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FUEL CHOICES
in the
HOUSEHOLD SECTOR

William Lin
Eric Hirst
Steve Cohn

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FUEL CHOICES IN THE HOUSEHOLD SECTOR

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Abstract

This study analyzes residential fuel choices for five major end-uses: space heating, water heating, cooking, air conditioning, and food freezing. Market share models for electricity, gas, oil and "other/none" are estimated using 1970 cross-section data for 48 states, sensitive to changes in fuel prices, equipment prices, income, demographic, and climatic variables.

Logit models are estimated jointly for each end use by incorporating a set of linear restrictions derived from the requirement that the sum of market shares equal unity.

Our estimated saturation elasticities with respect to own fuel prices are generally greater than unity. Cross-price elasticities are generally less than the own-price elasticities and usually less than unity. Saturation elasticities with respect to equipment prices are generally greater than unity.

A comparison of fuel price and equipment price elasticities suggests that households are more reluctant (require a higher internal rate of return) to invest in energy-efficient household equipment if the investment requires a change in fuel than if the investment requires no fuel switching. Also, higher interest rates are required for investments in energy-efficient appliances (ranges, freezers) than for investments in basic household equipment (space heating systems).

Introduction

The purpose of this study is to analyze residential fuel choices (equipment ownership) for five major end-uses: space heating, water heating, cooking, air conditioning, and food freezing. The market-share models constructed here for electricity, gas, oil and "other/none" are estimated from 1970 cross-section data for 48 states. Market shares are assumed to be sensitive to changes in fuel prices, equipment prices, income, demographic, and weather variables.

The results of this study are being used in a comprehensive economic-engineering computer model to simulate energy use in the residential sector from 1970 to 2000. The present version of the residential energy use model[8] estimates market shares as functions of fuel prices and incomes, but not equipment prices. This insensitivity to equipment price means that assumed changes in equipment efficiency are treated in the simulation model either as if there were no corresponding change in equipment price or as if the change in equipment price was immaterial. Neither assumption is plausible. Because the models developed here are sensitive to both fuel and equipment prices, we are now able to more properly evaluate the energy impacts of engineering design changes that affect both operating costs and capital costs.

Table 1 shows the distribution of household fuel use by fuel and end-use. Clearly, space heating is the most important end-use, accounting for more than half the total. Water heating is the second most important end-use. The five end-uses investigated in this study account for more than 80% of total fuel use. The remainder is used for refrigeration, lighting, and small appliances.

Fuel choice market shares differ from one region to another. Table 2 shows the U.S. average and state high and low market shares for each fuel/end-use combination. For example, for space heating, states in the New England region rely heavily on oil while some states in the Northwest and Southeast rely primarily on electricity.

Past studies[2,3,18] attempted to estimate the proportion of residential customers choosing a particular fuel for a particular end-use using different model specifications.[*] Wilson,[18] in his work focusing on electric appliance choices, specified the "percentage of homes with at least one unit of appliance X" as the dependent variable in the appliance market share equations. He then fit these equations with both log-log and linear forms, sensitive to prices of electricity and gas, income and climatic conditions (heating degree days) for cooking, water heating, space heating, food freezing, and air conditioning. He concluded that:

1. The price of electricity is the primary determinant of electricity consumed by households.

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[*] Reviews of these studies can be found in Ref. 13 and 14.

Table 1. Household Fuel Use by Fuel and End-Use, 1970

	Electricity	Gas	Oil	Other[a]	Total
	(10^{18} Joules)				
Space heating	0.84	3.92	3.39	0.82	8.97 (56)[b]
Water heating	0.88	0.98	0.28	0.07	2.21 (14)
Refrigeration	0.91				0.91 (6)
Freezing	0.31				0.31 (2)
Cooking	0.39	0.32		0.03	0.74 (5)
Air conditioning	0.70				0.70 (4)
Other	1.62	0.45			2.07 (13)
Total	5.65	5.67	3.67	0.92	15.91
	(35)[b]	(36)	(23)	(6)	

[a] Other fuels include coal, coke, and LPG.

[b] Numbers in parentheses are percentages of the grand total, 15.9 × 10^{18} J.

Source: Ref. 8.

Table 2. Regional Variations in Equipment Choice: 1970

	U.S. Average	High	Low
	(percent)[a]		
Space Heating			
Electricity	7.7	40.4 (TN)	1.8 (NY)
Gas	55.2	86.0 (CA)	1.8 (ME)
Oil	26.0	91.8 (ME)	0.2 (OH)
Water Heating			
Electricity	25.4	84.6 (OR)	7.6 (LA)
Gas	55.2	87.3 (CA)	4.8 (ME)
Oil	9.8	49.8 (RI)	0.1 (GA)
Cooking			
Electricity	40.7	89.9 (WA)	16.1 (LA)
Gas	49.2	72.7 (NJ)	5.8 (ID)
Air Conditioning			
Room	25.0	40.3 (TN)	2.3 (ME)
Central	10.7	41.7 (AZ)	0.5 (ME)
Food Freezing			
Electricity	28.2	58.6 (ND)	11.0 (RI)

[a] These figures are percentages of occupied housing units in the U.S. or the state that use the indicated fuel for the specified end-use, determined from the 1970 Census of Housing.

2. Residential demand for electricity is price elastic in the aggregate and for specific end-uses.
3. Cross-elasticity of residential demand for electricity with respect to the price of natural gas is substantial.

While Wilson's model is straightforward and yields unique single-valued price and income saturation elasticities, it nevertheless has two shortcomings:

1. It does not include oil price or equipment cost variables.
2. The specification does not ensure that predicted saturations remain within the range of 0 to 1.

Anderson's model[2] for predicting relative market shares of residential fuel choices involves equations of the following form:

$$\ln\left(\frac{S_i}{S_j}\right) = \alpha_{ij}^{0} + \beta_{ij}^{1} \ln P_i + \beta_{ij}^{2} \ln P_j + \beta_{ij}^{3} \ln Y + \beta_{ij}^{4} \ln HS + \gamma_{ij}^{1} SHU + \gamma_{ij}^{2} NU + \gamma_{ij}^{3} W + U_{ij} \quad .$$

where

S_i = fraction of total installations that consume energy type i,

P_i = price of fuel i,

Y = per capita income,

HS = average household size,

SHU = fraction of households in single-family detached housing units,

NU = fraction of households in nonurban housing units,

W = mean December or July temperature,

U_{ij} = random error term, and

α, β, and γ are unknown parameters.

For a given j, the above equation is estimated for each i (except for i=j). The equations are estimated jointly under the constraint that β^2_{ij} be the same in all equations. Therefore, all cross-price elasticities with respect to a given price change are identical. For instance, the cross-price elasticities of demands for natural gas and for oil with respect to electricity price are assumed to be the same. Prices of competing fuels other than i and j do not enter the equation. We share the view expressed by Hausman[7] that this econometric specification imposes rather strong assumptions on the structure of fuel-share equations.

As noted by Anderson himself, the estimated regression coefficients for this model vary depending upon the choice of j^{th} fuel in the denominator of the dependent variable. Therefore, the estimated price and income elasticities are not unique and occasionally show large differences. For example, in the space heating equation the β coefficients of electricity price range from -1.05 to -3.20 and the β coefficients of income range from -1.03 to +1.65. Saturation elasticities were then computed using β coefficients that lie somewhere in the middle of the ranges.

Baughman and Joskow's appliance choice model[3] is similar to Anderson's although a semi-log rather than a log-log formulation was used. Again, the appliance choice equations were estimated simultaneously under the constraint that all the cross-price β coefficients are identical for any explanatory variable j.

The remainder of this report proceeds as follows. In Section 2, the structure of our fuel choice models and our data sources are discussed. Estimated results, for both 1970 and 1960, are presented in Section 3. In Section 4, fuel and equipment price saturation elasticities, implicit rates of return for residential investments in household equipment, and trade-off relationships between equipment costs and annual operating costs are discussed. Finally, key results of this study, differences and improvements over past studies, limitations of our models, and future efforts to overcome these limitations are summarized in the last section.

Structure of the Models

Specification

A conditional logit model,* similar to the one originally formulated by McFadden,[10] is used to estimate fuel choice decisions facing residential customers:

$$ln \left(\frac{S_i^k}{1-S_i^k} \right) = \alpha_i^k + \sum_{j=1}^{J} \beta_{ij}^k X_j + U_i^k$$

---

*Both semi-log and log-log forms are estimated; the results agree closely. The semi-log results are described in the text because they yield unique implicit interest rates for residential investments (discussed in Section 4). The log-log results are given in Appendices C and D.

where

i = type of fuel ($i=1,2,3,4$: electricity, gas, oil, other/none),

k = end-use function ($k=1,2,3,4,5$: space heating, water heating, cooking, air conditioning, freezing),

S_i^k = fraction of occupied housing units that use fuel i for end-use k,

X_j = a set of explanatory variables including prices of major fuels (electricity, gas, fuel oil), prices of household equipment, per capita income, demographic variables and climatic variables (heating degree days, cooling degree days),

U = random error term, and

α and β are unknown parameters.

There are several merits to this formulation. First, prices of competing fuels enter directly into the market share equations. Second, this formulation does not require that all cross-price coefficients be identical for any explanatory variable j. Third, this formulation yields unique single-valued saturation elasticities.* Finally, this formulation ensures that the estimated market share for any fuel i remains between 0 and 1 for any set of independent variables.

The sum of market shares, S_i^k, over the four fuel choices i must equal 1.0, i.e.

$$\sum_{i=1}^{4} S_i^k = 1.0 \quad .$$

---

*This applies to both the log-log and semi-log models.

Incorporation of this "extraneous" information* into the estimation procedure improves the efficiency of the estimators (smaller standard errors for the regression coefficients). Therefore, the following constraints are applied to both the semi-log and log-log models:

$$\sum_{i=1}^{4} \beta_{ij}^{k} S_i^k (1 - S_i^k) = 0 \quad .$$

The derivation of these linear restrictions is given in Appendix A. Note that the constraints are applied to the market shares for the nation as a whole, not to the market shares for each state. These constraints ensure that changes in the exogenous variables leave the sum of market shares over the four fuel choices equal to 1.0.

The restricted least squares estimates were obtained using Zellner's three-stage least squares procedure.[17] This procedure estimates all market share equations jointly with linear restrictions across equations.

Data

The input data were taken from a cross-section of 48 states for the year 1970. Alaska and Hawaii were not included because of incomplete data. The District of Columbia was combined with Maryland.

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[*] Extraneous information refers to information obtained either from economic theory or logic in regard to the magnitudes or signs of the β coefficients, or the values of linear combinations of the coefficients.

Definitions of variables, sources of data, and their units of measurement are listed in Appendix B. Fractions of occupied housing units in each state that use each fuel for each end-use were computed from data in the *1970 Census of Housing*.[16] To simplify our analysis, we combined "other" fuels (bottled gas, coal, wood) with households that have no equipment for the end-use (none) into an "other/none" category. This "other/none" category generally accounted for less than 10% of the totals for each end-use in each state, except for air conditioning.

Number 2 fuel oil price data for 1970 were reported by the U.S. Department of Agriculture,[15] American Gas Association,[1] Independent Natural Gas Association of America[9] and McGraw-Hill, Inc.[11] Adjustments need to be made before these various data sets can be meaningfully compared. City retail fuel oil prices reported by AGA and INGAA must be appropriately weighted (e.g. by population) to derive state fuel oil prices. Wholesale oil prices reported by McGraw-Hill, Inc. are available only for 23 states and thus require geographical interpolation to obtain estimates for the other states. The McGraw-Hill wholesale prices must be converted into retail prices; we multiplied wholesale prices by 1.65 to account for retail markups.

A comparison of these oil price data sets (after the above adjustments) showed that:

1. USDA and AGA estimates agree well ($R^2 = 0.67$),

2. McGraw-Hill estimates differ considerable from USDA and AGA estimates (R^2 = 0.036 with USDA data and R^2 = 0.003 with AGA data), and
3. AGA and INGAA estimates, using the same set of 48 city observations, show low correlation (R^2 = 0.28).*

The low correlations between McGraw-Hill estimates and USDA and AGA estimates are striking; this is only slightly due to errors in the interpolation process. Confining our comparison only to the 23 states in which there is at least one price reported by McGraw-Hill yields slight improvements in correlation (R^2 = 0.12 with USDA data and R^2 = 0.03 with AGA estimates).

A closer look at the AGA and INGAA estimates reveals that the two data sets agree well. Only three cities show large discrepancies: Boise (Idaho), Omaha (Nebraska), and Orlando (Florida). Excluding these three cities from our comparison yields a high correlation (R^2 = 0.61) for the remaining 45 observations.

Thus we find reasonably good agreement among the USDA, AGA, and INGAA data. Because much of our equipment cost data (for ranges, air conditioners, and freezers) were obtained from the USDA and because the USDA oil price data provide complete 48 state (not cities) oil prices, we use USDA fuel oil prices.[15]

---

*Both AGA and INGAA reported fuel oil prices by city for all 48 states. However, the particular cities sampled within each state differ for the two sources; AGA surveys more cities. Thus, we compare the two sets of data based on the same set of 48 city observations, common to both data sets.

Equipment price data were very difficult to obtain. For space heating, we examined engineering cost estimates from ORNL[5] and Gordian Associates;[6] the former provides costs of electric, gas, and oil furnaces for 13 cities and the latter gives comparable figures for 9 cities. The ORNL engineering estimates were used in this study to approximate state data; we interpolated among the ORNL estimates for the 35 states not included.* We were unable to find regional estimates of residential water heater prices; therefore, equipment prices do not enter our market share equations for water heating. State-level equipment price data for freezers and electric and gas ranges were obtained from USDA. Regional price variations for central air-conditioners are not available; only room air-conditioner price data from USDA were used in our analysis.[15]

All price and income variables are deflated by the cost of living index, developed by Anderson.[2] Other variables used in our analysis and their sources are listed in Appendix B.

Estimated Results

Estimated results obtained with semi-log models are shown in Table 3 and those obtained with log-log models are shown in Appendix C for 1970. Comparable results using 1960 data are shown in Appendix D.

---

*For space heating equipment, Delene used a reference single-family home (1800 ft^2) and sized the equipment according to the climate in each city. We converted Delene's cost estimates from 1974 to 1970 costs and obtained slightly better results (higher *t* statistics) with these estimates than with the estimates from Gordian Associates.

**In this study we chose the following types and sizes from ref. 15 to represent the "average" equipment for each end use: (1) freezers--upright, 16 ft^3; (2) ranges, electric and gas--4 surface burners with oven; (3) room air conditioners--7,000 to 10,000 Btu/hour.

Table 3. Constrained Estimates of the Residential Fuel Choice Models: Semi-log, 1970[a]

End Use	Eq. No.	Dependent Variable	Constant	Fuel Price			Equipment Price				Y	HDD	R^2
				P_e	P_g	P_o	P_{oe}	P_{er}	P_{gr}	P_{ra}			
Space Heating	1.1	$\ln (S_e/1 - S_e)$	−1.677 (−0.974)	−16.000 (−8.505)^b	0.035 (2.198)^c	6.449 (1.794)^c	0.059 (0.774)				0.011 (0.764)	−0.0001 (−4.011)^b	0.83
	1.2	$\ln (S_g/1 - S_g)$	−2.997 (−1.431)^d	5.841 (2.553)^c	−0.250 (−12.950)^b	0.410 (0.094)	0.179 (1.936)^c				0.050 (2.721)^b	−0.0001 (−2.388)^c	0.90
	1.3	$\ln (S_o/1 - S_o)$	1.976 (0.740)	−1.097 (−0.376)	0.337 (13.710)^b	−7.442 (−1.335)^d	−0.363 (−3.076)^b				−0.017 (−0.731)	0.0002 (4.240)^b	0.65
	1.4	$\ln (S_{on}/1 - S_{on})$	−2.655 (−1.312)^d	−0.968 (−0.438)	−0.056 (−2.978)^b	8.787 (2.080)^c	0.215 (2.400)^c				−0.099 (−5.622)^b	−0.0001 (−2.237)^c	0.79
Water Heating	2.1	$\ln (S_e/1 - S_e)$	5.538 (3.779)^b	−20.000 (−8.578)^b	0.105 (4.973)^b	0.484 (0.101)					−0.094 (−4.556)^b		0.78
	2.2	$\ln (S_g/1 - S_g)$	−4.253 (−3.105)^b	12.290 (5.636)^b	−0.223 (−11.300)^b	3.662 (0.818)					0.092 (4.763)^b		0.81
	2.3	$\ln (S_o/1 - S_o)$	−6.729 (−2.273)^c	2.337 (0.497)	0.414 (9.713)^b	−21.610 (−2.236)^c					0.028 (0.659)		0.61
	2.4	$\ln (S_{on}/1 - S_{on})$	−2.134 (−1.649)^d	6.123 (2.972)^b	−0.013 (−0.703)	10.290 (2.434)^c					−0.084 (−4.583)^b		0.65
Cooking	3.1	$\ln (S_e/1 - S_e)$	2.657 (1.476)^d	−9.528 (−4.469)^b	0.068 (2.585)^c			−2.114 (−2.808)^b	2.593 (3.692)^b		−0.047 (−2.515)^c		0.60
	3.2	$\ln (S_g/1 - S_g)$	−3.161 (−1.719)^c	8.132 (3.805)^b	−0.099 (−3.782)^b			1.158 (1.534)^d	−2.070 (−2.939)^b		0.093 (4.450)^b		0.48
	3.3	$\ln (S_{on}/1 - S_{on})$	−4.347 (−3.037)^b	2.861 (1.687)^c	0.093 (4.441)^b			2.391 (3.992)^b	−1.169 (−2.091)^c		−0.072 (−4.917)^b		0.65
												CDD	
Air Conditioning	4.1	$\ln (S_{ra}/1 - S_{ra})$	−2.092 (−1.716)^c							−1.064 (−2.180)^c	0.053 (3.657)^b	0.0009 (7.590)^b	0.48
	4.2	$\ln (S_{ca}/1 - S_{ca})$	−7.462 (−3.519)^b							0.557 (0.655)	0.065 (2.592)^c	0.0011 (5.306)^b	0.57
	4.3	$\ln (S_{on}/1 - S_{on})$	3.481 (2.371)^c							0.638 (1.084)	−0.071 (−4.043)^b	−0.0012 (−8.346)^b	0.68
								P_{ff}	URBI	FSHU			
Food Freezing	5.1	$\ln (S_{ff}/1 - S_{ff})$	−0.752 (−0.727)	−2.170 (−1.698)^c				−0.453 (−1.936)^c	−0.012 (−2.212)^c	2.734 (4.434)^b	0.013 (0.638)		0.61
	5.2	$\ln (S_{on}/1 - S_{on})$	0.752 (0.727)	2.170 (1.698)^c				0.453 (1.936)^c	0.012 (2.212)^c	−2.734 (−4.434)^b	−0.013 (−0.638)		0.61

---

[a] Figures in parentheses are computed t-ratios; R is the correlation coefficient between the actual and estimated dependent variable. See Appendix B for definitions of variables.

[b] Significant at 1% level.

[c] Significant at 10% level.

[d] Significant at 20% level.

Most fuel price coefficients have correct signs and are significant at 10% level in both the semi-log and log-log models. Electricity price is not a significant determinant of space heating choice for oil and other/none. Bottled gas is the most important fuel in the other/none category and its price is highly correlated with natural gas price; therefore it is not surprising to see negative signs for gas price coefficients in equations 1.4 and 2.4.

Due to multicollinearity (correlation coefficient of almost 1.0) among the space heating equipment price variables, we were not able to separate the effects of individual equipment price variables on space heating fuel choice.* Therefore, we used only oil furnace cost data in the fuel choice equations for space heating. The individual "true" equipment price coefficients are then derived from a linear simultaneous equation system, discussed in Appendix E. The correct signs and high statistical significance (except in the electric heating equation) of the oil furnace price coefficients are encouraging, in light of the limited available data. For cooking and freezing, where we have complete equipment price data, all equipment price coefficients are significant. This suggests that, to the extent possible, one should include equipment price variables in fuel choice equations.

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*The estimated costs are broken by Delene (Ref. 5) into a fixed and a variable component. The fixed portion is generally an equipment cost and is assumed to be independent of the location in the country. The variable portion consists of labor and overhead costs which are different in various parts of the country, according to the labor index published by Robert Snow Means Company, Inc. (Ref. 12).

The statistical significance of the heating-degree-day variable in explaining space heating fuel choices is interesting. The results imply that electric space heating is less common in colder climates where total heating requirements are high; instead, oil is more popular than electricity and gas in such areas. We suspect that in colder climates annual operating cost is likely to dominate fuel choice decisions. The high operating costs associated with electric space heating thus are less favorable. Conversely, we believe that in warmer climates, where the cost of home heating is low, capital cost is more important than operating costs. The relatively lower capital cost of electric heaters thus is more favorable.

For all end uses, higher incomes favor natural gas over electricity, oil and other fuels. As expected, other/none is the least preferred option wherever per capita income is high. For space heating, higher incomes favor electricity over oil, although this can only be stated weakly due to the insignificance of income coefficients in equations 1.1 and 1.3. Household customer's preference between electricity and oil for water heating shows a tendency to favor oil over electricity. For air conditioning, higher incomes lead to higher market shares for both room and central air conditioners.

Confirming Anderson's results,[2] electricity price was found to be an insignificant factor affecting the choice for air conditioning. Instead, cooling-degree-days, income and equipment cost were found to be the primary determinants.

Electricity price is a significant determinant of food freezer ownership. Fraction of single-family detached units (FSHU) and proportion of urban population (URBI) are two significant demographic variables affecting ownership of freezers. According to the 1970 housing census, only 22 percent of urban households had food freezers, compared to 48 percent of rural households.

Although our cross-sectional analysis of fuel choices using 1970 data yields good results, this does not ensure that it will provide good predictions of the future. Problems might occur if the parameters characterizing the cross-sectional relation change over time. To determine whether significant structural changes occured between 1960 and 1970, we estimated the log-log models using 1960 data.

The 1960 results (see Appendix D) generally show similar patterns to that of 1970, especially for income and climatic variables. During this decade, household response to fuel price changes, measured in terms of price elasticities, are more consistent for own-price than for cross-price effects. This suggests that consumer preferences have not greatly changed.

Fuel price elasticities for space heating derived from the 1960 and the 1970 results are shown in Table 4.* The only major differences between 1960 and 1970 are: a decrease in cross-price elasticities with respect to prices of oil and electricity and a decrease in the own-price elasticity for oil.

---

*The formula to compute price elasticity is given in Section 4. Similar comparisons can be made for other end uses, based on results given in Appendices C and D.

Table 4. Comparison of Fuel Price Saturation Elasticities for Space Heating: 1960 and 1970

Fuel Choice	Electricity price	Gas price	Oil price
Electricity			
1970	−2.63	0.44	1.37
1960	−2.99	0.60	5.13
Gas			
1970	0.39	−1.57	0.03
1960	0.61	−1.59	0.81
Oil			
1970	0.03	3.51	−1.09
1960	0.37	1.98	−1.80
Other/None			
1970	−0.18	−0.74	1.45
1960	−0.38	0.13	0.55

Sources: Appendices C and D.

Interpretation of Results

Fuel and Equipment Price Elasticities

Based on our semi-log model, the market share elasticity of fuel i for end-use k with respect to the jth explanatory variable is:

$$\frac{\partial S_i}{\partial X_j} \cdot \frac{X_j}{S_i^k} = \beta_{ij}\, \overline{X}_j\, (1-S_i^k)$$

where $\overline{X}_j$ is sample mean value of the explanatory variable X_j; β_{ij} and S_i^k have the same meaning as defined earlier.

Computed fuel price elasticities and comparisons with results from other studies are shown in Table 5. In general, our estimated fuel price saturation elasticities fall within the range provided by previous studies. For example, the own-price elasticity of electricity for space heating ranges

Table 5. Fuel Price Saturation Elasticities

Fuel Choice	This study (log-log)			This study (semi-log)			Wilson			Anderson			Baughman & Joskow		
	P_e	P_g	P_o	P_e	P_g	P_o	P_e	P_g	P_o	P_e	P_g	P_o	P_e	P_g	P_o
Space Heating															
Electricity	−2.63	+0.44	+1.37	−3.19	+0.38	+1.09	−4.88	+1.20		−2.04	+2.21	+0.55	−2.08	+2.12	+3.30
Gas	+0.39	−1.57	+0.03	+0.57	−1.33	+0.03				+0.17	−1.80	+0.55	+0.23	−1.48	+3.30
Oil	+0.03	+3.51	−1.09	−0.18	+2.95	−1.01				+0.17	+2.21	−1.58	+0.23	+2.12	−7.21
Water Heating															
Electricity	−2.63	+1.05	+0.28	−3.22	+0.93	+0.07	−3.22	+2.10		−1.95	+2.20	+0.26	−1.77	+2.98	+1.00
Gas	+0.88	−1.38	+0.31	+1.19	−1.19	+0.30				+0.66	−1.79	+0.26	+0.97	−2.43	+1.00
Oil	+0.79	+5.20	−4.13	+0.46	+4.42	−3.56				+0.66	+2.20	−2.39	+0.97	+2.98	−9.50
Cooking															
Electricity	−1.05	+0.42		−1.22	+0.48		−1.98	+0.91		−0.63	+1.51		−0.78	+1.18	
Gas	+0.72	−0.56		+0.89	−0.60					+0.43	−1.56		+0.77	−1.15	
Food Freezing															
Electricity	−0.30			−0.34			−0.94			−0.57					

Source: Wilson, Ref. 18; Anderson, Ref. 2; Baughman and Joskow, Ref. 3.

from -2.0 by Anderson to -4.9 by Wilson. Our results, -2.6 for the log-log model amd -3.2 for the semi-log model, lie between the two extremes. Results obtained with our log-log model generally agree well with those from our semi-log model. According to the log-log model, 2.6% of the households that use electricity to heat their homes will switch to other fuels as a result of 1% increase in electricity price.

Electricity price saturation elasticities obtained from our study generally are smaller (in absolute value) than those obtained by Wilson, but are larger than those reported by Anderson and Baughman and Joskow. Gas price saturation elasticities, on the other hand, are smaller than those reported by Anderson and Baughman and Joskow. Different specifications of the models may contribute to these differences. Inclusion of equipment price variables in our models may also have an effect.

An important feature of our results is that there is no restriction imposed to equate cross-price saturation elasticities with respect to a given fuel price variable. For example, our semi-log model yields cross-price elasticities of electricity and oil with respect to gas price for space heating of 0.38 and 2.95, while Anderson's study shows 2.21 and Baughman and Joskow's gives 2.12.

Computed equipment price saturation elasticities for space heating, cooking, air conditioning, and freezing are shown in Table 6. Except for two cases, all equipment price

Table 6. Equipment Price Saturation Elasticities

Fuel Choice	Electric equipment price[a]	Gas equipment price	Oil equipment price
Space Heating			
Electricity	−2.30	+1.65	+1.63
Gas	+0.82	−2.24	+2.87
Oil	+1.27	+4.75	−10.66
Cooking			
Electricity	−2.88	+3.04	
Gas	+1.35	−2.08	
Air Conditioning			
Room	−1.76		
Central	+1.10		
Food Freezing			
Electricity	−0.79		

[a] Price of room air-conditioner for air conditioning.

saturation elasticities are elastic, suggesting a more than proportional change in equipment market shares in response to a percentage change in equipment price. For example, our results imply that 2.3% of the households that use electricity to heat their homes will switch to other fuels as a result of 1% increase in the price of electric heating systems, other things being equal.

Implicit Rates of Return

Consider our semi-log model in its general form:

$$ln \left(\frac{S_i^k}{1-S_i^k} \right) = \beta_{i0} + a_{i1}^k P_1 + a_{i2}^k P_2 + a_{i3}^k P_3 + A_{i1}^k C_1^c + A_{i2}^k C_2^c + A_{i3}^k C_3^c + \ldots$$

where,

P_1 = Price of electricity

P_2 = Price of gas

P_3 = Price of oil

C_1^c = Equipment cost of electric system

C_2^c = Equipment cost of gas system

C_3^c = Equipment cost of oil system

a_{ij} = Coefficient of the j^{th} fuel price variable in the i^{th} fuel-share

A_{ij} = Coefficient of the j^{th} equipment cost in the i^{th} fuel-share

To maintain a constant saturation for fuel i:

$$a_{ij}^k P_j + A_{ij}^k C_j^c = K, \text{ where } K \text{ is a constant.}$$

This amounts to:

$$C_j^c = \frac{1}{A_{ij}^k}\left[K - \frac{a_{ij}^k}{Q_j^k} \cdot C_j^o\right]$$

where,

Q_j^k = average household consumption of j^{th} fuel for end use k

C_j^o = annual operating cost for end use k with j^{th} fuel (= $P_j \cdot Q_j^k$)

An incremental change in C_j^c with respect to a change in C_j^o can be expressed as:

$$\frac{\Delta C_j^c}{\Delta C_j^o} = - \frac{a_{ij}^k}{A_{ij}^k} \cdot \frac{1}{Q_j^k}$$

Since annual payment = (principal) $\cdot \frac{r(1+r)^t}{(1+r)^t-1}$ where r = implicit rate of return, and t = equipment lifetime, then:

$$\frac{\Delta C_j^c}{\Delta C_j^o} = - \frac{a_{ij}^k}{A_{ij}^k} \cdot \frac{1}{Q_j^k} = \frac{(1+r)^t-1}{r(1+r)^t}$$

Given the estimated a_{ij}^k, A_{ij}^k (Table 3) and the assumed Q_j^k (Table 7), we compute the additional capital cost that the average household is willing to pay for energy-efficient equipment in exchange for a one dollar reduction in annual operating cost. This, along with the equipment lifetime assumption, allows us to compute implicit rates of return for residential investments. The equipment lifetime assumptions are:[8]

space heating	15 years
water heating	7 years
cooking	13 years
food freezing	14 years

Our results (Table 8) suggest that own-fuel rates of return are generally lower than cross-fuel interest rates. This is intuitively reasonable because it is easier to adopt

Table 7. 1970 Average Household Fuel Consumption

End-Use	Electricity	Gas	Oil
	(10^9 Joules/household)		
Space Heating	172.7	111.9	205.9
Water Heating	54.9	28.1	44.7
Cooking	15.2	10.3	
Food Freezing	18.1		

Source: Ref. 8.

Table 8. Implicit Interest Rates: Semi-log, 1970

End-Use	Fuel Choice	$\Delta C_j^c/\Delta C_j^o$[a]			Implicit Interest Rates (%)		
		Electricity	Gas	Oil	Electricity	Gas	Oil
Space Heating	Electricity	7.94	3.16	6.65	9.0	31.0	12.5
	Gas	3.93	8.10	*b*	24.5	9.0	*b*
	Oil	*b*	9.00	*b*	*b*	7.0	*b*
Cooking	Electricity	4.51	3.09		20.5	31.0	
	Gas	5.60	4.93		15.0	18.0	
Food Freezing	Electricity	4.09			23.5		

[a] Payback period.

[b] Implicit interest rate is not shown here because the fuel price coefficients are not significant.

an energy-efficient equipment without switching fuel. The only exception is gas cooking. However, this is due to the low t-ratio for the electric range price coefficient in equation 3.2 (Table 3), which tends to underestimate the coefficient. Had the coefficient been only slightly larger (>1.32), the implied rate of return for investment in electric ranges to induce households to switch from gas to electricity would have been greater than 18%.

The own-fuel interest rates for cooking and freezing are generally larger than those for space heating. As noted in Appendix E, consumers in 1970 paid 9-12% interest for home improvement loans and 18% for conventional charge accounts. Thus it is not surprising to see that households require lower rates of return for investments in space heating systems.

Note also that this derivation of interest rates assumes constant fuel prices over time. Between 1965 and 1970, residential fuel prices (in constant dollar terms) declined at average annual rates of: 5.4% for electricity, 3.6% for gas, and 1.1% for oil. Hence, "real" rates may be lower than those shown in Table 8.

Indifference Curves

The above analysis can answer the following question: What is the maximum additional capital cost that residential customers are willing to pay to adopt an energy-efficient system in exchange for a reduction of X dollars in annual operating cost? Of course, the maximum additional cost, in the eyes of residential customers, depends on the energy-saving capability of the new system and whether customers have to switch from one fuel to another.

If an investment requires no fuel switching, residential customers generally are willing to pay a higher additional cost to adopt a new technology. For example, Fig. 1 shows that if a new heating system can reduce electricity consumption and electricity bills for space heating by $50 per year (in 1970) and no fuel switching is required to adopt it, households are willing to pay $397 more than existing electric heating systems cost. This suggests that households will accept such a new technology if their investments will be paid back in less than 8 years.

Consumers would not pay such a high price for improved heating equipment if they had to switch from one fuel to another, e.g., from electricity to gas. Consider an improved gas heating system that provides the same fuel cost reduction relative to conventional gas systems as did the improved electric heating system of Fig. 1. Consumers would be willing to pay no more than $158 extra for the improved gas system than for a conventional gas system to switch from electricity to gas (Fig. 2). This is less than half the "price" they would pay if no fuel switching is involved, reflecting difficulties involved in switching from electricity to gas.

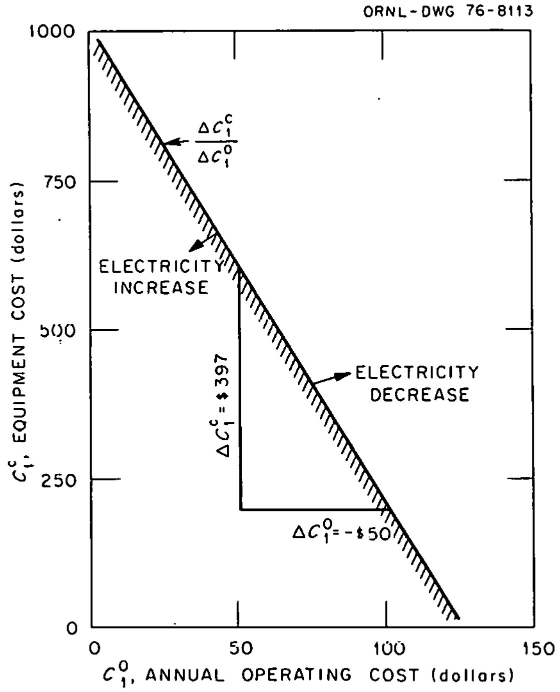

Fig. 1. Indifference Curve for Residential Investment: Electric Heating, No Fuel Switching.

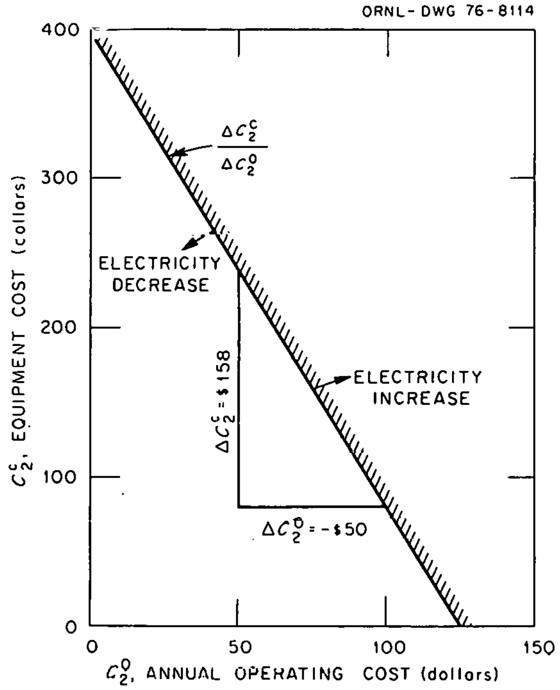

Fig. 2. Indifference Curve for Residential Investment Electric Heating, Fuel Switching from Electricity to Gas.

Summary and Conclusions

This study analyzed market shares for each major fuel (electricity, gas, oil, other/none) based on 1970 cross-sectional data for space heating, water heating, cooking, air conditioning and food freezing in the residential sector. Residential fuel choice models were developed and estimated simultaneously by incorporating additional information in the form of linear constraints.

Most of the regression coefficients in our equations have correct signs and are statistically significant. Equipment cost variables are generally significant. Most own-price elasticities for major fuels in each end-use (except gas price in gas cooking and electricity price in air conditioning) are elastic, implying that residential customers, in the long run, respond significantly to a fuel price change.

Based upon the estimated coefficients for fuel and equipment prices, we computed implicit rates of return for investments in household equipment and the associated discounted payback periods revealed from market behavior in 1970.

For cooking, households have a three to five years payback period, depending on whether the investment requires fuel switching or not. The corresponding rates of return range from 15% to 31%. For freezing, our results show a four year payback period (24% rate of return). For space heating, discounted payback periods range from three to nine years, and the corresponding rates of return range from 7% to 31%. In general, when an investment requires fuel switching, residential customers have shorter payback periods and demand higher rates of return.

The specification of our fuel choice models differs from past studies in several ways:

1. Equipment costs, along with fuel prices, are incorporated in the market share equations wherever possible.
2. Our models are more flexible than others in that they do not constrain the cross-price coefficients of the same explanatory variables to be equal. These constraints, used in previous studies, are unnecessarily restrictive.
3. Fuel price elasticities estimated with our models are unique and single-valued, whereas previous studies provide a wide range of choices.

Limitations of this study, however, should be noted in interpreting our results:

1. The weakest part of our input data are equipment costs, particularly those for space heating. Equipment cost data for space heating, although approximated from engineering estimates for 13 cities, yield good results. This is encouraging considering the limited amount of information available. Since we were not able to obtain equipment cost data for water heating by state, we excluded equipment cost variables in the water heating market share equations.
2. Because we used only cross-section data in our analysis, we have no information concerning the dynamics of changes in equipment ownership; the price and income elasticities derived here should be interpreted as long-run values. Additional work is required, using time series data, to evaluate changes in equipment ownership over time.
3. Several assumptions have been tested to complete a system of linear simultaneous equation to derive equipment cost coefficients for space heating. We

tentatively concluded that an equal cross-price coefficient assumption works best. Future efforts should continue to verify or to modify this conclusion.

4. We estimated both 1970 and 1960 log-log models. But our comparison of the two sets of results relied mostly on general patterns of the β coefficients and elasticity estimates. This may be less than satisfactory in light of the possibility of using more rigorous statistical tests.

Appendix A. Derivation of the linear constraint implied by the fuel choice models

Assume

$$ln\left(\frac{S_i}{1-S_i}\right) = b_{io} + \sum_{j=1}^{n} b_{ij} \cdot x_j \quad , i=1, \ldots n \tag{1}$$

and

$$\sum_{i=1}^{n} S_i = 1 \tag{2}$$

From eq. (1),

$$\frac{\partial S_i}{\partial X_j} = S_i \, (1-S_i) \, b_{ij} \tag{3}$$

and from eq. (2),

$$\sum_{i=1}^{n} \frac{\partial S_i}{\partial X_j} \partial X_j = 0 \tag{4}$$

But eq. (4) implies

$$\sum_{i=1}^{n} \frac{\partial S_i}{\partial X_j} = 0 \tag{5}$$

Substituting eq. (3) into eq. (5) yields

$$\sum_{i=1}^{n} S_i \, (1-S_i) \, b_{ij} = 0$$

The same linear constraint can be derived for the log-log model in a similar fashion.

Appendix B. Definition and unit of measurement of variables used in econometric models

Variable	Definition	Unit of measurement	Data sources
Saturation ratio of fuel i for end use k	S_i^k	fraction	1
Saturation of room air conditioner	S_{ra}	fraction	1
Saturation of central air conditioner	S_{ca}	fraction	1
Saturation of food freezer	S_{ff}	fraction	1
Deflated electricity price (average)	P_e	hundred mills/kWhr	2
Deflated gas price	P_g	hundred mills/MMBtu	3
Deflated oil price	P_o	dollar/gallon	4
Deflated per capita income	Y	hundred dollars	5
Heating degree days	HDD	Days	6
Cooling degree days	CDD	Days	7
Deflated price of oil space heating equipment	P_{oe}	hundred dollars	8
. . . for electric range	P_{er}	hundred dollars	4
. . . for gas range	P_{gr}	hundred dollars	4
. . . for room air conditioner	P_{ra}	hundred dollars	4
. . . for food freezer	P_{ff}	hundred dollars	4
Metropolitan cost of living index	MCLI	percent	9
Proportion of urban population	URBI	percent	5
Fraction of single housing detached units	FSHU	fraction	1

Data sources

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6. U.S. Dept. of Commerce, National Oceanic and Atmospheric Administration, National Climatic Center, *Monthly Heating Degree Days by State and Season, 1931-1973,* November 1973.

7. U.S. Dept. of Commerce, National Oceanic and Atmospheric Administration, *Climatological Data, National Summary,* December 1970.

8. J. G. Delene, *A Regional Comparison of Energy Resource Use and Cost to Consumer of Alternate Residential Heating System,* Oak Ridge National Laboratory, ORNL/TM-4689, November 1974.

9. K. P. Anderson, *Residential Energy Use: An Econometric Analysis,* Rand Corporation, Report R-1297-NSF, October 1973.

Appendix C. Constrained Estimates of the Residential Fuel Choice Models: log-log, 1970[a]

End Use	Eq. No.	Dependent Variable	Constant	Fuel Price			Equipment Price				ln Y	HDD	R^2
				ln P_e	ln F_g	ln P_o	ln P_{oe}	ln P_{er}	ln P_{gr}	ln P_{ra}			
Space Heating	1.1	ln $(S_e/1 - S_e)$	−8.770	−2.845	0.479	1.480	0.897				0.308	−0.0001	0.85
			(−1.715)^c	(−9.275)^b	(2.355)^c	(2.251)^c	(0.699)				(0.597)	(−4.708)^b	
	1.2	ln $(S_g/1 - S_g)$	−4.108	0.866	−3.496	0.066	2.389				2.160	−0.0001	0.93
			(−0.688)	(2.416)^c	(−14.720)^b	(0.086)	(1.593)^d				(3.583)^b	(−3.664)^b	
	1.3	ln $(S_o/1 - S_o)$	1.992	0.041	4.749	−1.474	−5.324				−1.018	0.0003	0.69
			(0.268)	(0.091)	(16.070)^b	(−1.542)^d	(−2.852)^b				(−1.358)^d	(6.148)^b	
	1.4	ln $(S_{on}/1 - S_{on})$	5.171	−0.198	−0.830	1.632	3.722				−3.634	−0.0001	0.82
			(0.819)	(−0.523)	(−3.[illegible]04)^b	(2.009)^c	(2.346)^c				(−5.701)^b	(−2.653)^c	
Water Heating	2.1	ln $(S_e/1 - S_e)$	3.896	−3.525	1.412	0.387					−3.629		0.88
			(1.292)	(−8.699)^b	(5.[illegible]07)^b	(0.428)					(−4.909)^b		
	2.2	ln $(S_g/1 - S_g)$	−1.759	1.958	−3.077	0.691					3.683		0.86
			(−0.655)	(5.421)^b	(−12.490)^b	(0.856)					(5.592)^b		
	2.3	ln $(S_o/1 - S_o)$	−26.560	0.875	5.760	−4.580					0.529		0.76
			(−4.372)^b	(1.072)	(10.340)^b	(−2.510)^c					(0.355)		
	2.4	ln $(S_{on}/1 - S_{on})$	14.220	1.201	−0.153	1.810					−3.073		0.81
			(5.314)^b	(3.339)^b	(−0.529)	(2.253)^c					(−4.686)^b		
Cooking	3.1	ln $(S_e/1 - S_e)$	−0.427	−1.763	0.702			−4.311	4.341		−0.954		0.51
			(−0.149)	(−4.217)^b	(1.778)^c			(−2.363)^c	(2.957)^b		(−1.573)^d		
	3.2	ln $(S_g/1 - S_g)$	−2.481	1.411	−1.104			1.863	−3.312		2.079		0.33
			(−0.753)	(3.230)^b	(−2.675)^c			(0.978)	(−2.161)^c		(2.802)^b		
	3.3	ln $(S_{on}/1 - S_{on})$	−0.843	0.786	1.159			6.229	−2.373		−1.777		0.56
			(−0.385)	(2.416)^c	(3.772)^b			(4.389)^b	(−2.078)^c		(−3.886)^b		
												CDD	
Air Conditioning	4.1	ln $(S_{ra}/1 - S_{ra})$	−7.686							−2.287	1.948	0.0009	0.48
			(−3.427)^b							(−2.110)^c	(3.603)^b	(7.603)^b	
	4.2	ln $(S_{ca}/1 - S_{ca})$	−14.240							1.843	2.501	0.0010	0.58
			(−3.698)^b							(0.990)	(2.693)^c	(5.258)^b	
	4.3	ln $(S_{on}/1 - S_{on})$	10.880							1.100	−2.636	−0.0011	0.67
			(4.029)^b							(0.843)	(−4.048)^b	(−8.288)^b	
								ln P_{ff}	ln URBI	lnFSHU			
Food Freezing	5.1	ln$(S_{ff}/1 - S_{ff})$	2.173	−0.421				−0.992	−0.604	1.716	0.122		0.59
			(1.097)	(−1.773)^c				(−1.696)^c	(−1.779)^c	(4.529)^b	(0.165)		
	5.2	ln $(S_{on}/1 - S_{on})$	−2.173	0.421				0.992	0.604	−1.716	−0.122		0.59
			(−1.097)	(1.773)^c				(1.696)^c	(1.779)^c	(−4.529)^b	(−0.165)		

[a] Figures in parentheses are computed t-ratios; R is the correlation coefficient between the actual and estimated dependent variable. See Appendix B for definitions of variables.

[b] Significant at 1% level.

[c] Significant at 10% level.

[d] Significant at 20% level.

Appendix D. Constrained Estimates of the Residential Fuel Choice Models: log-log, 1960^a

End Use	Eq. No.	Dependent Variable	Constant	Fuel Price			Equipment Price				ln Y	HDD	R^2
				ln P_e	ln P_g	ln P_o	ln P_{oe}	ln P_{er}	ln P_{gr}	ln P_{ra}			
Space Heating	1.1	ln $(S_e/1 - S_e)$	3.837 (0.510)	−3.044 (−7.683)^b	0.616 (2.822)^b	5.224 (5.329)^b	−1.623 (−0.813)				0.439 (0.830)	−0.0003 (−6.527)^b	0.88
	1.2	ln $(S_g/1 - S_g)$	6.146 (0.903)	1.072 (2.993)^b	−2.792 (−14.140)^b	1.416 (1.597)^d	−0.028 (−0.015)				1.779 (3.718)^b	−0.0002 (−5.948)^b	0.87
	1.3	ln $(S_o/1 - S_o)$	−9.364 (−1.335)^d	−0.552 (−1.494)^d	2.936 (14.420)^b	−2.664 (−2.913)^b	−2.240 (−1.204)				0.192 (0.390)	0.0003 (7.780)^b	0.86
	1.4	ln $(S_{on}/1 - S_{on})$	−1.414 (−0.254)	−0.498 (−1.697)^c	0.172 (1.065)	0.813 (1.121)	3.009 (2.038)^c				−2.776 (−7.092)^b	−0.00004 (−1.186)	0.82
Water Heating	2.1	ln $(S_e/1 - S_e)$	−0.823 (−0.325)	−3.115 (−7.030)^b	1.171 (4.365)^b	1.969 (1.765)^c					−1.369 (−2.042)^c		0.75
	2.2	ln $(S_g/1 - S_g)$	1.926 (0.884)	1.699 (4.451)^b	−2.012 (−8.709)^b	1.058 (1.101)					2.220 (3.846)^b		0.74
	2.3	ln $(S_o/1 - S_o)$	−29.20 (−9.018)^b	−0.440 (−0.775)	2.679 (7.805)^b	−7.773 (−5.442)^b					1.454 (1.695)^c		0.79
	2.4	ln $(S_{on}/1 - S_{on})$	10.670 (7.434)^b	0.783 (3.117)^b	0.219 (1.437)^d	1.347 (2.127)^c					−2.968 (−7.809)^b		0.96
Cooking	3.1	ln $(S_e/1 - S_e$	−1.723 (−0.687)	−1.536 (−3.716)^b	0.769 (2.506)^c			−4.485 (−2.179)^c	4.757 (2.687)^b		−0.782 (−1.466)^d		0.54
	3.2	ln $(S_g/1 - S_g)$	−2.555 (−0.924)	0.878 (2.106)^c	−1.283 (−4.151)^b			1.878 (0.906)	−4.087 (−2.292)^c		2.509 (3.841)^b		0.37
	3.3	ln $(S_{on}/1 - S_{on})$	0.287 (0.180)	0.740 (2.822)^b	1.075 (5.523)^b			3.328 (2.550)^c	0.058 (0.052)		−1.994 (−5.881)^b		0.69
												CDD	
Air Conditioning	4.1	ln $(S_{ra}/1 - S_{ra})$	−7.346 (−4.257)^b							−1.474 (−1.237)	1.555 (3.592)^b	0.0011 (8.457)^b	0.68
	4.2	ln $(S_{ca}/1 - S_{ca})$	−9.356 (−4.653)^b							1.610 (1.160)	1.064 (2.109)^c	0.0006 (4.125)^b	0.35
	4.3	ln $(S_{on}/1 - S_{on})$	7.268 (4.429)^b							1.005 (0.887)	−1.517 (−3.683)^b	−0.0010 (−8.502)^b	0.71

[a] Figures in parentheses are computed t-ratios; R is the correlation coefficient between the actual and estimated dependent variable. See Appendix B for definitions of variables.

[b] Significant at 1% level.

[c] Significant at 10% level.

[d] Significant at 20% level.

Appendix E. Linear simultaneous equation system

Because space heating equipment costs are almost perfectly correlated, we cannot estimate individual effects separately. Consequently, we use only oil furnace costs in the market share equations and then derive individual equipment cost coefficients using a linear simultaneous equation system containing 12 equations.

Consider our fuel choice model in its general form:

$$ln \left(\frac{S_i}{1-S_i} \right) = \beta_{i0} + \beta_{i1} X_1 + \beta_{i2} X_2 + \beta_{i3} X_3 + \dots \quad , \tag{1}$$

where,

X_1 = estimated cost of electric furnace,

X_2 = estimated cost of gas furnace,

X_3 = estimated cost of oil furnace, and

β's = the "true" coefficients of X_1, X_2, and X_3.

Let the linear relationships among X_1 , X_2 and X_3 be expressed as:

$$X_1 = k_1 X_3 \tag{2}$$

$$X_2 = k_2 X_3 \quad . \tag{3}$$

where k_1 and k_2 are known constants, obtained from Delene's results.[5]

Substituting equations (2) and (3) into (1), we have

$$ln \left(\frac{S_i}{1-S_i} \right) = \beta_{i0} + \hat{\beta}_i X_3 + \ldots \qquad (4)$$

where,

$$\hat{\beta}_i = k_1 \beta_{i1} + k_2 \beta_{i2} + \beta_{i3} \qquad (5)$$

$i = 1, 2, 3$ and 4.

Because these four equations are linearly dependent, we drop the fourth one from the system.

In addition to the above three restrictions, the three constraints used in the Zellner's three-stage least squares estimation procedure are applied to the system. They are:

$$\sum_{i=1}^{4} S_i (1-S_i) \beta_{ij} = 0 \qquad (6)$$

for $j = 1, 2$ and 3 .

An additional six equations are needed to complete the system. Numerous assumptions were tested to determine which yield the most reasonable results, in terms of "goodness of fit" and reasonableness of the derived implicit rates of return, discussed in Section 4. As in previous works[2,3] we first assumed that the cross-price saturation elasticities for a given explanatory variable were identical. This gave correct signs for each β coefficient, but only partially satisfactory results in terms of computed implied rates of return. In addition, signs of two β coefficients were not consistent with those of fuel price variables in the other/none market share equation.

We then replaced three of the six assumed constraints by imposing identical cross-price implicit rates of return, but the results were unsatisfactory. Finally, we assumed identical cross-price β coefficients and allowed the own-price β coefficient to vary parametrically from a 9% implied rate of return up to 20%. That is,

$$\beta_{12} - \beta_{21} = 0$$
$$\beta_{13} - \beta_{31} = 0$$
$$\beta_{23} - \beta_{32} = 0$$
$$\beta_{11} = \delta_1$$
$$\beta_{22} = \delta_2$$
$$\beta_{33} = \delta_3$$

where δ_1, δ_2, and δ_3 are the parameters for the own-price β coefficients. The result is more than 1,000 solutions to the system of 12 equations.

A screening process was then applied to select those solutions which gave correct signs for the β coefficients. Seventy five solutions which met this requirement, were further tested. The final solution* for space heating equipment price

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*Alternative plausible solutions differ only slightly from this one; other solutions yield unreasonable rates of return.

coefficients, which gave high R^2, correct signs, and reasonable implicit rates of return,* is:

$\beta_{11} = -0.141$ $\beta_{12} = 0.104$ $\beta_{13} = 0.097$

$\beta_{21} = 0.104$ $\beta_{22} = -0.290$ $\beta_{23} = 0.352$

$\beta_{31} = 0.097$ $\beta_{32} = 0.352$ $\beta_{33} = -0.790$

$\beta_{41} = -0.348$ $\beta_{42} = -0.034$ $\beta_{43} = 0.584$

Our estimated implied rates of return for residential investment based upon the above β coefficients are reasonable (Table 8). Admittedly, our assumption to equate cross-price β coefficients may be too restrictive. The lower limit of 9% is plausible since mortgage rates in 1970 averaged 9.05%.[4] Our correspondence with a local credit union, savings and loan association, and Sears Roebuck indicate that in 1970 residential customers generally paid 9% interest rate for home improvement loans with a first mortgage; 12% with a second mortgage; and 18% for easy-payment credit account. Therefore, we believe the range chosen for our analysis is reasonable.

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*Implicit rate of return is defined as the minimum acceptable internal rate of return to residential customers from investing in new energy efficient equipment. The benefit component in computing the rate of return is the annual operating cost reduction and the cost component is the additional capital cost for the new system.

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