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Shock Certification of Replacement Subsystems and Components in the Presence of Uncertainty

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Abstract

In this paper a methodology for analytically estimating the response of replacement components in a system subjected to worst-case hostile shocks is presented. This methodology does not require the use of system testing but uses previously compiled shock data and inverse dynamic analysis to estimate component shock response.

In the past component shock responses were determined from numerous system tests; however, with limitations on system testing, an alternate methodology for determining component response is required. Such a methodology is discussed. This methodology is mathematically complex in that two inverse problems, and a forward problem, must be solved for a permutation of models representing variabilities in dynamics.

Two conclusions where deduced as a result of this work. First, the present methodology produces overly conservative results. Second, the specification of system variability is critical to the prediction of component response.

Introduction

In this paper a methodology for predicting the dynamic shock response of replacement components in a system is presented. Due to the present limitations on system testing, it is not presently possible to determine the response of replacement components using experimental analysis alone. Therefore, an alternate methodology for determining component responses is needed.

It has been proposed to replace system test capability with modeling and simulation. Ideally, if validated finite element models of a system existed, the determination of component response could be performed without any system tests. A model of a replacement component could be integrated into a model of the system, and simulations could be performed to predict component responses during shock encounters. Unfortunately, the level of modeling capability required to construct these types of models is only now being developed.

Without future modeling capability, an interim methodology for determining component response is needed. This paper presents such a methodology. This methodology requires a balanced use of

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both laboratory experiments and analysis. This methodology is complex in that both inverse and forward dynamic analysis is required.

In this paper a model is *narrowly* defined to be a mathematical relationship between responses measured on a component and a force excitation on the system. Models are derived by curve fitting measured data to a mathematical formula relating output response to input excitation.

In the past component responses were determined by physically applying loads to the system and measuring the response of components. If these loads were measured and fed into a model of the same system, modeled response should mimic measured response. Conversely, by using this model and inverse dynamic analysis, the load required to produce a prescribed component response can be deduced. Taking this one step further, the load required to produce component accelerations with shock response spectra that envelope the shock response spectra from previous system tests can be deduced. Thus, assuming that the worst-case response of a component is one that envelopes the shock response spectra measured during previous system tests, modeling and inverse dynamics analysis could be used to determine a worst-case excitation for that component. This worst-case excitation could then be fed into models of the system with replacement components to approximate the worst-case replacement component responses.

The above statements represent a methodology for approximating the worst-case response of a component when system testing is not available. The steps required to implement this methodology are given below.

step 1: Using laboratory tests, produce experimentally derived input-output models of the system with orignal and replacement components.

step 2: Using a model of the system with a orignal component, determine an excitation that will produce a component acceleration response with a shock response spectra that envelopes the spectra from past system tests.

step 3: Apply the excitation derived in Step 2 to a model of the system with the replacement component.

Variability in Component Dynamics

The above methodology must be applied to a set of models representing the variability in the dynamics of the system. A principal source of variability in component dynamics is owing to variability in hardware assembly. This variability can change the dynamics of a component significantly.

As illustrated in Figure 1, mounting hardware contains components that can slip and slap under large loads. This slipping and slapping produces responses that are highly dependent on component spacing and preload. Small changes in this spacing can cause large variations in the response of the component. Component mounting hardware consist of a band, an upper strap (also called a blast cap), an upper pad that is glued to the upper strap, a lower pad that is glued to the lower strap, a lower strap, a base plate, and two bolts with lock nuts. Attached to the lower strap is a keeper and an alinement pin. The keeper is spot welded to the lower strap and fits into a slot on the lower side of a component. The alinement pin fits into a hole in the base plate.

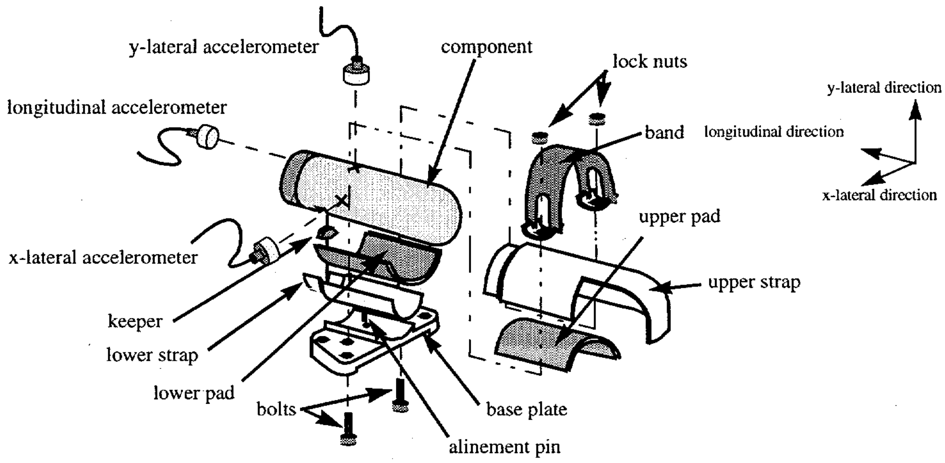

Figure 1. Assemble drawing of the mounting bracket for a component

Notice that there are many interfaces in the mounting hardware where slip and slap can occur:

- The component can slip on the pads.
- The lower strap can slip on the bracket.
- The upper strap can slip in the band.
- The keeper can slap against the key of the component.
- The pin can slap against the alinement hole in the base plate.

Micro and macro slipping and slapping can significantly alter the acceleration response of a component [2]. In Figure 2 two time responses of the component can be seen. These responses were produced by applying a one third millisecond, 500g enforced haversine acceleration at the bottom of the base plate. Notice that the peak acceleration levels vary by a factor of two.

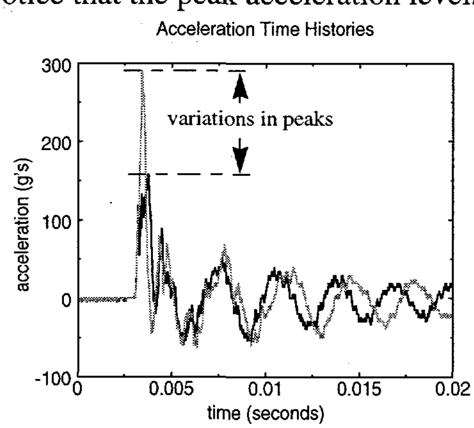

Figure 2. Acceleration time history responses of component for small variations in assemble

Aspects of the Methodology

In this section aspects of the above methodology for estimating component responses will be presented.

How input-output models are constructed

To use the above methodology, experimentally derived input-output models of orignal and replacement components in the system must be developed. These models are derived from data measured during laboratory testing.

Figure 3 is an illustration of the experimental apparatus used to obtain impulse response data for a component. Using an inverse Hopkinson bar, shock loads are applied to the exterior of the system. From measured loads and responses, frequency response functions are derived. From frequency response functions, impulse response functions are determined. From impulse response functions, numerical models are produced.

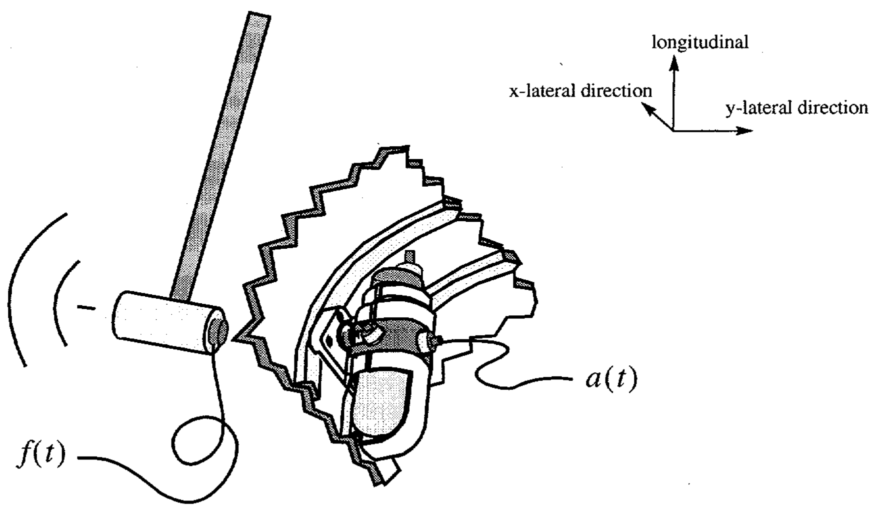

Figure 3. Inverse Hopkinson bar tests

To obtain a numerical model from impulse response functions, the Eigensystem Realization Algorithm (ERA) [3,4] is used. This algorithm results in a recursive relationship of the form

$$x(i+1) = Ax(i) + Bf(i) \tag{1a}$$

$$a(i) = Cx(i) + Df(i) , \tag{1b}$$

where $x(i) \in R^{Nx1}$, $A \in R^{NxN}$, $B \in R^{Nx1}$, $C \in R^{1xN}$, $D \in R^{1x1}$, $f(i)$ represents the forcing excitation, and $a(i)$ is equal to the acceleration time response. The index, i, is a time index where $t = i\Delta t$, and Δt is a time step.

In this paper models of the orignal component in the system will be represented as

$$x^j_{old}(i+1) = A^j_{old}x^j_{old}(i) + B^j_{old}f(i) \tag{2a}$$

$$a^j_{old}(i) = C^j_{old}x^j_{old}(i) + D^j_{old}f(i) \quad , \tag{2b}$$

and models of the replacement component in the system will be represented as

$$x^j_{new}(i+1) = A^j_{new}x^j_{new}(i) + B^j_{new}f(i) \tag{3a}$$

$$a^j_{new}(i) = C^j_{new}x^j_{new}(i) + D^j_{new}f(i) \ , \tag{3b}$$

where the superscript denotes one of a set of models used to represent variability in response.

How a Worst-Case Force Excitation is Constructed

A worst-case excitation is defined to be a load that produces a component acceleration response with a shock response spectra that envelopes the shock response spectra of old system test data. To determine this excitation, two inverse problems must be solved.

- In the first inverse problem, an acceleration time history must be found that produces a shock response spectra which envelopes the shock response spectra of past system tests.
- In the second inverse problem, a force excitation, $f(i)$, must be found that can produce the time history determined in the first inverse problem.

The first inverse problem involves finding a time history that will produce a shock response spectra which envelopes old shock response spectra data. For the y-lateral direction, the acceleration time history is assumed to be given by

$$a(t) = \sum_{i=1}^{N} A_i e^{-\alpha_i t} \sin(\omega_i t + \theta_i) \quad . \tag{4}$$

Notice that this time history is completely described by specifying the parameters A_i, ω_i, θ_i, α_i and N. These parameters also completely describe the shock response spectra of this time history.

To find a shock response spectra that will envelope old shock data, the parameters A_i, ω_i, θ_i, α_i and N must be recursively adjusted until a desired spectra is obtained. A code developed by Dave Smallwood and Jerome Cap [5] is used to perform this task.

The second inverse problem is the more difficult problem to solve. The second inverse problem involves determining a force excitation into a forward model that will produce the time histories determined in the first inverse problem. To solve the second inverse problem, the forward model will be inverted to produce a model that produces the input of the forward model when driven by the output of the forward model. This model is called the inverse model.

Since a model of any real physical system must be both stable and causal [6], Equation 2a,b must be stable and causal. By definition, stability is a condition that constrains the response of the model to be bounded for any bounded input, and causality is a condition that constrains the response of a model to be non-zero only after the time of input application. The forward model of any real physical system is both stable and causal; whereas, its inverse is stable but not necessarily causal. For the 2a,b forward model, the inverse model is given by

$$X'_{old}(i+1) = (A'_{old} - B'_{old}(D'^{-1}_{old})C'_{old})X_{old}(i) + B'_{old}(D'^{-1}_{old})y'_{old}(i) \tag{5a}$$

$$f(i) = -(D'^{-1}_{old})C'_{old}X_{old}(i) + (D'^{-1}_{old})y'_{old}(i) \quad . \tag{5b}$$

Without a statement as to the causality or stability of its dynamics, Equation 5a,b alone is only a partial representation of inverse dynamics. To show this, consider the discrete impulse response of a model

$$a_1(i) = c^i \qquad for \qquad i \geq 0$$
$$0 \qquad otherwise,$$

where $c > 1.0$. This is the impulse response of an unstable causal model. The z transform of this response is given by

$$\mathcal{Z}(a_1(i)) = \sum_{i=0}^{\infty} c^i z^{-i} = \frac{1}{1 - cz^{-1}}.$$

Notice that this represents a model with a single pole outside the unit circle. Now consider the noncausal stable impulse response given by

$$a_2(i) = -c^i \qquad for \qquad i < 0$$
$$0 \qquad otherwise,$$

where again, $c > 1.0$. The z transform of this response is given by

$$\mathcal{Z}(a_2(i)) = \sum_{i=-\infty}^{-1} -c^i z^{-i} = \frac{1}{1 - cz^{-1}},$$

which is the exact same z-domain representation as the unstable causal response above. Therefore, two different systems can have the exact same z-domain representation. Moreover, since the recursive relationship for a model can be derived from its z-domain representation, two different models can also have the exact same recursive relationship. In order to know what dynamics Equations 5a,b represents, the causality of model dynamics must be know or deduced.

Noncausality in inverse models is due to phase distortion in forward models, and this phase distortion comes from the zeros of the forward model that are outside the unit circle. Consider the forward model

$$H(z) = -\frac{1}{c}\frac{1 - cz^{-1}}{1 - \frac{1}{c}z^{-1}},$$

where again, $c > 1.0$. This model has a pole at $1/c$ and a zero outside the unit circle at $c > 1.0$.

Moreover, the magnitude of this model is given by

$$|H(z)| = \left| -\frac{1}{c} \frac{1 - cz^{-1}}{1 - \frac{1}{c}z^{-1}} \right| = 1.0$$

for all frequencies. Only the phase of this model changes with frequency. Models of this type are called allpass models. Similar to a pure time delay, a causal allpass model has an inverse that is all noncausal, stable, and allpass. A causal allpass model will consist of zeros outside the unit circle and corresponding poles inside the unit circle. A dynamics model of any real structure can be represented in terms of a causal allpass model cascaded with another causal model called a minimum phase model.

Phase distortion in a structural system occurs because of dispersive and non-dispersive propagation delays. These delays are represented in the allpass portion of a model. For example the non-collocated response of a semi-infinite bar of steel caused by an applied end force is a constant times a pure time delay. A model of a pure time delay is causal and allpass. When this allpass model is inverted, the inverse model is still allpass; however, the inverse of a time delay is a time lead which is all noncausal.

Reverberation in the model of a real structure is represented using minimum phase models. These models are all causal, stable models which have all of their poles and zeros inside the unit circle. For example, if the bar in the above example were not semi-infinite, but finite in length, then a model of the collocated response of the bar caused by an end load would have all of its poles and zeros inside the unit circle. There would be no time (phase) delays in this type of system, and therefore, there would be no zeros with magnitude greater than one. A model with all of its zeros and poles inside the unit circle is a minimum phase model. When a minimum phase model is inverted, the result is a minimum phase model.

A dynamic model of a real physical structure can be represented as the cascade of an allpass model with a minimum phase model. In Figure 4 the poles and zeros of a forward model, labeled Model A, are plotted in the complex plane. The poles are represented by x's, and the zeros are represented by o's. This model represents the dynamics of a real system, and therefore, all of the poles are inside the unit circle, but the zeros can be anywhere. Since the response from Model A is only real, poles and the zeros are symmetric about the real axis. The forward model, Model A, pole-zero plot can be represented by the union of a minimum phase pole zero plot, Model C, and an allpass pole zero plot, Model B).

Model B contains the zeros of Model A that are outside the unit circle and a set of corresponding poles. The poles of Model B are inside the unit circle on a line from the origin to a zero and at a distance from the origin which is the inverse of the distance from its corresponding zero from the origin. This model represents phase distortion caused by dispersive or non-dispersive wave propagation in the physical system.

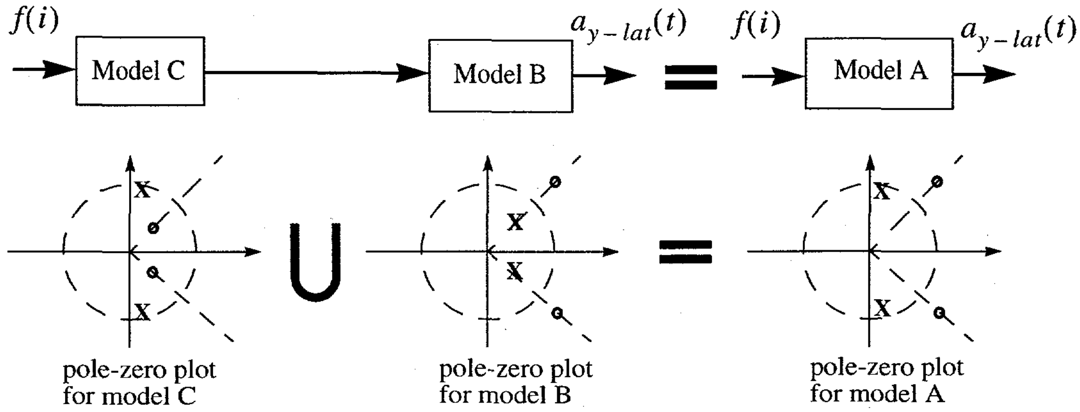

Figure 4: The representation of a model by the cascade of an allpass and a minimum phase model

Model C contains the Model A zeros and poles that were inside the unit circle and contains additional zeros at the location of Model B poles. All of Model C's poles and zeros are inside the unit circle; therefore, Model C is minimum phase. When Model B is cascaded with Model C, poles and zeros cancel and Model A results. The inverse of Model C is all causal, and the inverse of Model B is all noncausal.

To solve for the response of Model A, the recursive relationship for Model B and Model C are used. Model B is solved forward in time as with any causal system, but Model C must be solved backwards in time. Solving for the response of Model C can be difficult since it requires, moving the input backwards in time, convolving the time shifted input with Model C backwards in time and then time shifting the result backwards in time. As a result, if Model C can be neglected, the entire process of solving for an inverse response becomes simplified.

Results

A model of a real physical system will have an allpass portion which represents phase distortion caused by propagation delays and a minimum phase portion which represents reverberation. For the geometry shown in Figure 3, the allpass portion of the model representing component dynamics contributes little to the total response since the propagation time between the point load and component response are small compared to the settling time of the reverberant response. Therefore, the allpass portion of the dynamics can be neglected.

Following the steps outlined above, predictions of the response of the new component to worst-case excitations were determined. These predictions were determined by using a set of models to represent variability in orignal and replacement component dynamics.

In Figure 5 the shock response spectra data for the orignal component and the results of the first inverse problem are shown. These results contain no variability since a model was not needed to construct them.

In Figure 6 the variability of the frequency response function between the load and the y-lateral acceleration is given. The variability in this frequency response function is typical of variability in all of the frequency response function used to produce the following results.

In Figure 7, an estimate of replacement component response can be seen. The response of the new component is not one response but a set of responses that lie within these bounds.

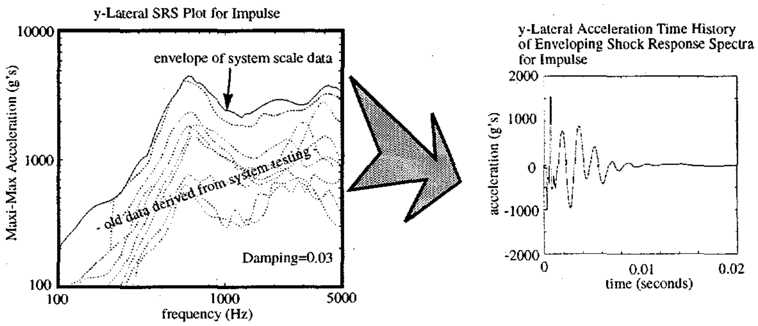

Figure 5. Results from the first inverse problem

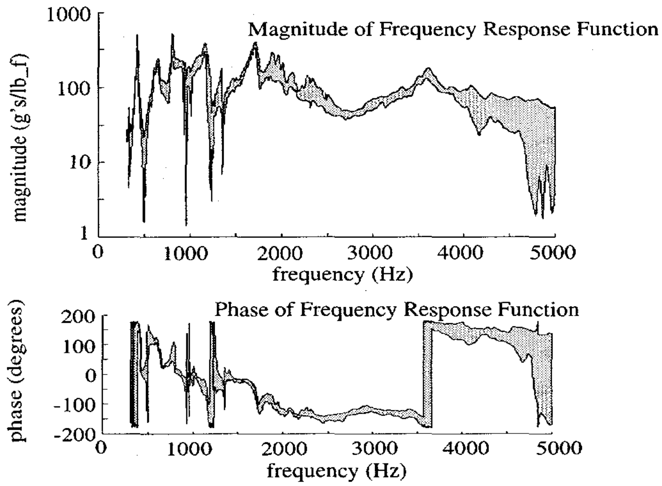

Figure 6. Variability in frequency response functions

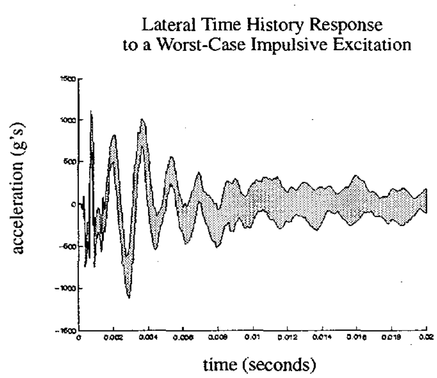

Figure 7. Time response of replacement component

Conclusions

In this paper an interim methodology to predict the time responses of a replacement component subjected to a worst-case excitation was presented. In constructing and executing this methodology, two conclusions were drawn:

- Any model used to predict the time response of a replacement component should include a representation of variability.
- The use of this methodology tends to produce response bounds which are overly conservative.

The first conclusion results from the fact that modeling results show significant variability. The second conclusion results from the fact that if this methodology were used not to predict responses for a replacement component but to predict responses for the orignal component, the prediction would envelope the orignal response data. Thus, this methodology produces more and more conservative results each time it is applied.

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