COO-2682-4

REINFORCED CONCRETE CONSTITUTIVE RELATIONS

Progress Report
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DISCLAIMER

REINFORCED CONCRETE CONSTITUTIVE RELATIONS

ABSTRACT

A constitutive relation and failure criterion for concrete material under general three-dimensional stress states has been developed using the work-hardening theory of plasticity. The formulation has all the required properties of concrete and gives a close estimate to experimental stresses for complete general stress states.

In order that the results of research be readily usable in the analysis of suboceanic structures such as the large shells proposed for adoption in the Ocean Thermal Energy Conversion program (OTEC), corresponding computer codes have also been developed to reflect this material response.

The proposed material model has been applied to several selected concrete and reinforced concrete shell structures. The finite-element subroutine for the NONSAP program has been modified and applied to analyze plain and reinforced concrete shell specimens of cone-, cylinder- and dome-shapes under monotonically increasing axial load condition. The finite-element solid program (EPFFEP) has also been developed and applied to study the behavior of concrete cylindrical hulls under hydrostatic loading conditions. The analytical results will be compared with corresponding experimental data. Preliminary results indicate a close agreement to the experimental failure load.

REINFORCED CONCRETE CONSTITUTIVE RELATIONS

1. Introduction

1.1 Problem

Large submersible shells and other components of reinforced concrete, whose dimensions will be many times larger than such elements studied previously, have been envisioned as part of a total concept to utilize the solar energy available in the oceans. The analysis and design of these structural components pose a challenge due to several factors, some of which may be entirely novel due to the location of the plant. The uncertainties, at this stage, of the manner of supporting and loading the components, and of making provisions for connections with other units add to the complexities of the problem. The procedure for analysis must, of necessity, possess the versatility to account for any combination of these and other normal features. A fundamental concern, however, is the nature of response to loading by the materials used in construction under suboceanic conditions.

1.2 Scope

In this work, the emphasis is on developing constitutive relations for reinforced concrete, considering the possible effects of hydrostatic pressure. Since it is conceivable that several structural components may be built on shore, the constitutive relations neglecting such pressure are also desired. The capability to handle loadings in a generalized manner is, therefore, an inherent aim in the development of these relations.

In order that the results of research be readily usable in the analysis, a corresponding computer code will also be developed to reflect material response. This code, when in the form of a subroutine, will be adaptable for use in an existing or postulated larger finite element analysis computer program.

1.3 Objectives

The overall objective of the program is to devise the means necessary to achieve a satisfactory analysis of the undersea structures.

More specifically, the work is to include the following:

1. setting up constitutive relations for the individual materials, concrete and steel,
2. combining these relations for reinforced concrete and expressing the constitutive equations explicitly in matrix form suitable for use in finite element analysis,
3. writing a corresponding computer code in the form of a subroutine, which can be adaptable for use in an existing or postulated large finite element analysis computer program,
4. studying the elastic-plastic-fracture behavior of some typical concrete and reinforced concrete shell structural problems using the computer model
5. verifying the contemplated constitutive relationships by comparing the analytical solution with experimental results.

2. Progress as of March 20, 1976

The progress of the research is summarized in the following four phases corresponding to the five specific objectives stated in Sec. 1.3.

2.1 Development of Constitutive Relations for Concrete

A constitutive relation and failure criterion for concrete material under general three-dimensional stress states has been formulated using the work-hardening theory of plasticity. The formulation considers the weak tensile strength, strain-hardening, and fracture behavior of concrete. The resulting stress-strain relationships are compared with existing biaxial loading experimental results and good agreement is generally observed. This theoretical development was presented in ASCE Engineering Mechanics Journal, Vol. 101, No. EM4, August 1975 (Constitutive Relations for Concrete).

In a subsequent progress, the stress-strain relationships are expressed explicitly in matrix form for the general case of three-dimensional stress states and then reduced to the special cases of plane stress, plane strain and axisymmetric stress conditions, suitable for use in finite element analysis. The matrix formulation was presented in ASCE Engineering Mechanics Journal, Vol. 101, No. EM6, December 1975 (Constitutive Equations and Punch-Indentation of Concrete).

In summary, the proposed stress-strain relations and the failure criterion for concrete are found to give a close estimate to relevant experimental data, and also reflect all the required characteristics of concrete concerning nonlinearity, irreversibility, convexity, deformability and normality, etc. The formulation has the further advantage that it contains several of the earlier proposed stress-strain relations as special cases. In particular, the present formulation contains the von Mises yield criterion widely used in metal plasticity as a special case. This makes the formulation most desirable for computer applications to reinforced concrete structures since the same constitutive relationships are used for both materials: steel and concrete.

The main objection is that the proposed criterion is not very simple, but this could hardly be expected when it has to embody all the characteristics of concrete mentioned above. However, the present stress-strain relations and the failure criterion are in feasible mathematical forms convenient for direct applications with respect to computer solutions.

2.2 Development of Finite Element Subroutine

The OTEC structure in the construction and operation configuration consists of a series of ring-stiffened cylindrical or cone-shaped reinforced concrete shells covered with spherical caps. The analysis of such a configuration requires the availability of a modern computer and a computer program that has been designed to perform such analysis in an efficient manner. Computer programs written for shell analysis tend to be based on finite difference or finite element analysis techniques. Many such computer programs are now available for our research.

We decided to use the NONSAP program which is a general purpose nonlinear finite element analysis program that was originally developed by the University of California at Berkeley in 1974. Considerable engineering skill is required in order to add our concrete constitutive model to the program. For the past several months, we have familiarized ourselves with the fundamentals of this program. Several typical shell problems of von Mises material were performed using the NONSAP code and some of the results, along with the general description of the NONSAP program are presented in Appendix "A".

The inclusion of our newly developed elastic-plastic-strain hardening-fracture material model to the NONSAP program has been achieved for the case of uncracked concrete. Several cone-shaped plain concrete shells were analyzed by using the modified NONSAP code and some typical results are presented in Appendix B. The inclusion of kinematics of concrete fracture and crushing, along with reinforced steel bars is now underway. The results corresponding to this development will be summarized in the form of "the modified NONSAP User's Guide for Reinforced Concrete". This report will serve as an introduction to the modified NONSAP program and its capabilities. Other reports containing a collection of solutions representing several real-world applications of the modified NONSAP program will be issued later in the continuation phase of this research.

Due to the complexity of the OTEC structure, some local grid point refinement is necessary to account for local details that will influence the solution. A computer program, Elastic-Plastic-Fracture-Finite-Element-Program(EPFFEP) has been developed for the numerical solutions. The program, EPFFEP, using the constant strain triangle element, has been used successfully to solve some plane stress, plane strain and axisymmetric problems subjected to force, displacement or mixed boundary conditions. Some applications of the program on punch indentation problems are reported in the forthcoming paper "Nonlinear Analysis of Concrete Splitting Tests", Journal of Computers and Structures, to be published in June 1976.

The EPFFEP User's Guide is now under preparation. This report will serve as an introduction to EPFFEP and its capabilities. Other reports concerning a collection of solutions representing some real-world applications of EPFFEP will be issued in the continuation phase of this research.

2.3 Numerical Applications

Probably the most important, the most unique feature and yet the most potentially critical aspect of the application of the finite element method to concrete and reinforced concrete structures lies in the fact that concrete will crack under certain tensile stress or strain conditions. It has been increasingly recognized in recent times that the failure for concrete under polyaxial stresses may be a function of the dual criteria of fracture and flow. Under triaxial compression, concrete can flow like a ductile material but it fractures in a brittle manner under certain tension-compression or isotropic tension states. Preliminary results indicate that such a dual representation of concrete behavior is necessary in certain cases in order to arrive at a good correlation between theory and experiments.

Before we refine our failure criterion using such a dual representation, extensive numerical solutions on selected reinforced concrete structures are needed. These numerical results will furnish the analytical basis for the load-deformation response, internal stresses, and crack propagation of the reinforced concrete specimens throughout the entire range of elastic and inelastic loading up to ultimate load. Comparison between analytical and experimental results will help to evaluate the theory.

Two types of numerical solution are currently underway, using the present computer model. The modified NONSAP program is applied to analyze plain and reinforced concrete shell specimens of cone-, cylinder- and dome-shapes described in Sec. 2.4. The validity of the present theory in the tension-compression region can be studied by comparing the analytical solution with experimental results.

The EPFFEP program is applied to study the behavior of concrete cylindrical hulls under hydrostatic loading. Experimental studies conducted at the Naval Civil Engineering Laboratory (NCEL) have shown that concrete is an effective construction material for undersea pressure-resistant structures. These previous studies have established experimentally the relationship between implosion pressure, concrete strength, wall thickness, strain distribution and strain magnitude in cylindrical concrete hulls subjected to external hydrostatic loading. The analytical solution of this problem can be used to demonstrate the validity of the present theory in the triaxial compression region of the material model.

2.4 Experiments

Shell specimens of cone-, cylinder- and dome-shapes have been made and tested under monotonically increasing axial load conditions. Load-deformation response, internal stresses and crack propagation were traced experimentally through the elastic, inelastic and ultimate ranges. Data from this experimental study are currently being reduced and analyzed. These data can be used to demonstrate the validity of the present computer model and method of solution. A brief description of the dimensions and testing procedure for these concrete shells is presented in Appendix C.

It has been possible to prepare polymer-impregnated concrete (PIC) whose mechanical behavior and fracture characteristics can be tailored to range from strong linear elastic, but brittle; to tough and ductile, of elastic-perfectly plastic type. Some of the shell specimens have been impregnated with such monomers, which are then polymerized within the concrete pore system to give a composite comprising two interpenetrating networks--polymer and cement. Such composite specimens are ideal for the purpose of comparison with various material model now available in the NONSAP program, including, of course, our general material model of elastic-plastic-strain hardening-fracture type.

Data from this experimental study on modified concrete can be used to demonstrate the general validity of the theory which ranges from brittle and abrupt type of tensile failure of concrete to ductile and flow type of behavior of reinforcing steel.

3. Progress Schedule

The estimated overall percent completion of the project as of March 22 is 76%. The funds expended to date is estimated to be 70%. A detailed progress schedule is given below.

Research Phases	Name of Investigator	1975 M	J	J	A	S	O	N	D	1976 J	F	M	A	M	J	J	A	Estimated % Complete
Detailed Work Plan	Chen			50	100													100
Literature Survey	Chen	5	10	20	40	60	80	90	100									95
Kinematics of Concrete Fracture	Chen	10	20	30	40	50	60	70	80	90	100							90
Steel Concrete Interaction	Chen	5	10	30	50	60	70	80	90	100								60
Numerical Solutions of Shells Using Kalnin's Program	Updike	5	10	15	20	30	50	70	80	90	100							50
Development of Finite-Element Subroutine for NONSAP Program	Chang	2	5	10	15	20	30	50	60	70	80	90	100					80
Development of Finite-Element Solid Program (EPFFEP)	Suzuki	2	5	10	15	20	30	50	60	70	80	90	100					85
Experiments	Mehta	1	5	10	20	30	40	50	60	70	80	100						90
Data Analysis	Chen and Staff							10	20	30	40	60	80	100				70
Final Report	Chen and Staff												10	30	50	100		40
Overall % Completion		3	6	18	30	39	46	62	65	70	78	84	89	93	95	100		76

Estimated Schedule ___ Estimated % Complete Funds-Expended Estimate 70%

4. Personnel

(1) Project Director: Dr. W. F. Chen

He has devoted approximately 25 percent time to this project since the beginning of the agreement and is expected to devote the same amount of time during the remainder of the current term.

(2) Faculty Associates: Dr. D. P. Updike and Dr. L. Tall

They have devoted approximately 10 percent time each on this project during the initial phase of the program and will not devote any time during the remainder of the current term.

(3) Research Associate: Dr. H. Suzuki

He has been and will be 100 percent time involvement to the project.

(4) Research Assistant: Mr. H. C. Mehta

He has been 50 percent time involvement during the academic year and will be 100% during the summer.

5. Reports, Publications and Seminars

1. Chen, A. C. T. and Chen, W. F., "On Structural Concrete and Work-Hardening Theories of Plasticity," Proceedings, Eleventh Annual Meeting, Society of Engineering Science, Inc., (George J. Dvorak, Editor), Duke University, Durham, North Carolina, pp. 304-305, November 11-13, 1974.

2. Chen, A. C. T. and Chen, W. F., "Constitutive Relations for Concrete," Journal of the Engineering Mechanics Division, ASCE, Vol. 101, No. EM4, August 1975, pp. 465-481 (COO-2682-1).

3. Chen, A. C. T. and Chen, W. F., "Constitutive Equations and Punch-Indentation of Concrete," Journal of the Engineering Mechanics Division, ASCE, Vol. 101, No. EM6, December 1975, pp. 889-906 (COO-2682-2).

4. Chen, A. C. T. and Chen, W. F., "Nonlinear Analysis of Concrete Splitting Tests," Computer and Structures, June 1976, to appear (COO-2682-3).

5. Chen, W. F., "Reinforced Concrete Constitutive Relations," Proceedings, Third Ocean Thermal Energy Conversion Workshop, Houston, Texas, May 8-10, 1975, pp. 99-102 (COO-2682-5).

6. Chen, W. F., "Constitutive Relations for Concrete," Seminar to be delivered at Penn State University on May 4, 1976, University Park, Pennsylvania.

7. Chen, W. F. and Chang, T-Y., "Developments in Concrete Constitutive Relations," submitted for possible presentation at the 14th International Congress of Theoretical and Applied Mechanics, Delft, The Netherlands, August 30-September 4, 1976.

8. Chen, W. F. and Chang, T-Y., "Analysis of Reinforced Concrete Conical Shells Under Axial Load," submitted for possible presentation at the 18th Solid Mechanics Conference, Wisla, Poland, September 7-14, 1976.

9. Anderson, J. H., "Sea Solar Power: A New Energy Source on the Horizon," Public lecture and demonstration on March 29, 1976, at Lehigh University, co-sponsored by the present OTEC program.

Appendix A Description of NONSAP Program

NONSAP is a general purpose nonlinear finite element analysis program that has originally been developed by the University of California at Berkeley in 1974. The existing version of this program contains, however, a limited number of element types and material models. Development effort is being undertaken to add additional element types as well as our nonlinear material models for reinforced concrete so that a wide range of nonlinear structural problems can be treated.

The present version of NONSAP can perform both static and dynamic analyses of truss or two-dimensional solids with large displacements. Since isoparametric elements are used, the program can analyze either two-dimensional continuum, plate or shell structures. Material models consist of elastic, elastic-plastic, Mooney-Rivlin materials (plane stress only). The program uses a step-by-step incremental solution procedure with equilibrium iteration method to solve the nonlinear static problems and direct time integration technique to solve dynamic problems.

Several highlights of the program are outlined as follows:

1. Structural configurations--The program handles truss, two-dimensional solids including continuums, plates and shells with large displacements. Problems involving three-dimensional deformations are presently under development at the University of Akron under the direction of Dr. T. Y. Chang (see Table 1).
2. Material models
   . Elastic-isotropic or anisotropic
   . Elastic-plastic. Incremental stress-strain relationship with von Mises yield criterion. Both kinematic and isotropic

hardening rules are available.

. Mooney-Rivlin stress-strain relationship (for plane stress only).

. Elastic-plastic-fracture. Incremental stress-strain relationship proposed by Chen for reinforced concrete material is in progress and will be available.

3. Problem size

The program utilizes the dynamic core allocation with compacted matrix storage. The present version of NONSAP performs all the analysis in core and, therefore, the problem size that can be handled varies in accordance with available core space. For example, for static analysis, typical problem sizes can be specified as the following (also shown in Fig. 1):

Available Core (in Octal)	Problem Size (Degrees of Freedom)
200 K	750
300	1820
400	3000

Out-of-core solution package will be prepared in the future so that any size of problem can be handled.

4. Analysis type

. Static analysis

. Transient dynamic analysis

. Buckling analysis (under development at University of Akron)

Table 1 Element Library in NONSAP

Element Description	Linear Analysis	Nonlinear Analysis: Material Nonlinear	Nonlinear Analysis: Geometric Nonlinear	Static	Dynamic
Truss	Yes	Yes	Yes	Yes	Yes
Two-Dimensional Element (Plane and Axisymmetric Cases)					
Isotropic	Yes		Yes	Yes	Yes
Orthotropic	Yes		Yes	Yes	Yes
Variable Tangent Modulus		Yes	Yes	Yes	Yes
Curve Description Nonlinearity		Yes	Yes	Yes	Yes
Plasticity (von Mises for metal)		Yes	Yes	Yes	Yes
Plasticity (Drucker for soil)		Yes	Yes	Yes	Yes
Plasticity (Chen for concrete)		Yes	*	Yes	**
Mooney-Rivlin		Yes	Yes	Yes	Yes
Three-Dimensional Element					
Isotropic	Yes	**	**	Yes	Yes
Curve Description Nonlinear		Yes		Yes	Yes

*under development
**future development

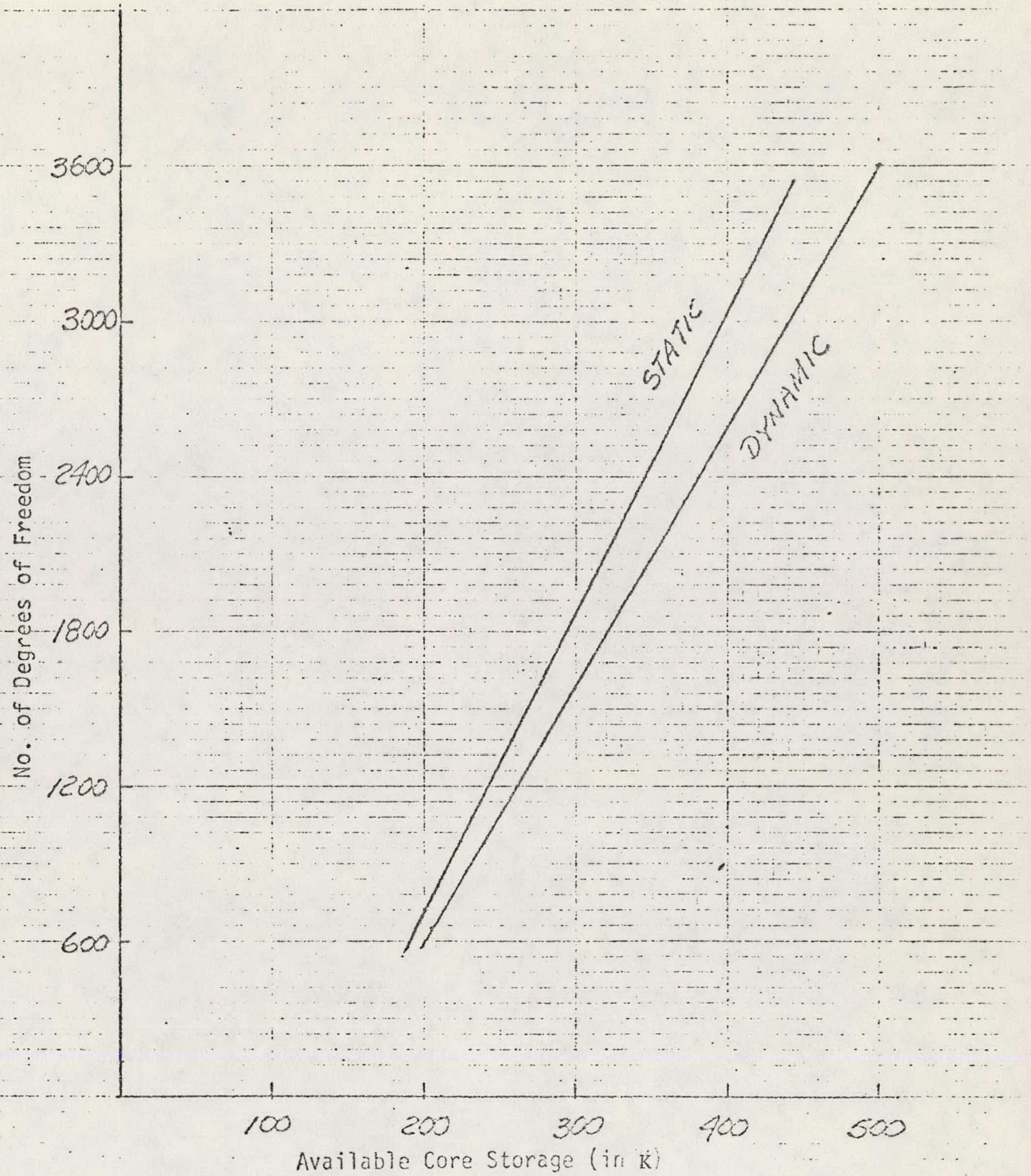


Fig. 1 Core Storage vs. No. of Degrees of Freedom

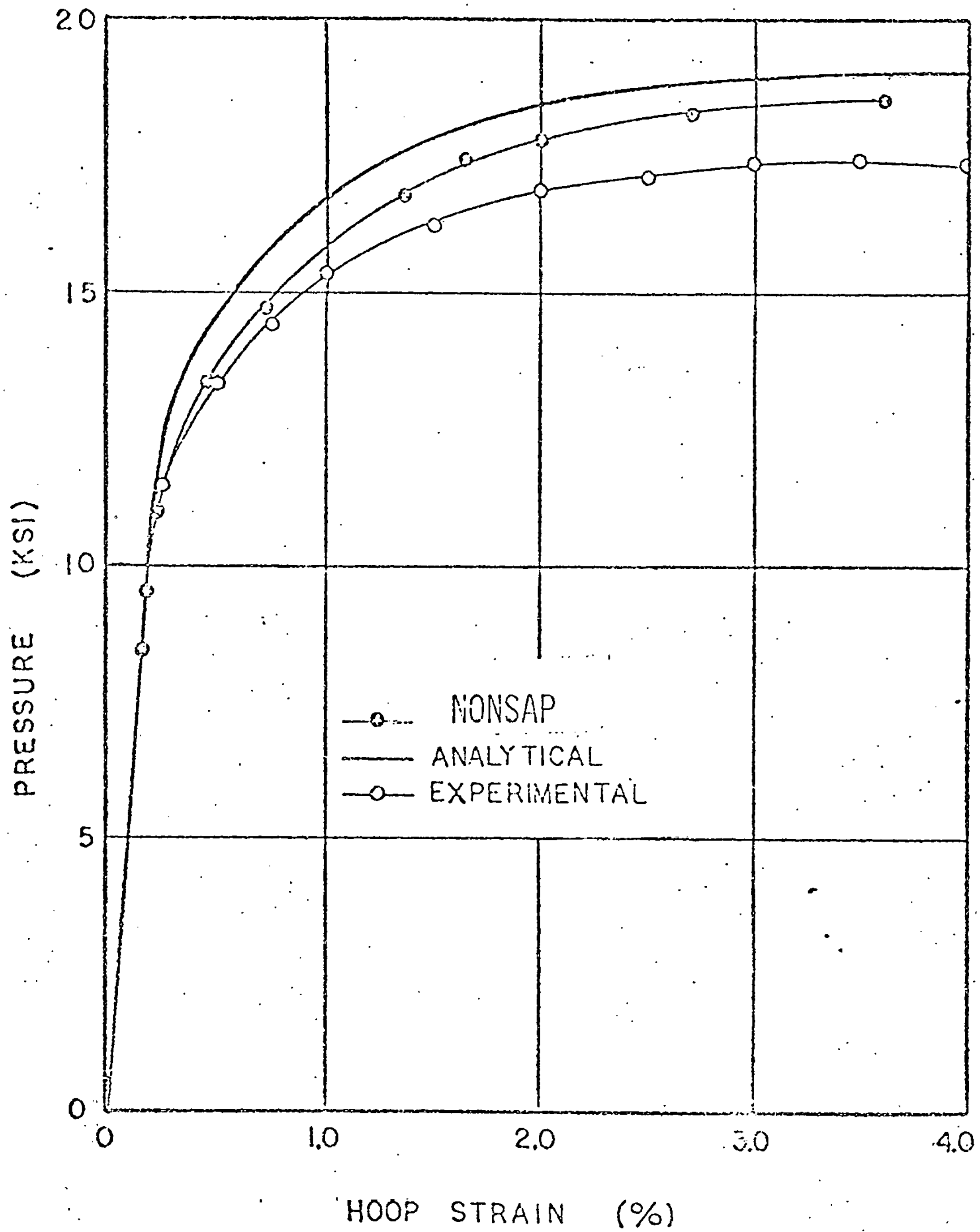


Fig. 2. Elastic-Plastic Response of a Thick-Walled Cylinder (von Mises Material)

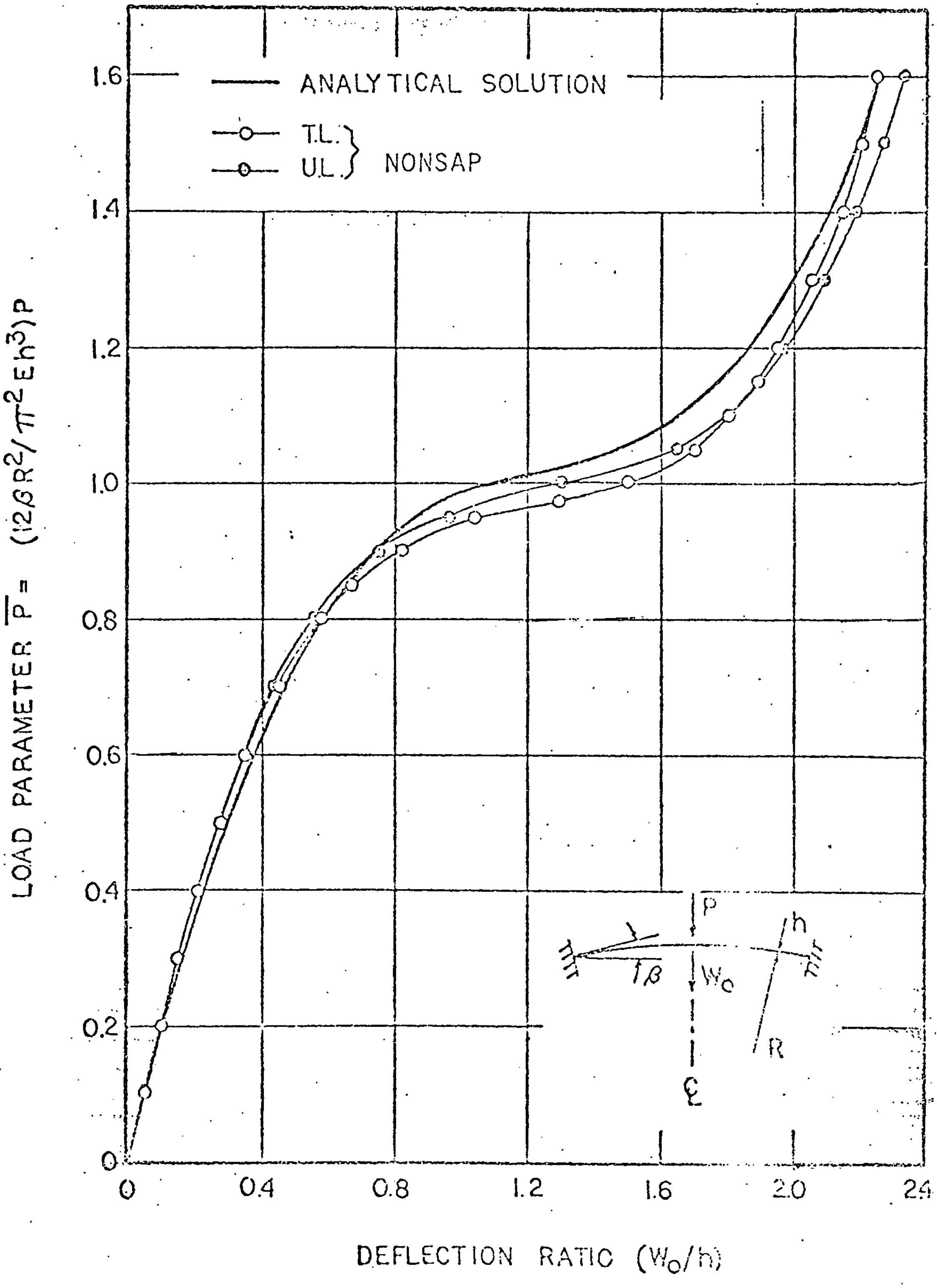


FIG. 3 FINITE DEFORMATION OF A SPHERICAL CAP
(von Mises Material)

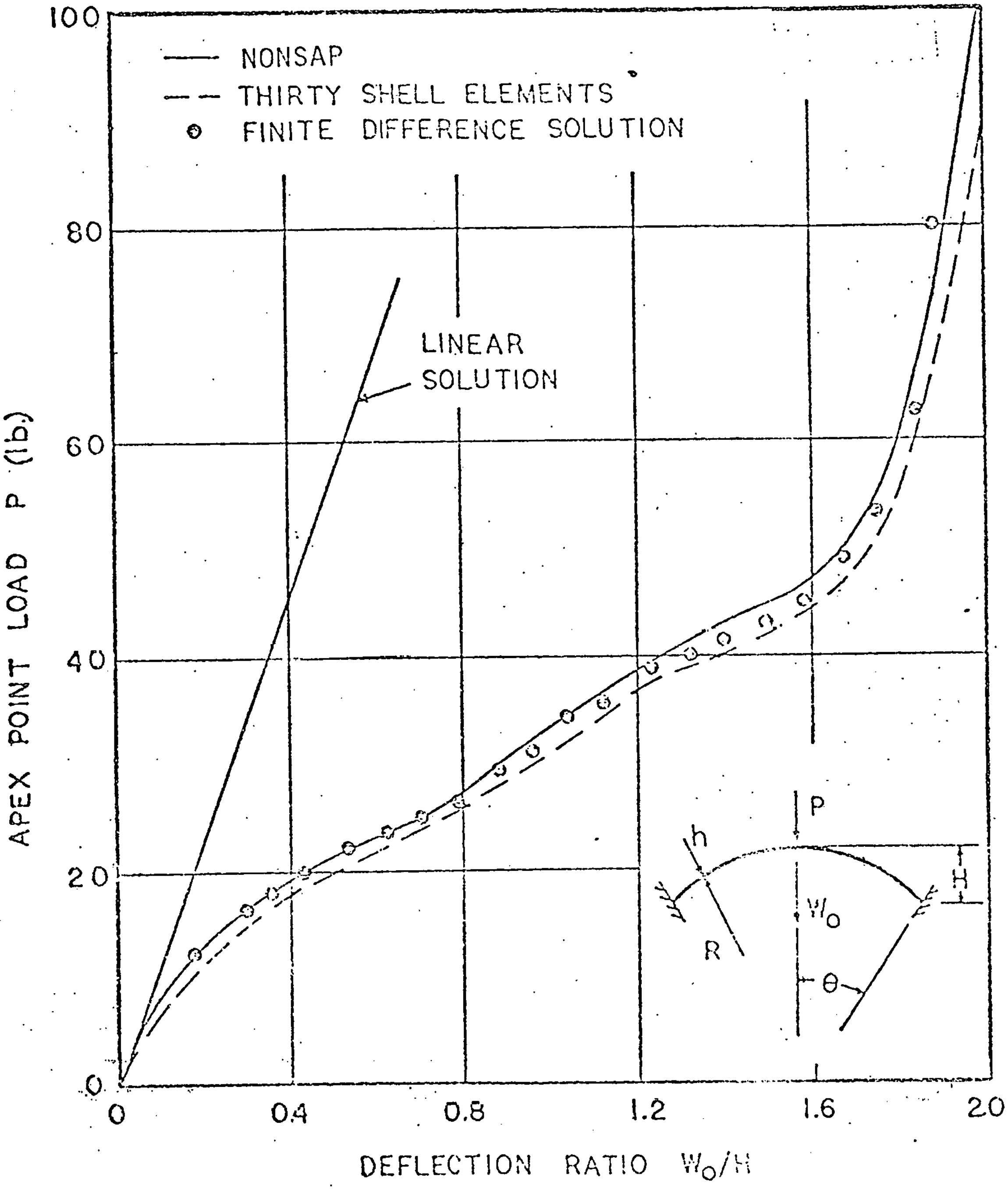


FIG. 4 LARGE DEFORMATION OF A CYLINDRICAL SHELL
(von Mises Material)

Appendix B <u>Comparison of Results</u>

METAL BLOCK

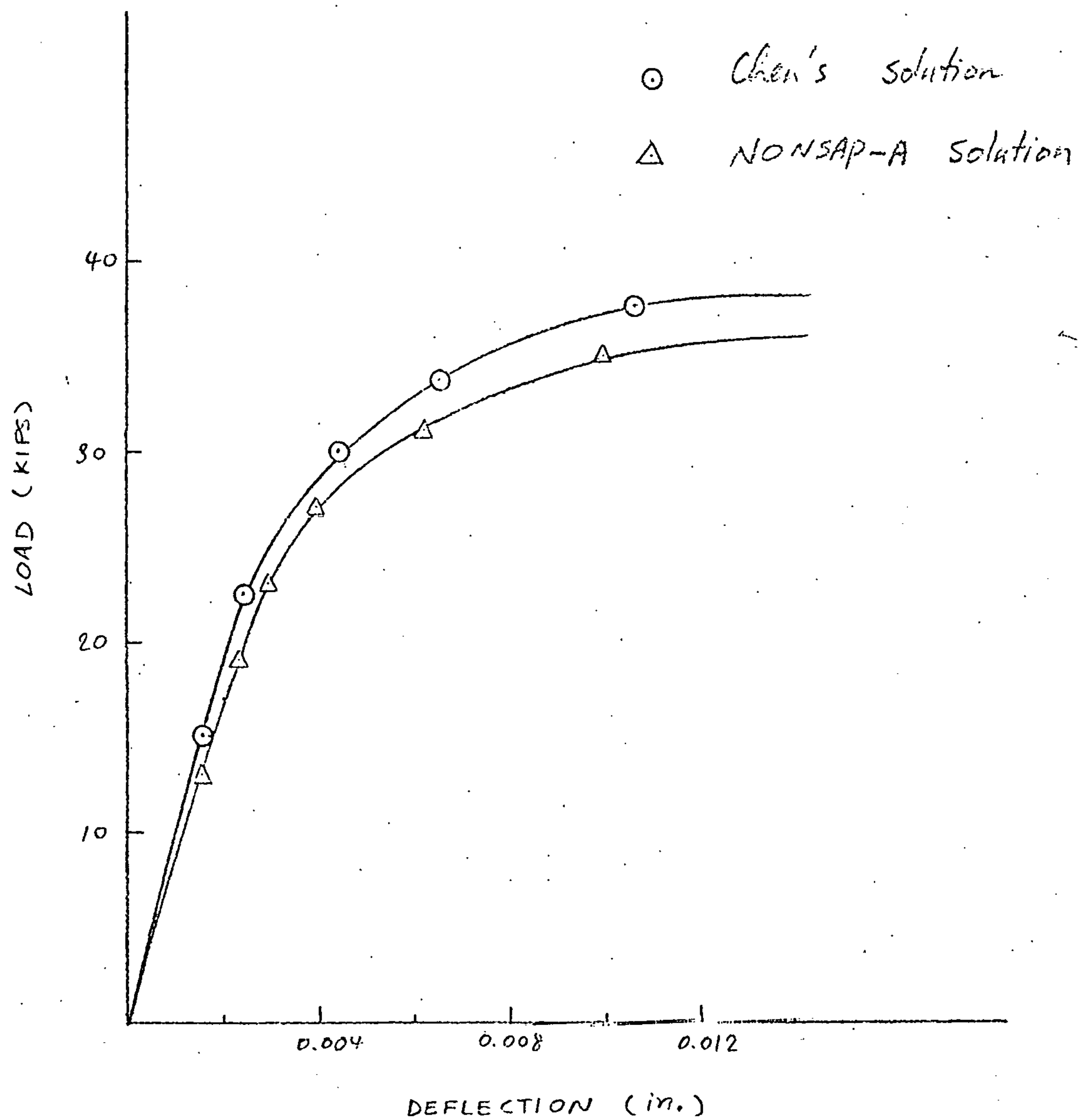


Fig. 5 Load-Deflection Curve for a Metal Block Using NONSAP and EPFFEP Programs

CONCRETE BLOCK

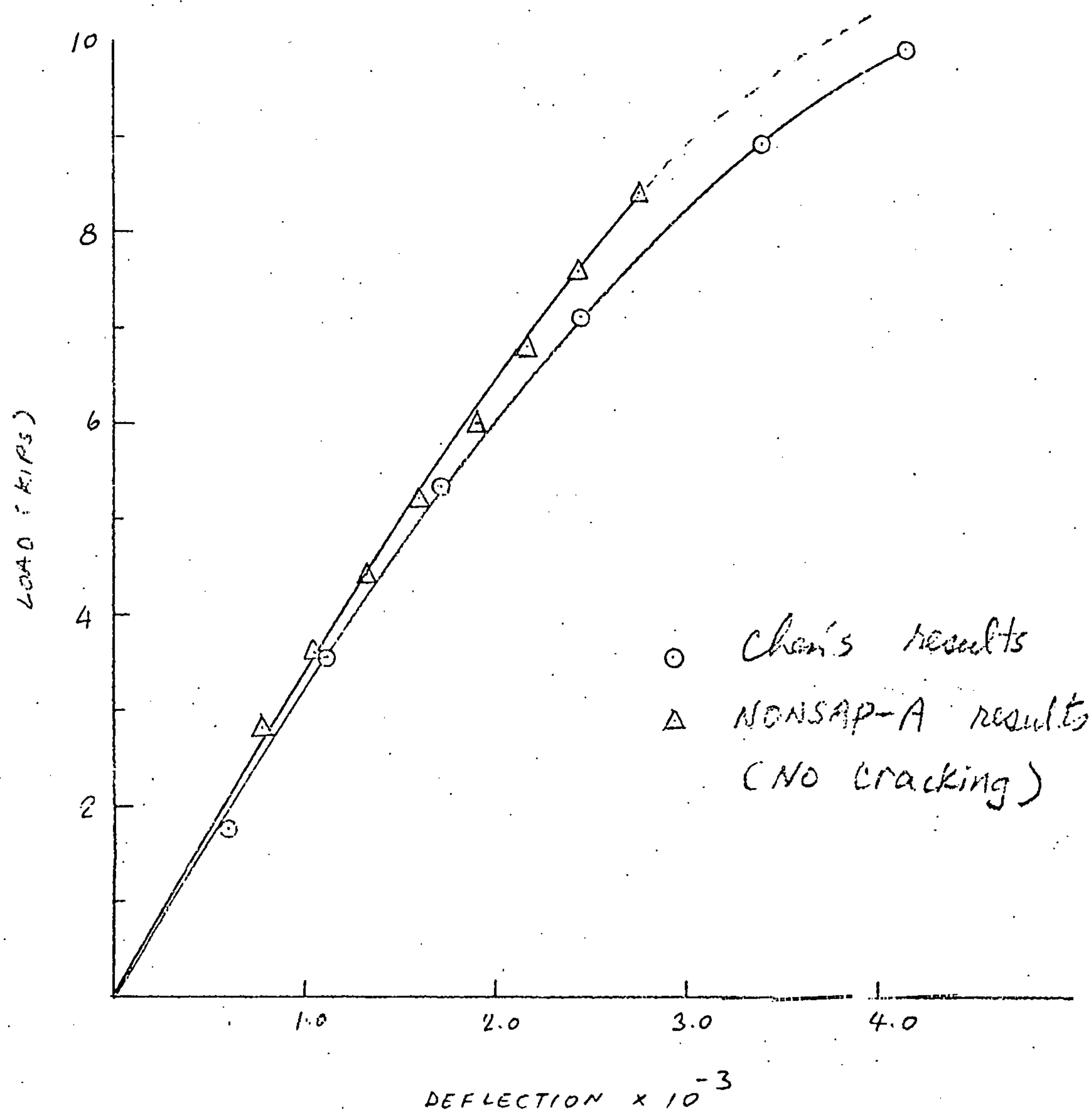
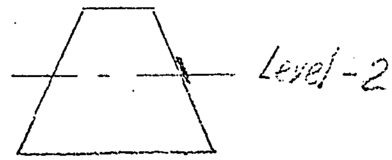


Fig. 6 Load-Deflection Curve for a Concrete Block Using NONSAP and EPFFEP Programs



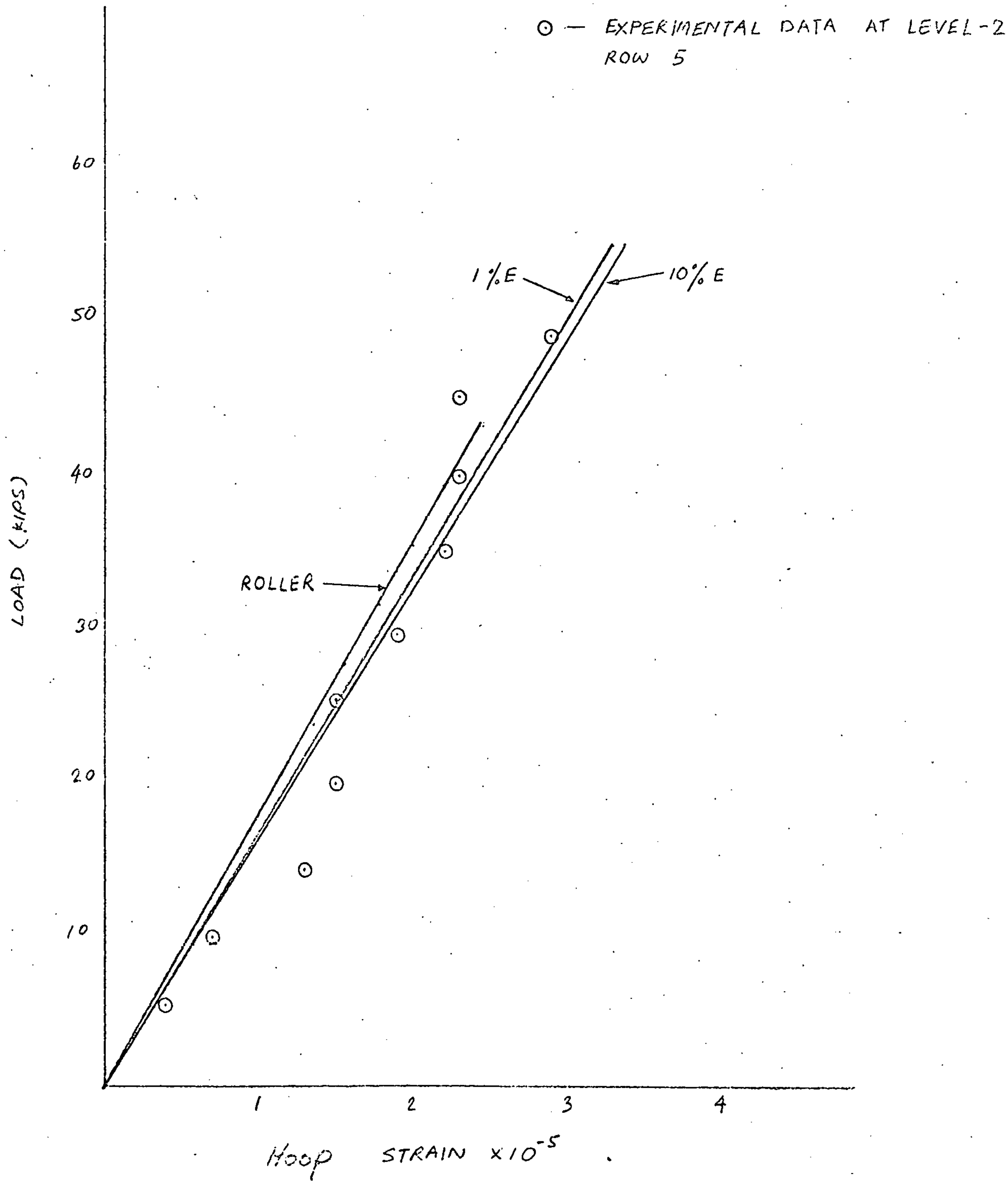


Fig. 7 Load-Hoop Strain Curve for a Plain Concrete Shell Using NONSAP Program

Appendix C <u>Dimensions and Testing Procedure for Concrete Shells</u>

(1) <u>Cone (a) and (B)</u> (see the attached figure)

(a) Dimension: D1 = 32-1/4", D2 = 18", H = 25", T = 1-1/2"

(B) Dimension: D1 = 20.5", D2 = 12", H = 13-1/4", T = 3/4"

<u>Type of Loading</u>: Compression

<u>End Conditions</u>:

(a) Fixed at D1 end, simply supported at D2 end

(b) Simply supported at D1 end, fixed at D2 end

(c) Simply supported at both ends

(d) Fixed at both ends

(2) <u>Cylinder</u>

<u>Dimension</u>: D = 8", H = 10", T = 0.5"

<u>Type of Loading</u>: Compression

<u>End Conditions</u>: Same as in <u>cone</u> except (a), (b) are same.

(3) <u>Dome</u>

<u>Dimension</u>: D1 = 20", T = 1"

<u>Type of Loading</u>: Compression at crown

<u>End Condition</u>: Simply supported at base

<u>Material Properties</u>:

(a) σ_c = 4000-5000 psi (regular concrete)

E_c = 2.8-3.5x10^6 psi

$\epsilon_{ultimate}$ = .003-.004 (nonlinear)

(b) σ_c = 16000-20000 psi (PIC)

E_c = 6.0-7.0x10^6 psi

Linear, elastic to failure, $\epsilon_{ultimate}$ = .0035-.005

Curve deviates at 70% of ultimate load.

(c) σ_c = 8000-10,000 psi (ductile PIC)

E_c = 4.0-4.5x10^6 psi

Nonlinear $\epsilon_{ultimate}$ = .008-.012

Curve deviates from linearity at 50% of ultimate load.

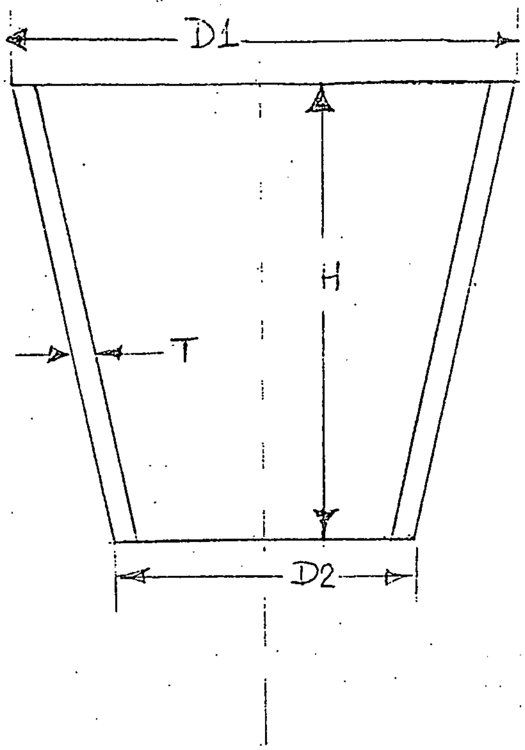
(d) Composite material: mesh at center of thickness

(i) Cone A (BIG) use 1/4"φ steel mesh (both ways) at 3" CIC: mesh at center of thickness.

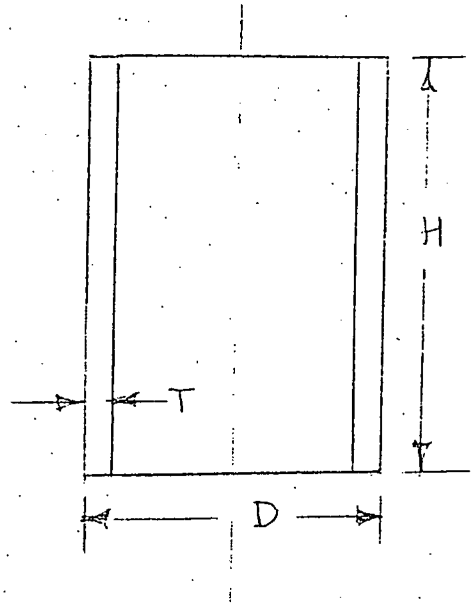
(ii) Cone B (small) use 1/16"φ steel mesh (both ways) at 0.5" CIC. Also same for cylinder dome.

<u>Experimental Data</u>: Determine stress-strain curves in two directions (axial, horizontal) both inside and outside walls of the shell at center, inflection points and points of maximum stresses.

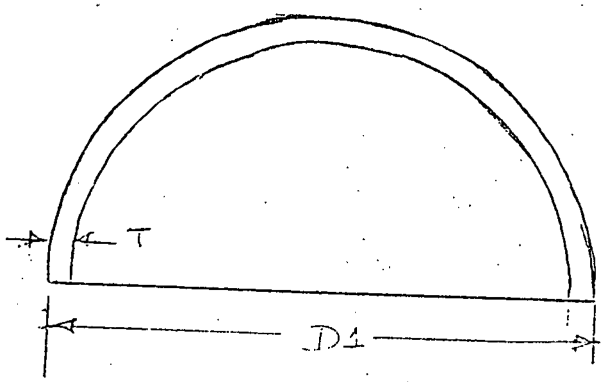
Provide imperfection in the roundness of A cos 2θ where A = ±1/32, θ = angle around the circumference. See paper "Strength of Reinforced Concrete Chambers under External Pressures," by O. Buyukozturk and P. V. Marcal.



CONE



CYLINDER



DOME