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AN ELASTIC-PERFECTLY PLASTIC FLOW MODEL FOR FINITE ELEMENT ANALYSIS OF PERFORATED MATERIALS

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ABSTRACT

This paper describes the formulation of an elastic-perfectly plastic flow theory applicable to equivalent solid [EQS] modeling of perforated materials. An equilateral triangular array of circular penetrations is considered. The usual assumptions regarding geometry and loading conditions applicable to the development of elastic constants for EQS modeling of perforated plates are considered to apply here. An elastic-perfectly plastic [EPP] EQS model is developed for a collapse surface that includes fourth-order stress terms. The fourth order yield function has been shown to be appropriate for plates with a triangular array of circular holes. A complete flow model is formulated using the consistent tangent modulus approach based on the fourth order yield function.

EQS-EPP method is used to obtain a limit load solution for a plate subjected to transverse pressure and fixed at the outer edge. This solution is compared to a solution obtained with an EPP-FEA model in which each penetration in the plate is modeled explicitly. The calculated limit load from using the EQS-EPP model is 8% lower than the limit load calculated by the explicit model.

NOMENCLATURE

P	Pitch of pattern, mm
d	Diameter of penetrations, mm
h	Minimum ligament width, (P-d), mm
μ	Ligament efficiency, h/P
σ_i, ε_i	Stress and strain vectors for i = xx, yy, zz, xy, zx, yz components, MPa and mm/mm
EQS	Equivalent solid
S_y	Yield stress of material, MPa
$S_0 = \mu S_y$	Effective yield stress of EQS material, MPa
D	Elastic matrix, MPa
T	Tangent modulus matrix, MPa

Superscripts E and p refer to elastic and plastic components. Bold type is used to distinguish vectors and matrix quantities.

INTRODUCTION

Elastic analysis procedures are well developed for the equivalent solid [EQS] treatment of perforated materials. These methods have proven to be very useful in the analysis of perforated plates found in pressure vessel applications such as tubular heat exchangers.

Extension of EQS elastic methods to elastic-perfectly plastic [EPP] material response has not progressed as far owing in part to complex nature of the yield surfaces needed to describe yielding of perforated materials and to a sense that elastic methods are sufficient for design of perforated materials.

Incentive for further development of EPP-EQS solution capability for perforated plates has recently increased as improvements in computer capabilities have made contemplation of EPP finite element analysis [FEA] a viable tool for design calculations providing significant fabrication cost savings through use of thinner sections.

This paper develops an EPP-EQS theory suitable for incorporation into a standard elastic-plastic FEA program such as ABAQUS (1997). The theory is based on the fourth order collapse surface developed by Gordon et al. (1999). The theory is implemented into the ABAQUS program by way of a user defined plasticity module. Verification of the procedure is provided by comparing the EPP-EQS solutions with an EPP-FEA solution for which each penetration is modeled explicitly. As additional verification and to show practicality of the method to more complex problems, a perforated plate subjected to transverse pressure and clamped on its outer edge is analyzed using the proposed method. The EPP-EQS solution is compared to a solution developed using an FEA model where each hole is modeled explicitly. The collapse load calculated by the proposed method agrees within 8% of the collapse load calculated by the explicit model.

ELASTIC-PLASTIC FLOW THEORY

Consider a yield surface defined by the equation

$$\sigma_{eff} = \{\frac{1}{4}\{P(\sigma_{xx} + \sigma_{yy})^4 + Q[(\sigma_{xx} - \sigma_{yy})^2 + 4\tau_{xy}^2]^2$$

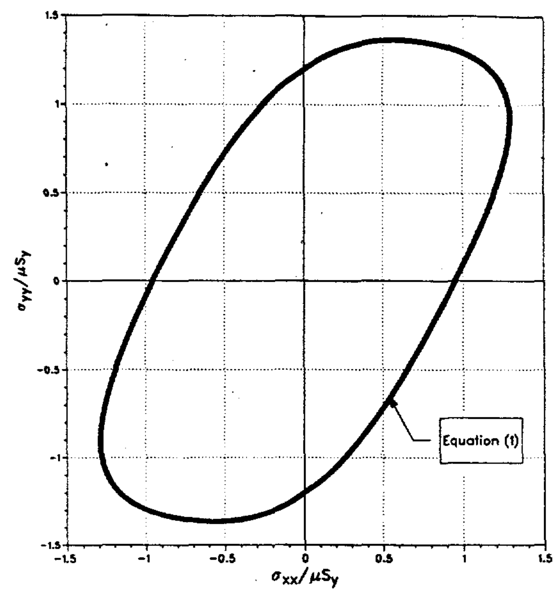
$$+ R(\sigma_{xx} + \sigma_{yy})^2[(\sigma_{xx} - \sigma_{yy})^2 + 4\tau_{xy}^2]$$

$$+ T(\sigma_{xx}^2 - \sigma_{yy}^2)[(\sigma_{xx} - \sigma_{yy})^2 - 12\tau_{xy}^2]\}^{0.5}$$

$$+ Y[\sigma_{zz}^2 - \sigma_{zz}(\sigma_{xx} + \sigma_{yy})] + 3Z_1\tau_{yz}^2 + 3Z_2\tau_{zx}^2\}^{0.5} \quad (1)$$

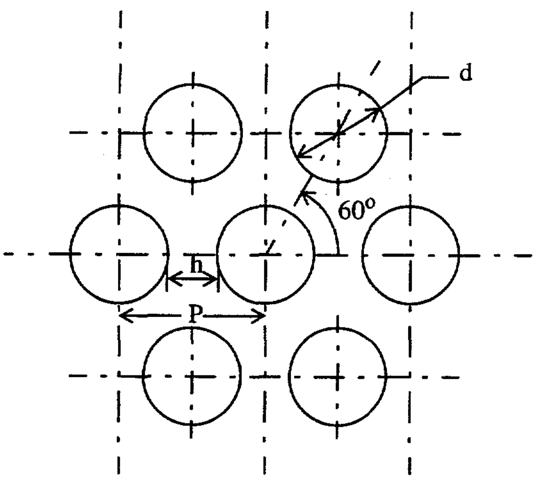
This surface is shown in Figure 1 in (σ_{xx}, σ_{yy}) stress space. The constants P, Q, R, T, Y, Z_1, and Z_2 which define this surface were obtained by Gordon, et al. (1999) for a perforated plate with a triangular pattern and h/P = 0.31733 as shown in Figure 2. An EPP flow theory is sought which is consistent with that yield surface.

Figure 1. Collapse Surface



EPP flow theories using the Von Mises yield function have been developed and are incorporated in virtually all EP-FEA programs. The basic concepts of plasticity provided in texts such as Mendelson (1968) are used here. The main concern is to provide a theory that is consistent with the requirements of modern FEA programming practices in that most programs require the so-called "consistent tangent modulus formulation" as developed Simo and Taylor (1985), Hinton and Owen (1984), Zienkiewicz and Taylor (1991), Dodds (1987), Keppel and Dodds (1990) and Sauve, Badie and Lovie (1998) among others. This means that we seek the relationship between the total stress and the total strain of the form $\mathbf{T} = \partial\sigma_i / \partial\varepsilon_j$ where **T** is the consistent tangent modulus matrix.

Figure 2. Triangular Penetration Pattern



$\mu = (P\text{-}d)/P = h/P$

The notation used in the following is based on the desire to have the result in matrix form directly applicable to FEA programs. For this reason a convention is adopted such that stresses and strains are six dimensional vectors of the form $\sigma = \{\sigma_{xx}, \sigma_{yy}, \sigma_{zz}, \tau_{xy}, \tau_{zx}, \tau_{yz}\}^T$ and $\varepsilon = \{\varepsilon_{xx}, \varepsilon_{yy}, \varepsilon_{zz}, \gamma_{xy}, \gamma_{zx}, \gamma_{yz}\}^T$.

The elastic matrix for perforated materials with a triangular array of holes is transversely isotropic and of the form

$$\mathbf{D}^{-1} = \begin{bmatrix} 1/E_p^* & -\nu_p^*/E_p^* & -\nu/E_z^* & 0 & 0 & 0 \\ & 1/E_p^* & -\nu/E_z^* & 0 & 0 & 0 \\ & & 1/E_z^* & 0 & 0 & 0 \\ & & & 2(1+\nu_p^*)/E_p^* & 0 & 0 \\ & \text{(Symmetric)} & & & & \\ & & & & 1/G_z^* & 0 \\ & & & & & 1/G_z^* \end{bmatrix} \qquad (2)$$

A flow rule based on the collapse surface of Equation (1) is developed by assuming that a yield function exists such that

$$F(\sigma_i) = \sigma_{eff} - S_0 \qquad (3)$$

The material yields when F = 0 and dF = 0.

The plastic strain increment is given by the flow rule

$$\Delta\varepsilon^p = \Delta\lambda \frac{\partial F}{\partial \sigma} = \Delta\lambda\, \phi \qquad (4)$$

where $\phi = \partial F/\partial \sigma = \partial \sigma_{eff}/\partial \sigma$ and $\Delta\lambda$ is a constant that is determined by the requirement that σ satisfy $F(\sigma) = 0$.

A strain increment $\Delta\varepsilon$ from stress and strain state $(\sigma_o, \varepsilon_o)$ will produce plastic deformation only if the elastic estimate of the end-of-step stress $\sigma^* = \sigma_o + \mathbf{D}(\Delta\varepsilon)$, leads to $F(\sigma^*)>0$. In that case, the end-of-step stress is

$$\sigma_F = \sigma_o + \mathbf{D}(\Delta\varepsilon - \Delta\lambda\, \phi) \qquad (5)$$

Thus the calculation of the end-of-step stress requires solution of seven simultaneous equations involving $\Delta\lambda$ and the six independent stress components σ. Defining a vector $\mathbf{x} = \{\sigma_{xx}, \sigma_{yy}, \sigma_{zz}, \tau_{xy}, \tau_{zx}, \tau_{yz}, \Delta\lambda\}^T$ and re-arranging Equation(5) gives

$$\mathbf{f}(\mathbf{x}_i) = \sigma_F - \sigma^* + \Delta\lambda \mathbf{D}\phi = 0 \qquad (6)$$

for i = 1,...,6 and

$$f_7 = \sigma_{eff}(\sigma_F) - S_o = 0 \qquad (7)$$

These may be solved iteratively by the Newton-Raphson method. Expand $\mathbf{f} = \mathbf{f}(\mathbf{x}_i)$ in a Taylor series about the n-th estimate of the solution $\mathbf{x}$

$$\mathbf{f}(\mathbf{x}) = \mathbf{f}(\mathbf{x}_n) + (\mathbf{x} - \mathbf{x}_n)\left[\frac{\partial f_i}{\partial x_j}\right] + \ldots \qquad (8)$$

Then, neglecting the higher-order terms and noting that if $\mathbf{f}(\mathbf{x}_{n+1}) \cong 0$, $\mathbf{x}_{n+1}$ is

$$\mathbf{x}_{n+1} = \mathbf{x}_n - \left[\frac{\partial f_i}{\partial x_j}\right]^{-1} \mathbf{f}(\mathbf{x}_n) \qquad (9)$$

The Newton-Raphson Jacobian matrix $[\partial f_i/\partial x_j]$ provides the return of the stress components to the yield surface as shown schematically in Figure 3.

CONSISTENT TANGENT MODULUS

The consistent tangent modulus operator **T** is required for efficient numerical implementation. For this purpose it is convenient to define an effective strain as an adjunct to the effective stress given by the yield function. Using the usual equalities of plastic work provides the relationships

$$\Delta W^{P} = \sigma_{eff} \Delta\varepsilon^{p}_{eff} \qquad (10)$$

Setting $\sigma\Delta\varepsilon^{p} = \Delta W^{p}$ and using Equation (4) results in $\Delta\lambda = \Delta\varepsilon^{p}_{eff}$. Defining $\varepsilon^{E}_{0} = \mathbf{D}^{-1}\sigma_0$ and setting $\varepsilon = \varepsilon^{E}_{0} + \Delta\varepsilon$, Equation (5) can be written as

$$\sigma + \Delta\varepsilon^{p}_{eff}\, \mathbf{D}\, \phi = \mathbf{D}\, \varepsilon \qquad (11)$$

Differentiating Equation (11) with respect to all variables and manipulating the equation as described in Zienkiewicz and Taylor (1991) gives

$$\mathbf{D}^{-1} d\sigma + \Delta\varepsilon^{p}_{eff}\, d\phi + d\varepsilon^{p}_{eff}\phi = d\varepsilon \qquad (12)$$

At this point it is convenient to define **D*** as

$$\mathbf{D}^{*} = [\mathbf{D}^{-1} + \Delta\varepsilon^{p}_{eff}\, \partial\phi/\partial\sigma]^{-1} \qquad (13)$$

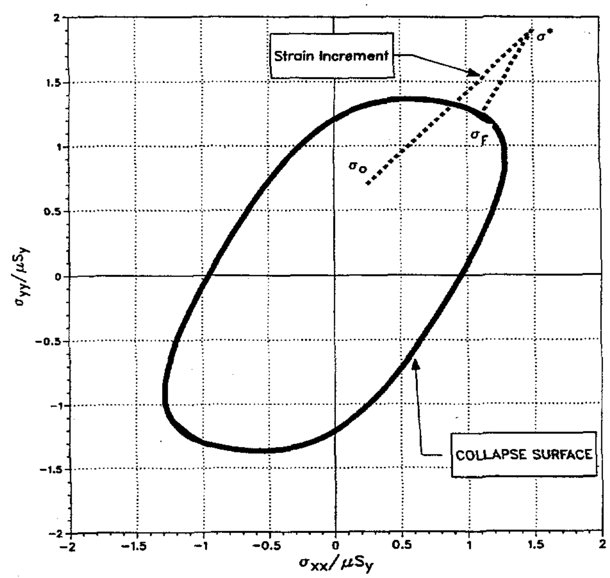
At yielding dF = 0 so that $(\partial\sigma_{eff}/\partial\sigma)d\sigma = 0$. From Equation (12),

$$d\varepsilon^{p}_{eff} = \phi^{T}\ \mathbf{D}^{*} d\varepsilon / \phi^{T} D^{*} \phi \qquad (14)$$

Again manipulation of Equation (12) gives the desired final consistent tangent modulus

$$[\mathbf{T}] = \left[\mathbf{D}^{*} - \frac{\mathbf{D}^{*}\ \phi\ \phi^{T}\ \mathbf{D}^{*}}{\phi^{T}\ \mathbf{D}^{*}\ \phi}\right] \qquad (15)$$

Figure 3. Return Mapping Procedure



NUMERICAL IMPLEMENTATION

The EQS plasticity algorithm is as follows:

1. Apply an increment of strain, $\Delta\varepsilon$, and consider it to be elastic. Calculate $\sigma*$ using $\sigma* = \sigma_0 + \mathbf{D}\Delta\varepsilon$.

2. If $\sigma*$ produces a σ_{eff} that is less than yield, the step is elastic and the consistent tangent modulus is **D** per Equation (2). The plastic strain increments are zero for this step. Return from the algorithm.

3. If $\sigma*$ produces $\sigma_{eff} > S_o$, find the appropriate σ_F that is on the yield surface using the return mapping procedure to solve the non-linear equations given in Equations (6 and 7). This requires determination of the Jacobian matrix $[\partial f_i / \partial x_j]$ as described in Equation (9). Note that $\Delta\lambda$ is equal to the effective plastic strain increment.

4. Calculate the flow vector ϕ for σ_F and the plastic strain increments per Equation (4).

5. Calculate the $[\partial\phi_i/\partial\sigma_j]$ matrix, D* from Equation (13) and **T** from Equation (15). Return from the algorithm.

The flow vector ϕ, the derivatives of the flow vector for the formation of the Newton-Raphson Jacobian Matrix, and the $[\partial\phi_i/\partial\sigma_j]$ matrix are tedious to obtain for the fourth order yield function. These relations were obtained using MATHEMATICA (1997).

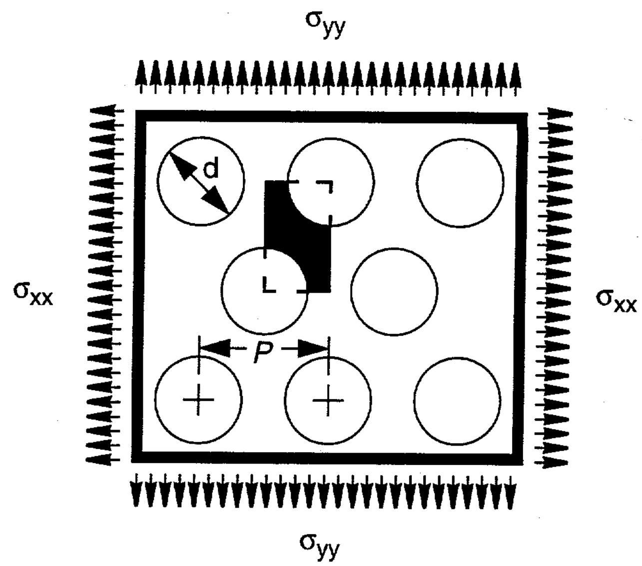
EXAMPLE PROBLEMS

Numerical results are presented in this section. The problems were selected for their utility in both verifying that the EPP-EQS theory can be used to accurately predict plastic collapse of perforated materials and for demonstrating the efficiency with which the method can be obtained using a standard EPP-FEA program. We used ABAQUS for these problems. The EPP-EQS method was used by way of the user module UMAT capability of ABAQUS defined according to the algorithm described in the previous section. Solutions from the EPP-EQS models were compared to solutions from an FEA analysis for which the penetrations were modeled explicitly. These explicit models allowed us to confirm that the EQS-EPP solutions were correct.

Example 1. Collapse Surface

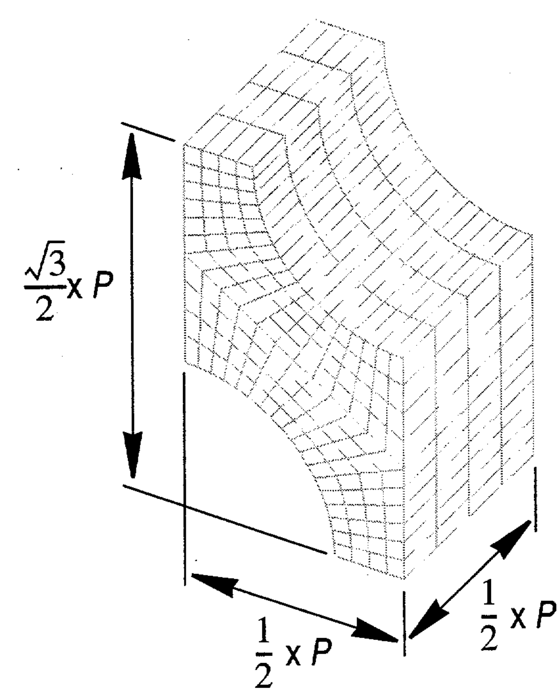
The first and fourth quadrants of the collapse surface in the (σ_{xx},σ_{yy}) plane were generated using ABAQUS with the user specified EPP-EQS module. A simple two-by-four rectangular element mesh was defined for this study. Boundary conditions were applied to produce 19 different σ_{xx}/σ_{yy} or σ_{yy}/σ_{xx} ratios. The unit cell for this analysis is shown in Figure 4.

Figure 4. Unit Cell Verification Problems



The ligament efficiency of 0.31733 was chosen for this study. All problems were run until the collapse load was obtained. The collapse load is defined as the load for which a small increase in load produces an unbounded increase in deflection. Numerically this is reflected by a large difficulty in achieving a converged solution for a small increase in load.

Figure 5. FEA Model for Unit Cell Analysis



Solutions for the explicit models were obtained for the same boundary conditions. The EQS model and the explicit unit cell model are shown in Figures 5 and 6. Generalized plane strain solutions were generated by Gordon et al. (1999) so that the results are independent of thickness dimensions. The solutions are compared in Figure 7. The largest difference between the EPP-EQS solutions and the explicit model solutions is 5%. The EPP-EQS solutions were obtained in a fraction of the computer time required to obtain the explicit solutions.

Figure 6. EQS FEA Unit Cell Model

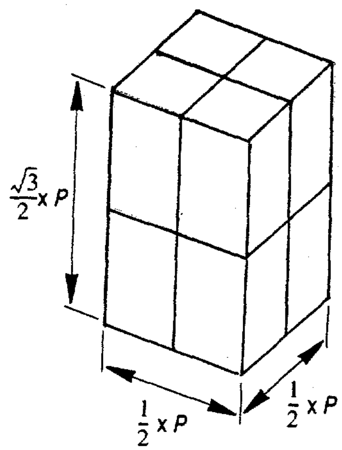
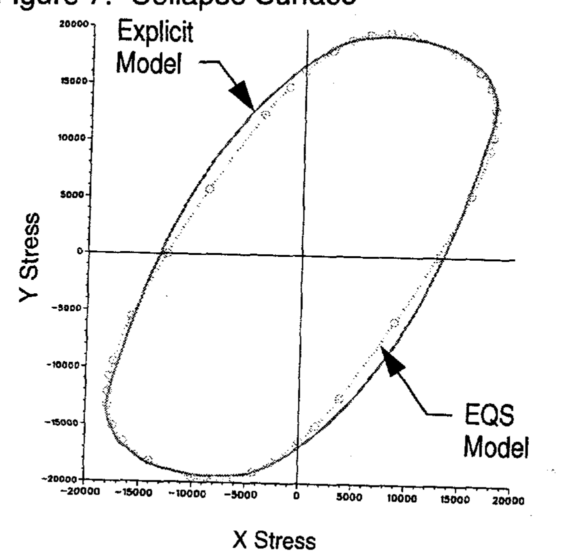


Figure 7. Collapse Surface



Example 2. Transverse Pressure on a Plate

Consider a rectangular plate of finite length and infinite width which has the short sides clamped. The plate has a triangular array of penetrations parallel to the transverse z-direction and pressure is applied to the top surface. The center of the plate is shown in Figure 8, such that symmetry boundary conditions are applied along the y = 0 and $\sqrt{3}P/2$ surfaces and along the x = 0 surface. Pressure is applied on the z = 21.33 P surface. The associated EPP-EQS model is shown in Figure 9. Both of these models are solved using the ABAQUS 20 node hexagon reduced element three-dimensional solid element. The explicit model has 24,432 elements and the EPP-EQS model has 7,182 elements.

Figure 8. Clamped Plate Geometry

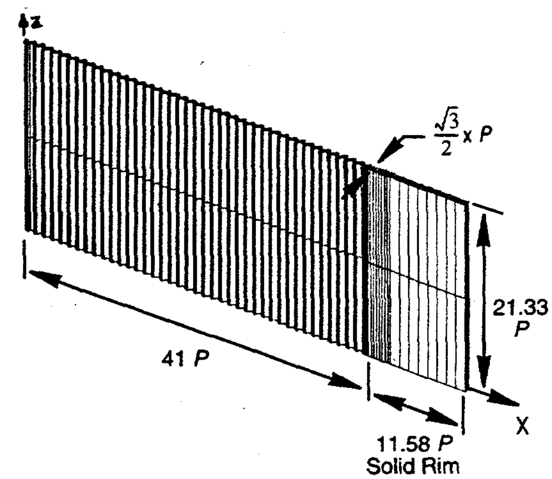
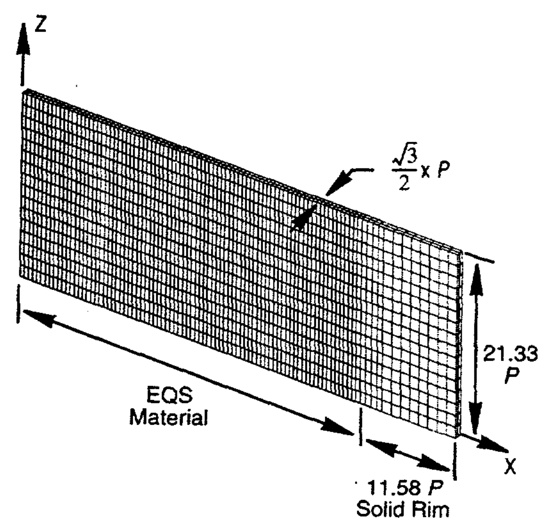


Figure 9. EPP-EQS Model for the Slice Problem



The deformed geometry for each model is seen by comparing Figures 10 and 11. These models respond identically as shown in the load deflection diagram shown in Figure 12. The calculated collapse load from each model agrees within 8%.

Figure 10. Deformed Geometry for the Explicit Model

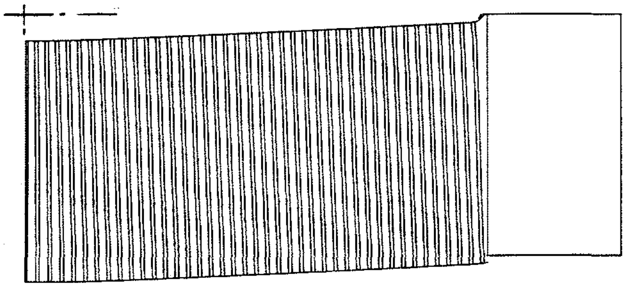


Figure 11. Deformed Geometry for the EQS Model

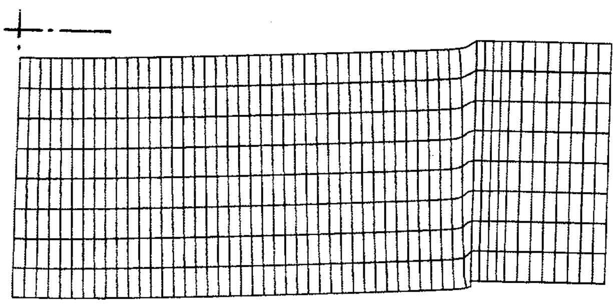
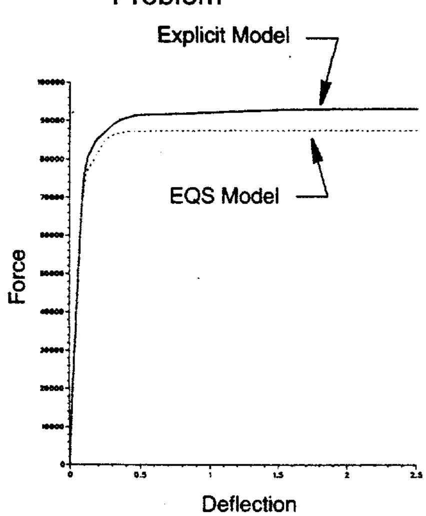


Figure 12. Load Deflection Plot for the Slice Problem



The utility of the EPP-EQS model is demonstrated by the fact that very similar results were obtained for both models even though a significantly simpler finite element mesh is required for EPP-EQS model than in the explicit model. As the number of penetrations and the complexity of the overall structure increases, the simplicity afforded by the EQS model is apparent.

CONCLUSIONS

A consistent tangent modulus was obtained for a fourth order yield function appropriate to describe the collapse surface of a perforated plate with a triangular array of penetrations. An algorithm was developed for implementing the theory into a practical finite element program for the purposes of calculating limit loads for perforated plates. The method was demonstrated to be accurate with 8% of limit loads calculated using finite element models where the penetrations were included explicitly. The method was shown to greatly reduce the complexity of limit load calculations for perforated plates over the conventional finite element modeling practice.

ACKNOWLEDGEMENT

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