

The first fermi in a high energy nuclear collision.

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At very high energies, weak coupling, non-perturbative methods can be used to study classical gluon production in nuclear collisions. One observes in numerical simulations that after an initial "formation" time, the produced partons are on shell, and their subsequent evolution can be studied using transport theory. At the initial formation time, a simple non-perturbative relation exists between the energy and number densities of the produced partons, and a scale determined by the saturated parton density in the nucleus.

An outstanding problem in high energy scattering is the problem of initial conditions for particle production¹. In perturbative QCD, for processes which involve a hard scale $Q^2 \gg \Lambda_{QCD}^2$, the hard and soft contributions can be factorized. The soft contributions are lumped into non-perturbative, process independent parton distribution functions, while the hard contributions are computed for each physical process of interest. For a fixed hard scale of interest Q^2 , there is a center of mass energy \sqrt{s} beyond which this approach in particular, and the operator product expansion (OPE) in general breaks down². However, since the parton densities in this regime are large, weak coupling classical methods may be applicable³. Wilson renormalization group methods have been developed for this high parton density regime⁴.

At small x , classical parton distributions in a nucleus can be computed in a model with a dimensionful scale μ^2 proportional to the gluon density per unit transverse area. In this model, parton distributions saturate at a scale $Q_s \propto g^2 \mu$. For Au-Au collisions at RHIC, one can estimate $Q_s \sim 1$ GeV, and at the LHC, the saturation scale will be $Q_s \sim 2-3$ GeV. Most of the gluons produced therefore have transverse momenta $k_t \sim Q_s$, and since this scale for RHIC and LHC is at least marginally a weak coupling scale, these classical methods may be applied to study the production and initial evolution of partons at RHIC and LHC.

These classical methods were first applied to nuclear collisions by Kovner, McLerran and Weigert⁵. For an interesting alternative approach, see Ref.⁶. Assuming boost invariance, and matching the equations of motion in the forward and backward light cone, they obtained the following initial conditions for the gauge fields in the $A^\tau = 0$ gauge: $A_\perp^i|_{\tau=0} = A_1^i + A_2^i$, and

$A^\pm|_{\tau=0} = \pm \frac{i g}{2} x^\pm [A_1^i, A_2^i]$. Here $A_{1,2}^i(\rho^\pm)$ ($i = 1, 2$) are the pure gauge transverse gauge fields corresponding to small x modes of incoming nuclei (with light cone sources $\rho^\pm \delta(x^\mp)$) in the $\theta(\pm x^-)\theta(\mp x^+)$ regions respectively of the light cone.

The sum of two pure gauges in QCD is not a pure gauge—the initial conditions therefore give rise to classical gluon radiation in the forward light cone. For $p_t \gg \alpha_S \mu$, the Yang–Mills equations may be solved perturbatively to quadratic order in $\alpha_S \mu/p_t$. After averaging over the Gaussian random sources of color charge ρ^\pm on the light cone, the perturbative energy and number distributions of physical gluons were computed by several authors^{5,7}. In the small x limit, it was shown that the classical Yang–Mills result agreed with the quantum Bremsstrahlung result of Gunion and Bertsch⁸.

In Ref.⁹, we suggested a lattice discretization of the classical EFT, suitable for a non-perturbative numerical solution. Assuming boost invariance, we showed that in $A^\tau = 0$ gauge, the real time evolution of the small x gauge fields $A_\perp(x_t, \tau)$, $A^\eta(x_t, \tau)$ is described by the Kogut–Susskind Hamiltonian in 2+1–dimensions coupled to an adjoint scalar field. The lattice equations of motion for the fields are then determined straightforwardly by computing the Poisson brackets. The initial conditions for the evolution are provided by the lattice analogue of the continuum relations discussed earlier in the text. We impose periodic boundary conditions on an $N \times N$ transverse lattice, where N denotes the number of sites. The physical linear size of the system is $L = a N$, where a is the lattice spacing. It was shown in Ref.¹⁰ that numerical computations on a transverse lattice agreed with lattice perturbation theory at large transverse momentum. For details of the numerical procedure, and other details, we refer the reader to Ref.¹⁰.

In our numerical simulations, all the relevant physical information is compressed in $g^2 \mu$ and L , and in their dimensionless product $g^2 \mu L$ ¹¹. The strong coupling constant g depends on the hard scale of interest; $\mu \propto A^{1/6}$ depends on the nuclear size, the center of mass energy, and the hard scale of interest; L^2 is the transverse area of the nucleus. Assuming $g = 2$ (or $\alpha_S = 1/\pi$), $\mu = 0.5$ GeV (1.0 GeV) for RHIC (LHC), and $L = 11.6$ fm for Au -nuclei, we find $g^2 \mu L \approx 120$ for RHIC and ≈ 240 for LHC. (The latter number would be smaller for a smaller value of g at the typical LHC momentum scale.) As will be discussed later, these values of $g^2 \mu L$ correspond to a region in which one expects large non-perturbative contributions from a sum to all orders in $\sim 6 \alpha_S \mu/p_t$, even if $\alpha_S \ll 1$. We should mention here that deviations from lattice perturbation theory, as a function of increasing $g^2 \mu L$, were observed in our earlier work¹⁰.

In Ref.¹², we computed the energy density ε as a function of the proper

time τ . This computation on the lattice is straightforward. To obtain this result, we computed the Hamiltonian density on the lattice for each ρ^\pm , and then took the Gaussian average (with the weight μ^2) over between 40 ρ trajectories for the larger lattices and 160 ρ trajectories for the smallest ones.

The dependence of $\varepsilon\tau$ as a function of τ was investigated in our numerical simulations. For larger values of $g^2\mu L$, $\varepsilon\tau$ increases rapidly, develops a transient peak at $\tau \sim 1/g^2\mu$, and decays exponentially there onwards, satisfying the relation $\alpha + \beta e^{-\gamma\tau}$, to the asymptotic value α (equal to the lattice $dE/L^2/d\eta!$). This behavior is satisfied for all $g^2\mu L \geq 8.84$, independently of N . One can interpret the decay time $\tau_D = 1/\gamma/g^2\mu$ as the appropriate scale controlling the formation of gluons with a physically well defined energy. In other words, τ_D is the “formation time” in the sense used by Bjorken¹³.

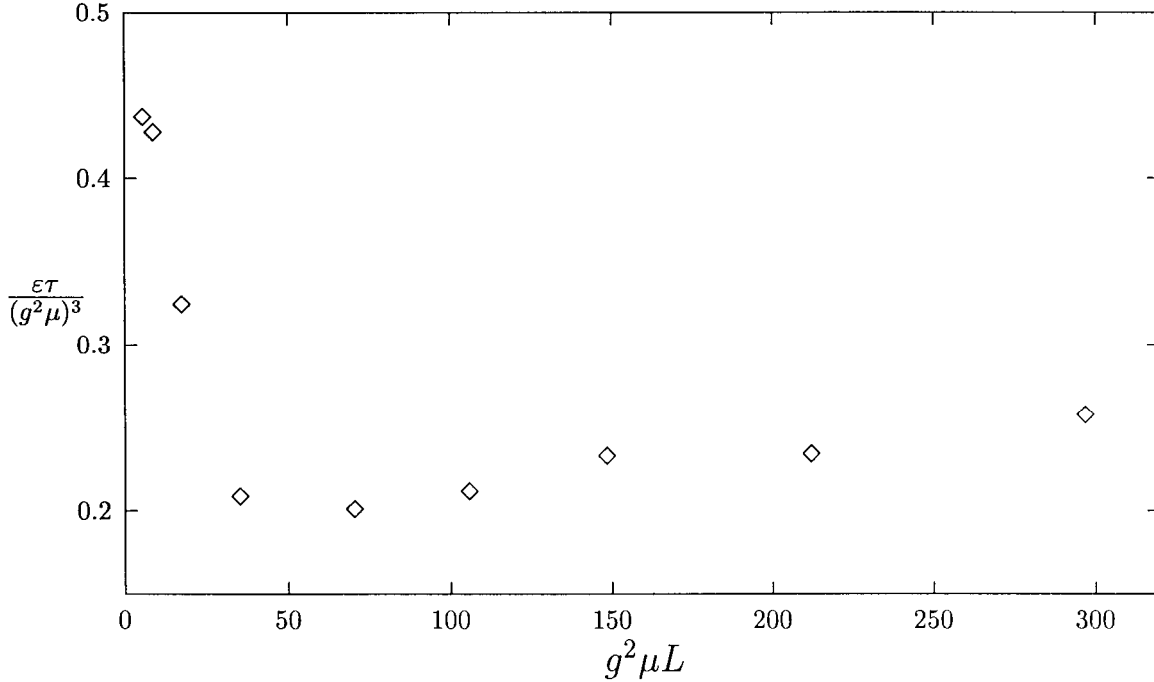


Figure 1: $\varepsilon\tau/(g^2\mu)^3$ extrapolated to the continuum limit: f as a function of $g^2\mu L$. The error bars are smaller than the plotting symbols.

The physical energy per unit area per unit rapidity of produced gluons can

be defined in terms of a function $f(g^2\mu L)$ as

$$\frac{1}{L^2} \frac{dE}{d\eta} = \frac{1}{g^2} f(g^2\mu L) (g^2\mu)^3. \quad (1)$$

The function f here is obtained by extrapolating our results for finite lattice spacings to the continuum limit. In the region of physical interest for heavy ion collisions, f varies very slowly. It changes by $\sim 25\%$ for nearly an order of magnitude change in $g^2\mu L$. The saturation scale $Q_s \sim 6\alpha_S\mu$ —one can therefore re-write our result for the energy density in terms of Q_s .

Doing so, we confirmed that our results are consistent with an estimate by A. H. Mueller¹⁴ for the number of produced gluons per unit area per unit rapidity. He obtains $dN/L^2/d\eta = c(N_c^2 - 1)Q_s^2/4\pi^2\alpha_S N_c$, and argues that the number c is a non-perturbative constant of order unity. If most of the gluons have $p_t \sim Q_s$, then $dE/L^2/d\eta = c'(N_c^2 - 1)Q_s^3/4\pi^2\alpha_S N_c$ which is of the same form as our Eq. 1. In the $g^2\mu L$ region of interest, our function $f \approx 0.23$ – 0.26 . We obtain $c' = 4.3$ – 4.9 . Since one expects a distribution in momenta about Q_s , it is very likely that c' is at least a factor of 2 greater than c —thereby yielding a number of order unity for c as estimated by Mueller. This coefficient can be determined more precisely when we compute the non-perturbative number and energy distributions¹⁵.

In Ref.¹², we estimated the initial energy per unit rapidity of produced gluons at RHIC and LHC energies. We did so by extrapolating from our SU(2) results to SU(3) assuming the N_c dependence to be $(N_c^2 - 1)/N_c$ as in Mueller's formula. At late times, the energy density is $\varepsilon = (g^2\mu)^4 f(g^2\mu L) \gamma(g^2\mu L)/g^2$, where the formation time is $\tau_D = 1/\gamma(g^2\mu L)/g^2\mu$ as discussed earlier. We find $\varepsilon^{RHIC} \approx 66.49$ GeV/fm³ and $\varepsilon^{LHC} \approx 1315.56$ GeV/fm³. Multiplying these numbers by the initial volumes at the formation time τ_D , we obtained the classical Yang–Mills estimate for the initial energies per unit rapidity E_T to be $E_T^{RHIC} \approx 2703$ GeV and $E_T^{LHC} \approx 24572$ GeV respectively.

Compare these numbers to results presented recently by Kajantie¹⁶ for the mini-jet energy (computed for $p_t > p_{sat}$, where p_{sat} is a saturation scale akin to Q_s) in the pQCD mini-jet approach¹⁷. He obtains $E_T^{RHIC} = 2500$ GeV and $E_T^{LHC} = 12000$. The remarkable closeness between our results for RHIC is very likely a coincidence. Kajantie's result includes a K factor of 1.5—estimates range from 1.5–2.5¹⁸. For the latest estimates from our Finnish colleagues, see the preprint of Eskola et al.²⁰. If we pick a recent value of $K \approx 2$ ¹⁹, we obtain as our final estimate, $E_T^{RHIC} \approx 5406$ GeV and $E_T^{LHC} \approx 49144$ GeV.

We can also boldly estimate the number of produced gluons at central rapidities. As mentioned in the preceding text, the value of the constant c in the expression for the number distribution is currently being computed

numerically. We obtain for Au–Au collisions in one unit of rapidity the result that $N_{RHIC} = 714 \cdot c$ and $N_{LHC} = 2855 \cdot c$. Given that the corresponding constant for the energy density was larger, we would anticipate that it is more likely that $c = 2\text{--}2.5$. Taking the higher value, we obtain $N_{RHIC} = 1785$ and $N_{LHC} = 7138$. Again, these values are very close to those of Eskola et al.²⁰. Note that they consider Pb–Pb collisions and their results include a K factor of 2. The purpose of this simple exercise is primarily to confirm that our numbers are not wildly divergent from mini-jet calculations. Our results are typically a factor of two larger (more at LHC) but this is easily understood because our results include all momentum modes.

The number density of these out-of-equilibrium gluons can be related to the equilibrium entropy: $S_{glue} = 3.6 \cdot N_{glue}$. This is particularly so at the LHC, where because $Q_s^2 \gg \Lambda_{QCD}^2$, elastic scattering dominates. The equilibrium entropy of gluons is, to within a factor of two, (which can be quantified in one's thermal+hydro model of choice), the entropy of pions.

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