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PAPER

Robot Positioning based on Point-to-Point Motion Capability*

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Abstract

This paper presents an optimal search method for determining the base location of a robot manipulator so that the robot can have a designated point-to-point(PTP) motion capabilities. Based on the topological characterization of the manipulator workspace and the definitions of various p-connectivity, a computational method is developed for enumerating various PTP motion capabilities into quantitative cost functions. Then an unconstrained search by minimizing the cost function yields the task feasible location of the robot base. This methodology is useful for placement of mobile manipulators and robotic workcell layout design.

1. Introduction

The characterization of a manipulator's ability to move through its workspace is useful for task planning. Such a characterization can be addressed as 1) determination of maximal task capable subspaces, and 2) identification of moveability of a given task trajectory. While the former is particularly relevant to path planning, this work focuses on automatic placement of robot base in such a way to assure specified moveability, for which the latter analysis has a direct relevance. Although many authors have tackled robot placement problem based on various task capabilities[1]-[5], little have taken complete account of the manipulator's global moveability in its workspace.

Moveability of a robotic manipulator can be studied by topological analysis of workspace and obstacles. Schiller has introduced a description of obstacle's influence on the robot workspace, commonly known as 'obstacle shadows', and demonstrated its applicability for robot motion planning and placement problems[6]. In the sequel, Borrel and Legeois have introduced the notion of 'aspect' which partitions the configuration space of a manipulator according to continuous trajectory motion capability[7]. Chedmail and Wenger have further characterized the *manipulator workspace based on various moveability of robot manipulators in the presence of obstacles[8]-[10]. Based on such analytical results, Reynier has presented a efficient method for placing the robot base in such a way to assure continuous trajectory motion[11]. Park has presented a complete method which is computationally coherent with the topological analysis[12][13]. Currently, however, no significant work has been reported on robot placement for point-to-point (PTP) motion capabilities.

To this end this paper presents a base placement method applicable to PTP motion. In this regards, a computational formalism is presented for enumerating the task capability into a cost function for optimization.

2. Topology of Workspace

In manipulator kinematic analysis, operation space, W, is referred to as the unbounded space spanned by the spatial coordinates of the end effector, $\underline{x}\in R^m$, where m is the number of operational coordinate variables. Configuration space, Q, of a manipulator is referred to the compact space spanned by joint variables, $\underline{\theta}\in R^n$, where n is the number of joint variables[14]. For an n-d.o.f. manipulator, $\underline{\theta}=\{\theta_1,\ldots,\theta_n\}$ and

$$Q \equiv \{\underline{\theta} \mid \forall i,\ \theta_{i,\min} \le \theta_i \le \theta_{i,\max},\ \underline{\theta} \in \mathrm{R}^n\}.$$

A point in configuration space is mapped to a point in operation space by the geometric operator $f{:}Q\rightarrow W$. Conversely, the inverse geometric operator $f^{-1}{:}W\rightarrow Q$ is a mapping from operation space to configuration space.

The inverse mapping from W to Q is generally not unique due to the multiple solutions to the inverse kinematics problem. To resolve this ambiguity, it is convenient to subdivide the configuration space into subsets, $QU_j\in Q$, consisting each of points for which a

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single valued inverse kinematics exists. Here, the subscript j is the index number indicating a unique inverse kinematics solution out of multiple solution sets, thus $j \in J$, $J=\{1,2,\ldots,j_{\max}\}$, where $j_{\max}$ is the multiplicity of inverse kinematics solutions. Then, for two different points $\underline{y},\underline{z} \in Q$ and $\underline{y} \in QU_j$,

$$f(\underline{y}) \neq f(\underline{z}) \quad \text{if} \quad \underline{z} \in QU_j .$$

For type 1 manipulators, QU_js are referred as 'aspects', which are defined as the connected components of singularity free subspaces of Q [7]. Therefore, each aspect is separated by singularity hypersurfaces. For type 2 manipulators, it can change posture within an aspect, and thus QU_j is defined as further subdivisions of aspects known as uniqueness domains[10][15].

In the absence of joint limits and obstacles, each aspect maps to the same maximal workspace by f. Therefore, any number of task points can be traversed by the end effector with arbitrary postures at each point. However, if there are joint limits, each aspect generally maps to different subsets of reachable workspace. Therefore, some points in the workspace may not be reached with arbitrarily chosen postures. This impose limitations on the trajectory feasibility of the workspace, because it is generally undesirable for a robot to change postures while tracking a continuous trajectory. Thus, continuous trajectory motion is feasible only within an aspect or a uniqueness domain. In the presence of obstacles, the possible collision further limits the moveability of the workspace. For an obstacle, denoted o, and the body of the manipulator, denoted $R(\underline{\theta})$ at configuration $\underline{\theta}$, configuration space obstacle, $QO(o)$ is defined as[6]

$$QO(o)=\{\underline{\theta} \mid R(\underline{\theta}) \cap o \neq \varnothing\} .$$

The obstacle shadow $WO(o)$ is the maximal union of $WO(b)_j$ defined as

$$WO(o)_j = \{\underline{x} \mid \underline{x} = f(\underline{\theta}), \underline{\theta} \in QO(o) \cap QU_j\}$$

and $$WO(o) = \bigcup_{\forall j \in J} WO(o)_j .$$

Complementarily, the configuration free space, Qf, is defined as

$$Qf(o) = \{\underline{\theta} \mid R(\underline{\theta}) \cap o = \varnothing\} ,$$

and the free workspace, $Wf(o)$, is the maximal union of the image of Qf under f, defined as

$$Wf(o)_j = \{\underline{x} \mid \underline{x} = f(\underline{\theta}),\ \underline{\theta} \in Qf(o) \cap QU_j\} ,$$

and $$Wf(o) = \bigcup_{\forall j \in J} Wf(o)_j .$$

To simplify the notation, the obstacle o is omitted in the subsequent.

Configuration space obstacle divides Q into path connected subspaces $Qf_i \in Q$, where $i \in I$, $I=\{1,2,\ldots,i_{\max}\}$ and i_{max} is the number of path connected configuration free subspaces. In general, the following relationship holds: $Qf = \bigcup_{i \in I}(Qf_i)$ where $\forall i,k \in I$ and $i \neq k$, $Qf_i \cap Qf_k = \phi$. Within a Qf_i, any number of discrete points can be traversed by the end effector in a sequence of motion, with at least one posture at each point. Each Qf_i is mapped to a subspace, Wf_i, by

$$Wf_i = f(Qf_i)$$

and the entire free workspace is the union of all Wf_i, thus

$$Wf = \bigcup_{\forall i \in I} Wf_i .$$

Although Qf_i are disjoint sets, the corresponding Wf_i may overlap partially or entirely as illustrated in Fig. 1.

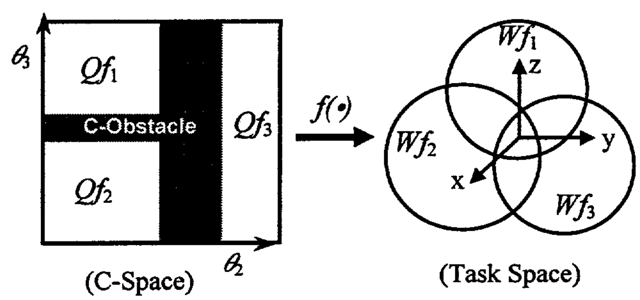

Fig.1. Kinematic mapping from C-space to Operation space

Not all discrete points in a path connected region in Wf may be traversed by the end effector. Furthermore, a proper choice of posture is required at each of the points in a Wf_i. Consequently, various moveability characteristics are defined based on what postures are achievable at the task points in Wf, namely P_1, P_2, P_3 and P_4 connectivity[8][9], as followings,

P_1-connectivity : Two points $\underline{x}_1,\underline{x}_2 \in Wf$ are connected in the sense of P_1, if they can be joined by the end effector with at least one posture at each point.

P_2-connectivity : An arbitrary number of points $\underline{x}_1,\ldots,\underline{x}_n \in Wf$ are connected in the sense of P_2, if they can be joined by the end effector in an arbitrary sequence of movements and in at least one posture at each point.

P_3-connectivity : Two points $\underline{x}_1,\underline{x}_2 \in Wf$ are connected in the sense of P_3, if they can be joined by the end effector with whatever the initial or final configuration.

P_4-connectivity : Two points $\underline{x}_1,\underline{x}_2 \in Wf$ are connected in the sense of P_4, if they can be joined by the end effector with whatever the initial and final configurations.

Fig. 2 illustrates the various PTP connectivity.

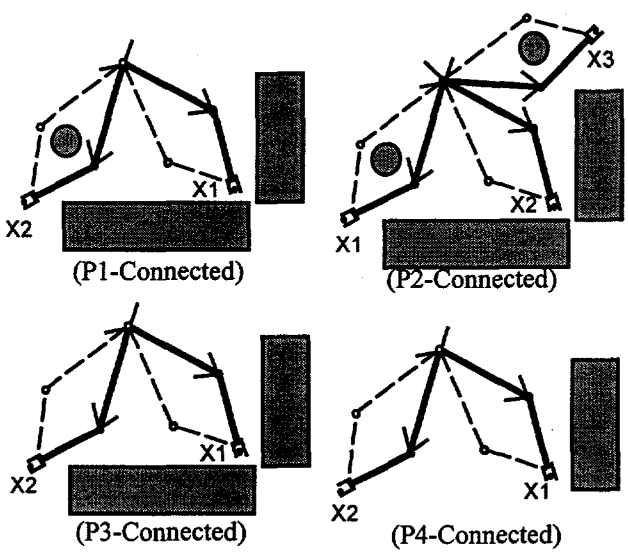

Fig. 2. Various P-connectivity

The maximal p-connected subspaces can be determined from topological characterization of the workspace. According to Chedmail and Wenger[8][9], the necessary and sufficient conditions for a subspace of free workspace, $Wf_p \subset Wf$, to satisfy various P-connectivity are

$P_1(Wfp)\ :\ Wfp \subset (\cap_{k\in K}(\cup_{j\in I_k} Wf_j)) \cap (\cup_{k\in K}(\cap_{j\in I_k} Wf_j))$

where topological subsets, Wf_k, of Wf are defined for indices $\{I_k/k\in K\} = \{I_1, I_2, ..., I_p\}$ of all possible of subsets of I.

$P_2(Wfp)\ :\ \exists\, i \in I,\ Wfp \subset Wf_i$

$P_3(Wfp)\ :\ Wfp = (\cap_{i\in I'} Wf_i) - (\cup_{k\notin I'} Wf_k)$ for some sets I' of indices in I.

$P_4(Wfp)\ :\ \exists\, i \in I$ such that $Wfp = Wf_i - (\cup_{k\neq i} Wf_k)$.

3. Robot Placement for PTP Motion

The current base placement problem is considered for an environment with stationary obstacles. The task trajectories are prescribed as a collection of discrete points in operation space, and the requirements for PTP motion capabilities are given. In this regards, presented in this section is a computational method for characterizing the workspace based on PTP motion capability and extending the result for robot base placement problem.

In this work, three dimensional configuration free space spanned by joint angles, θ_1, θ_2 and θ_3, is constructed using approximate characterization method similar to that described in Faverjon[17]. It proceeds by finding legal ranges of joint angles from the most proximal link to the most distal link, and the resulting configuration free spaces are coded in octree data structure[16].

3.1. Characterization of PTP Motion Capability

The various p-connectivity can be tested by constructing free subspaces and analyzing the topology of the given subspaces in relevance to these free spaces. In practice, construction of both Qf_i and Wf_i are time consuming processes, and should be minimized. Thus, in this section, a method is presented for grouping the maximal subsets of an arbitrary discrete trajectory according to various p-connectivity, without requiring complete construction of Wf_i.

All topological subspaces of Wf can be constructed from elementary subspaces called the 'basic components', which are categorized according to the multiplicity of inclusion of multiple inverse kinematics solutions in Qf. The basic components are disjoint subspaces defined for indices $\{I_k/k\in K\} = \{I_1, I_2, ..., I_p\}$ of all smallest subsets of I. For instance, when there are three path connected configuration subspaces Qf_i in Qf, $i\in I$ and $I=\{1,2,3\}$, then the basic components are defined for $\{I_1, I_2, ..., I_7\}$ with $I_1=\{1\}$, $I_2=\{2]$, $I_3=\{3\}$, $I_4=\{1,2\}$, $I_5=\{2,3\}$, $I_6=\{1,3\}$, $I_7=\{1,2,3\}$. Consequently, as shown in Fig. 3, the basic components are $b_1=Wf_1-Wf_{12}-Wf_{13}$, $b_2=Wf_2-Wf_{12}-Wf_{23}$, $b_3=Wf_3-Wf_{13}-Wf_{23}$, $b_{12}=Wf_{12}-Wf_{123}$, $b_{13}=Wf_{13}-Wf_{123}$, $b_{23}=Wf_{23}-Wf_{123}$ and $b_{123}=Wf_{123}$, where $Wf_{ij}=Wf_i\cap Wf_j$ and $Wf_{ijk}=Wf_i\cap Wf_j\cap Wf_k$.

A discrete task trajectory, $T_d\subset Wf$, is given as a set of discrete points, $\underline{x}_j$, specifying a location of the end effector,

$$(T_d = \cup_{j\in J}\, \underline{x}_j) \subset Wf$$

where $J=\{1,2,\ldots,j_{max}\}$ and j_{max} is the maximum number of task points. Each $\underline{x}_j$ is then mapped by inverse kinematics operator to multiple images $\underline{\theta}^l_j$, where $l=1,2,\ldots,l_{max}$ and l_{max} is the multitude of inverse kinematics solutions. At each of the obtained joint angle, manipulator is tested for collisions with obstacles, and corresponding basic components are identified as following.

- If $\underline{\theta}^l_j$ causes collision, assign *NULL* to the lth element, then $b[l] = NULL$.
- If $\underline{\theta}^l_j$ causes no collision, then identify which Qf_i it belongs to.

The second test is accomplished by an octree operator *Find*(Oct_1, Oct_2) which identifies wether an octree entity Oct_1 is included in another octree entity Oct_2. Therefore, upon operation on $\underline{\theta}^i_j$ and Qf_i, it identifies wether $\underline{\theta}^i_j$ belong to Qf_i. As a result, an ordered list, (b[1], b[2], ..., b[l]), is obtained. Then the corresponding task point, $\underline{x}_j$, is stored into a proper basic components by *Group*( ) operator,

$$G_{I_k} = Group(b[1], b[2], \ldots, b[l_{max}]).$$

Here, *Group*( ) operator returns a pointer to the basic component group, G_{I_k}. The index I_k spans all possible basic components and denoted with non-repeated index of free space.

For example, when l_{max}=4 and i_{max}=3, *Group*(1,1,2,3) = G_{123} and Group(1,*NULL*,1,3) = G_{13}, etc. As a result, the following verification process is accomplished.

If (all b[i] ∊ Qf_1), then (store $\underline{x}_j$ in Group G_{100}).
If (all b[i] ∊ Qf_2), then (store $\underline{x}_j$ in Group G_{200}).
If (all b[i] ∊ Qf_3), then (store $\underline{x}_j$ in Group G_{300}).
If ((some b[i] ∊ Qf_1) and (some b[i] ∊ Qf_2)), then (store $\underline{x}_j$ in Group G_{120}).
If ((some b[i] ∊ Qf_1) and (some b[i] ∊ Qf_3)), then (store $\underline{x}_j$ in Group G_{130}).
If ((some b[i] ∊ Qf_2) and (some b[i] ∊ Qf_3)), then (store $\underline{x}_j$ in Group G_{230}).
If ((some b[i] ∊ Qf_1) and (some b[i] ∊ Qf_2) and (some b[i] ∊ Qf_3)), then (store $\underline{x}_j$ in Group G_{123}).

After all task points are identified and grouped for basic components, the p-connected task points are determined by the following set operations,

(P_4) P_4 connected task points are identified as
$$P_4(T_d) = G_{Ip},\ I_p = \{ijk \mid i=1,2,\ldots,i_{max},\ \text{and } j=k=0\}.$$
(P_3) P_3 connected task points are identified as
$$P_3(T_d) = G_{Ip},\ I_p=\{I_k \mid \forall k \in K\}.$$
(P_2) P_2 connected task points are identified as
$$P_2(T_d) = G_{Ip} = \{\cup_{j,k\in K'} G_{ijk},\ j \neq k,\ i=1,2,\ldots,i_{max}\}.$$
(P_1) P_1 connected task points are identified as
$$P_2(T_d) = G_{Ip} = \{\cup_{j,k\in K'} G_{ijk},\ j \neq k \ \text{ for } i=1,2,3\}$$
$$\text{and } \{\cup_{\forall i,j,k} G_{ijk},\ j \neq 0 \text{ or } k \neq 0\}$$
These characterization procedure is illustrated in Fig. 3.

3.2. Optimal Search

The problem of base positioning is summarized as follows: Given 1) the geometric data of the robot and environment, 2) the discrete task trajectory, 3) the desired PTP motion characteristics, find a location, $\underline{x}_d$, of the robot base which renders the desired motion capability.

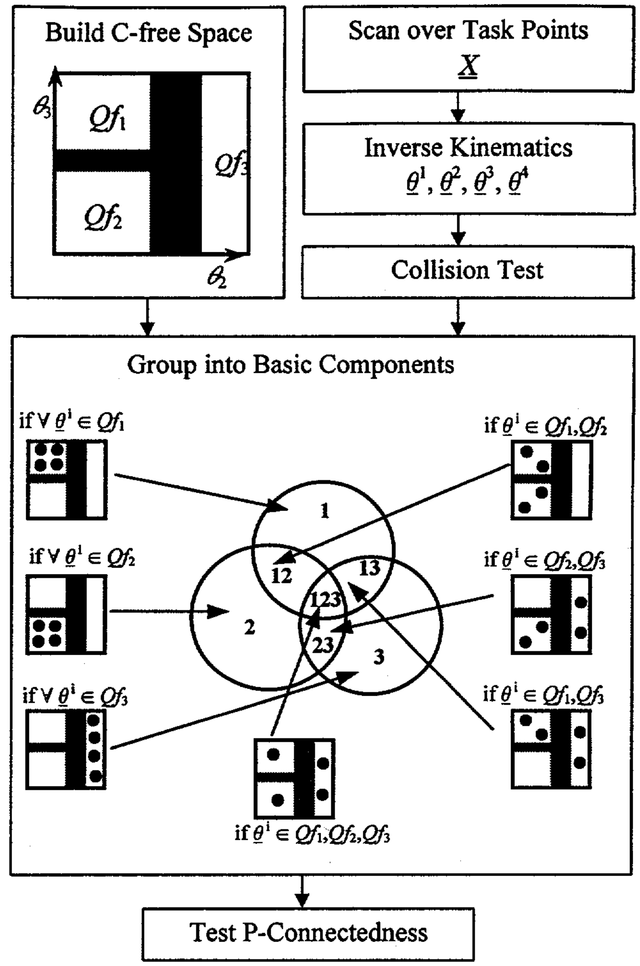

Fig. 3. Grouping the trajectory into basic components

We propose to implement an optimal search based on downhill simplex method[18]. To facilitate the previous results into the search process, the following cost function is proposed:

$$F_{opt}(\underline{x}_{base}) = \max_{Ip}[Area(G_{Ip})].$$

Here, the function Area( ) returns the number of the task points stored in the topological subgroup, G_{I_p}.

Then the optimal search for the task feasible base location is accomplished through the following schema.

1) Initialize the simplex: Base locations are chosen at arbitrary locations for initial simplex.

2) Assess basic components: The task points are grouped into basic components as described in section 3.2.

3) Evaluation of cost function: Through set operations between the basic groups, the motion feasible portion of the task points are found. The cost function, defined as above, reflects the size of the largest basic group.

4) Optimization: Update the simplex point according to the procedure described in the previous chapter, and steps 1) to 3) are repeated until termination condition is met. The result gives the optimal base location.

The optimal search process is illustrated in Fig. 4. The results of optimization gives the portion of each p-connectivity as well as the task feasible base location of the manipulator. Furthermore, the complete map of possible postures at each point of the task space can be obtained.

4. Case Study

The method has been demonstrated for a 3 d.o.f. manipulator, whose Denavit-Hartenberg parameters are given in Table 1. Its task is to perform PTP motions among the various task points spanning over the rectangular shaped horizontal plane of size 6m x 6m

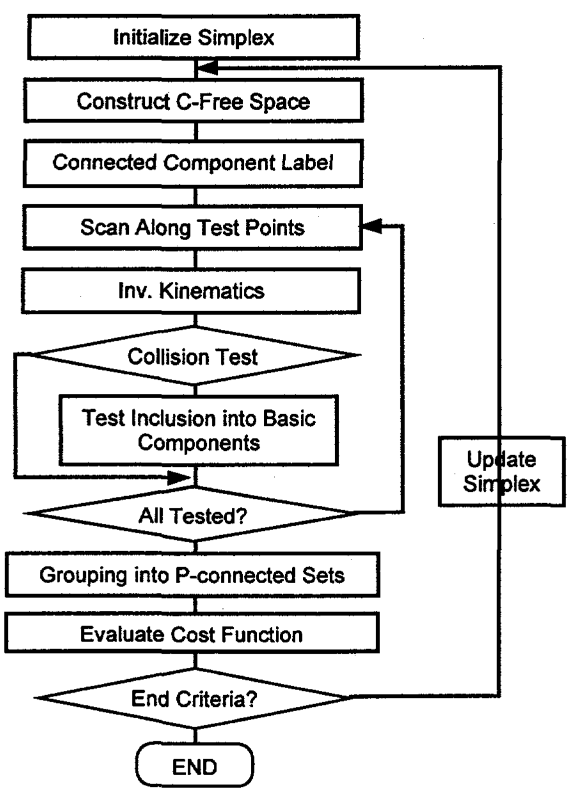

Fig. 4. Optimal search for base positioning

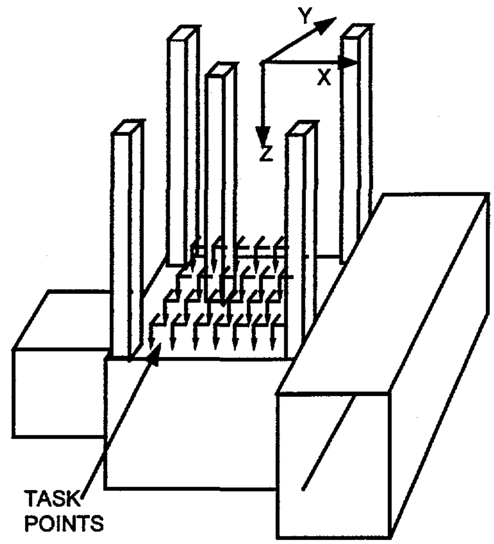

Fig. 5. Task environment for case study

placed at the grid points with 1m intervals in both directions as shown in Fig. 5. The task points are coded into octree entities of unit pixel resolution appended with a vector representing the orientation of the end effector, which is constrained in *z*-direction of world coordinate. The environment consists of obstacles represented with nine rectangular blocks also as shown in the figure. The configuration free space is built with the collision detection accuracy of 3°, which is converted to octree with resolution of 128 pixels in all directions. The optimization variables are chosen to be $\underline{X}=(x,y,z,\alpha)$ which is a position of the robot base and the its orientation with respect to the vertical plane.

Simulation is performed for P2 connectivity, which is defined as the capability to move between arbitrary numbered points. As shown in Fig. 6, the final location is obtained to be (x=0.45m, y=-0.62m, z=3.3m, α=43.54°).

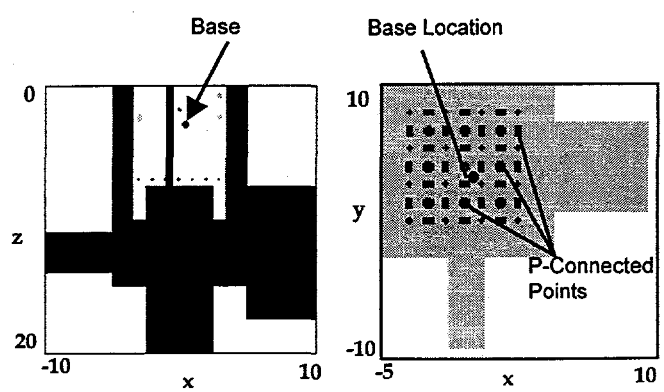

Fig. 6. Test result for for P2 connectivity

Total coverage of 35 points is achieved shown as black points in the figure. As can be seen in the figure, the optimization ended at 60 calls to the cost function.

In this method, computation time is greatly affected by the resolution of octree construction, thus it is important to choose this factor with care. The complexity of the environment also has significant influence on time. As it is confirmed by the result, the selected resolution appears to be sufficiently accurate for the given task environment. However, for different task environment, it may be necessary to carefully select the octree resolutions.

Table 1. D-H parameters of the manipulator

joint i	α	a_i	d_i	q_i
1	0	0	0	θ_1
2	-90°	1 m	1 m	θ_2
3	90°	2 m	0	θ_3
4	0	1.5 m	0	0

5. Summary and Conclusions

This paper presents a computational procedure for robot placement based on PTP motion characteristics. On such an application of theoretical works, practically no previous works have been found. Testing for PTP motion capabilities of multiple task points are more involved computationally because the trajectory between the points can have infinite number of possibilities. The formalism presented here adopts a topological characterization of free space to identify task capable portion of task space, rather than searching for feasible paths. This formulation is coherent with the existing techniques on workspace analysis and provides complete solution.

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