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# Direct Doubler-Main Ring $\bar{p}\,p$ Collisions*

Consider the following situation:

A $\bar{p}$ beam is produced from source protons accelerated in the main ring and focused on a target. The $\bar{p}$ beam is captured in the doubler, its bunch structure intact. The $\bar{p}$ beam is accelerated in the doubler. A proton beam is accelerated in the main ring. What luminosity results from ensuing $\bar{p}p$ collisions?

The $\bar{p}$ efficiency yield, i.e. the number of antiprotons per proton per GeV/c momentum bite is

$$\eta_{\bar{p}} = N_{\bar{p}}/N_p(\Delta p) = \frac{d^2N}{d\Omega dp}\,\pi\,\eta_{ta}\left(\frac{\varepsilon_v\varepsilon_h}{r^2}\right),$$



where $\eta_{ta}$ is the target efficiency,

$(d^2N/d\Omega dp)$ is the $\bar{p}$ production cross-section, $\bar{p}$'s per steradian per GeV/c,

$\varepsilon_{v,h}$ are the vertical and horizontal betatron emittances,

and $r$ is the 1/2-spot size of the proton beam incident on the target.

The proton bunch structure is maintained in the production process. The target efficiency is given by

$$\eta_{ta} = \eta_g \frac{\ell_t}{L_{coll}}\, e^{-(\ell_t/L_{coll})},$$



where $\ell_t$ is the target length,

$L_{coll}$ is the nuclear collision length of the target material,

and $\eta_g$ is the geometric efficiency of the target,



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## DISCLAIMER

$$\eta_g = \left\{ 1 - \frac{2}{\pi}\left[ \tan^{-1} y - \frac{1}{y}\ln(1+y^2) \right] \right\}^2 ,$$

with

$$y = \frac{(\varepsilon_v\ \varepsilon_h)^{1/2}\ell_t}{r^2} .$$

Consider the production of antiprotons from a 200 GeV proton source. Collecting the $\overline{p}$ beam in the energy region of 30 GeV implies a production cross section, $(d^2N/d\Omega dp) \simeq 0.5$ $\overline{p}$'s per steradian per GeV/c.

The normalized proton emittance for $N_p = 2\mathrm{x}10^{13}$ protons is

$$E = \beta\gamma\varepsilon = 2\mathrm{x}10^{-5} \text{ rad-m},$$

with $\beta$, $\gamma$ the usual relativistic parameters.

Thus, assuming a vertical crossing mode between doubler and main ring, we should accept a horizontal emittance for the antiprotons of this value. However, for an optimized luminosity in a vertical crossing mode, we can accept a considerably larger vertical emittance. In fact, we can estimate just how much vertical emittance from

$$2a_v \lesssim \frac{\ell_B \alpha}{2} ,$$

where $a_v$ is the beam vertical 1/2-size (95%),

$\ell_B$ is the total bunch length (95%),

and $\alpha$ is the full vertical crossing angle.

Note that because the relative beam velocity is $(2\beta c)$, the effective bunch length of one beam seen by the other is only $(\ell_B/2)$. In terms of the normalized vertical emittance, $E_v$, this constraint can be written

$$E_v \lesssim \frac{\ell_B{}^2 \alpha^2 \gamma_{\bar{p}}}{16\, \beta_v} \quad ,$$

where $\gamma_{\bar{p}}$ is the $\bar{p}$ energy at collision, in proton mass units, and $\beta_v$ is the vertical focusing function at the collision point. Taking $\ell_B = 41.2$ cm (assuming an invariant bunch area, $A = 0.2$ eV-sec, a peak voltage, $V = 4$MV, a transition energy, $\gamma_{tr} = 17$, and a momentum, $p = 1000$ GeV/c), $\alpha = 27$ mrad, $\gamma_{\bar{p}} = 1066$, and $\beta_V = 25$m, we have for the maximum $E_v$,

$$E_v \lesssim 3.3\text{x}10^{-4} \text{ rad-m} \quad .$$

Even though the 30 GeV beam size corresponding to this emittance and a $\beta$-value of $\beta_v = 100$m, the lattice maximum, is

$$\text{total width} = 2\ (E_v\beta_v/\gamma)^{1/2}$$

$$= 2\ (3.3\text{x}10^{-4}\text{x}100/32)^{1/2} = 6.4 \text{ cm},$$

we adopt this emittance for the vertical acceptance of the 30 GeV $\bar{p}$ beam. This aperture constraint could be alleviated by either choosing a stronger focusing doubler lattice or by increasing the $\bar{p}$ production energy.

To optimize the target efficiency, we take a spot size about 1/2 mm (r = 0.25 mm). For a target length roughly the collision length of an iridium target, $\ell_t = 5$ cm, we have that the geometric parameter, $y = 2.03$, and the geometric efficiency is $\eta_g = 0.65$. The target efficiency is therefore

$$\eta_{ta} = \eta_g/e = 0.24.$$

Thus, the $\overline{p}$ efficiency yield is

$$\eta_{\overline{p}} = \frac{(0.5)\ \pi\ (0.24)\ (2\times10^{-5})\ (3.3\times10^{-4})}{(32)^2\ (0.25\times10^{-3})^2} = 3.9\times10^{-5}\ .$$

The momentum spread that can be accepted is related to the bunch structure of the proton source beam and the desired invariant longitudinal area, $A_{\overline{p}}$ , of the antiproton bunch:

$$\Delta p = \frac{4c\ A_{\overline{p}}}{\pi \ell_p}\ ,$$

where $\ell_p$ is the proton bunch length, which is just the bunch length for the captured antiproton beam at the production energy. The proton bunch length is given by

$$\ell_p = c\left[\frac{8\ A_p^2\ |\eta_p|}{\pi^3\ p\ f_p^2\ Vh}\right]^{1/4}\ .$$

These correspond to main ring parameters: V = 4 MV, h = 1113, $\eta_p \simeq 1/\gamma_{tr}^2 \simeq 3.5\times10^{-3}$, $f_p$ = 47.75 kHz, p = 200 GeV/c, and $A_p$ = 0.1 eV-sec. Therefore, $\ell_p$ = 43.4 cm. If we assume that twice the invariant area can be accommodated for the $\overline{p}$ beam, $A_{\overline{p}}$ = 0.2 eV-sec, and we have $\Delta p$ = 0.176 GeV/c. Thus, the number of $\overline{p}$'s produced per proton, $\overline{\eta}$, is given by

$$\overline{\eta} = (N_{\overline{p}}/N_p) = (3.9\times10^{-5})\ (0.176) = 6.9\times10^{-6}\ .$$

If we match horizontal size and bunch length in the two colliding bunches, we can write for the luminosity, assuming Gaussian bunches in all dimensions,

$$L = \frac{4n \; f_{rev} \; N_{1B} \; N_{2B}}{\pi \; \alpha \; a_h \; \ell_B} \quad ,$$

where $\ell_B$ is the total bunch length (95%),

$a_h$ is horizontal 1/2-size (95%),

$n$ is the number of bunches,

$f_{rev}$ is the revolution frequency,

$N_{1B}$, $N_{2B}$ are the number of particles per bunch in beams 1 and 2 respectively,

and $\alpha$ is the total crossing angle.

The parameter values are taken to be $f_{rev}$ = 47.75 kHz, n = 1113, $\alpha$ = 27 mrad, $\ell_B$ = 41.2 cm, and

$$a_h = (E_h \, \beta_h/\gamma)^{1/2} = (2\mathrm{x}10^{-5}\mathrm{x}25/1066)^{1/2} = 0.68 \text{ mm},$$

where we have assumed $\beta_h$ = 25m for the 1000 GeV beam.

The number of particles in the proton beam is $2\mathrm{x}10^{13}$, corresponding to an emittance E = $2\mathrm{x}10^{-5}$ rad-m. The number per bunch is therefore $N_p = (2\mathrm{x}10^{13})/1113 = 1.8\mathrm{x}10^{10}$. Although not necessary, if we presume that the same character proton beam is used for $\bar{p}$ production as is used for collisions, then the number of antiprotons per bunch is

$$N_{\bar{p}} = N_p \, \bar{\eta} \quad ,$$

and the $\bar{p}p$ luminosity is proportional to the pp luminosity, and can be written

$$L_{\bar{p}p} = \frac{4n \; f_{rev} \; N_p^2 \; \bar{\eta}}{\pi \; \alpha \; a_h \; \ell_B} = \bar{\eta} \; L_{pp} \quad .$$

With the above parameters, there results for the pp luminosity,

$$L_{pp} = 2.9 \times 10^{29}\ \mathrm{cm}^{-2}\ \mathrm{sec}^{-1},$$

and for the $\bar{p}p$ luminosity,

$$L_{\bar{p}p} = 2.0 \times 10^{24}\ \mathrm{cm}^{-2}\ \mathrm{sec}^{-1}.$$

The following comments reflect some peculiarities and potential improvements in the given scheme.

1. The energies of both the source proton beam and the produced antiproton beam could be increased. This would increase the $\bar{p}$ production cross section as well as alleviate the aperture problem when the $\bar{p}$ beam is captured in the doubler.

2. More horizontal emittance of the $\bar{p}$ beam could be accepted. The $\bar{p}$ yield would go up linearly while the effective beam size for luminosity calculations increases much more slowly, roughly by the square root. The luminosity would therefore increase.

3. Although a smaller crossing angle increases the pp luminosity, the $\bar{p}p$ luminosity would actually decrease, since the $\bar{p}$ yield is essentially proportional to the square of the crossing angle.

4. Those colliding beam modes employing common dipole magnets to bring two proton beams into collision cannot be used in the case of protons colliding against antiprotons. Such common dipoles would instead cause the p and $\bar{p}$ beams to diverge. In the $\bar{p}p$ case, a septum magnet arrangement, with all its accompanying risks, would have to be attempted. On the other hand, the large crossing angle mode, with no common magnets, accommodates pp and $\bar{p}p$ collisions equally well.