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DESIGN OF DOUBLE HELIX CONDUCTORS FOR SUPERCONDUCTING AC POWER TRANSMISSION†

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ABSTRACT

Coaxial cable conductors in the form of helical tape windings have been proposed in order to make Nb_3Sn cables which have flexibility and the ability to take up thermal contraction. For ac power transmission the axial magnetic fields which occur in a simple helical construction produce a number of undesirable consequences. It has been shown that these problems can be avoided by using double layer windings of opposite helicity, with 45^{o} as the optimum helix angle. However, smaller values than this are desirable for mechanical reasons, and this paper extends the theory to include pitch angles $< 45^{o}$. Measurements on short cable models are shown to be in reasonable agreement with calculation. The effect of current flow around the superconductor tape edges, which occurs in helical windings, is analyzed and it is shown that appreciable ac loss can arise if laminated tape with non-superconductive edges is used indiscriminately.

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INTRODUCTION

The original BNL conceptual design of a Nb_3Sn superconducting coaxial cable for ac power transmission envisioned segmented tape conductors wound in simple helical form.[1] The aim was to provide flexibility and the ability to take up thermal contraction. However, helical current flow produces axial magnetic fields which lead to several undesirable consequences. It has been shown, for example, that non-zero axial flux can generate large eddy current losses in metal enclosures (such as cryostat walls) external to the cable proper.[2] In addition, with simple helical conductors this flux would produce large voltage drops along the length of the outer conductor, necessitating high voltage insulation on the outside of the cable.[3] Finally, axial magnetic fields in the inner core region of the cable can produce significant eddy current heating in normal metals which are present there for mechanical support and current stabilization.[3]

The authors referred to have pointed out that it is not possible to eliminate all of these problems simultaneously by a simple adjustment of the inner and outer helix pitch angles. Double windings of opposite helicity have been proposed in place of the simple helices. Although it was initially suggested that the tapes of a double helix be transposed periodically to ensure balanced currents,[2] this would be difficult, if not impractical, for comparatively fragile Nb_3Sn tapes. It has been shown that a simple overlay of the helices

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will work as well if the winding pitch angle is approximately 45°.[3] In this paper we calculate the behavior for smaller pitch angles. This is of some importance since smaller angles are desired in order that the radial contractions of the conductors and dielectric are properly matched upon cooldown. We show that for pitch angles in the range 20° to 45° the problems discussed above can be reduced to acceptable levels.

In addition, we discuss the current flow perpendicular to the tape axes which occurs in the individual segments of a double helix. The losses which arise because of this are analyzed as a function of pitch angle and tape construction.

DESCRIPTION OF CURRENTS AND FIELDS

The cable design considered is shown in Fig. 1. The conductors are numbered 1 to 4 from inner to outer. Dielectric insulation is located in the annular space of the coax, i.e., between 2 and 3. The region inside conductor 1 contains mechanical support for the tapes and refrigerant. It is referred to below as the core region. Positive direction for I_1 and I_2 is opposite to that for I_3 and I_4. Current flow between individual tape segments is assumed negligible in the superconductive state. The helicities of conductors 1 and 2 are opposite and likewise for 3 and 4, in order to reduce the axial fields in the annular space of the coax and in the core region. The helicities of conductors 2 and 3 are shown the same but this convention is not essential. If we denote the cable current by I, then:

$$I_1 + I_2 = I \qquad (1)$$

$$I_3 + I_4 = I \qquad (2)$$

In addition, each helix pair is connected at its ends, so that the voltage drops per meter of conductor satisfy

$$V_1 = V_2 \qquad (3)$$

$$V_3 = V_4 \qquad (4)$$

These voltages can be expressed as functions of the currents and the various pitches and diameters. Magnetic fields are determined by superposition of the fields of each of the four currents. For a helical current shell i an axial field. $H_{zi} = I_i/P_i$, is produced inside the shell, and an azimuthal field $H_{\theta i} = I_i/2\pi r$, outside, where P_i is the helix pitch length and r is the distance from the cable axis (mks units). In what follows the P_i are taken as positive quantities, and the algebraic signs of the H-components are written explicity. V_1, V_2, V_3, V_4 may be written down from Faraday's law as follows:

$$V_1 = (j\omega \mu_o \pi/4) D_1^2 P_1^{-1} (I_1 P_1^{-1} - I_2 P_2^{-1} + I_3 P_3^{-1} - I_4 P_4^{-1}) + (j\omega \mu_o/2\pi) I_1 \ln D_2/D_1 \qquad (5)$$

$$V_2 = (-j\omega\mu_o\pi/4) D_2^2 P_2^{-1} [(D_1^2/D_2^2) I_1 P_1^{-1} - I_2 P_2^{-1} + I_3 P_3^{-1} - I_4 P_4^{-1}] \quad (6)$$

$$V_3 = (-j\omega\mu_o\pi/4) P_3^{-1} (I_1 P_1^{-1} D_1^2 - I_2 P_2^{-1} D_2^2 + I_3 P_3^{-1} D_3^2 - I_4 P_4^{-1} D_3^2) + (j\omega\mu_o/2\pi) I_4 \ln D_4/D_3 \quad (7)$$

$$V_4 = (j\omega\mu_o\pi/4) P_4^{-1} (I_1 P_1^{-1} D_1^2 - I_2 P_2^{-1} D_2^2 + I_3 P_3^{-1} D_3^2 - I_4 P_4^{-1} D_4^2) \quad (8)$$

where D_i is the diameter of the ith conductor, ω is the angular frequency, and $j = \sqrt{-1}$. In Eqs. (5) and (6), voltages due to flux linkages outside conductor 2 are not written down since they cancel in Eq. (3). Flux linkages and resistance in the superconducting surfaces are neglected in the calculation. In reference 3 these voltages were written in terms of the coefficients of inductance of the conductors. Eqs. (1) to (4), with the voltages given by Eqs. (5) to (8), enable us to solve for I_1, I_2, I_3 and I_4 as functions of the pitches, diameters, and I.

Approximate values of azimuthal megnetic fields can be calculated quickly by assuming equal helix currents. $I_1 = I_2 = I_3 = I_4 = I/2$. As can be shown by numerical calculation the individual helix currents may differ from this by about 10% for parameters corresponding to practical cases. For calculations involving these fields the equal current assumption is adequate. Referring to Fig. 2, the magnetic field at the surface of the inner conductor (outside 2) is entirely azimuthal, and $= I/\pi D_2$. We shall call this reference field H_θ. In the annulus between 1 and 2 the azimuthal field component is $\approx H_\theta/2$ and there is an axial component $H_z \approx (H_\theta \tan \varphi)/2$, where φ is the pitch angle, i.e., the angle between the tape axis and the coax axis. There is a small axial field in the core region, H_o, which cannot be calculated accurately under the equal current assumption. It will be calculated exactly in the next section. The fields for the outer conductor are similar and will not be considered further.

VOLTAGE IN OUTER CONDUCTOR; CORE FIELD

The purpose of the double helix is to reduce to insignificant levels the quantities H_o, the axial magnetic field in the core region, and Φ, the total axial flux across the cable cross section. These are given by:

$$H_o = I_1 P_1^{-1} - I_2 P_2^{-1} + I_3 P_3^{-1} - I_4 P_4^{-1} \quad (9)$$

and

$$\Phi = (\pi\mu_o/4) (I_1 P_1^{-1} D_1^2 - I_2 P_2^{-1} D_2^2 + I_3 P_3^{-1} D_3^2 - I_4 P_4^{-1} D_4^2) \quad (10)$$

Using Eqs. (1) to (4) H_o and Φ can be expressed as follows:

$$H_o = - I_1 (P_1 D_2^2 + P_2 D_1^2)^{-1} [2/\pi^2) P_1 P_2 \ln D_2/D_1 - (D_2^2 - D_1^2)] \quad (11)$$

$$\Phi = (\pi\mu_o/4) I_4 (P_3+P_4)^{-1}[(2/\pi^2)P_3P_4 \ln D_4/D_3 - (D_4^2 - D_3^2)] \quad (12)$$

The conditions for the pitches if $H_o = 0$ and $\Phi = 0$, are obtained by setting the bracketed quantities equal to zero. As was pointed out in reference 3, they imply a pitch angle $\varphi = \tan^{-1} \pi D/P \approx 45^o$. In practice it will be necessary to employ smaller pitch angles in order that a tight coaxial structure result upon cool-down. Therefore, we are interested in the magnitude of H_o and Φ for other pitch angles. In practice Nb_3Sn tapes are thin (less than 0.2 mm, typically), and we may treat $D_2 - D_1 = \Delta D_1$ and $D_4 - D_3 = \Delta D_4$ as small quantities. Also, to simplify the discussion let $P_1 = P_2 = \alpha_1 \pi D_1$ and $P_4 = P_3 = \alpha_4 \pi D_4$. Deviations from this assumption would lead to terms in ΔP in the expressions which follow. As these are unimportant in practice, they have not been included. For H_o we get:

$$H_o \approx (I_1/\alpha_1 \pi D_1)(\Delta D_1/D_1)(\alpha_1^2 - 1) \quad (13)$$

As $I/\alpha_1 \pi D_1$ is the axial field for single helix conductors, this field is reduced by $(\Delta D_1/2D_1)(\alpha_1^2 - 1)$ for the double helix. Taking reasonable limits for $\Delta D_1/D_1 \lesssim 10^{-2}$ and $45^o \leq \varphi_1 \leq 20^o$ $(1 \leq \alpha_1 \leq 2.7)$ we find that H_o^1 is less than 3% of the single helix value. Eddy current losses in support metals in the core region[3] are thus reduced by more than 10^{-3}, i.e., to an insignificant level.

The quantity Φ determines the longitudinal voltage in the outer conductor, $V_4 = j\omega \Phi/P_4$. With the same assumptions as before we obtain

$$V_4 \approx (\mu_o \omega/4\pi) I_4 (\Delta D_4/D_4)(1 - \alpha_4^{-2}) \quad (14)$$

We see that for all pitch angles less than 45^o and $\Delta D_4/D_4 \leq 10^{-2}$, $I \leq 10^4$, and $\omega = 377$, V_4 is less than 2×10^{-3} V/m, which is entirely acceptable in practice.

MEASUREMENTS OF FIELDS IN A MODEL COAXIAL CABLE

Ac magnetic fields were measured in coaxial models made in the form of simple helical conductors, using uninsulated tapes. These showed spiral current flow; i.e., the impedance to spiral flow is much less than contact resistance between adjacent tapes. When problems due to axial flux in single helical conductors became understood double helix models were then made. Uninsulated tapes were used again. The overlying double helix layers were wound in direct contact. The Nb_3Sn tapes were clad with a thin (0.03 mm) copper layer for stabilization; this copper served in effect to separate the superconducting layers electrically. In addition to being easy to make, this method of construction is important since it leads to low radial heat resistance in the conductors of the double helix.

Results are given in Table I for a coaxial double helix model. The cable is shorted at one end through a copper ring. Measured quantities were proportional to current (1 kA to 5 kA) and were independent of frequency (except for V_4) (40 to 150 Hz). In addition to H_o, Φ, and V_4 which have been defined previously, the quantities H_A, the axial field in the annular space between 2 and 3 (Fig. 1) and Φ_i, the axial flux across the cross section of the inner double helix were also measured. Φ and Φ_i are measured by means of coils wound around the outer and inner conductors respectively. H_A and H_o are measured by means of calibrated pick up coils in the annular and core regions respectively. Formulas for H_o, Φ, and V_4 have been given. The following expressions for Φ_i and H_A can be derived also:

$$\Phi_i \approx (u_o/4) I_1 \Delta D_1 (\alpha_1 + \alpha_1^{-1}) \tag{15}$$

$$H_a \approx [\pi(D_3^2 - D_2^2)]^{-1} [I_1 \; \Delta D_1 (\alpha_1 + \alpha_1^{-1}) + I_4 \Delta D_4 (\alpha_4 + \alpha_4^{-1})] \tag{16}$$

In evaluating these quantities it is reasonable to let $I_1 = I_4 = I/2$, although as was mentioned previously this is not exactly true. For the model of Table I the values calculated numerically from Eqs. (1) to (8) are $I_1 = .94(I/2)$ and $I_4 = .92(I/2)$.

The model is 800 mm long. The Nb_3Sn tapes are 3.2 mm wide and 0.13 mm thick. The average gap between adjacent tape segments is about 0.1 mm. Pitch and diameter data are given in the table. It is difficult to determine the quantities ΔD_1 and ΔD_4 since the tape thickness is comparable in magnitude to these quantities and since the double helix layers are not in uniform contact. Since there are five measured quantities and only two ΔD's we have picked values for ΔD_1 and ΔD_4 which give the best agreement between all the measured and culculated quantities. The values thus chosen for ΔD_1 and ΔD_4 are physically reasonable. The difference between them seems somewhat large and may be due to experimental error to variations in pitch, and to the fact that the calculations assume the tape thickness to be zero whereas it is comparable to the ΔD's.

LOSSES IN CRYOGENIC ENCLOSURES

It is likely that the coaxial cable of each phase of a superconducting transmission line will be enclosed in a pipe to contain the supercritical helium refrigerant. For pitch angles other than 45° the axial flux, Φ, will lead to eddy current losses in the pipe.
The calculation of these losses is complicated by the fact that the induced currents may be large enough to alter significantly the value of I calculated previously. The result of a calculation taking this effect into account is as follows:

$$P/\ell \approx \frac{(\mu_o\omega/4\pi)(2\delta_x^2 D_x/t^*)(\alpha_4 - \alpha_4^{-1})^2 \; (\Delta D_4)^2 \; I_4^2}{(D_x^2 - D_4^2)^2 + (2\delta_x^2 D_x/t^*)^2} \qquad (17)$$

where P/ℓ is the loss per unit length of coax; D_x, δ_x are the diameter and skin depth of the enclosure; $t^* = t$, the enclosure wall thickness when $t \lesssim \delta_x$ and $t^* = \delta_x$ when $t \gtrsim \delta_x$. This expression is greatest when the terms in the denominator are equal. The maximum acceptable value of P/ℓ is of the order 10^{-2} W/m for typical superconducting cable designs. Substituting for numerical evaluation $\Delta D_4 \lesssim 2 \times 10^{-4}$ m, $(\alpha_4 - \alpha_4^{-1})^2 \leq 6$ (i.e., $\varphi \leq 20^o$), $I \leq 10^4$ A, $D_x \geq 0.1$ m, and $P/\ell \leq 10^{-2}$ W/m we find $\delta_x^2 /t^* \geq 0.1$ m. For pure metals, $t^* \approx \delta_x$ $<<$ 10 mm, and it would be necessary to keep close to $\alpha = 1$ ($\varphi = 45^o$). However, for virtually all alloy constructional materials this condition, which is conservative in view of the approximations made, is easy to satisfy. For example, assuming a stainless steel enclosure with $D_x = 0.1$ m, $t = 2$ mm, $\delta_x = 30$ mm ($\rho = 20$ $\mu\Omega$ cm). $I = 10^4$A, $\varphi = 20^o$ and $\Delta D_4 = 0.2$ mm we find $P/\ell \leq 2.4 \times 10^{-3}$.

LOSSES DUE TO EDGE CURRENTS

In contrast to the behavior in an ohmic conductor, current flow in superconducting tapes is confined to a much narrower surface layer (except for currents close to the critical current). The surface current density is perpendicular to the direction of the surface magnetic field and is equal to it in magnitude (in mks units). For conductor 1(Fig. 2)there is little or no field on the inner surface. $H_o \approx 0$. Hence, the current, I_1, flows entirely on the outer tape surfaces, parallel to the tape axes. The magnetic field in the space between 1 and 2 is therefore perpendicular to the tape axes of 1. On the other hand for conductor 2 there is a component of field parallel to the tape axes of magnitude $H_\theta \sin\varphi$ on each surface, and therefore, a circulation of current around the tape segments. Similar remarks may be made for conductors 3 and 4 and it is not necessary to consider them explicitly. If conductor 2 consists of a solid superconductor the curculating currents only cause a slight increase in the hysteretic loss. However, as indicated in Fig. 2, the situation which commonly occurs in practice is the existence of a core or substrate of different material which extends to the tape edge. This results when wide sheets of Nb_3Sn, for example, are slit into narrower tapes. The substrate usually consists of unreacted superconductive Nb or Nb-1% Zr or a resistive alloy such as Hastelloy. If the substrate is resistive the axial field $H_\theta \sin\varphi$ will cause a fraction of the circulating current to cross through the substrate and the rest to flow along the inner superconducting surfaces (i.e., at the superconductor - substrate boundary, Fig. 2). If, on the other hand, the substrate is itself superconducting the circulating current will flow entirely through the edge region of the substrate. In either case significant additional losses can result, as we now show.

Normal metal substrates. This problem can be solved by an appropriate application of standard eddy current formulas.[5] The superconductive surfaces impose the boundary condition $E \approx 0$ at the interfaces; that is, there is little or no current flow parallel to the interface in the normal metal. The current flow in the resistive layer is therefore identical to that in a slice of thickness a taken from an infinite slab of width b with a field $H_a \sin \varphi$ applied along the slab. The losses per unit area of the inner coaxial conductor are:

$$P/A = (2\mu_0\, \rho\omega)^{\frac{1}{2}}\, (a/b) H^2 \quad \text{for } b \gg \delta \tag{18}$$

$$P/A = (\mu_0^2\, \omega^2/12\rho)\; ab^2 H^2 \quad \text{for } b \leq \delta \tag{19}$$

where ρ is the substrate resistivity and H is the rms value of the parallel field ($= H_a \sin \varphi$). For the component of field perpendicular to the tape axes, currents cross the substrates only at the ends of the cable, and $P/A \approx 0$.

For normal metals of high conductivity the above formulas give unacceptably large losses. For metals of poor conductivity the skin depth will be $\gtrsim$ 10 mm and therefore comparable to the tape width. The second of the above expressions applies, therefore. Compared to the usual a^3 dependence for a normal metal slab with either one or no superconducting face[5] the loss now varies as ab^2; that is, it is enhanced by the factor b^2/a^2. For numerical illustration let $a = 12\ \mu m$ (1/2 mil), $b = 6$ mm (1/4") $H_a = 5 \times 10^4$ A/m, $\varphi = 30^\circ$, and $\rho = 2 \times 10^{-8}\ \Omega m$ (approximate value for Nb-1%Zr in the normal state). Then $P/A = 25\ \mu W/cm^2$. This is obviously too large compared with hysteretic losses in Nb_3Sn of 10 $\mu W/cm^2$.[4] It is apparent from Eq. (15) that this number can be brought down to an acceptable value by reducing a, b, φ, and increasing ρ. An alternative solution is to employ tape segments which are fully enclosed by a superconductive layer.

Fig. 2 does not show the normal metal cladding usually present for superconductor stabilization. This can produce significant losses if the metal is too thick. However, these losses are not greatly affected by the helix angles and accordingly this problem is not discussed here.

Superconductive Subtrates. Losses for the case of a superconductive substrate (say Nb or Nb-1%Zr between 6 K and 9K) can easily be calculated provided one again makes the simplifying assumption that no currents flow parallel to the superconductor - substrate interfaces. In this case the current pattern in the substrate is given by the critical state model for a section of thickness a taken from an infinite slab of width b. Provided flux does not penetrate to the center of the tape during a cycle, the loss per unit area of the coaxial cable at 60 Hz is (in mks units)

$$P/A = 2.85 \times 10^{-4} \, (a/b) \, (H^3/J_c) \qquad (20)$$

where J_c is the critical current density of the substrate, and $H = H_\theta \sin \varphi$ is the magnitude of the axial field. Assuming

$$J_c = J_o(1 - T/T_c) \qquad (21)$$

with $J_o = 10^{10}$ A/m^2 and $T_c = 9$ K, parameters which correspond to either Nb or Nb-1%Zr, one can calculate the loss for various temperatures. The maximum loss for the substrate in the superconducting state will occur when magnetic flux penetrates to the center of the tape at peak amplitude. If $H_\theta \sin \varphi = 25$ A/mm, and b = 6 mm, $J_c = 1.2 \times 10^7$ A/m^2. This will occur, according to Eq. (21) at 9.88 K. At this temperature the contribution to the cable loss will be ~ 60 μW/cm^2, i.e., higher than for the substrate in the normal state (see above). Reducing the temperature by 0.2 K (to ≤ 8.8 K), however, already brings this loss to the acceptable level of ≤ 3 μW/cm^2. If the maximum temperature of the cable is not allowed to exceed 8.8 K, edge losses would therefore be acceptable for a Nb substrate provided other effects such as flux jumping, etc., associated with flux entering the edge do not degrade the overall performance of the Nb_3Sn.

The losses due to this effect can be eliminated if the tape segments are fully enclosed, i.e., the edges are Nb_3Sn. Attempts to produce such tapes are being made both commercially and in our laboratory.

CONCLUSIONS

Model calculations and experiments have been carried out for double helix, tape wrapped, superconducting cables. The cables were made by a simple overlay of opposing helices. Over a wide range of pitch angles, $20^\circ \leq \varphi \leq 45^\circ$, the effects of axial flux in producing ac losses in normal metals, which are present both outside and in the core region of the cable, are reduced to insignificant amounts. Voltage drop along the outer conductor is likewise reduced to an acceptably small value.

Finally, the flow of current perpendicular to the axes of the tape segments of a double helix is discussed. This can lead to significant losses in cables made with presently available commercial tapes. There are several possible options for eliminating or reducing this problem: the tapes may be fully enclosed by the Nb_3Sn layer; the substrate may be made highly resistive; finally if the substrate is made of superconductive material other than Nb_3Sn, the cable must be operated at temperatures not too close to the transition temperature of the substrate.

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Fig. 1. Double helix coaxial cable schematic. $I_1+I_2 = I_3+I_4 = I$, the single phase line current.

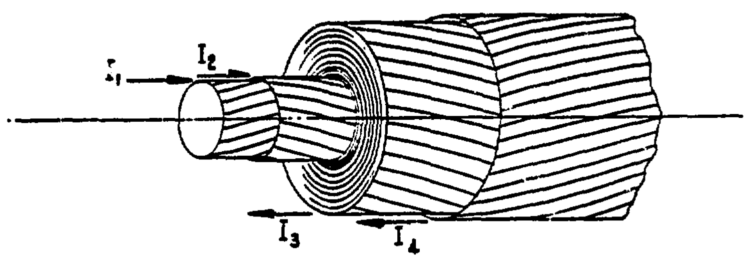

Fig. 2. Magnetic fields on inner conductor tape segments.

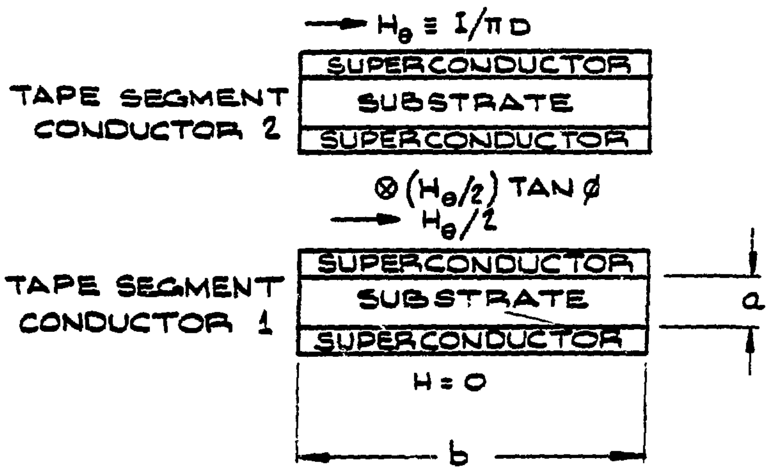

TABLE I

Measured and calculated fields in a double helix model coax. D_1 = 19.2 mm, D_4 = 28.9 mm, P_1 = 112 mm(φ_1 = 28°), P_4 = 242 mm (φ_4 = 21°). Values of ΔD_1 = .17 mm and ΔD_4 = .35 mm were picked to give best fit. I = 1000 A.

	Measured	Calculated
$\Phi(Wb/m^2)$	10.5×10^{-8}	11.5×10^{-8}
$\Phi_f(Wb/m^2)$	5.9×10^{-8}	6.0×10^{-8}
H_o (A/m)	92 (1.2 Oe)	91
H_A (A/m)	590(7.4 Oe)	500
V_4 (V/m)	0.22×10^{-3}	0.18×10^{-3}