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NON-PERTURBATIVE GLUODYNAMICS OF HIGH ENERGY HEAVY-ION COLLISIONS

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The dynamics of low-x partons in the transverse plane of a high-energy nuclear collision is classical, and therefore admits a fully non–perturbative numerical treatment. We report results of a recent study estimating the initial energy density in the central region of a collision. Preliminary estimates of the number of gluons per unit rapidity, and the initial transverse momentum distribution of gluons, are also provided.

In heavy-ion experiments, planned at RHIC later this year, gold ions are expected to collide at $\sqrt{s} = 200$ GeV per nucleon. A few years later $\sqrt{s} = 5.5$ TeV per nucleon will be attained in heavy ion collisions at LHC. In the central region of these collisions, a combination of very high center-of-mass energy with a very large number of participating valence quarks will likely give rise to a novel regime of QCD, one characterized by a very high parton density. This regime does not easily lend itself to a description based on conventional approaches. Collisions involving large transverse momenta can be adequately described in terms of pairwise scattering of individual partons comprising the colliding systems. Final-state interactions of secondary partons formed therein can be safely neglected [1]. However, as the parton density grows, final-state interactions of secondary partons must be taken into account. This requirement is only partially satisfied by multiple scattering or by classical cascade descriptions, which ignore the coherence of the secondary field configuration [2].

The coherence of the secondary partons is incorporated naturally into the classical effective field theory approach of McLerran and Venugopalan (MV) [3]. If the parton density in the colliding nuclei is high at small x, classical methods are applicable. It has been shown recently that a RG-improved generalization of this effective action reproduces several key results in small-x QCD: the leading $\alpha_s \log(1/x)$ BFKL equation, the double log GLR equation and its extensions, and the small-x DGLAP equation for quark distributions [4].

Briefly, the model is based on the following assumptions. Partons in a nucleus are separated into high-x and the low-x components. The former corresponds to valence quarks and hard sea partons. These high-x partons are considered recoilless sources of color charge. For a large Lorentz-contracted nucleus, this results in a static Gaussian distribution of their color charge density ρ in the transverse plane:

$$P([\rho]) \propto \exp\left[-\frac{1}{2g^4\mu^2}\int \mathrm{d}^2 r_t \rho^2(r_t)\right].$$

The variance μ^2 of the color charge distribution is the only dimensional parameter of the model, apart from the linear size L of the nucleus. For central impact parameters, μ is given in terms of single-nucleon structure functions[5]:

$$\mu^2 = \frac{A^{1/3}}{\pi r_0^2}\int_{x_0}^1 \mathrm{d}x \left(\frac{1}{2N_c}q(x,Q^2) + \frac{N_c}{N_c^2-1}g(x,Q^2)\right),$$

with the separation scale $x_0 \equiv Q/\sqrt{s}$, $r_0 = 1.12$ fm, and N_c the number of colors. It is assumed, in addition, that the nucleus is infinitely thin in the longitudinal direction. Under this simplifying assumption the resulting gauge fields are boost-invariant.

The small x fields are then described by the classical Yang-Mills equations

$$D_\mu F_{\mu\nu} = J_\nu \tag{1}$$

with the random sources on the two light cones: $J_\nu = \sum_{1,2}\delta_{\nu,\pm}\delta(x_\mp)\rho_{1,2}(r_t)$. The two signs correspond to two possible directions of motion along the beam axis z. As shown by Kovner, McLerran and Weigert (KMW), low x fields in the central region of the collision obey sourceless Yang-Mills equations (this region is in the forward light cone of both nuclei) with the initial conditions in the $A_\tau = 0$ gauge given by

$$A^i = A_1^i + A_2^i; \quad A^\pm = \pm\frac{ig}{2}x^\pm[A_1^i, A_2^i]. \tag{2}$$

Here the pure gauge fields $A_{1,2}^i$ are solutions of (1) for each of the two nuclei in the absence of the other nucleus.

Equation (1) with the initial condition (2) can now be solved, in order to obtain the resulting gluon field configuration at late proper times. Since the initial condition depends on the random color source, averages over realizations of the source must be performed. This aspect of the solution resembles the classical thermal theory, wherein an average is performed over initial conditions

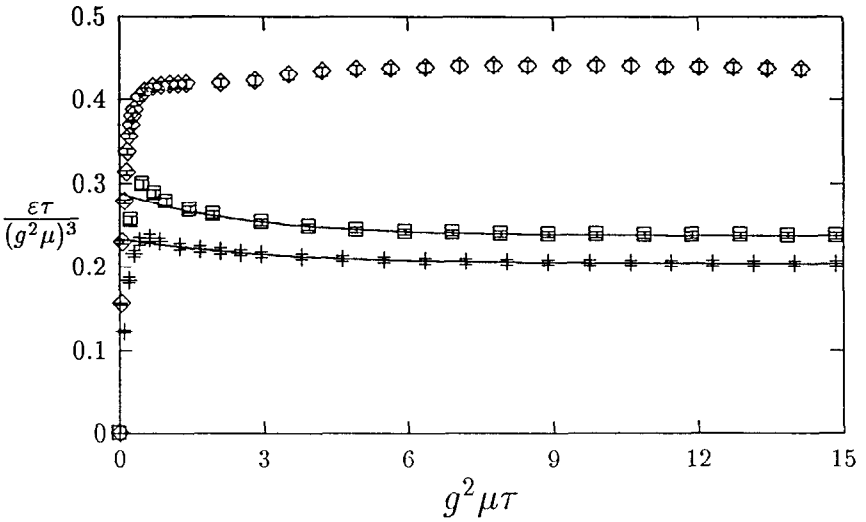

Figure 1: Transverse-plane energy density per unit rapidity versus proper time for the values 5.66 (diamonds), 35.36 (plusses), and 297 (squares) of $g^2\mu L$. Both the energy density and the proper time are expressed in units of $g^2\mu$. The solid lines are fits of the data to the form $\alpha + \beta\exp(-\gamma\tau)$.

drawn from the canonical ensemble. In fact, the analogy between the MV effective theory and the classical thermal theory goes further. This analogy can be made explicit by considering the perturbative solution of (1) obtained by KMW. They showed that in perturbation theory the gluon number distribution by transverse momentum (per unit rapidity) suffers from an infrared divergence and argued that the distribution must have the form

$$n_{k_\perp} \propto \frac{1}{\alpha_s}\left(\frac{\alpha_s\mu}{k_\perp}\right)^4 \ln\left(\frac{k_\perp}{\alpha_s\mu}\right) \tag{3}$$

for $k_\perp \gg \alpha_s\mu$. We can now draw a parallel between μ and the temperature T of the thermal system. In particular, the log term suggests that the perturbative description breaks down for $k_\perp \sim \alpha_s\mu$. Likewise, the perturbative thermal theory loses validity at the non-perturbative scale $k \sim g^2T$.

It is therefore clear that a fully non-perturbative study of MV model is necessary. The model is discretized on a lattice in the transverse plane and the lattice field equations solved numerically. Boost invariance and periodic boundary conditions in the transverse plane are assumed. Technical details of the lattice formulation can be found in Ref. [8]. The quantity $g^2\mu$ and the

linear size L of the nucleus are the only physically interesting dimensional parameters of the MV model[7]. Any dimensional quantity q can then be written as $(g^2\mu)^d f_q(g^2\mu L)$, where d is the dimension of q. All the non-trivial physical information is contained in the dimensionless function $f_q(g^2\mu L)$. On a lattice, q will generally depend also on the lattice spacing a; we will seek to remove this dependence by taking the continuum limit $a \to 0$. Finally, we estimate the values of the dimensional parameter $g^2\mu L$ which correspond to key collider experiments. Assuming Au-Au collisions, we take $L = 11.6$ fm (for a square nucleus!) and estimate the standard deviation μ to be 0.5 GeV for RHIC and 1 GeV for LHC[5]. Also, we have approximately $g = 2$ for energies of interest. The rough estimate is then $g^2\mu L \approx 120$ for RHIC and $g^2\mu L \approx 240$ for LHC. Clearly, there is some variation in $g^2\mu L$ due to the various uncertainities in this estimate. The expression we will derive is a non–perturbative formula, from which one can deduce the number or energy of produced gluons for a particular choice of $g^2\mu L$.

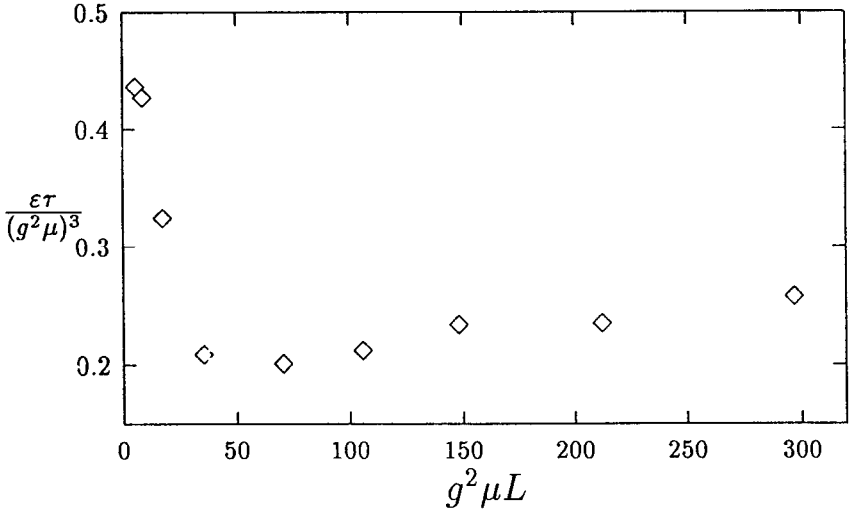

Figure 2: Transverse-plane energy density (in units of $g^2\mu$) per unit rapidity versus $g^2\mu L$. The error bars are smaller than the plotting symbols.

Results of a numerical investigation (for SU(2) only) are as follows. We first compute the energy per unit transverse area per unit rapidity, deposited in the central region by the colliding nuclei. As Figure 1 illustrates, this quantity tends to a constant at late proper times. It is this asymptotic value of the energy density that we wish to determine. If we express the energy density in

units of $g^2\mu$ and extrapolate our numerical findings to the continuum limit, we find that the energy density depends on the dimensionless parameter $g^2\mu L$ as described in Figure 2. Note the very slow variation of this dimensionless function in the entire range of $g^2\mu L$ values, which includes both our RHIC and LHC estimates. Using this plot, and assuming, in accordance with Ref.[11], the $(N_c^2-1)/N_c$ dependence of the energy on the number of colors N_c, we arrive at the values of 2700 GeV and of 25000 GeV for the transverse energy per unit rapidity at RHIC and at LHC, respectively[9].

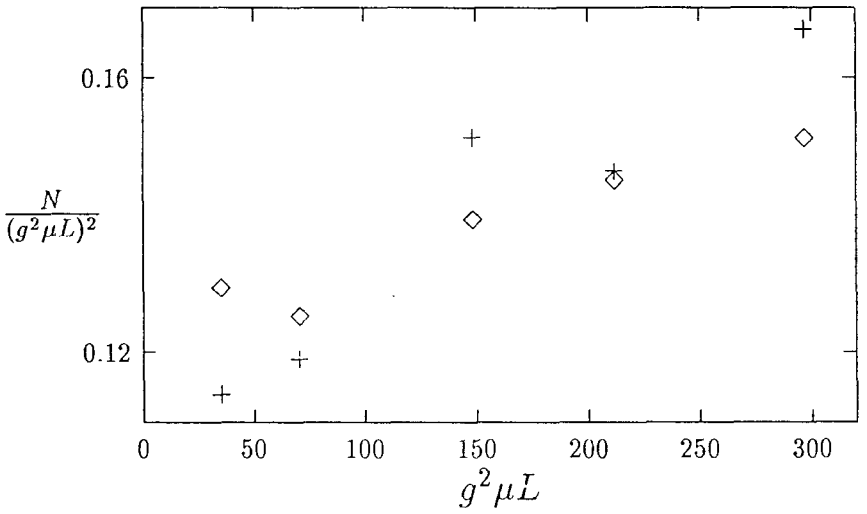

Figure 3: Transverse-plane gluon number density per unit rapidity versus $g^2\mu L$ from Coulomb gauge fixing (diamonds) and from relaxation (plusses).

Next, we report our preliminary estimates of low-x gluon multiplicities in the central region. Determination of the total number of produced gluons is of considerable interest: this quantity may be directly related to the number of produced hadrons[10]. Further, the momentum distribution of gluons in the transverse plane can be used as initial data for a Boltzmann-type equation describing evolution of the gluon gas towards thermal equilibrium[11].

The particle number is a well-defined notion in a free field theory whose Hamiltonian in momentum space has the form

$$H_f = \frac{1}{2}\sum_k \left(|\pi(k)|^2 + \omega^2(k)|\phi(k)|^2\right). \qquad (4)$$

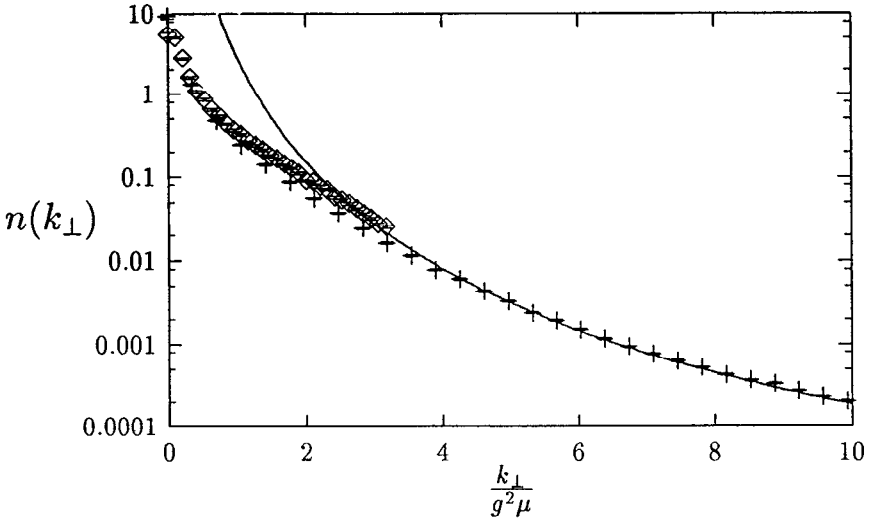

Figure 4: Transverse-plane gluon number (per unit rapidity) distribution for $g^2\mu L = 35.5$ (plusses) and for $g^2\mu L = 297$ (diamonds). The transverse momentum is expressed in units of $g^2\mu$. The solid line is a fit of the $g^2\mu L = 35.5$ data to the perturbative expression (3).

where $\phi(k)$ is k-th momentum component of the field, $\pi(k)$ is its conjugate momentum, and $\omega(k)$ is the corresponding eigenfrequency. The average particle number of the k-th mode is then

$$n(k) = \omega(k)\langle|\phi(k)|^2\rangle = \sqrt{\langle|\phi(k)|^2|\pi(k)|^2\rangle}, \tag{5}$$

where, in our case, the average $\langle\rangle$ is over the initial conditions. Obviously, any extension of this notion to interacting theories should reduce to the standard free-field definition of the particle number in the weak-coupling limit. However, this requirement alone does not define the particle number uniquely outside a free theory. We therefore use two different generalizations of the particle number to an interacting theory, each having the correct free-field limit. We verify that the two definitions agree in the weak-coupling regime corresponding to late proper times in the central region. We note, however, that the theory in question may have low-lying metastable states. If, for a small value of $g^2\mu L$, the system finds itself in the vicinity of a metastability, then the system is far from linearity, and both our definitions of the number make little sense. We therefore restrict our attention to values of $g^2\mu L$ for which energies of metastable minima are much lower than the average energy of a configuration. In all such cases the two definitions give results close to each other.

Our first definition is straightforward. We impose the Coulomb gauge condition in the transverse plane: $\vec{\nabla}_\perp \cdot \vec{A}_\perp = 0$ and substitute the momentum components of the resulting field configuration into (5). At this point, there are two possibilities open to us. We can assume $\omega(k)$ to be the standard massless (lattice) dispersion relation and use the middle expression of (5) to compute $n(k)$. Alternatively, we can determine $n(k)$ from the rightmost expression of (5); the middle expression of (5) can then be used to obtain $\omega(k)$.

Our second definition is based on the behavior of a free-field theory under relaxation. Consider a simple relaxation equation for a field in real space,

$$\partial_t \phi(x) = -\partial H / \partial \phi(x), \tag{6}$$

where t is the relaxation time (not to be confused with real or proper time) and H is the Hamiltonian. For a free field ($H = H_f$) the relaxation equation has exactly the same form in the momentum space with the solution $\phi(k,t) = \phi(k,0)\exp(-\omega^2(k)t)$. The potential energy of the relaxed free field is $V(t) = (1/2)\sum_k \omega^2(k)|\phi(k,t)|^2$. It is then easy to derive the following integral expression for the total particle number of a free-field system:

$$N = \sqrt{\frac{8}{\pi}} \int_0^\infty \frac{\mathrm{d}t}{\sqrt{t}} V(t). \tag{7}$$

Now (6) can be solved numerically for interacting fields. Subsequently, $V(t)$ can be determined, and N can be computed by numerical integration. Note that in a gauge theory the relaxation equations are gauge-covariant, and the relaxed potential $V(t)$ is gauge-invariant, entailing gauge invariance of this definition of the particle number. This is an attractive feature of the relaxation method. On the other hand, this technique presently only permits determination of the total particle number and cannot be used to find the number distribution.

Our findings are summarized in Figures 3 and 4. We consider these results preliminary, since we are yet to perform a careful extrapolation to the continuum limit, as we did in the case of the energy. In the case of the energy measurement, the systematic error related to a finite lattice cutoff was of the order of 10%. For the particle number, which is better behaved in the ultraviolet than the energy, this systematic error should be smaller.

As Figure 3 shows, our two definitions of the particle number agree on a 20% level in a wide range of values of $g^2\mu L$, which includes the RHIC and the LHC regimes. If we write the particle number per unit rapidity as $N = (g^2\mu L)^2 f_N(g^2\mu L)$, then $f_N(g^2\mu L) = 0.14 \pm 0.03$ in that range.

We now estimate the number of gluons produced in one unit of rapidity, at central rapidities, at RHIC and LHC. We extrapolate to SU(3) in a manner

analogoues to that of the energy estimate. For $g^2\mu L \approx 116$ (RHIC) and $f_N = 0.13$–0.15, we obtain $N_g = 778$–897. For $g^2\mu L \approx 232$, (LHC) we obtain $N_g = 3100$–3600. For the same range of f's, a $g^2\mu L = 150$ value for RHIC would give $N_g = 1300$-1500, and $g^2\mu L = 300$ for LHC would give $N_g = 5200$–6000. Since N_g depends quadratically on $g^2\mu L$, and the latter is not known with with great precision, the range of the prediction is significant. What would be more interesting though is the slope f of the ratio of the two, for which we have a prediction up to 20% at present. Varying the energies, and sizes of the nuclei, should enable one to extract this quantity. This point, and comparisons to predictions from other models, will be discussed further in a forthcoming paper [12].

Finally, Figure 4 shows how N is distributed among various momentum modes, for two extreme cases: $g^2\mu L \approx 300$ and $g^2\mu L \approx 35.5$. For comparison, we also show a fit of the high-momentum tail ($k_\perp \gg g^2\mu$ of the $g^2\mu L \approx 35.5$ distribution to the perturbative expression (5). On the low-$k_\perp$ end of the spectrum our fully non-perturbative result deviates significantly from the perturbative prediction and remains finite at $k_\perp = 0$. Note that the deviation first occurs for $k_\perp$ of the order of the non-perturbative scale $g^2\mu$.

In summary, our numerical implementation of MV model allows one to take into account non–perturbative effects at high parton density in the central region. We have derived non–perturbative formulae which relate the energy and number of produced gluons to the gluon density and the size of the incoming nuclei. Varying the energy and the size of nuclei should enable us to test the predictive power of these formulae. Our treatment can be made more accurate by switching from the SU(2) to the true physical SU(3) gauge group, by relaxing the assumption of exact boost invariance, and by replacing periodic boundary conditions by more realistic ones. We plan to address these issues in future work.

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