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ANGLE OF CRACK PROPAGATION FOR A VERTICAL HYDRAULIC FRACTURE

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ABSTRACT

Using the strain-energy-density-factor (S) theory, the positive fracture angle $+\theta_o$ (the initial fracture angle of crack propagation) of a near-vertical crack is predicted by using the opening- and sliding-mode stress-intensity factors in the presence of the overburden pressure, the least *in situ* horizontal principal stress, and the borehole fluid pressure. The crack spreads in the positive θ_o direction (counterclockwise) in the plane for which S is a minimum, S_{min}. It was verified that $S_{min} \geq S_c$. The quantity S_c is defined as the critical value of S, and remains essentially constant.

Of interest is the numerical example for calculating fracture angle and the critical uniform borehole fluid pressure required to initiate fracture at such an angle for the present LASL Dry Hot Rock Geothermal Energy Program.

I. INTRODUCTION

Hydraulic fracturing is an important part of the Los Alamos Scientific Laboratory's (LASL's) Dry Hot Rock Geothermal Energy Program. Water is injected into a hot granite in order to produce hydraulic fracturing and the final goal is to circulate water through the fracture and recover it as steam or hot water.[1]

For the depths of practical interest in heat recovery, all of the fractures created are close to the vertical direction. For example, several vertical fractures have been made in the well GT-2 of the LASL program. Each of them seems to be roughly parallel to the others and to intersect the well over a height of a few meters. (The well was drilled with a slight deviation from the vertical direction.)

A major problem is to intersect one or more of those fractures and then to create a large fracture, in order to produce a sufficient heat transfer between rock and fluid. As such, fracture angle of crack propagation from a vertical fracture needs to be examined. A theoretical treatment is proposed to estimate the fracture angle and the critical uniform borehole fluid pressure required to initiate fracture at such an angle. Numerical examples will be given for the present LASL Dry Hot Rock Geothermal Energy Program.

II. MODEL

Consider an elastic solid of infinite dimensions with a vertical interior crack (Fig. 1). The crack, with height 2a and infinite length in the direction of the (x,z) plane, is perpendicular to the (x,y) plane, the plane of Fig. 1.

Define σ_1, σ_3 as the vertical principal total stress and the horizontal one applied at infinite distance from the crack. They represent the overburden pressure and the least *in situ* horizontal principal stress. A uniform borehole fluid pressure P is applied inside the crack. This pressure can be monitored as the sum of the pumping pressure measured at the surface and the pressure resulting from the hydrostatic head of the fluid column extending to the surface. Deformation is allowed only in the (x,y) plane and a plane-strain problem is then considered.

Suppose that P is increased inside the crack. What will the fracture angle of the direction of crack propagation be at some critical pressure? Could

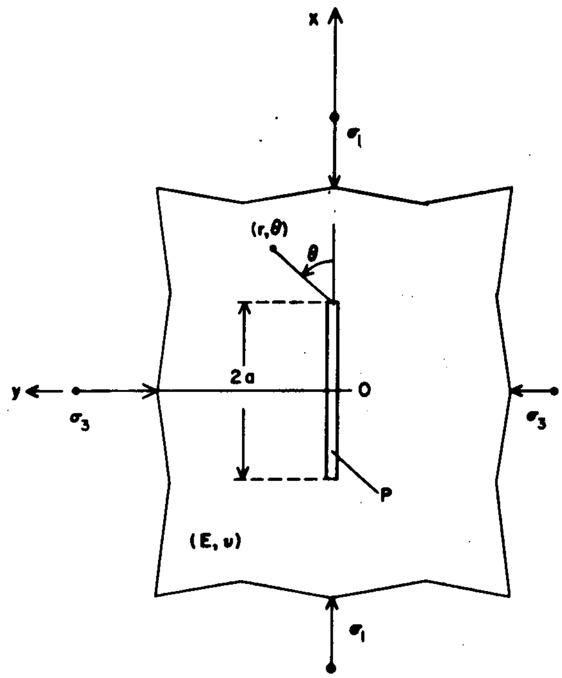

Fig. 1. A vertical interior crack subject to σ_1, σ_3, and P.

the fracture angle incline with respect to the vertical direction, possibly allowing the intersection of this crack with others?

III. THE STRAIN-ENERGY-DENSITY THEORY

Two theories have been developed to explain crack propagation. The maximum principal stress theory specifies that the fracture develops along the path which is perpendicular to the maximum principal stress. The minimum strain-energy-density theory[2] states that crack propagation initiates along the path of minimum strain-energy density and the onset of such crack propagation is governed by a critical strain-energy-density factor, S_c. By using the latter theory, Hsu and Forman[3] predict the fracture angle of a crack at a hole at an arbitrary angle. Predicted fracture angles agree well with experimental ones as cited in Ref. 3.

It is very difficult and inconvenient to use the maximum stress theory for predicting fracture angle of crack propagation in combined loading situations as in Fig. 1. At each instant, fracture angle will depend on the energy state and the material properties in a region ahead of the crack tip. As such, this requires a more general treatment of the problem by using the minimum strain-energy-density theory.

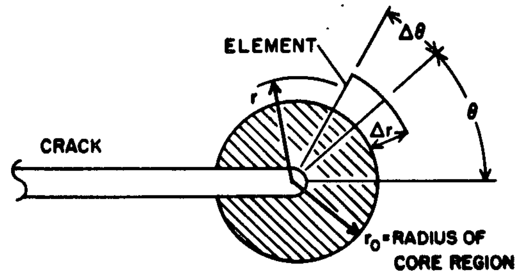

Fig. 2. Crack-tip element.

A. The Strain-Energy-Density Factor, S

The material in the immediate vicinity (inside the core region of radius r_o) of the crack tip will behave differently from that of the rest of the body. The stresses in the material next to the crack tip (inside the core region of radius r_o) are exceedingly high and the mechanical properties of the material are not known there. Hence, of concern are elements in the regions away from the crack tip (or outside the core region of radius r_o). The amount of energy[2] stored in one of these elements, for instance an incremental area of $\Delta A = r\Delta\theta\Delta r$ as shown in Fig. 2, is called the local strain-energy-density function and derived as

$$\frac{dW}{dA} = \frac{1}{r}\left(a_{11}k_1^2 + 2a_{12}k_1k_2 + a_{22}k_2^2 + a_{33}k_3^2 + \ldots\right) \quad (1)$$

where the coefficients a_{11}, a_{12}, ... a_{33} for plane strain are

$$\left.\begin{aligned} a_{11} &= \frac{1}{16\mu}\left[(3 - 4\nu - \cos\theta)(1 + \cos\theta)\right], \\ a_{12} &= \frac{1}{16\mu}(2\sin\theta)\left[\cos\theta - (1 - 2\nu)\right], \\ a_{22} &= \frac{1}{16\mu}\left[4(1 - \nu)(1 - \cos\theta) + (1 + \cos\theta)(3\cos\theta - 1)\right], \\ a_{33} &= \frac{1}{4\mu} \;. \end{aligned}\right\} \quad (2)$$

Here ν and μ are the Poisson's ratio and the shear modulus of elasticity ($\mu = E/2(1+\nu)$). E is the Young's modulus. The dW/dA in Eq. (1) is inversely proportional to the radial distance measured from the crack tip. The dW/dA becomes exceedingly large as r is made smaller and smaller, reaching a limit on the boundary of the core region $r = r_o$. The intensity of this energy field, which varies along the periphery of circle $r = r_o$, is called S, the strain-energy-density factor,

$$S = a_{11}k_1^2 + 2a_{12}k_1k_2 + a_{22}k_2^2 + a_{33}k_3^2 , \quad (3)$$

which depends on θ, locating the position of the element ΔA through the coefficients a_{11}, ..., a_{33}. Here k_1, k_2, k_3 are the stress-intensity factors for the opening, edge-sliding, and tearing modes, respectively. For the current problem, only k_1 and k_2 are needed. Hence Eq. (3) reduces to

$$S = a_{11}k_1^2 + 2a_{12}k_1k_2 + a_{22}k_2^2 . \quad (4)$$

B. Strain-Energy-Density-Factor Theory

The theory[2] states that crack initiation takes place first along the line $\theta = \theta_o$ where S (or a crack-resistance force) is a minimum. The necessary and sufficient conditions for S to be a minimum at the angle θ_o are

$$\left(\frac{\partial S}{\partial \theta}\right)_{\theta_o} = 0, \quad \left(\frac{\partial^2 S}{\partial \theta^2}\right)_{\theta_o} > 0 . \quad (5)$$

In addition, crack extension occurs along θ_o only if S is at least equal to a critical value, S_c:

$$S(k_1,k_2,k_3) \geq S_c, \quad \text{for } \theta = \theta_o . \quad (6)$$

Here the difference between S and S_c is analogous to the one between k and k_c. Thus, an intrinsic material parameter S_c is also a measure of the resistance of a material against fracture.

IV. APPLICATION OF EQS. (5) AND (6)

We first calculate S associated with a preexisting fracture having an initial crack angle β with respect to the vertical direction and subject to the state of stresses, σ_1, σ_3, and P. The first and second derivatives of S will then be obtained.

A. The Principle of Superposition (Fig. 3)

The stress intensity factors k_1 and k_2 are sums of two terms, one denoted by "*" and associated with the vertical total stress σ_1, the other denoted by "**" and associated with the horizontal net stress, $(P - \sigma_3)$;

$$k_1 = k_1^* + k_1^{**} , \quad k_2 = k_2^* + k_2^{**} . \quad (7)$$

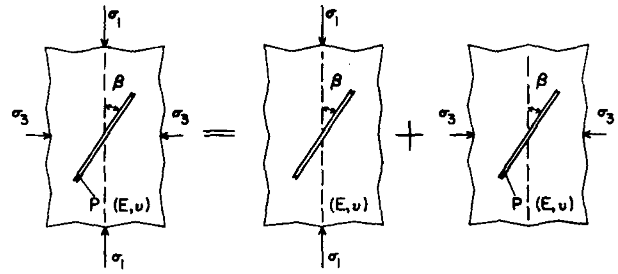

Fig. 3. Solution obtained based on the principle of superposition.

Here k_1^* and k_2^* are obtained from Ref. 4.

$$\left.\begin{aligned} k_1^* &= -\sigma_1\sqrt{a}\,\sin^2\beta , \\ k_2^* &= -\sigma_1\sqrt{a}\,\sin\beta\cos\beta . \end{aligned}\right\} \quad (8)$$

Considering $-(P - \sigma_3)$ as a tensile stress and the angle of loading with respect to the crack as $\left(\frac{\pi}{2} - \beta\right)$, k_1^{**} and k_2^{**} are

$$\left.\begin{aligned} k_1^{**} &= (P - \sigma_3)\sqrt{a}\,\cos^2\beta , \\ k_2^{**} &= (P - \sigma_3)\sqrt{a}\,\sin\beta\cos\beta . \end{aligned}\right\} \quad (9)$$

So, for the problem under consideration in Fig. 1, k_1 and k_2 are

$$\left.\begin{aligned} k_1 &= (P - \sigma_3)\sqrt{a}\,\cos^2\beta - \sigma_1\sqrt{a}\,\sin^2\beta , \\ k_2 &= (P - \sigma_3)\sqrt{a}\,\sin\beta\cos\beta - \sigma_1\sqrt{a}\,\sin\beta\cos\beta . \end{aligned}\right\} \quad (10)$$

B. Strain-Energy-Density Factor, S

From Eq. (4) and Eq. (10), S is

$$S = a\Big[a_{11}\{(P - \sigma_3)\cos^2\beta - \sigma_1\sin^2\beta\}^2 + a_{12}\sin 2\beta\,\{(P - \sigma_3) - \sigma_1\}\,\{(P - \sigma_3)\cos^2\beta - \sigma_1\sin^2\beta\} + a_{22}\left(\frac{\sin 2\beta}{2}\right)^2 \{(P - \sigma_3) - \sigma_1\}^2\Big] . \quad (11)$$

It is convenient to define the dimensionless parameter α.

$$\alpha = (P - \sigma_3)/\sigma_1 . \quad (12)$$

Then, from Eq. (2), Eq. (11), and Eq. (12), S becomes

$$S = \frac{\sigma_1{}^2 a}{16\mu} f(\alpha,\beta,\theta) \quad , \tag{13}$$

For convenience, write $S_c\big|_{\beta\neq 0}$ as S_c. Once the fracture angles θ_o are inserted into Eq. (13), S must be equal to S_c at least in order to initiate new crack at the tip,

$$\begin{aligned} \text{where } f(\alpha,\beta,\theta) = {} & (3 - 4\nu - \cos\theta)(1 + \cos\theta)(\alpha\cos^2\beta - \sin^2\beta)^2 \\ & + 2\sin\theta(\cos\theta - (1 - 2\nu))(\alpha - 1)(\alpha\cos^2\beta - \sin^2\beta)\sin 2\beta + (4(1 - \nu)(1 - \cos\theta) \\ & + (1 + \cos\theta)(3\cos\theta - 1))\left(\frac{\sin 2\beta}{2}\right)^2 (\alpha - 1)^2 \ . \end{aligned} \tag{14}$$

This expression is not valid for the case of $\beta = 0$, i.e., a strictly vertical fracture. For the problem under consideration, we will have to deal with small values of β, i.e., a near-vertical pre-existing fracture.

Derivatives of S are

$$S_c \leq \frac{\sigma_1{}^2 a}{16\mu} f(\alpha,\beta,\theta_o) \quad . \tag{18}$$

From Eqs. (17) and (18), one obtains

$$\sigma_1{}^2 a\, f(\alpha,\beta,\theta_o) \geq 4(1 - 2\nu) k_{1c}^2 \quad . \tag{19}$$

$$\begin{aligned} \frac{8\mu}{\sigma_1{}^2 a}\frac{\partial S}{\partial\theta} = {} & \sin\theta(\cos\theta + 2\nu - 1)(\alpha\cos^2\beta - \sin^2\beta)^2 \\ & - (\cos 2\theta - (1 - 2\nu)\cos\theta)(\alpha - 1)(\alpha\cos^2\beta - \sin^2\beta)\sin 2\beta \\ & + \sin\theta((1 - 2\nu) - 3\cos\theta)\left(\frac{\sin 2\beta}{2}\right)^2 (\alpha - 1)^2 \ . \end{aligned} \tag{15}$$

$$\begin{aligned} \frac{8\mu}{\sigma_1{}^2 a}\frac{\partial^2 S}{\partial\theta^2} = {} & (\cos 2\theta + (-1 + 2\nu)\cos\theta)(\alpha\cos^2\beta - \sin^2\beta)^2 \\ & + (2\sin 2\theta + (-1 + 2\nu)\sin\theta)(\alpha - 1)(\alpha\cos^2\beta - \sin^2\beta)\sin 2\beta \\ & + (-3\cos 2\theta + (1 - 2\nu)\cos\theta)\left(\frac{\sin 2\beta}{2}\right)^2 (\alpha - 1)^2 \ . \end{aligned} \tag{16}$$

C. Conditions for Crack Propagation Along Fracture Angle θ_o, Where $S(k_1,k_2)\big|_{\theta=\theta_o} \geq S_c$

Find θ_o based on $\frac{\partial S}{\partial\theta} = 0$ such that $\left(\frac{\partial^2 S}{\partial\theta^2}\right)_{\theta_o} > 0$.

Here the positive and negative of θ_o are defined as fracture angles of crack extension with respect to the initial crack line, in counterclockwise and clockwise directions (Fig. 4). From Ref. 2, $S_c\big|_{\beta=0}$ is, with k_{1c} as the fracture toughness of the opening mode,

$$S_c\big|_{\beta=0} = \frac{1 - 2\nu}{4\mu} k_{1c}^2 \quad . \tag{17}$$

Based on experimental results,[3] $S_c\big|_{\beta\neq 0} \simeq S_c\big|_{\beta=0}$.

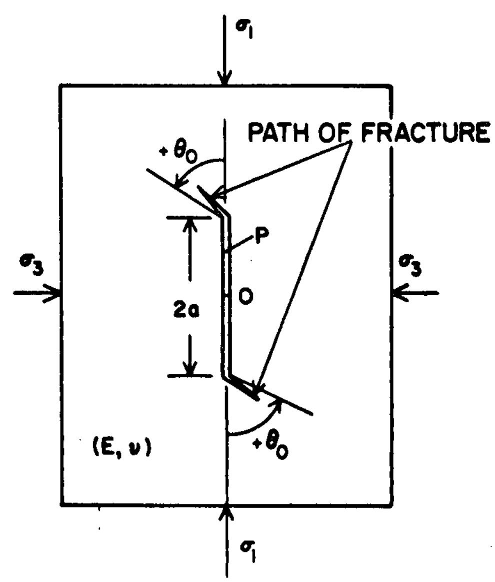

Fig. 4. Positive fracture angle $+\theta_o$.

Here, σ_1 is the overburden pressure, with $\sigma_1 = \rho gh$ and ρ = mean density of the rock. So,

$$(\rho gh)^2 a\ f(\alpha,\beta,\theta_o) \geq 4(1 - 2\nu)k_{1c}^2 \quad . \qquad (20)$$

For a given material, the term on the left side of Eq. (20) depends on a depth h, an initial crack length 2a, an initial crack angle β, and $\alpha = \frac{(P - \sigma_3)}{(\rho gh)}$.

V. NUMERICAL EXAMPLES

The problem concerning the well GT-2 of the LASL program can now be studied as a particular case of the general theory developed in Sec. IV.

As we deal with a nearly vertical crack, a low β angle is chosen. For example, $\beta = 0.02$ rad. A depth, h, of practical interest in the well GT-2 is 3000 m. With $\rho \simeq 2400\ kg/m^3$, $\sigma_1 = 72$ MPa (10,300 psi). Suppose now there is a pore pressure P_o equal to the hydrostatic head. Hence, the matrix (effective) stress $(\rho gh - P_o)$ is 42 MPa (6000 psi). Field data showed that $0 \leq (P - \sigma_3) \leq 3$ to 4 MPa (420 to 560 psi). An equivalent statement is that $0 \leq \alpha \leq 0.05$ to 0.1.

Typical material constants for granite are $\mu = 2.7 \times 10^4$ MPa, $\nu = 0.25$, and $\gamma = 10^2\ Nm/m^2$. Here γ is the energy required to create unit area of new surface. Then $k_{1c} \equiv K_{1c}/\sqrt{\pi}$ can be calculated, based on Ref. 5, as

$$k_{1c} = \sqrt{(2E\gamma/\pi)} = \sqrt{[4(1+\nu)\mu\gamma]/\pi} = 2.07\ MN/m^{3/2}$$

for the initial fracture size, 2a = 10 m.

A. Fracture Angle of Crack Propagation

For $\alpha = 0$, $\theta_o \simeq 80$ degrees. Hence, a crack would initiate almost horizontally. For $\alpha = 0.05$ to 0.1, $\theta_o \simeq 32$ to 34 degrees.

Rewrite Eq. (20) as, in the presence of P_o,

$$f(\alpha,\beta,\theta_o) \geq \frac{4(1 - 2\nu)k_{1c}^2}{a(\rho gh - P_o)^2} \quad .$$

With the above-mentioned data, Table I lists the values of $f(\alpha,\beta,\theta_o)$. Equation (20) becomes

$$f(\alpha,\beta,\theta_o) \geq 10^{-3} \quad . \qquad (21)$$

TABLE I

THE VALUES OF $f(\alpha,\beta,\theta_o)$

$P - \sigma_3$	$\alpha = \frac{P - \sigma_3}{\sigma}$	θ_o	$f(\alpha,\beta,\theta_o)$
0	0	80°	8.4×10^{-4}
3-4 MPa	0.05-0.1	32-34°	58.0×10^{-4}

For $\alpha = 0$, Eq. (14) becomes

$$f(0,\beta,\theta_o) \simeq \beta^2(4(1-\nu)(1 - \cos\theta_o) + (1 + \cos\theta_o)(3\cos\theta_o - 1)).$$

Inserting $\theta_o \simeq 80$ degrees into the above form, $f(0,\beta,80°)$ is equal to 8.4×10^{-4}. Substituting this value into the left term in Eq. (21), $8.4 \times 10^{-4} < 10^{-3}$. For this case, the given parameters are either not or hardly sufficient for causing crack initiation in a direction of $\theta_o = 80$ degrees. At such a depth, a slight increase of P over σ_3 is probably required in order to initiate the fracture.

For $\alpha = 0.05$, $f(0.05,\beta,32°) \simeq 58 \times 10^{-4}$. However, for such parameters, a crack is easily initiated.

For the low values of α, $f(\alpha,\beta,\theta)$ increases with the increase of α. So, crack initiation is easier when α is increased (i.e., $(P - \sigma_3)$ is increased).

B. Further Propagation of the Crack

As soon as cracking in the θ_o direction is initiated, the energy state in a region ahead of the crack tip varies and the whole problem has to be reconsidered. Under compression σ_1, the crack eventually follows a vertical path after a curved path for crack initiation.

Nevertheless this curved path for crack initiation of the pre-existing vertical crack could possibly permit the intersection of nearby cracks. Rigorously speaking, in the presence of nearby vertical cracks, k_1 and k_2 have to be resolved at the tip of the vertical crack of interest.

VI. CONCLUSION

Considering a rock of very low porosity and permeability, Eq. (20) can be applied to predict the initiation of cracks from a given initial hydraulic fracture. In order to produce larger positive fracture angles at the tip of the pre-existing vertical crack, P needs to be slowly increased up to a value barely above P_c. With such large angles, this curved path for crack initiation could easily permit the interaction of nearby cracks.

For depths of interest in the well GT-2 of the LASL's program, hydraulic fractures are formed as nearly vertical ones in the direction of the axis of the wellbore. For this particular case, a new crack initiates from a pre-existing vertical fracture with a fracture angle of 32 degrees with respect to the vertical direction for $\alpha = 0.05$. This could possibly permit the intersection of this extending crack with nearby vertical hydraulic fractures.

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