eff} is the effective multiplication factor and β_{eff} is the effective delayed neutron fraction], and $\overline{\nu}_p$ is the average number of prompt neutrons released per fission. It is also assumed that the source neutrons that initiate the fission chains correspond to the equivalent-fundamental-mode (EFM) source neutrons (Spriggs *et al.*, 1999) and, as such, have the same probability of causing a fission as the fission neutrons. When a neutron interaction results in a fission event, then the number of prompt neutrons released from that fission is sampled from the neutron number distribution (i.e., P(ν) vs. ν). In our point model we have chosen to use Frehaut's Gaussian model because of its general applicability to a large number of isotopes and a wide range of $\overline{\nu}_p$ (Frehaut, 1989).

Although Frehaut's model is an adequate representation to the neutron number distribution for most isotopes, there were two inconsistencies that had to be corrected prior to its use in the point Monte Carlo model. First, the probabilities of releasing 0 through 7 neutrons (which we arbitrarily set to be our maximum number of neutrons released per fission) did not add up to 1.0. And second, the neutron number probabilities, multiplied by their corresponding ν values and then summed, did not yield the same average $\overline{\nu}_p$ used to generate the original neutron number probabilities. Table 1 presents an example of this for an input value of $\overline{\nu}_p = 2.42$.

Table 1. Frehaut's unmodified multiplicity distribution

$\overline{\nu}_p = 2.42$	
ν	**Probability**
0	0.0436
1	0.1720
2	0.3313
3	0.3051
4	0.1296
5	0.0337
6	0.0048
7	0.0003
Total	1.0205
$\overline{\nu}_p$	2.4680

The problem now becomes to preserve $\overline{\nu}_p$ while producing a normalized probability distribution. To correct these two deficiencies, the shape of the probability distribution had to be altered slightly. This was accomplished as follows. First, the distribution was normalized before computing the new $\overline{\nu}_p$. Then the values of the probabilities for $\nu=0$ and $\nu=7$ were adjusted to retain the normalization while increasing or decreasing the computed $\overline{\nu}_p$ to match the input value. In some cases, the adjustment to the tail probabilities, Eq. (2), exceeded their magnitude and the remainder of the adjustment was applied to probability associated with the next inner ν value.

$$\begin{aligned} P_{\nu_A} &= P_{\nu_A} + \Delta\overline{\nu} \\ P_{\nu_B} &= P_{\nu_B} - \Delta\overline{\nu} \end{aligned} \quad , \tag{2}$$

where P_{ν_X} is the probability of having ν_X neutrons emerge from fission,

$$\Delta\overline{\nu} = \frac{\overline{\nu}_{actual} - \overline{\nu}_{computed}}{\nu_B - \nu_A}, \tag{3}$$

and $\nu_A < \nu_B$. This approach of adjusting values in the tails was adopted to lessen the impact on the shape of the curve because the last values modified will be those with the highest probabilities. The results of this methodology appear in Table 2.

Table 2. Corrected distribution

$\overline{\nu}_p = 2.42$		
ν	**Normalized**	**Corrected**
0	0.0428	0.0425
1	0.1685	0.1685
2	0.3246	0.3246
3	0.2990	0.2990
4	0.1270	0.1270
5	0.0330	0.0330
6	0.0047	0.0047
7	0.0003	0.0006
Total	1.0000	1.0000
$\overline{\nu}_p$	2.4184	2.4200
Correction	0.0016	

2.2 Validation of Point Model

The point Monte Carlo code was run for a user specified number of source neutrons ($1 < S < 2x10^9$). The chain length distribution was then obtained by dividing the number of times a chain of a particular length was observed by the number of source neutrons specified in the problem. One means of verifying the accuracy of the chain length distribution was to compute the multiplication of the system, *M*, by integrating over the entire distribution. The system multiplication is identically equal to

$$M = \frac{1}{1 - k_{eff}} \quad , \tag{4}$$

where k_{eff} is the system multiplication factor. *M* is also identically equal to the average chain length. In a multiplied fixed source Monte Carlo algorithm, the calculated *M* will only match that predicted in Eq. (4) when the source neutrons are distributed as the fission source fundamental mode. Because the point model assumes that the source neutrons are equivalent fundamental mode source neutrons, then the multiplication calculated by integrating of the chain length distribution agreed with Eq. (4).

We also validated the point solution by comparing its chain length distribution with the chain length distribution obtained using a 3D Monte Carlo code, MC++, developed at the Los Alamos National Laboratory (Lee *et al.*, 1997). MC++ tracks particles across any arbitrary three-dimensional geometry using an Eulerian mesh. When computing the chain-length distribution, the code is run in the *multiplied fixed source* mode using a user specified source distribution corresponding to either a point source, a uniform source in a shell, or a uniform source in the fissile material. Each neutron in MC++ is tracked until it is either captured or escapes, while new neutrons are introduced after fission and (*n,xn*) events. When computing the chain length distribution, MC++ uses a similar chain generation mechanism based on Frehaut's distributions. MC++ also computes either k_{eff} or the prompt multiplication factor, *K*, depending on whether the specific cross section library provides $\overline{\nu}$ or $\overline{\nu}_{prompt}$. However, the current code does not store the spatial locations of the subsequent fission

source points from the final cycles. Hence, a fundamental mode source cannot be simulated. Therefore, the number of source particles run in MC++ had to be converted to an equivalent fundamental source strength in order to perform the comparison. Hence, the g^* factor for the given source distribution used in the simulation had to be determined. This was accomplished by taking the ratio of the calculated multiplication to that predicted using Eq. (4) based on a k-eigenvalue solution. That is,

$$g^* = \frac{M_{computed}}{M_K} \quad , \tag{5}$$

where the k-eigenvalues reported in this paper were based on ν_{prompt}. To compare the two chain length distributions, it was necessary to multiply the MC++ chain lengths by their probability of occurrence, divide the value by g^*, and sum to find the multiplication. One such comparison is shown in Figure 1 for a spherical metal uranium system in which two different fixed source distributions were assumed in MC++: a point source, and a uniform spatial distribution. Using the

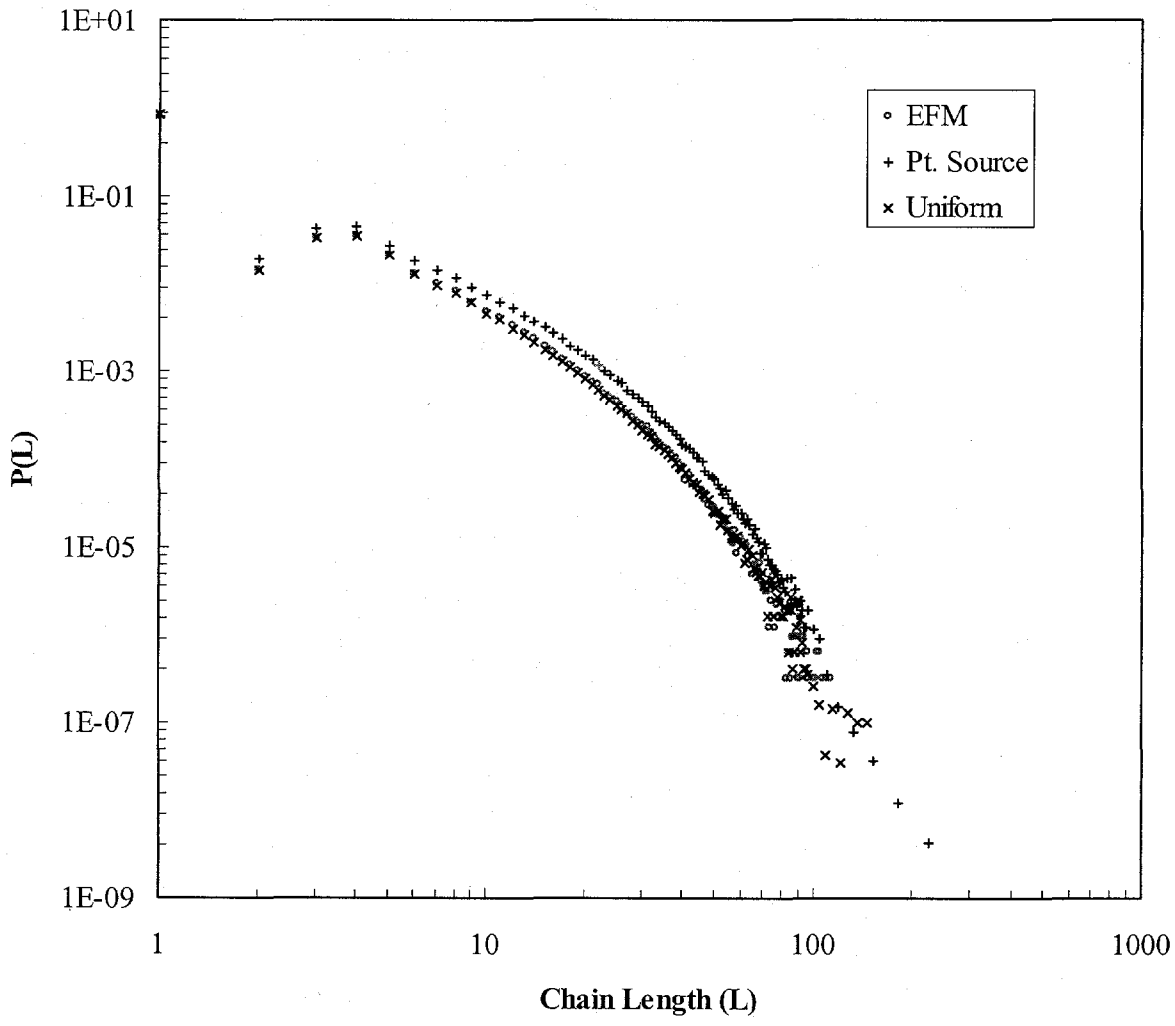

Figure 1 Distribution comparison between point model and MC++

values of K=0.5 and $\overline{\nu}_p$=2.84 as calculated per MC++ using a one-group cross section set corresponding to an analytical benchmark (Sood, 1998), the corresponding point model chain length distribution was also calculated.

Figure 1 clearly shows that the spatial distribution of the neutron source has an impact on the chain length distribution. As expected, the probability of seeing a chain of only 1 neutron decreases as the average importance of the source neutrons increases beyond that of EFM neutrons. However, the area under the curve, M, is the same when the probabilities are reported per EFM source neutron. The reduction in the contribution from shorter chain lengths is equally compensated by an increase in the probability of observing chains of length greater than one. This shift in the chain length distribution makes physical sense because the point source would, of course, have a lower probability of seeing a chain length of 1 while having a greater probability for longer chains as compared to an EFM source.

Although there are small differences in the chain length distribution as predicted by the point model and the 3D solution, the point model yields a reasonably accurate representation of the chain length distribution. When coupled with the significantly faster running time of the point model code, we have chosen to study the chain length distribution as a function of K using the point model. These results are presented in the following section.

3 Results

Figure 2 shows the results of the chain length distribution in a plutonium system for five different prompt multiplication factors. As can be observed, the chain length distribution near delayed critical (i.e., K=0.999) follows the classical solution obtained from the Galton-Watson problem for K=1.0; that is, the probability, $P(L)$, of obtaining a chain length L falls off as

$$P(L) \sim \frac{C}{L^{1.5}} \tag{6}$$

where C is a proportionality constant that is dependent on the neutron number distribution.

When K is less than 1.0 (i.e., below prompt critical), it is theoretically impossible for a prompt neutron chain to grow to infinity, and the chain must eventually die out. This effect is clearly demonstrated in Fig. 2 in which we observe that the maximum neutron chain length for K=0.999 is ~100 million neutrons. We also note that when K<1.0, the maximum neutron chain length decrease dramatically.

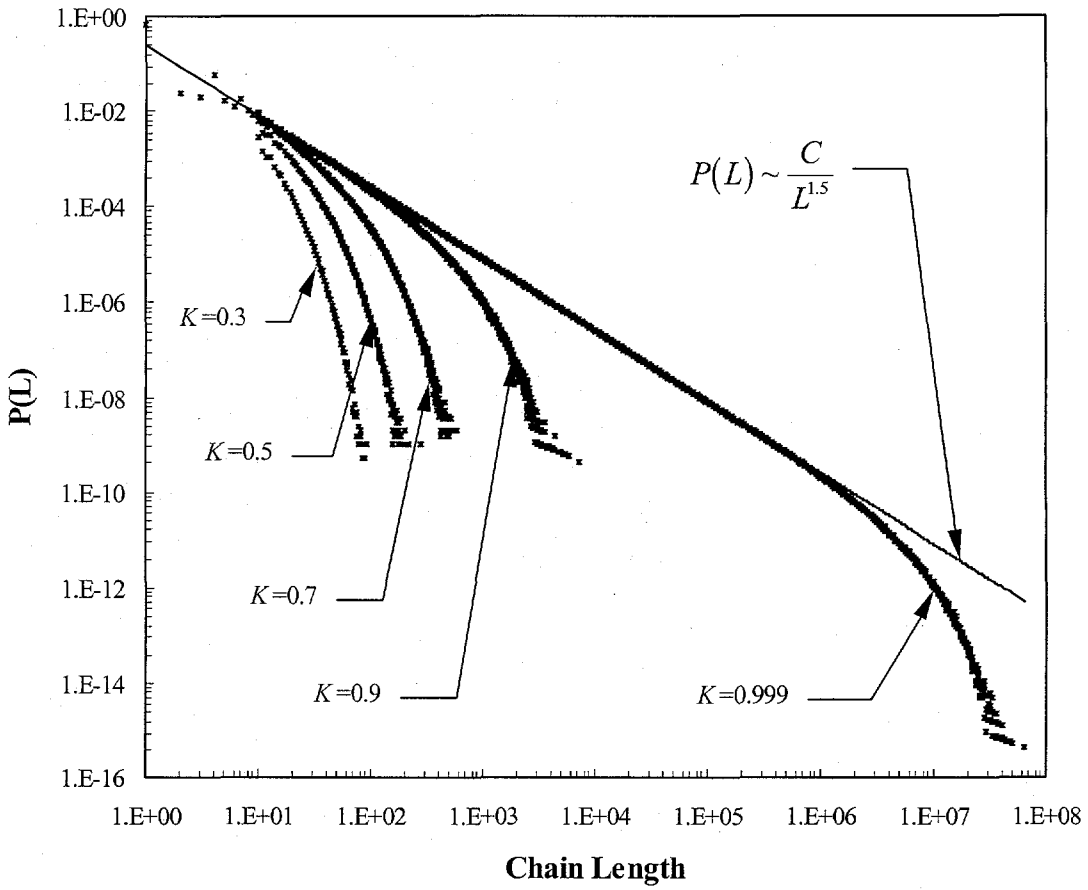

Figure 2. Chain length distribution for a plutonium system.

4 Discussion

When performing neutron noise measurements, such as Rossi-α and variance-to-mean techniques, one attempts to achieve high neutron-detection efficiency by placing a detector in a region of high flux or by distributing several detectors throughout the system and then summing their signals together. Even under the best of circumstances, it is usually difficult to achieve detector efficiencies greater than 10^{-4}. Hence, with an efficiency this low, the probability of detecting several fission neutrons belonging to the same fission chain is very low for those fission chains that spawn less than 10^5 neutrons. In the vicinity of delayed critical, this, of course, is not a problem; chains as long as 10^8 neutrons occur with reasonable frequency. With a chain of that length, it is expected that, on an average, 1000 neutrons would be detected. However, if K were to drop much below 0.7, we note that the maximum chain length would decrease to 1000 or less. Consequently, with a detector efficiency of 10^{-4}, one would not expect to detect very many correlated pairs of counts for this combination of K and detector efficiency.

The ability to perform a neutron noise measurement with a low-efficiency detector in a system that is far subcritical is further hampered by the presence of a normal background of uncorrelated counts (i.e., random counts from other source neutrons and/or unrelated fission chains spawning elsewhere

in the system). If the uncorrelated background is significant, such as in a plutonium system, the correlated counts may only represent a small percentage of the total signal. Hence, the ability to resolve the correlated counts from the uncorrelated counts becomes a real challenge.

5 Conclusions

Although the point Monte Carlo model described in this report is useful for gaining a basic understanding of neutron noise theory, the 3D Monte Carlo code, MC++, is a better choice for modeling and analyzing most real world problems. Yet, while MC++ can be generally applied to any system for which the cross sections are available, tracking a large neutron population can become a problem. Unlike the point model, which only follows one source neutron at a time (or one per processor when running in parallel), MC++ actually follows all of the source neutrons simultaneously. To see extremely rare chains, it requires at least as many neutrons as the inverse of the probability. To see them with any frequency means even more initial source neutrons. While MC++ can handle this in some degree by batching the source neutrons into separate cycles, it will still run into memory problems when K nears 1.0 and longer chain lengths become more frequent. Because MC++ stores a variety of attributes for each particle, memory problems begin when attempting to simultaneously track ~5 million neutrons on LANL's Bluemountain platform. No such memory limits associated with K have been seen for the point model, but speed is naturally affected in both.

In conclusion, both the point Monte Carlo code and the 3D Monte Carlo code, MC++, have provided additional insight into neutron noise theory. In addition to allowing us to study the spatial effects of the initial source distribution, MC++ also provides the means for studying energy and angular distribution effects of the source. For its part, the point model has been shown to be invaluable for exploring the chain length distributions as a function of K, particularly for values of K in the vicinity of prompt critical in which a large number of source particles are necessary to establish an accurate distribution. In combination, both Monte Carlo simulations of prompt fission chains have provided some means of determining when a neutron noise technique can be performed successfully with low-efficiency detectors.

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