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A Collapse Surface for a Perforated Plate with an Equilateral Triangular Array of Penetrations

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ABSTRACT

This paper describes the development of incipient yield and subsequent collapse surfaces for a plate containing a large number of small circular penetrations arranged in an equilateral triangular array. The collapse surface developed here is appropriate for formulating a generic elastic-plastic flow theory for perforated materials. A unit cell is defined to characterize the mechanical response of an equilateral triangular array of penetrations. An elastic-perfectly plastic [EPP] finite element analysis [FEA] computer program is used to calculate the EPP response of the unit cell. A sufficient number of load cases are solved to define the complete incipient yield and collapse surfaces for the unit cell. A fourth order yield function is defined by squaring the Von Mises quadratic yield function and retaining only those terms that are required for the symmetry dictated by the triangular array. Curve fitting is used to determine the coefficients of the fourth order function to match the incipient yield and collapse data calculated for the unit cell by FEA.

The incipient yield function in the plane of the plate incorporating the penetration pattern is shown to be almost rhomboidal in shape while the collapse curve is more elliptical. The fourth order yield function which passes through the incipient yield data possess regions where the surface is concave - a concern when developing a plasticity theory based on the function. Fitting the coefficients of the fourth order function to the collapse data results in a curve which is shown to be always convex thus having all positive outward normal vectors which is a required property for the development of plasticity flow theories.

NOMENCLATURE

P	Pitch of pattern, mm
d	Diameter of penetrations, mm
h	Minimum ligament width, (*P*-d), mm
μ	Ligament efficiency, h/*P*
σ_i, ε_i	Stress and strain for i = xx, yy, zz, xy, zx, yz components, MPa and mm/mm
EQS	Equivalent solid
S_y	Yield stess of material, MPa
$S_0 = \mu S_y$	Effective yield stress of EQS material, MPa
P, Q, R, T, Y , Z_1, Z_2	Curve fit coefficients

INTRODUCTION

There are many applications of flat perforated plates in the design of heat exchangers and pressure vessels. Calculation of stresses and deformations for perforated plates is greatly simplified by using a model that replaces the perforated region with an equivalent solid [EQS] material for which the structural response is identical to the structural response of the actual perforated material.

The EQS method has been well developed for the elastic response of perforated materials for hole patterns typically used in modern heat exchanger design such as equilateral triangular and square arrays of circular penetrations. The work of Slot (1972) and Slot and O'Donnell (1971) are typical of the fundamental works in this area. The technology for use of elastic methods in design is summarized by Paliwal and Saxena, (1993) and Ukadgaonker, et al. (1996). The elastic methods have matured to a sufficient state that elastic EQS procedures are incorporated in the ASME Boiler and Pressure Vessel Code Sections VIII and III. Jones (1978) and Jones et al. (1998) have applied these methods for application to elastic FEA programs. Code procedures are in the process of being updated based on the efforts of Osweiller (1991).

With the recent advances in computer technology, elastic-plastic [EP] analysis has become practical for general structures using finite element analysis [FEA]. EP-FEA can provide a more realistic assessment of a structure allowing for better balance between strength requirements which typically call for thicker sections to lower mechanical stresses and fatigue requirements which typically call for thinner sections in order to lower cyclic thermal stresses.

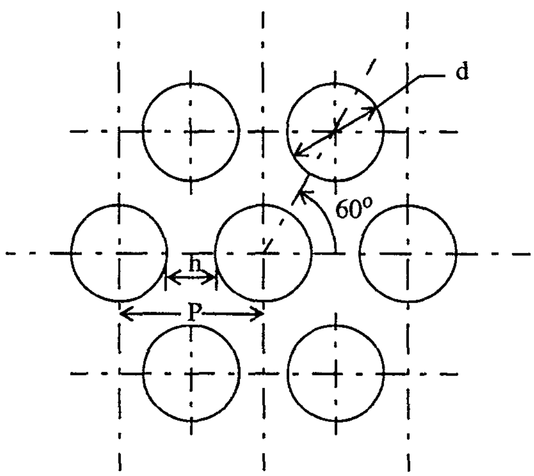
In this paper, equivalent solid plate methods are developed for use in EPP-FEA of perforated plates. A collapse surface is obtained that is appropriate for the development of limit load analyses by way of EPP-FEM of flat perforated plates with an equilateral triangular array of circular holes. The collapse surface is shown to satisfy the fundamental properties required by the periodicity of the triangular pattern. Requirements for the yield surface are given in general terms so that sufficient fundamental data can be obtained for any specific hole pattern. An example is given for a pattern with a ligament efficiency (μ) of 0.31733 where ligament efficiency is h/P, P is the pitch of the pattern, and h is the minimum ligament width.

LITERATURE

In a series of papers, O'Donnell and Porowski (1973) and Porowski and O'Donnell (1974), developed a very practical way of EPP solutions to perforated plates based on the "cut-out" factor concept. Porowski and O'Donnell defined the cut-out factor as the lowest limit load possible for a perforated material for all possible ratios of in-plane stress bi-axiality ratios, σ_{xx}/σ_{yy} or σ_{yy}/σ_{xx}. The cut-out factor was typically limiting for a biaxial stress ratio of -1.0 and generally quite conservative for positive biaxiality ratios.

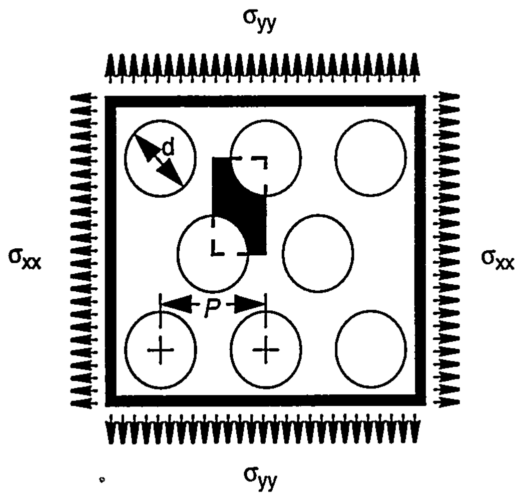
With more widespread use of computers, EP-FEA and EPP-FEA solutions were developed for perforated materials – i.e. Kichko et al. (1981), O'Donnell et al. (1979), Slot and Branca (1974), and Pai and Hsu (1975). These papers extended the unit cell concept used to generate the elastic EQS solutions to develop EP or EPP response of the perforated material given loadings in the x and y directions of the pattern. Figure 1 shows a typical equilateral triangular pattern and Figure 2 shows a unit cell. Typically these papers represented the yield surface using an anistropic plastic yield function such as in Hill (1956).

Figure 1. Triangular Penetration Pattern



$\mu = (P-d)/P = h/P$

Figure 2. Unit Cell



Jones and Gordon (1979) developed an EPP-EQS theory based on curve fitting the initial yield locus and subsequent yield surfaces for a work hardening material. Curve fitting was used because the quadratic yield locus representation given in Hill (1956) was observed to be insufficient to actually fit the calculated yield loci predicted by FEA of the unit cells.

In addition to the analytic work, results of a number of experimental efforts have been reported in the literature that support the yield surface shapes calculated by FEA for triangular patterns. The work of Litewka and Swacyuk (1981), Shiratori and Ikegame (1969) and Konig (1986) are typical of these experimental works.

Reinhardt (1998) proposed the use of a fourth order yield function to represent the surfaces observed both experimentally and computationally. Reinhardt (1998) demonstrated that by squaring the Hill (1956) quadratic yield function, a fourth order relationship is obtained:

$$\sigma_{eff}(\sigma_i) = \{Ps_1^4 + Qs_2^4 + Rs_1^2s_2^2 + Ss_1^3s_2 + Ts_1s_2^3 + Us_1^2s_3^2 + Vs_2^2s_3^2 + Ws_1s_2s_3^2 + Xs_3^4\}^{0.25} \qquad (1)$$

where, for convenience, s_1, s_2, s_3 are defined as

$$\begin{Bmatrix} s_1 \\ s_2 \\ s_3 \end{Bmatrix} = \begin{Bmatrix} (\sigma_{xx} + \sigma_{yy})/2 \\ (\sigma_{xx} - \sigma_{yy})/2 \\ \tau_{xy} \end{Bmatrix}$$

This function has sufficient generality to represent the periodicity properties required by an equilateral triangular pattern. The paper by McClellan and Mou (1997) further qualified the fourth order yield function with specific examples.

INCIPIENT YIELD SURFACE

In this paper, starting with the yield function proposed by Reinhardt (1998), a function is developed that can be used in a EPP-FEA program for the purposes of calculating a limit load for a perforated plate. Consider the function provided in Equation (1). As demonstrated by Reinhardt (1998) the periodicity of the triangular pattern requires that the same yield stress be obtained at every 60° around the pattern. Using coordinate transformations, Reinhardt (1998) showed that the following relationships had to be observed in the coefficients

$$S = 0,\ U = R,\ W = -3T,$$
$$V = 2Q,\ X = Q \qquad (2)$$

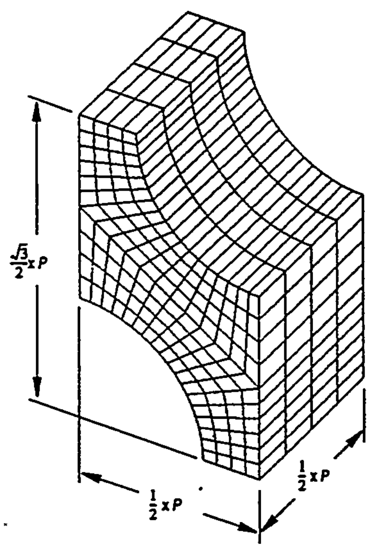
Accordingly, a yield function of the form

$$\sigma_{eff} = [Ps_1^4 + Q(s_2^2+s_3^2)^2 + Rs_1^2(s_2^2 + s_3^2) + Ts_1s_2(s_2^2-3s_3^2)]^{0.25} = S_o \qquad (3)$$

is sufficient to represent the yielding of the pattern. By calculating the EQS stress that brings the highest stressed point in the perforated material to yield for a sufficient number of load cases, the constants P, Q, R, and T solutions can be obtained. Calling these points the incipient yield stress points and using a unit cell representation for a pattern, incipient yield stresses for the cell loaded in the x-direction (S_{xx}), y-direction (S_{yy}), the equibiaxial case (S_b), and the pure-shear case (S_s) can be obtained by elastic analysis such as available in Slot and O'Donnell (1971). Elastic FEA solutions for a unit cell of the pattern as in Figure 2 is also a valid approach to obtaining the incipient yield stresses.

The FEA approach was used here for a plate with a ligament efficiency of 0.31733. The ABAQUS (1997) program was used to compute a (σ_{xx}, σ_{yy}) interaction surface for the incipient yield points based on the unit cell FEA model which is shown in Figure 3.

Figure 3. FEA Model of Unit Cell



The boundary conditions for the analyses are described in Figures 4 and 5. By changing the prescribed displacements, δ_x and δ_y, solutions for various (σ_{xx}, σ_{yy}) values can be obtained. FEA was used to obtain the solutions in Table 1. The in-plane loading cases were run as generalized plane strain problems making the solutions independent of the plate thickness. The EQS values of σ_{xx} and σ_{yy} were calculated by dividing the vector forces from FEA, resulting from the displacement boundary condition, by the area of the face on which the force was calculated.

The incipient yield surface for the data in Table 1 is shown in Figure 6. The data in Table 1 is fit to Equation (3) using MATHEMATICA (1997) to determine the coefficients P= 2.0439, Q = 79.8215, R = 70.5655, and T = 115.07. The predicted surface agrees very well with the computed surface although it is not always convex. This property is an important consideration in plasticity flow theory development as noted in Mendelson (1968).

Figure 4. Boundary Conditions for Normal X-Load

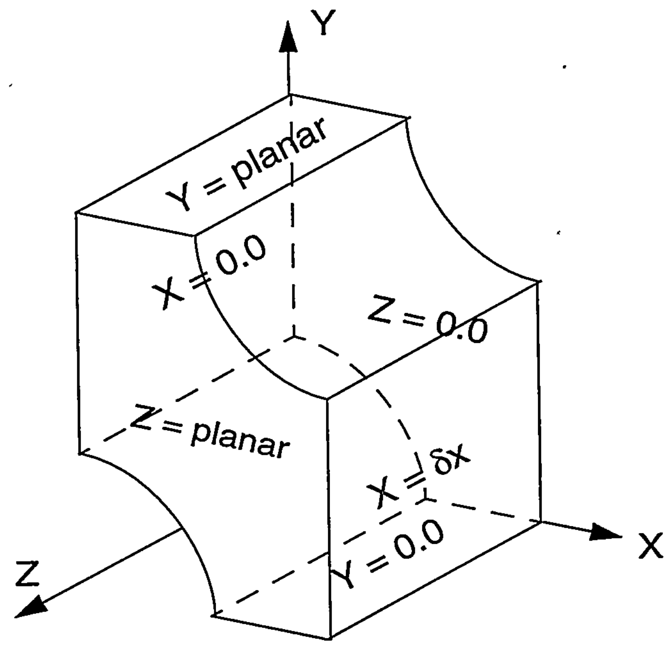


Figure 5. Boundary Conditions for Normal Y-Load

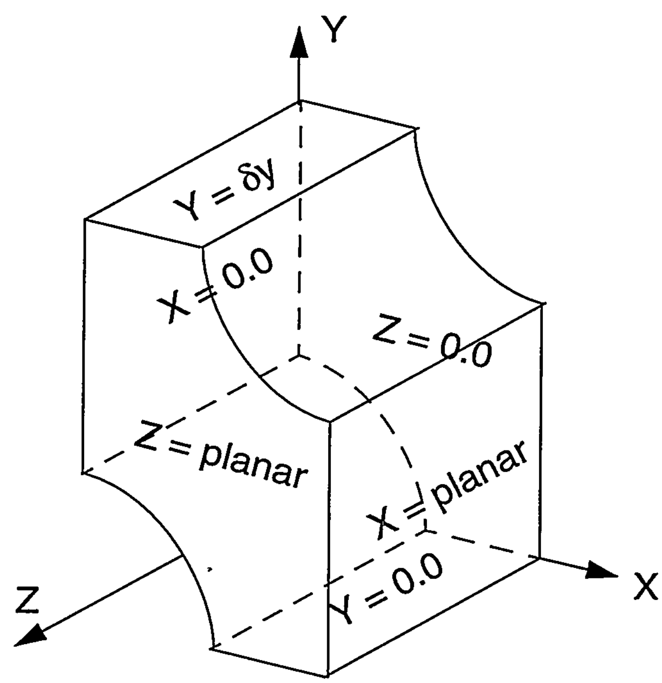
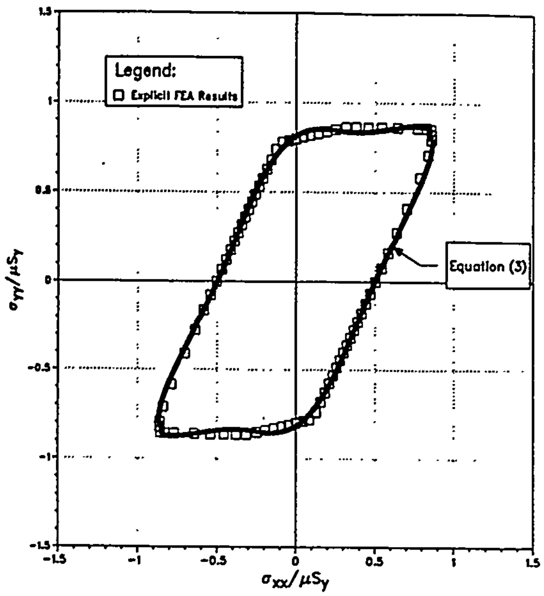


Figure 6. Incipient Yield Surface



COLLAPSE SURFACE

The goal of this study is to develop a surface appropriate for representation of the collapse or limit load of a perforated material. Consider a three-dimensional surface of the form

$$\sigma_{eff} = \{\frac{1}{4}\{P(\sigma_{xx} + \sigma_{yy})^4 + Q[(\sigma_{xx} - \sigma_{yy})^2 + 4\tau_{xy}^2]^2$$

$$+ R(\sigma_{xx} + \sigma_{yy})^2[(\sigma_{xx} - \sigma_{yy})^2 + 4\tau_{xy}^2]$$

$$+ T(\sigma_{xx}^2 - \sigma_{yy}^2)[(\sigma_{xx} - \sigma_{yy})^2 - 12\tau_{xy}^2]\}^{0.5}$$

$$+ Y[\sigma_{zz}^2 - \sigma_{zz}(\sigma_{xx} + \sigma_{yy})] + 3Z_1\tau_{yz}^2 + 3Z_2\tau_{zx}^2\}^{0.5}$$

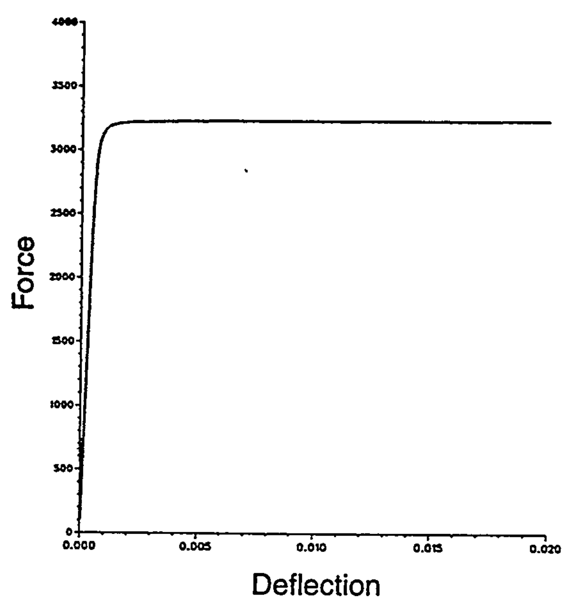
$$= S_o \qquad (4)$$

It is noted that to satisfy the periodicity of a triangular pattern, $Z_1 = Z_2$. However, Z_1 and Z_2 are allowed to be independent here to provide an improved approximation for the τ_{yz} and τ_{zx} yielding. A collapse surface is developed using the unit cell problems defined in Table 1 loaded until the collapse or limit load of the perforated material is reached. The ABAQUS (1997) program is used to analyze the FEA model of the unit cell shown in Figure 3 for a small strain, small deflection elastic-perfectly plastic model. The 20-node reduced integration hexagonal element was used.

Since the σ_{zz}, τ_{yz}, and τ_{zx} are independent of the other stresses, only three independent solutions are necessary to obtain the coefficients Y, Z_1 and Z_2. These solutions are given in Table 2. The remaining coefficients are obtained by using the same procedure as for incipient yield, in that δ_x and δ_y displacements in Figure 4 are varied to give a number of (σ_{xx}, σ_{yy}) combinations for $\sigma_{zz} = \tau_{yz} = \tau_{zx} = \tau_{xy} = 0.0$.

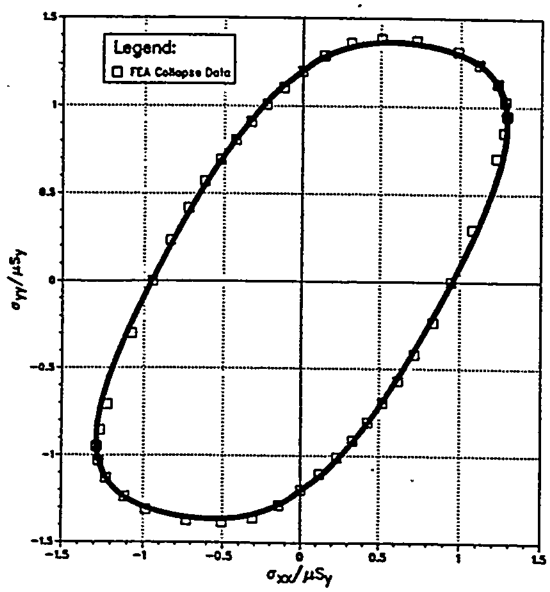
A typical load deflection curve is shown in Figure 7 for the σ_{xx} case. The collapse load for each load case is taken to be the last converged solution computed by ABAQUS and as seen in Figure 7 represents the load for which a small increase in load provides a very large increase in deflection.

Figure 7. Load Deflection Curve for the σ_{xx} Case



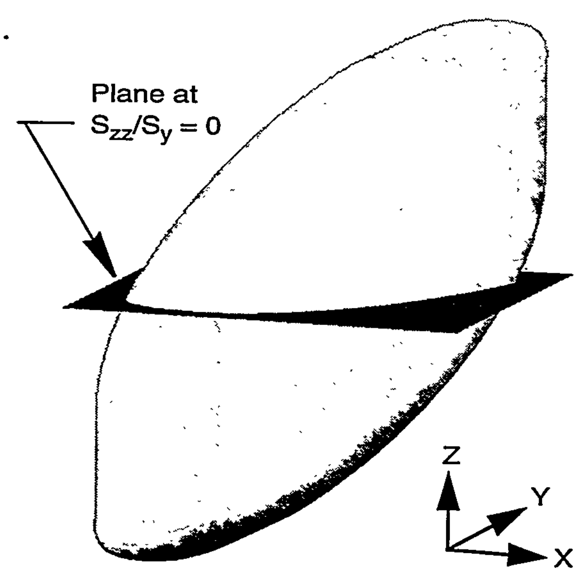
Collapse solutions to the (σ_{xx}, σ_{yy}) cases, shown in Table 3, give a collapse surface as shown in Figure 8 for the case where the only non-zero stresses are σ_{xx} and σ_{yy}.

Figure 8. Collapse Surface for $\sigma_{zz} = 0$ Plane



There are similar interaction curves between all six stress components such that a six dimensional surface in stress space coordinates can be generated. Figure 8 shows the collapse surface for $\sigma_{zz} = 0$ plane. Any particular value of σ_{zz} provides a unique σ_{xx} - σ_{yy} surface. Figure 9 shows a three-dimensional collapse surface in general (σ_{xx}, σ_{yy}, σ_{zz}) coordinates.

Figure 9. Collapse Surface for (σ_{xx}, σ_{yy}, σ_{zz})



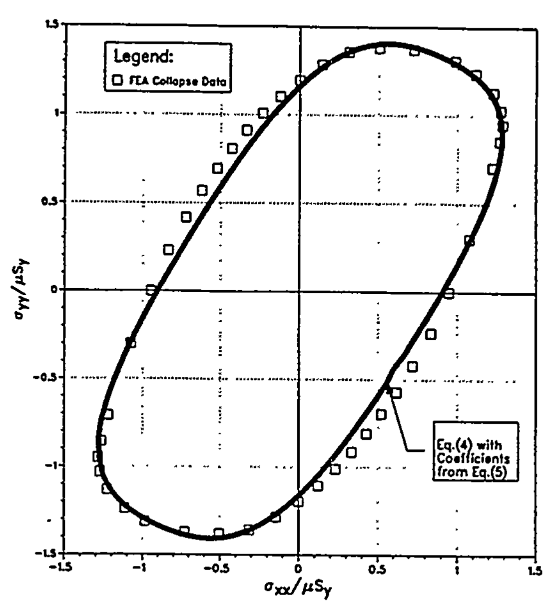
The MATHEMATICA (1997) program is used to fit Equation (4) to the collapse surface data given in Table 3 by first calculating the Q term to match the S_{xy} case given in Table 2. Coefficients P, R, and T are then fit to the data points shown in Figure 8.

For a ligament efficiency of 0.31733 the complete collapse surface is given by Equation (4) and the coefficients

$$
\begin{aligned}
P &= 0.52971 \\
Q &= 11.86453 \\
R &= 3.96229 \\
T &= 7.57493 \\
Y &= 0.30206 \\
Z_1 &= 1.000 \\
Z_2 &= 0.750
\end{aligned}
\qquad (5)
$$

The analytic surfaces given by Equation (4) and the coefficients in Eq (5) are compared to the explicit EPP-FEA data in Figure 10 for $\sigma_{zz} = 0$ plane. The maximum error between these surfaces is less than about 7%. The collapse surface is convex for all but very few locations on the surface. This was determined using MATHEMATICA (1997), by showing that the determinate of the Hessian matrix is positive definite. It is anticipated that these deviations from convexity will be impractical when implementing a plasticity algorithm to compute limit loads for an EQS material in a FEA program.

Figure 10. Analytic Collapse Surface



CONCLUSIONS

A collapse surface was developed using three-dimensional EPP-FEA of a unit cell representing an infinite array of equilateral penetrations in a flat plate. The collapse surface was analytically represented using a fourth order function that incorporates the periodicity dictated by the 60° periodicity of the hole pattern. A collapse surface for a triangular array of holes of ligament efficiency 0.31733 was developed and shown to be appropriate for development of an EPP-EQS theory for perforated plates. The analytic surface agrees to within 7% of the actual collapse surface obtained by EPP-FEA of the unit cell representing the penetration.

ACKNOWLEDGEMENT

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Table 1. Incipient Yield Data
(All stresses zero except σ_{xx}, σ_{yy})

Case	$\sigma_{xx}/\mu S_y$	$\sigma_{yy}/\mu S_y$
1	0.0000	0.7935
2	0.0456	0.8025
3	0.0896	0.8068
4	0.1399	0.8213
5	0.1914	0.8314
6	0.2470	0.8423
7	0.3128	0.8551
8	0.3784	0.8569
9	0.4538	0.8565
10	0.5399	0.8559
11	0.6420	0.8550
12	0.7690	0.8533
13	0.8273	0.8527
14	0.8545	0.8466
15	0.8596	0.8177
16	0.8589	0.7896
17	0.8388	0.7050
18	0.7853	0.5790
19	0.7019	0.4063
20	0.6359	0.2723
21	0.5809	0.1611
22	0.5389	0.0769
23	0.5003	0.0000
24	0.4669	-0.0667
25	0.4116	-0.1762
26	0.3645	-0.1560
27	0.3203	-0.3554
28	0.2772	-0.4395
29	0.2324	-0.5263
30	0.1825	-0.6225
31	0.1553	-0.6749
32	0.1248	-0.7333
33	0.0857	-0.7759
34	0.0445	-0.7843
35	0.0000	-0.7935

Table 2. Collapse Data for Out-of-Plane Normal and Shear Loads

Case	$\sigma_i/\mu S_y$	Coefficient
S_{zz}	1.8195	Y = 0.30206
S_{yz}	0.57770	Z_1 = 1.0000
S_{zx}	0.66721	Z_2 = 0.7500
S_{xy}	0.53881	Q = 11.86453

where S_{ij} means a load case where only the (i, j) stress component is non-zero.

Table 3. Collapse Surface Data
(All stresses zero except σ_{xx}, σ_{yy})

Case	$\sigma_{xx}/\mu S_y$	$\sigma_{yy}/\mu S_y$
1	0.0000	1.1965
2	0.1419	1.2854
3	0.3126	1.3584
4	0.5061	1.3837
5	0.7262	1.3710
6	0.9846	1.3116
7	1.1147	1.2374
8	1.2288	1.1303
9	1.2748	1.0310
10	1.2869	0.9489
11	1.2688	0.8551
12	1.2228	0.7078
13	1.0767	0.2988
14	0.9487	0.0000
15	0.8306	-0.2305
16	0.7205	-0.4170
17	0.6184	-0.5688
18	0.5224	-0.6958
19	0.4287	-0.8093
20	0.3331	-0.9108
21	0.2316	-1.0061
22	0.1218	-1.1045
23	0.0000	-1.1965

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