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IMPACT OF A STACKING RING ON  $p\bar{p}$  COLLISIONS IN ISABELLE

T.F. Kycia and M. Month

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## ABSTRACT

Some thoughts on  $p\bar{p}$  collisions in ISABELLE are presented. The proposed option using 200 GeV protons is reconsidered. It is contrasted with a new scheme which makes use of a stacking and accelerating ring (SAR). It is demonstrated that the two methods are competitive with the scheme using the SAR providing somewhat greater performance potential.

Although the maximum production rate of  $\bar{p}$ 's using 200 GeV protons is about a factor of 20 better than with 30 GeV protons, the use of a high current proton beam at 50 GeV becomes far more competitive. In fact, we will see that in the latter case, we might expect an increase of about an order of magnitude in  $\bar{p}$  production using for the source a 50 GeV, 30 A beam from the SAR (Stacking and Accelerating Ring).

There are essentially three steps in the production process: 1) preparation of the proton beam for  $\bar{p}$  production, 2) production of a  $\bar{p}$  pulse and 3) stacking of  $\bar{p}$  pulses. In terms of these three steps, we will contrast two cases: the  $\bar{p}$  option using a 200 GeV proton beam without a stacking ring and an option including the SAR, using a 50 GeV proton beam to produce  $\bar{p}$ 's.

#### OPTION I: 200 GeV PROTONS<sup>1</sup>, NO SAR

##### Preparation

For a 10 A stack and a momentum spread  $\Delta p/p = 0.7\%$  at injection, the invariant longitudinal area A is

$$A = \frac{\Delta p}{f h} = 1015.3 \text{ eV} \cdot \text{sec} \quad (1)$$

with the frequency of the ISA,  $f = 101.351 \text{ kHz}$ ,  $h = 2$  and the momentum

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1. C. Baltay, R. Chasman, H. W.J. Foelsche, H. Hahn, M. Month and A. van Steenbergen, Proc. IXth Intern. Conf. on High Energy Accelerators, SLAC, Stanford, California 1974, p. 572.

at injection  $p = 29.4 \text{ GeV/c}$ . At  $200 \text{ GeV/c}$ , the proton bunch is adiabatically damped and its length is approximately given by

$$\ell_p = c \left[ \frac{8A^2 \eta}{\pi^3 p f^2 v_h} \right]^{1/2} \quad (2)$$

where 
$$\eta = \gamma_{tr}^{-2} - \gamma^{-2} \quad (3)$$

Since at  $\gamma = 213.2$  ( $p = 200 \text{ GeV/c}$ ),  $\eta = 2.121 \times 10^{-3}$ , if we take a voltage,  $V = 40 \text{ kV}$ , we find  $\ell_p = 408.33 \text{ m}$ .

The number of protons in each bunch is just half the number in the  $10 \text{ A}$  ISA beam,  $N_{ppp} = 3.08 \times 10^{14}$ , where we have taken for the ISA circumference,  $C = 2960 \text{ m}$ .

### Production

The number of  $\bar{p}$ 's produced per proton pulse can be written:

$$N_{ppp}^- = \left[ \frac{d^2 N}{d\Omega dp} \right] N_{ppp} (\Delta\Omega) \Delta p \eta_{ta} \quad (4)$$

where  $d^2 N/d\Omega dp$  is the  $\bar{p}$  production rate, number of  $\bar{p}$ 's per steradian, per  $\text{GeV/c}$ , per proton,

$$\Delta\Omega = \frac{\pi E_h E_v \gamma_p}{\beta E_p \gamma_p^2} \quad (5)$$

is the production solid angle, with  $E_{h,v}$  the normalized horizontal and vertical  $\bar{p}$  emittance ( $\text{Area}/\pi$ ),  $E_p$  the normalized proton emittance, assumed circular, and  $\beta$  the transverse size focusing  $\beta$  factor for the source,

$\Delta p$  is the momentum acceptance of the  $\bar{p}$  beam, in  $\text{GeV/c}$  and finally,

$\eta_{ta}$  is the effective target efficiency due to its finite length, given by

$$\eta_{ta} = \eta_g \frac{\ell_t}{L_{coll}} e^{-\ell_t/L_{coll}} \quad (6)$$

or if the target and collision lengths,  $\ell_t$  and  $L_{\text{coll}}$  are equal,

$$\eta_{\text{ta}} = \eta_g / e \quad (7)$$

$\eta_g$  is the geometric efficiency factor,

$$\eta_g = \left[ 1 - \frac{2}{\pi} \left\{ \tan^{-1} y - \frac{1}{y} \ln (1+y^2) \right\} \right]^2 \quad (8)$$

with

$$y = \frac{\Delta\Omega \ell_t}{2\pi \epsilon_p^-} \quad (9)$$

$\epsilon_p^-$  is the  $\bar{p}$  emittance, averaged geometrically, that is:

$$\epsilon_p^2 = E_h E_v / \gamma_p^2 \quad (10)$$

We take an emittance  $E_p = E_v = 1.256 \times 10^{-5}$  rad-m. There is no value in taking a larger  $\bar{p}$  vertical emittance, since being unmatched to the proton beam, the excess would not contribute to the luminosity. However, taking a larger horizontal emittance would contribute to the luminosity by increasing the  $p$ - $\bar{p}$  length of collision. We take therefore  $E_h = 5E_v$ . With  $\gamma_p = 213.2$ ,  $\gamma_p^- = 31.4$ ,  $\beta = 0.6$  m, we find  $\Delta\Omega = 7.110 \times 10^{-5}$  radians. The production rate for 200 GeV protons can be taken to be 0.45 per steradian, per GeV/c, per proton. Taking a target length,  $\ell_t = 4.4$  cm, the geometric factor  $y = 0.557$ , leading to  $\eta_g = 0.971$  and  $\eta_{\text{ta}} = 0.357$ . Thus, we have:

$$N_{\text{ppp}}^- = 3.519 \times 10^9 \Delta p. \quad (11)$$

An acceptance in the ISA rings of  $\Delta p/p = 2.5\%$  gives for  $p = 29.4$  GeV/c,  $N_{\text{ppp}}^- = 2.586 \times 10^9$ .

The number of  $\bar{p}$ 's is linearly proportional to the available aperture.

### Stacking

The number of  $\bar{p}$  pulses that can be stacked in one of the ISABELLE rings is determined by the length of the  $\bar{p}$  beam after production. This is simply the length of the 200 GeV proton bunch used for production,

$\ell_p = 408.33$  m. The maximum number of buckets of length greater than  $\ell_p$  that can fit into the ISA circumference of 2960 m is just the integer closest to the ratio  $C_{ISA}/\ell_p$ , which is 7 in this case.

We take a peak momentum acceptance for the rf bucket of  $\Delta p/p = 2.5\%$  at  $p = 29.4$  GeV/c and a harmonic number for the  $\bar{p}$  capture rf system,  $h = 7$ . This gives a bucket area

$$A_{\text{bucket}} = \frac{2}{\pi} \frac{p}{hf} \left( \frac{\Delta p}{p} \right) = 659.54 \text{ eV} \cdot \text{sec}. \quad (12)$$

The longitudinal area of the rectangular  $\bar{p}$  bunch is simply:

$$A_{\bar{p}} = \frac{\Delta p \ell_p}{c} = 1000.41 \text{ eV} \cdot \text{sec}. \quad (13)$$

giving a capture efficiency,

$$\eta_{\text{cap}} = \frac{A_{\text{bucket}}}{A_{\bar{p}}} = 0.659. \quad (14)$$

We have neglected for our purposes here two other limiting aspects - the effect of the finite kicker rise time is to cause some particles to be deflected out of the aperture and the fact that the bunch is to be accelerated means that somewhat more momentum aperture will be required when the acceleration is performed. These are probably small efficiency factors and can be included in a more complete calculation.

The minimum voltage,  $V$  required to accomplish the capture is given by:

$$V = \frac{A_{\text{bucket}}^2 \pi^3 f^2 \eta h^3}{32p} = 57.03 \text{ kV}, \quad (15)$$

where we have taken  $\eta$  (at 29.4 GeV/c) =  $1.129 \times 10^{-3}$ .

If we call  $n_p$  the number of stacked pulses, then the total number of  $\bar{p}$ 's is:

$$N_{\bar{p}} = \eta_{\text{cap}} n_p N_{\text{ppp}} = 1.193 \times 10^{10}. \quad (16)$$

A convenient way of expressing  $\bar{p}$  production is as a comparison to the number of protons in a 10 A beam in ISABELLE. With  $N_p = 6.15 \times 10^{14}$  we have:

$$r_{\bar{p}p} = N_{\bar{p}}/N_p = 1.940 \times 10^{-5}. \quad (17)$$

## OPTION II: 50 GeV PROTONS, WITH SAR

### Preparation

The accumulation of 30 A of current in the SAR, which is a machine conceived to be similar, in fact simpler in structure, than the ISR, seems like a very reasonable assumption to make. Assuming that the stack density is 0.07%  $\Delta p/p$  per 1 A current, as in the ISA, we have a momentum spread of 2.1% for the 30 A stack. It should be kept in mind that the SAR is conceived as a conventional magnet ring, and horizontal aperture is therefore not a limiting factor. Nonetheless, it is desirable not to attempt to bunch such a stack. However, to accelerate 30 A to 50 GeV for the purposes of  $\bar{p}$  production, the method of phase displacement acceleration, as used in the ISR, is ideally suited. By allowing a longitudinal dilution equivalent to the adiabatic momentum damping, we can accelerate the beam in a rather short time and end up with a stack of  $\sim 2\%$   $\Delta p/p$  and  $\sim 30$  A at 50 GeV. This dilution is acceptable since the momentum spread of the source proton beam is not a significant factor in  $\bar{p}$  production.

Taking a circumference of the SAR similar to the AGS,  $C_{\text{SAR}} = 808$  m, we have that the number of protons in the 30 A beam is:

$$N_{\text{PPP}} = \frac{IC_{\text{SAR}}}{ec} = 5.04 \times 10^{14}. \quad (18)$$

### Production

With the same transverse emittances as assumed in the 200 GeV production case, and taking  $\gamma_p = 53.31$ ,  $\gamma_{\bar{p}} = 9.59$ , and  $\beta = 0.1$  m, we have from (5),  $\Delta\Omega = 1.142 \times 10^{-3}$  steradians. The production solid angle here is much larger than in the 200 GeV p case and perhaps some collecting horn could be used to focus this 9 GeV  $\bar{p}$  beam. The production rate for 50 GeV protons is somewhat smaller than with a 200 GeV proton source and can be estimated from Serphukov data<sup>2</sup> to be  $\sim 0.1 \bar{p}$ 's per steradian,

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2. For the production ratio,  $\bar{p}/\pi^-$ , see Serpukhov data in Phys. Lett. 30B, 506 (1975). The  $\pi^-$  spectrum is obtained from the "Wang"  $\pi^-$  formula (A. Thorndike, private communication).

per GeV/c, per proton. Given our choice of  $\beta$  at the target, the geometric factor, from (9),  $y = 2.73$ , leading to a target efficiency  $\eta_{ta} = 0.191$ . Thus, from (4), we have that the number of  $\bar{p}$ 's produced per proton pulse is:

$$N_{\bar{p}pp} = 1.101 \times 10^{10} \Delta p. \quad (19)$$

A momentum aperture of 3% at 9 GeV/c leads to a value for the number of  $\bar{p}$ 's per pulse of  $N_{\bar{p}pp} = 2.974 \times 10^9$ . Since this rectangular pulse of antiprotons must be captured in an rf bucket system, we must include an efficiency factor. The efficiency is simply the ratio of the area of a bucket of maximum  $\Delta p/p = 3\%$  to that of a rectangle of the same momentum width. From (12) and (13), we can infer

$$\eta_{cap} = \frac{A_{bucket}}{A_{rectangle}} = \frac{2}{\pi} \quad (20)$$

Including this capture efficiency, we have:

$$N_{\bar{p}pp} = 1.893 \times 10^9 \bar{p}'s \text{ per proton pulse.}$$

The antiproton bunch would be captured in the AGS and transferred back into the SAR for the final acceleration to 50 GeV from where it would be transferred to one of the ISABELLE rings, ISA I.

In order to be able to stack later on, we need a  $\bar{p}$  capture rf system in the AGS with harmonic number  $h = 1$ . With  $\Delta p/p = 3\%$ ,  $f = 371.29$  kHz,  $p = 9$  GeV/c, the bucket area, from (12) is,  $A_{bucket} = 462.95$  eV.sec and the required minimum voltage, from (15), and with  $\eta(9 \text{ GeV}) = 4.0 \times 10^{-3}$ , is  $V = 12.72$  kV.

### Stacking

The transfer of the single  $\bar{p}$  bunch of area,  $A = 462.95$  eV.sec, would be accomplished by matching into a low harmonic rf system in ISA I at  $p = 50$  GeV/c. In principle, bunching could be performed in the SAR and stacking in ISA I. But it is far more efficient to bunch

in one of the low frequency (i.e. large circumference, low  $\eta$ ) ISA rings. Thus, ISA I is conceived as a  $\bar{p}$  bunching ring, with the other ISABELLE ring, ISA II, being used to azimuthally stack the tight bunches. The bunches can be reduced in length by voltage increases. However, limitations on power or momentum aperture will limit the bunching. A further increase in bunching can be achieved by accelerating the injected 50 GeV bunch in ISA I, then bunching at a higher energy. Transfer of the tight bunch would then be done at the higher energy. At the increased energy, the adiabatic energy damping allows tighter bunching and therefore the stacking in ISA II of more bunches.

The maximum number of bunches can be found given the longitudinal area of each bunch and the allowed momentum aperture. In fact, we must simply compute the maximum harmonic number from:

$$h_{\max} = \frac{2}{\pi} \frac{p}{A f} \left( \frac{\Delta p}{p} \right) \quad (21)$$

where we take  $A$  the bucket area to be the same as the given bunch area. Thus, with a 2.5% momentum aperture, and with  $A = 462.95 \text{ eV} \cdot \text{sec}$ , we have  $h_{\max} = 17.0$  for  $p = 50 \text{ GeV}/c$ , and  $h_{\max} = 67.8$  for  $p = 200 \text{ GeV}/c$ .

For example, let us consider an rf system with  $h = 40$  ( $f_{\text{rf}} = 4.05 \text{ MHz}$ ),  $V_{\text{peak}} = 3.0 \text{ MV}$  and bunching at  $p = 200 \text{ GeV}/c$ . Using the small amplitude approximation, we have for the bunch length, from (2),  $\ell_p = 44.30 \text{ m}$ . The corresponding peak full momentum width is given by:

$$\frac{\Delta p}{p} = \left[ \frac{32 A^2 f^2 v h}{\pi \eta p^3} \right]^{\frac{1}{2}} = 1.99\% \quad (22)$$

The bunching factor,  $B$ , in this case is

$$B = \frac{\text{bunch length}}{\text{bunch separation}} = \frac{\ell_p h}{C_{\text{ISA}}} = 0.599. \quad (23)$$

The attainment of such bunching cannot be accomplished in one step from  $h = 1$  to  $h = 40$ . This is because to avoid dilution, we require  $Vh$  be held constant in the transition to a different harmonic system. This means that to keep the bunch matched before transfer an increase in  $h$  must involve a corresponding decrease in  $V$ . But there is a minimum  $V$ , from (15), required to contain the bunch area. However, with the high voltage of 3 MV (note that  $V_{\min}$  for  $h = 40$  at  $p = 200$  GeV/c is  $V_{\min} = 1.448$  MV), the matching can be accomplished in two steps. For example we can use the following sequence:

Single Bunch in ISA I (3 rf systems) → Azimuthal Stack  
of 40 bunches in ISA II (1 rf system)

- (1)  $h = 4, V = 3 \text{ MV} \rightarrow h = 25, V = 480 \text{ kV}$  ( $V_{\min} = 353.5 \text{ kV}$ )  
 $h = 25, V = 480 \text{ kV} \rightarrow h = 25, V = 3 \text{ MV}$  (Adiabatically)
- (2)  $h = 25, V = 3 \text{ MV} \rightarrow h = 40, V = 1.88 \text{ MV}$  ( $V_{\min} = 1.448 \text{ MV}$ )  
 $h = 40, V = 1.88 \text{ MV} \rightarrow h = 40, V = 3 \text{ MV}$  (Adiabatically)
- (3)  $h = 40, V = 3 \text{ MV}$  (ISA I)  $\rightarrow h = 40, V = 3 \text{ MV}$  (ISA II) (matched synchronous transfer)

The total number of  $\bar{p}$  bunches stacked is  $n_{\bar{p}} = 40$  in this case, and we therefore obtain:

$$N_{\bar{p}} = n_{\bar{p}} N_{\bar{p}pp} = 7.572 \times 10^{10}. \quad (24)$$

The ratio of  $\bar{p}$ 's to the number of protons in a 10 A beam in ISABELLE is thus:

$$r_{\bar{p}p} = N_{\bar{p}}/N_p = 1.231 \times 10^{-4}. \quad (25)$$

With this ratio we can relate the  $\bar{p}p$  luminosity,  $L_{\bar{p}p}$  to the corresponding  $pp$  luminosity,  $L_{pp}$ , by:

$$L_{\bar{p}p} = r_{\bar{p}p} L_{pp}. \quad (26)$$

It should be kept in mind that to reduce the crossing angle with bending magnets can be accomplished with common bending units in the pp case, while septum magnets are required for the  $\bar{p}p$  case. Let us consider the "low- $\beta$ " luminosity for ISABELLE, which does not require extra bending elements in the collision area. We have in this case  $L_{\bar{p}p}$  (low  $\beta$ ) =  $5.3 \times 10^{32} \text{ cm}^{-2} \text{ sec}^{-1}$ . Thus, we obtain for the  $\bar{p}p$  luminosity at 200 GeV,

$$L_{\bar{p}p} = 1.03 \times 10^{28} \text{ (option I, No SAR),}$$

or

$$L_{\bar{p}p} = 6.52 \times 10^{28} \text{ (option II, With SAR).}$$

A reduction of the crossing angle by a factor of 2 with septum magnets would increase the  $\bar{p}p$  luminosity above  $10^{29} \text{ cm}^{-2} \text{ sec}^{-1}$  in option II (i.e. with SAR).

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