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COMPARISON TO THE ASME CODE PRIMARY STRESS LIMITS

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Elastic-Plastic Analysis of the PVRC Burst Disk Tests with Comparison to the ASME Code Primary Stress Limits

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ABSTRACT

This paper provides a comparison between finite element analysis results and test data from the Pressure Vessel Research Council [PVRC] burst disk program. Testing sponsored by the PVRC over 20 years ago was done by pressurizing circular flat disks made from three different materials until failure by bursting. The purpose of this re-analysis is to investigate the use of finite element analysis (FEA) to assess the primary stress limits of the ASME Boiler and Pressure Vessel Code (1998) (hereafter the Code) and to qualify the use of elastic-plastic (EP-FEA) for limit load calculations. The three materials tested represent the range of strength and ductility found in modern pressure vessel construction and include a low strength high ductility material, a medium strength medium ductility material, and a high strength low ductility low alloy material. Results of elastic and EP-FEA are compared to test data. Stresses from the elastic analyses are linearized for comparison of Code primary stress limits to test results. Elastic-plastic analyses are done using both best-estimate and elastic-perfectly plastic (EPP) stress-strain curves. Both large strain-large displacement (LSLD) and small

strain-small displacement (SSSD) assumptions are used with the EP-FEA. Analysis results are compared to test results to evaluate the various analysis methods, models, and assumptions as applied to the bursting of thin disks.

The test results show that low strength-high ductility materials have a higher burst capacity than do high strength-low ductility materials. Linearized elastic FEA stresses and ASME Code primary stress limits provide excessive margins to failure for the burst disks for all three materials. The results of these studies show that LSLD EP-FEA can provide a best estimate analysis of the disks but the accuracy depends on the material stress-strain curve. This work concludes that SSSD EPP analysis methods provide a robust and viable alternative to the current elastic linearization method of satisfying the primary stress limits of the Code.

NOMENCLATURE

P_m	Primary Membrane Stress Intensity, MPa, [ksi]
$P_m + P_b$	Primary Membrane plus Bending Stress Intensity, MPa, [ksi]
S_y	Yield Strength, MPa, [ksi]

S_{ult}	Ultimate Strength, MPa, [ksi]
S_m	Material Strength Allowable, MPa, [ksi]
S_L	Limit Load Input Strength, MPa, [ksi]
p	Pressure, MPa, [ksi]
FEA	Finite Element Analysis
Code	ASME Boiler & Pressure Vessel Code
PVRC	Pressure Vessel Research Council
LL	Limit Load
LSLD	Large Strain, Large Deflection
SSSD	Small Strain, Small Deflection
EP	Elastic Plastic
EPP	Elastic-Perfectly Plastic

INTRODUCTION

The PVRC Subcommittee on Effective Utilization of Yield Strength conducted burst tests on disk specimens of a number of different steels to study the effect of yield stress on failure. The program considered the influence of yield strength, as related to strain-hardening exponent or to yield-to-tensile strength ratio, upon the bursting pressure of components containing strain-concentrating geometries. Cooper, Kottcamp, and Spiering (1971) reported the results of these tests and the results were further used by B.F. Langer (1971).

The results of these tests are of current interest to provide qualification of finite element computer programs that are used to compute limit load solutions for use in assessing primary stress limits of the ASME Code (1998). In this paper, three of the tests conducted by the PVRC were selected to compare the test results to analysis results. Each disk was analyzed three ways: a standard ASME Code primary stress evaluation using elastic analysis, EP-FEA using strain-hardening characteristics of the material, and EPP-FEA to compute a limit load for the disks.

The burst disks were 152 mm diameter flat circular plates 3 mm thick. The disks were clamped in a test fixture and hydrostatically pressurized to bursting. Three materials were used to make the plates: a low strength, high ductility stainless steel;

a medium strength, medium ductility carbon steel; and a high strength, low ductility low-alloy steel. Records of transverse deflection versus pressure and burst pressure were recorded for each test.

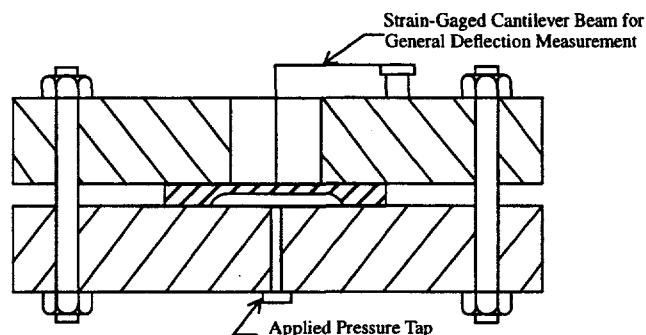
In this paper results from the tests are compared to results from FEA made using various assumptions of elastic-plastic material properties and strain-displacement relationships. Results from the analyses are used to assess the various assumptions relative to the test data. Linearized elastic stresses are used for comparison with ASME Code primary stress limits and these results are compared to the burst pressures for each test. Finally, observed burst pressures are compared to limit load solutions obtained by using EPP-FEA.

TESTING

A detailed summary of the test phase of this investigation is discussed below. Testing was sponsored by the PVRC and completed over twenty years ago as reported by Cooper, Kottcamp, and Spiering (1971).

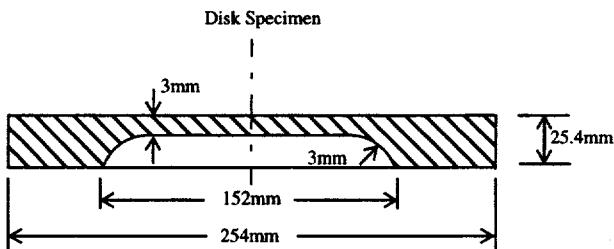
The test specimens were hydrostatically pressure tested to bursting in a test fixture as shown schematically in Figure 1. Pressure was increased from zero until the disks failed by catastrophic bursting. The tests were all performed at room temperature.

Figure 1: PVRC Test Fixture



The specimens tested by PVRC consisted of three 152mm diameter circular plates 3mm thick. The geometry associated with the specimens is shown in Figure 2.

Figure 2: Disk Specimen



The materials of the three specimens tested are 304 stainless steel (304 SS), ABS-C carbon steel, and A533 Grade B low alloy steel (A-533-B). Material properties associated with these materials are shown in Table 1.

Table 1: Material Properties

Material	Yield Strength MPa	Ultimate Strength MPa	Uniform Strain	Reduction of Area
304 SS	234	579	0.54	0.74
ABS-C	269	440	0.31	0.66
A-533-B	510	662	0.17	0.68

TEST RESULTS

Central deflection was measured with a cantilever beam device. Central deflection versus pressure data along with maximum burst pressure and failure location were provided in Cooper et al. (1971). The central deflection versus pressure records on each specimen are given in Figures 10, 11, and 12. In interests of safety, deflection measurements were stopped after about 50mm of deflection. The disks were then pressurized until

failure.

Burst pressure and failure location for each disk are as follows:

- 304 SS Burst Disk - 6800 psi (46.9 MPa) - Failure at the center of the disk by ductile rupture.
- ABS-C Burst Disk - 3750 psi (25.9 MPa) - Failure at the center of the disk by ductile rupture
- A-533-B Burst Disk - 5300 psi (36.5 MPa) - Failure at the built in edge of the plate by cracking.

ANALYSIS

The following FE analyses were conducted in this study:

- Elastic FEA of the burst disks including linearization of the elastic FEA results per Code methods.
- LSLLD EP-FEA of the burst disks using a bilinear true stress-true plastic strain curve.
- SSSD EPP-FEA of the burst disks using $1.5S_m$ as the limit load input strength parameter for the analysis. S_L is used as a yield strength in the EPP-FEA analysis.
- The elastic FEA was conducted using an in-house FEA program. The in-house program and ABAQUS give identical results for elastic problems.
- Stress linearization of the elastic FEA result was conducted using standard stress linearization routines consistent with the recommendations of the PVRC study by Hechmer and Hollinger (1998).
- The EP and EPP finite element analyses were conducted using the ABAQUS (1994) program and the FEA model generated using PATRAN (1990).

The finite element model used to conduct the elastic, the LSLLD EP and SSSD EPP FEA's of the

test specimen is a two-dimensional (2-D) axisymmetric model and is shown in Figure 3. An enlargement of the built-in section showing the mesh around the fillet is shown in Figure 4. The finite element model consists of:

- 23,966 eight node reduced integration quadrilateral elements (ABAQUS element CAX8R)
- 73,151 Nodes

Figure 3: Finite Element Model

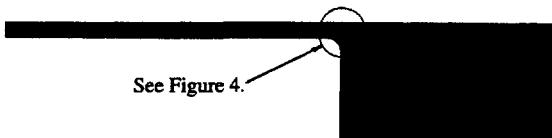
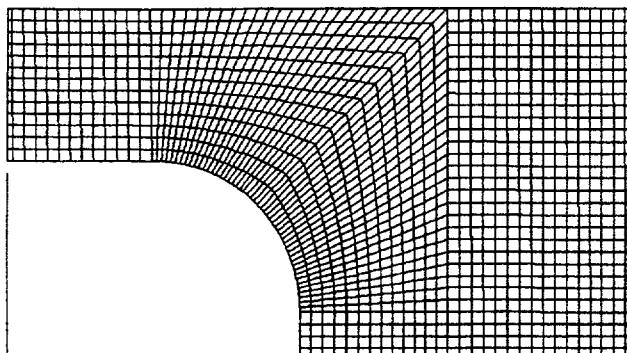
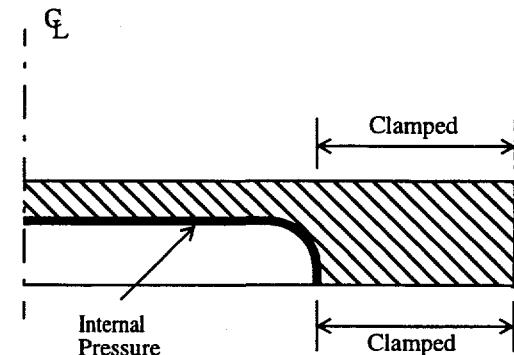


Figure 4: Close-up of Built-In Edge



Boundary conditions applied to the FE model are shown in Figure 5. The clamped in portion of the specimen is assumed to be fixed against all motion. There was no motion of these surfaces observed during the test. A symmetry boundary condition was used at the center of the models.

Figure 5: Boundary Conditions for FEA Models



Both the elastic and EP analyses used Young's modulus of 206×10^3 MPa and Poisson's ratio of 0.3. A bi-linear true-stress, true-strain curve was used for the LSLEP and the SSSD EPP analyses. These curves are shown in Figure 6. Tabular values are given in Table 2. ASME sets allowable strength (S_m) as the lower of $2S_y/3$ or $S_{ult}/3$. Table 3 provides this assessment for the three materials.

Table 2: True-Stress, True-Strain Values

Material	Yield Strength S_y , MPa	Engineering Ultimate S_{ult} , MPa	True Ultimate Strength MPa	True Uniform Strain
304 SS	234	579	892	0.432
ABS-C	269	440	576	0.270
A-533-B	510	662	774	0.157

Table 3: Material Allowable Strength

Material	$2S_y/3$ MPa	$S_{ult}/3$ MPa	S_m MPa
304 SS	156	193	156
ABS-C	179	147	147
A-533-B	340	221	221

Yield Criterion

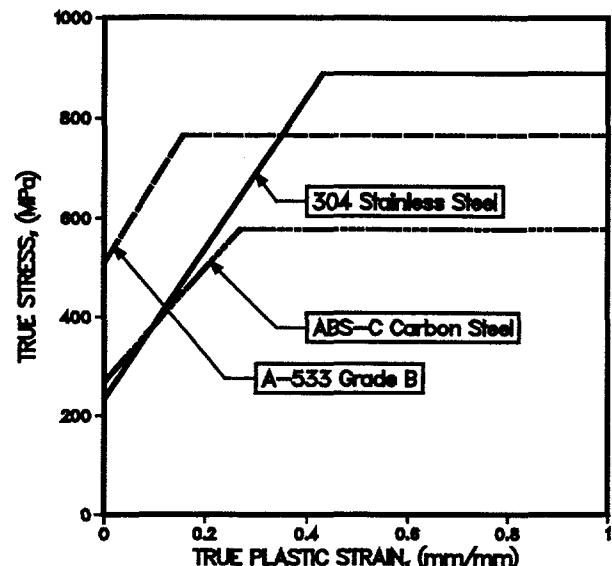
The elastic-plastic analyses are conducted assuming VonMises yield criterion and an isotropic hardening material law. Isotropic hardening is considered reasonable since the specimens were tested by application of monotonically increasing pressure to failure, with no reversal of load. Thus cyclic hardening concerns are limited to very local unloading regions and considered second order for these tests.

Failure Definition

The LSID and SSSD EP-FEA are carried out until the numerical solutions becomes unstable, e.g. when a converged solution for an additional increment of load is numerically difficult to obtain. At this point, the slope of the pressure-deflection curve for the disks becomes vanishingly small. The total pressure that causes this condition is the incipient failure pressure.

Failure in the tests was defined when the disks burst. Instead of trying to predict local rupture, tearing, or cracking from the FEA results, it is proposed here that a useful definition of failure is the pressure for which the structure approaches dimensional instability, i.e. unbounded deflection for a small increment in pressure. This condition is symptomatic of an ill-conditioned boundary value problem caused by the combined changes in geometry and material stiffness leading to a physical instability. This is often preceded by

numerical convergence problems. The numerical convergence problem is indicative of impending structural collapse and is characterized by the slope of the load-deflection approaching zero. A horizontal load-deflection plot means that the deflection grows unbounded with a small increase in load.

Figure 6: True Stress versus True Plastic Strain Curve**COMPARISONS**

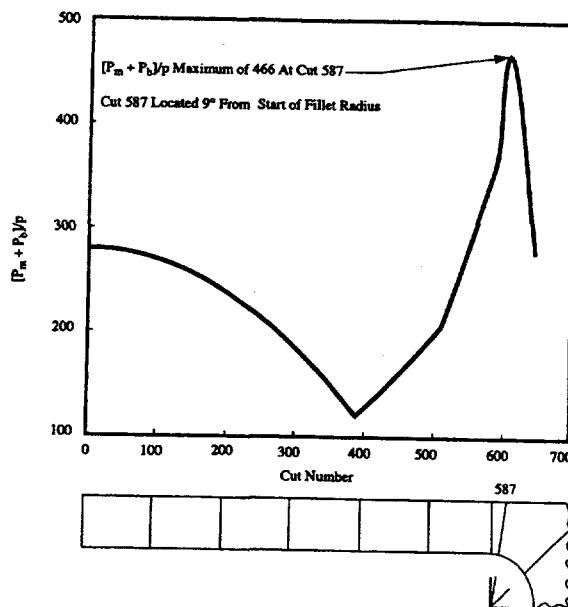
Analysis results are compared to appropriate test data in this section.

Elastic FEA Results

The FEA stress results were linearized to compute membrane plus bending stress intensity for comparison to Code primary membrane plus bending stress intensity allowables. The linearization cut lines (called stress classification lines or SCLs by Hechmer & Hollinger (1998)) are shown in Figure 7. Linearizations were made at a total of 642 SCLs in the burst disk. This number of SCLs was used to enable a clear understanding of

the distribution of $[P_m + P_b]/p$ and does not imply that this many SCLs are required for design evaluations. A plot of normalized membrane plus bending stress intensity divided by applied pressure versus cut line is shown in Figure 7. The worst case membrane plus bending stress intensity occurs at SCL 587, which is 9 degrees beyond the point of tangency of the 3 mm fillet. This location is at the built in edge where the bending moment due to the pressure is large. Since this moment is not needed for static equilibrium, it is a redundant moment and not required in a primary stress evaluation in the Code. However, an elastic FEA analyzes does not provide enough information by itself to determine whether or not a linearized stress is primary or secondary.

Figure 7: Linearized Elastic Stresses



From Figure 7 it is observed that the largest linearized stress is equal to $466p$. Dividing this result into the $1.5 S_m$ from Table 3 for the three materials gives the elastic based allowable pressures for three burst disks given in Table 4.

Table 4: Linearization Results.

Material	Allowable Pressure	Worst Case Location
304 SS	73 psi (0.503 MPa)	Built in edge (SCL 587)
ABS-C	69 psi (0.473 MPa)	Built in edge (SCL 587)
A-533-B	103 psi (0.710 MPa)	Built in edge (SCL 587)

The elastic results predict that the limiting location is always at the built in edge of the plate suggesting that the plates should all fail at this location regardless of material. Test results, however, show that the ductile 304 SS and ABS-C disks fail in the center while the A-533-B less ductile plate fails at the built in edge. Test failure locations are easily observed from the photographs taken from Cooper et al. (1971) repeated here in Figures 8 and 9. For this geometry, the largest $[P_m + P_b]/p$ location can be used to predict the location of the failure only for the A-533-B material.

Figure 8: Centerline Failure

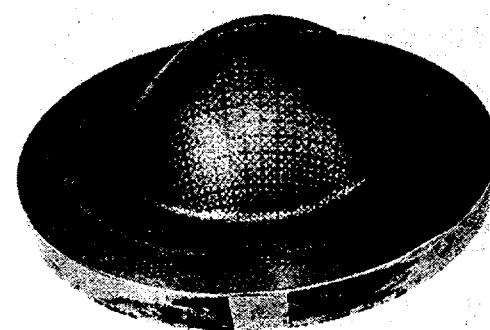
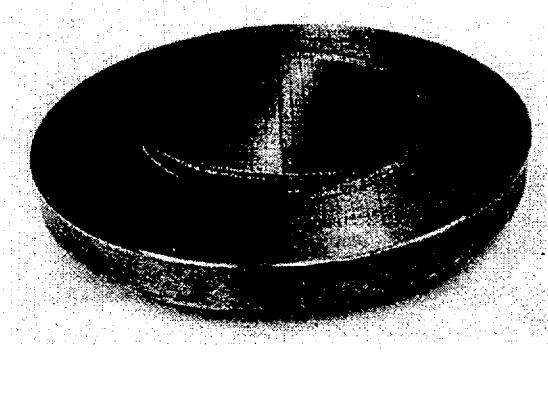


Figure 9: A-533-B Edge Failure

EP-FEA Results

A comparison between test and LSLLD EP-FEA results for applied pressure versus maximum center-line deflection response is provided in Figures 10, 11, and 12. From these figures it is observed that LSLLD EP-FEA predicts the essential character of the pressure deflection response of the specimens. The analytical results are considered to agree well with the test data especially in view of the simplified stress-strain curves used to characterize the material behavior of the three material tested and evaluated. Since the available stress-strain data for the actual materials is very limited, bi-linear true stress-true plastic strain curves shown in Figure 6 are used in the LSLLD EP-FEA.

A power-law representation was tried and the predicted failure pressures were not significantly changed. Since actual material stress-strain curves are not available there is no clear way to improve the material model at this point.

Burst pressures is obtained from the LSLLD EP-FEA by finding the pressure for which a small

increase in pressure results in a very large increase in deflection are compared to observed test burst pressures in Table 5.

Table 5: Burst Pressure Comparisons

Material	LSLLD EP-FEA Analysis Result	Test Result
304 SS	6,138 psi (42.3 MPa)	6,800 psi (46.9 MPa)
ABS-C	4,214 psi (29.1 MPa)	3750 psi (25.9 MPa)
A-533-B	5,675 psi (39.1 MPa)	5300 psi (36.5 MPa)

It is observed from Figure 8 that the low strength high ductility 304 SS disk failed at the center of the disk by ductile rupture. The same is true for the ABS-C disk. Figure 9 shows that the high strength low ductility A-533-B burst disk failed at the built in edge of the disk because the material does not possess sufficient ductility to permit the development of the strains necessary for the disk to deform to the point where failure in the membrane could occur.

Figure 13 shows the deformation plot for the largest converged load step for the 304 SS disk from the LSLLD EP-FEA. The insert on that figure allows comparisons with the deformation at the built in edge found for the A-533-B disk.

Figure 14 shows distributions of equivalent plastic strain (PEEQ) along cut-line A and B for the 304 SS and A-533-B disks for the largest converged load step. The deformation plot and PEEQ plot for the ABS-C disk is about the same as for the 304 SS disks. The largest PEEQ peak is at the built in edge for all three disks. However, the largest through-thickness average PEEQ is at the center of the 304 SS and ABS-C disks and at the built in edge for the A-533-B disk. These observations support the notion that the actual failure site may be predicted

by finding the location of the highest through-thickness average PEEQ.

EPP-FEA Results

Limit load (LL) can be calculated using SSSD EPP FEA programs. The pressure for the last converged solution from a SSSD EPP FEA problem is taken as an estimate of the LL. The results of these computations are given in Table 6.

Table 6: Limit Load Solutions

Material	LL Strength Parameter	Limit Load
304 SS	$1.5 S_m = 34,000 \text{ psi}$ (234 MPa)	204 psi (1.41 MPa)
ABS-C	$1.5 S_m = 32,000 \text{ psi}$ (221 MPa)	192 psi (1.32 MPa)
A-533-B	$1.5 S_m = 48,000 \text{ psi}$ (331 MPa)	299 psi (2.06 MPa)

In the above tabulation it is observed that the LL is proportional to the LL strength parameter, selected for this evaluation to equal $1.5S_m$. From the above tabulation it is observed that even though the 304 SS has the lowest yield strength, it has a higher LL strength parameter and therefore a higher LL than the ABS-C burst disk. The LL for the A-533-B is the greatest of all three specimens reflecting its higher strength.

DISCUSSION OF RESULTS

Calculated burst pressure are compared with test data for each of the specimens for the linearized elastic stress analysis results, the SSSD EPP LL results, and the LSLL EP results in Table 7. Table 7 gives the ratio of measured burst pressure divided by the calculated burst pressure for each disk.

Table 7: Ratio of Test over Predicted Burst Pressure

Material	Burst pressure, test	Elastic Linearized	SSSD EPP	LSLL EP
304 SS	6800 psi (46.9 MPa)	93	33	1.11
ABS-C	3750 psi (25.9 MPa)	54	20	0.89
A-533-B	53-0 psi (36.5 MPa)	51	18	0.93

From Table 7 it is seen that linearized elastic FEA results provide the largest margins to failure while limit analysis provides substantially lower but adequate margins to failure. LSLL EP-FEA provides a best-estimate analysis of burst pressure. However, it should also be noted that the EPP analyses are limited by S_{ult} because the slope assumed for the bi-linear stress strain curve is set by the uniform elongation strain.

The large failure margins associated with linearized elastic stress analysis are due primarily to the fact that the limiting location for that stress in the burst disk occurs near the built in edge and includes a significant contribution to from the redundant bending moment. The stress from a redundant moment may be secondary if equilibrium can be maintained in the absence of the moment. This is one of the major difficulties associated with using linearized elastic FEA to satisfy the primary stress limits of the Code. It is very difficult to differentiate primary from secondary stresses in redundant structures such as this one. As a result secondary stresses are commonly included in the Code primary stress evaluations. While this is always conservative, it can result in designs that are excessively thick.

SSSD EPP limit analysis results provide significantly reduced failure margins compared to

linearized elastic results because at the limit load, by definition, only primary stresses exist in the structure. Thus the plate is permitted to carry higher pressure and therefore have a reduced failure margin. However the reduced failure margins do not compromise the structural integrity of the plate since the margins are in excess of the minimum required margin to burst failure of 3.0 required by the Code. The reason that the SSSD EPP LL failure margins are still in excess of 3.0 is twofold: (1) the analysis does not account for increase in structural strength due to strain hardening, and (2) the analysis does not account for geometric strengthening as the plate deforms into a spherical shape. Geometric strengthening is quite large for thin flat plates because they deform to nearly a sphere before rupture. Other initial geometries may not experience such large geometric strengthening.

The failure margins associated with the LSLLD EP-FEA results are considered best-estimate and only as accurate as the stress-strain curve used in the analysis. The approximate bi-linear stress-strain curve used here produced -9.7% to 12.3% error relative to the measured test burst pressures. Improved stress-strain curves should enable improved estimates. Power-law models were used but since the data is not available to provide a credible power-law model, results are not reported here.

OBSERVATIONS

The following observations are made as a result of this work:

- (1) Burst pressures predicted for flat rupture disks by comparing linearized elastic FEA stress results to ASME Code primary stress limits are well below actual measured burst pressures.
- (2) LSLLD EP-FEA can be used to provide reasonable best-estimates of burst pressures. The accuracy of these predictions will depend on the accuracy of the material stress-strain curves.

(3) SSSD EPP FEA provides a conservative estimate of burst pressure.

(4) The failures observed in these tests are located at sites of large through-thickness averaged equivalent plastic strain calculated by LSLLD EP-FEA.

CONCLUSIONS

These observations lead to the conclusion that small strain small deflection elastic perfectly plastic FEA satisfies the intent of the ASME Code primary stress limits because as the limiting load is approached, only primary stresses exist in the structure. Using small strain small deflection elastic perfectly plastic FEA as a method to compute the limit load for a structure is thus a conservative alternative way to evaluate the ASME Code primary stress limits.

ACKNOWLEDGEMENTS

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Figure 10: Pressure-Deflection Curve for 304 SS.LSLD-EP-FEA

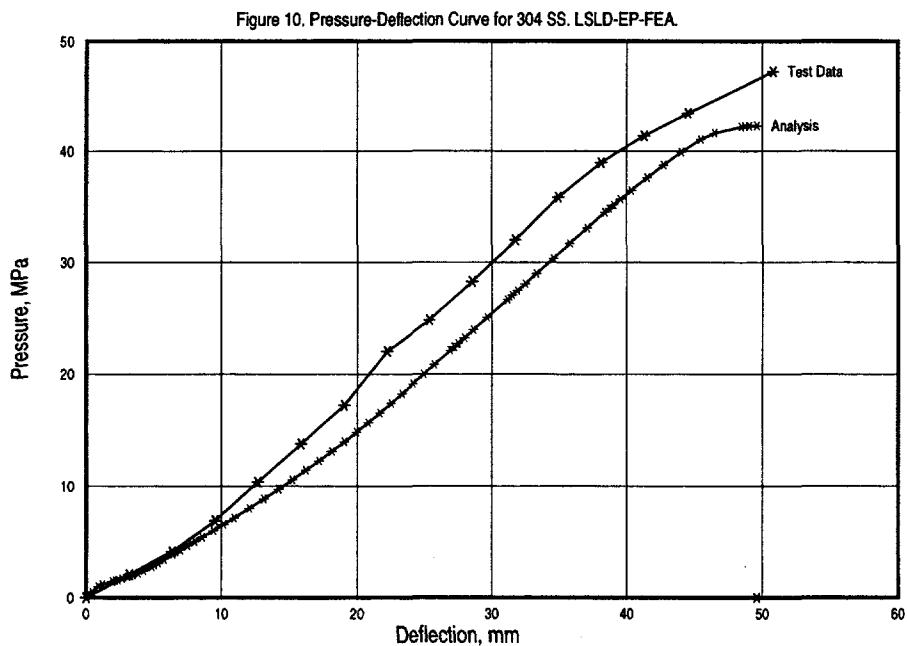


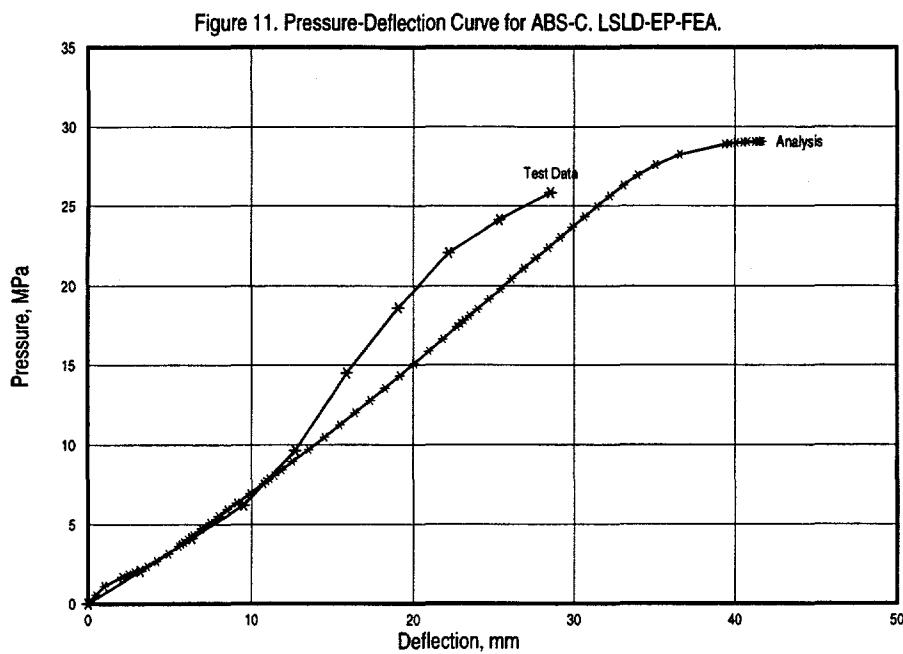
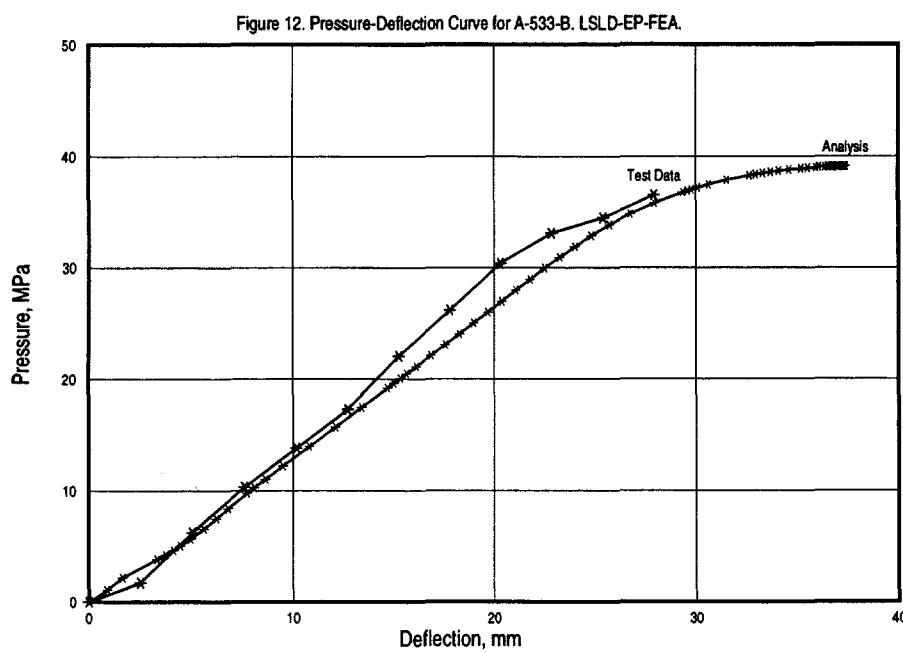
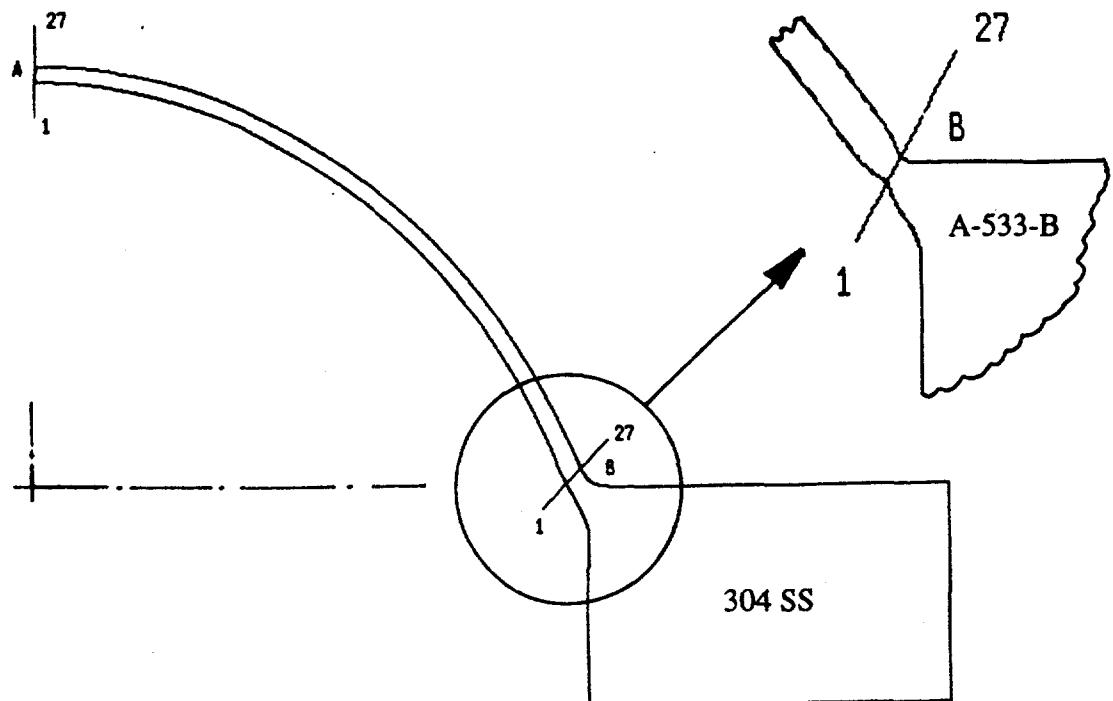
Figure 11: Pressure-Deflection Curve for ABS-C.LSLS-EP-FEA**Figure 12: Pressure-Deflection Curve for A-533-B.LSLD-EP-FEA**

Figure 13: Deformed Geometry and Cut-line Definitions**Figure 14: Effective Plastic Strain Distributions**