DETERMINATION OF NONDESTRUCTIVE INSPECTION RELIABILITY USING FIELD OR PRODUCTION DATA

EPRI NP-315

(Research Project 700-1)

Technical Report 8

October 1976

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ABSTRACT

This paper introduces a method which uses field or production data rather than specimen data for estimating the probability of rejecting a part containing an imperfection of a given size and for estimating the dependence of this probability upon the nondestructive inspection (NDI) level or levels. In this method the imperfection response features used to make the accept/reject decision are identified for the inspection process of interest. The correlations between the NDI signal characteristics used in making the accept/reject decision, and the relative size (severity) of the imperfection are estimated from destructive examination of material units containing selected imperfection response amplitudes. The destructive evaluation of several retired steam turbine rotors to characterize imperfections located using prototypical field NDI techniques is presently under way as part of EPRI Contract RP502. This method has a distinct advantage over using flawed specimens, both because of possible cost savings and because the uncertainty associated with how well the imperfections fabricated in specimens represent actual imperfections encountered in practice does not enter the analysis method.

CONTENTS

FIGURES

NOMENCLATURE

a	imperfection size
$\hat{a}$	apparent imperfection size
$\hat{a}_R$	apparent imperfection size rejection threshold
C_F	average cost of a failure
C_I	average cost to inspect a material unit
C_M	average cost to manufacture a material unit
C_R	average cost to repair a material unit
C_T	total life cycle cost of a material unit
$\langle C_T \rangle$	expectant total life cycle cost of a material unit
DAC	distance amplitude corrected
F_F	fraction of material units that fail
F_R	fraction of material units rejected by the inspection
IRP	imperfection rejection probability ($P(R\|a)$)
$P(A)$	probability of event A
$P(F)$	probability of failure event
$P(R)$	probability of the rejection event
$P(A \cap B)$	probability of the intersection of events A and B
$P(A\|B)$	probability of the event A given the event B
$P(D\|a)$	imperfection detection probability
$P(F\|a)$	imperfection failure probability
$P(R\|a)$	imperfection rejection probability (IRP)
$P(R\|D,a)$	conditional imperfection rejection probability
$pd(\hat{a}_R)$	probability distribution of rejection levels
$pd(a\|i)$	probability distribution of imperfection size given a response signal of magnitude i
$pd(a\|s)$	probability distribution of imperfection size given a detection response signal of amplitude s
$pd(a\|D,\hat{a})$	probability distribution of imperfection size given the detection event and the imperfection has an apparent size $\hat{a}$

$pd(\hat{a}\|D,a)$	probability distribution of apparent imperfection size given the detection event and the actual imperfection size is a
$pd(i\|a)$	probability distribution of imperfection response signal magnitude given an imperfection of size a
$pd(s\|a)$	probability distribution of detection signal amplitude given an imperfection of size a
PN(F)	probable number of failures per material unit
PN(R)	probable number of rejections per material unit
pn(a)	preinspection distribution of imperfection sizes
pn(i)	distribution of imperfection response signal magnitudes
pn(s)	distribution of imperfection detection response amplitudes
$pn(D,\hat{a})$	distribution of detected apparent imperfection sizes
s	imperfection response amplitude
s_D	imperfection response amplitude detection threshold
V_u	volume of the material unit

Section I

INTRODUCTION AND SUMMARY

A quantitative estimate of the imperfection rejection probability* (IRP) and the dependence of this probability on the inspection procedure are required to predict quantitatively the effect of a nondestructive inspection (NDI) and the effect of changes in the NDI procedure on the rejection rate, failure probability and total cost of material units that are subjected to this inspection (1, 2). Traditionally, the methods used to estimate the IRP involve the inspection of specimens containing fabricated imperfections of known size and counting the number of rejected versus unrejected imperfections (3-26). The cost of fabricating a sufficient number of test specimens (27) and the uncertain relevance of the imperfections fabricated in test specimens to imperfections experienced in actual production or in the field represent significant barriers to the full implementation of quantitative NDI procedures.

This paper outlines a method for estimating the imperfection rejection probability using field or production data rather than specimen data. When production or field experience exists, this approach has a distinct advantage over using intentionally flawed specimens and artificial inspections because of possible cost savings but more importantly because one does not have the uncertainty associated with how well the imperfections fabricated in specimens and experimental inspection conditions represent actual imperfections and inspections encountered in the field. When production or field experience does not initially exist, the approach developed should be useful in updating the initial reliability predictions as the experience is acquired.

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*The imperfection rejection probability (IRP) can be defined as the fraction of imperfections of a given size (severity) that would be rejected by an inspection of an arbitrarily large number of these imperfections.

The inspection uncertainty analysis introduced in this paper can also be used to estimate the IRP and the dependence of the IRP on the inspection procedure from significantly fewer flawed-specimen data than is required by the traditional rejection counting approach. This paper does not consider this aspect of the inspection uncertainty analysis in detail, but concentrates instead on developing a method for estimating the IRP without a flawed specimen program.

In the traditional approach for estimating the inspection reliability, the number of specimens containing imperfections in a given size range that are rejected by the inspection is divided by the total number of inspected specimens containing imperfections in the given size range to give a point estimate of the IRP. Because there are a finite number of specimens in the sample, the fraction of imperfections of a given size that would be rejected in an arbitrarily large sample of these imperfections (i.e., the IRP for these imperfections) may be larger or smaller than the point estimate. The point estimate and the sample size are usually combined to establish a lower bound confidence limit on the IRP. The reason one cannot use such a rejection-nonrejection counting approach to estimate the IRP from production or field data is the difficulty in establishing the total number of imperfections of a given size in the inspected population (i.e., the number of imperfections not detected by the inspection).

The analysis of the inspection process introduced in this paper and referred to as inspection uncertainty analysis extends beyond the traditional counting analysis and does not require the total number of imperfections of a given size in the inspected population in order to estimate the IRP. In inspection uncertainty analysis, the structure of the accept/reject decision and the signal characteristics used in making the accept/reject decisions are identified along with the signal levels or thresholds at which the decisions are made. Estimation of the correlation between the NDI response from an imperfection and the size (severity) of the imperfection then permits the estimation of the IRP and the dependence of the IRP on the decision thresholds.

Section II of this paper, as background information, outlines how a quantitative estimate of the IRP can be used to make accept/reject decisions that minimize the total expectant cost of a product, including failure and rejection costs. Section III discusses the structure of the accept/reject decision and the fact that accept/reject decisions are usually made up of a number of subdecisions. Section IV discusses the fact that the accept/reject decision is based on certain imperfection response features and introduces the underlying correlation functions between the nondestructive signal characteristics (imperfection response features) used in the decision and the actual imperfection size (severity). These correlation functions along with the decision levels (that is, the signal thresholds at which the decisions are made) can be used to determine the subdecision probabilities and subsequently the imperfection rejection probability (IRP) as a function of the imperfection size (severity). Section V outlines how these correlation functions can be determined from field or production quality control programs.

Section II

ACCEPT/REJECT DECISION

In nondestructive inspection, the decision to reject a material unit is based on the imperfection response to a NDI probing agent or combination of such responses exceeding specified thresholds and not on the imperfection size (severity) itself exceeding a specified size. Because of the uncertainty in the relative severity of an imperfection given the imperfection response, a NDI will have associated with it 1) a certain probability of accepting material units that contain imperfections that affect the integrity of the material unit and 2) a certain probability of rejecting material units that do not contain imperfections that affect the integrity of the material unit (1). The optimum accept/reject thresholds or inspection levels are those levels that result in a minimum total expectant undesirability or cost from both of these inspection errors (1). The optimum levels can be determined for a given inspection method from the cost of the rejected material units and the expectant cost resulting from the finite probability of defective material units existing in the field. For example, if, as is often the case in weldments, the material unit is repaired when it is rejected, then the total cost of a material unit that has passed the inspection is given by

$$C_T = C_M + C_I + (C_R + C_I) F_R (1-F_R)^{-1} + C_F F_F \qquad \text{(II-1)}$$

where C_M is the cost of manufacturing a material unit, C_I is the average cost to inspect a material unit, C_R is the average cost to repair a material unit, C_F is the average cost if a material unit fails, F_R is the fraction of material units rejected, and F_F is the fraction of material units that pass the inspection and fail. Here for simplicity we have assumed that the cost of inspection before repair is the same as after repair and the repair procedure returns the weldment to the same material state that would be expected from another new weldment. If these other complications are considered important then they can be added easily to the economic model;

however, for the illustration purposes of this section, additional complications can be ignored.

Assuming that the variance in the probability estimates is small, the expectant total cost of a material unit that has passed the inspection can be obtained by substituting P(R), the probability of rejection, and P(F), the probability of failure, for F_R and F_F respectively; i.e.,

$$<C_T> = C_M + C_I + (C_R + C_I) P(R) (1-P(R))^{-1} + C_F P(F). \qquad \text{(II-2)}$$

As the inspection levels decrease, $P(R) (1-P(R))^{-1}$ increases at an increasing rate while P(F) decreases at a decreasing rate, leading to finite inspection levels which results in a minimum cost. Figure 1 illustrates the dependence of the expectant total cost on the inspection size or level for a hypothetical situation involving turbine blades (2).

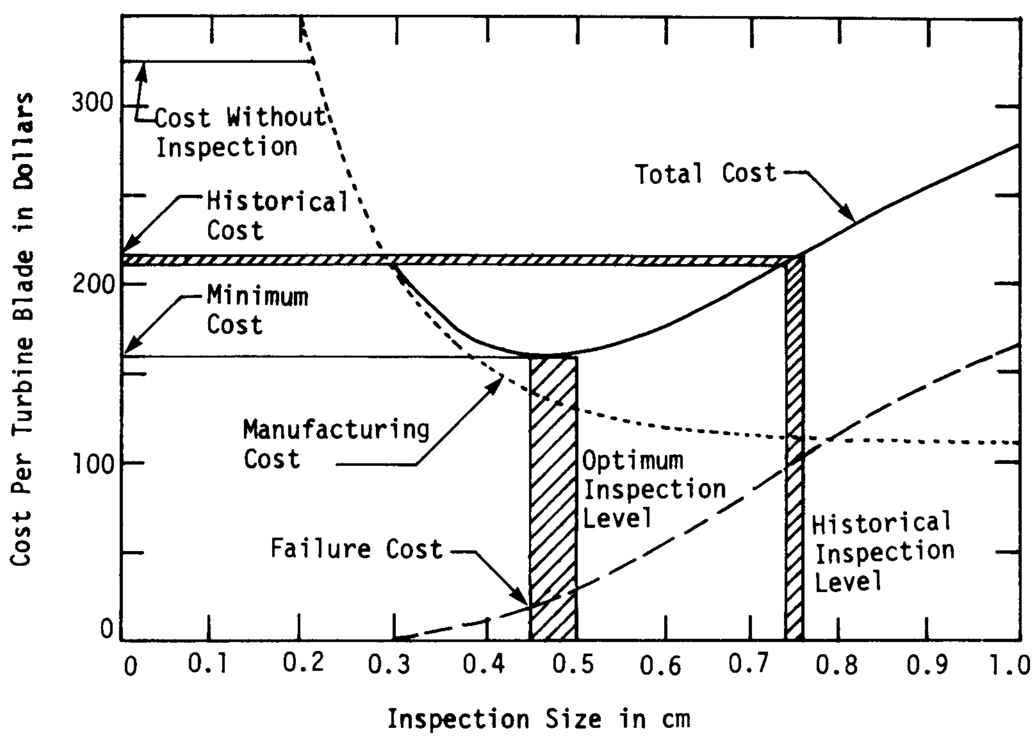

FIGURE 1 - Costs Per Turbine Blade As A Function Of Inspection Size. (From Reference 2.)

The probability of failure can be written as (1)

$$P(F) = 1-e^{-PN(F)} \tag{II-3}$$

where

$$PN(F) = V_u \int_0^\infty P(F|a)\ pn(a)\ (1-P(R|a))\ da, \tag{II-4}$$

$P(F|a)$ is the probability of failure given that an imperfection of size "a" passes the inspection, $(1 - P(R|a))$ is the probability that an imperfection of size "a" will pass the inspection given an imperfection of size "a" exists prior to the inspection, $pn(a)\ da$ is the probable number of imperfections of size between "a" and a + da per unit volume existing prior to inspection, and V_u is the volume of the material unit.

The probability of rejection can be written as

$$P(R) = 1-e^{-PN(R)} \tag{II-5}$$

where

$$PN(R) = V_u \int_0^\infty P(R|a)\ pn(a)\ da \tag{II-6}$$

and the imperfection rejection probability $P(R|a)$ is the probability of a rejectable indication given there exists an imperfection of size, "a", and is illustrated in Fig. 2.

It is evident from the above development that knowledge of the imperfection rejection probability $(P(R|a))$ is required to predict the failure probability, the rejection probability and eventually the total expectant cost associated with a material unit subjected to an NDI.

Reference 2 develops the methodology for optimizing the inspection levels given the dependence of the IRP on the inspection levels. The next sections develop a procedure for determining the IRP as a function of the inspection levels from the performance of the inspection in the field or in production.

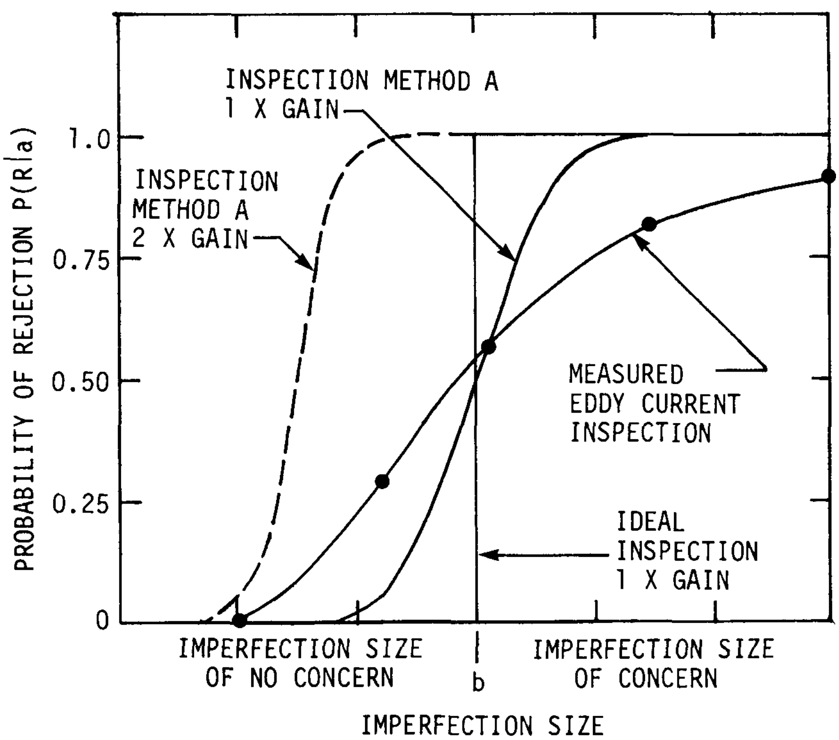

FIGURE 2 - Imperfection Rejection Probability
(From Reference 1.)

Section III

IMPERFECTION REJECTION PROBABILITY

The rejection decision is generally a composite event consisting of a number of subdecisions or events. For example, the rejection decision in an ultrasonic inspection may consist of a detection event, where the ultrasonic response signal exceeds a specified threshold; a sizing event, where the apparent imperfection size exceeds a specified threshold; a judgment, based on the indication location on the oscilloscope and the behavior of this indication as the transducer is moved, that the indication is not simply a structural feature; and finally, a calculation based on the apparent size of the imperfection that the failure probability exceeds a specified level.

The discussion in the remaining sections of this paper is easily generalized to any number of events; however, for illustrative purposes, let us assume the rejection event consists of only two events, a detection event and a subsequent evaluation which leads to the rejection event. The imperfection rejection probability $P(R|a)$ can then be written as a product of an imperfection detection probability $P(D|a)$ and an imperfection rejection probability given the detection event has occurred $P(R|D,a)$, i.e.,

$$P(R|a) = P(R|D,a)\ P(D|a). \qquad \text{(III-1)}$$

The various events that make up the rejection event will depend on features or a combination of features of the imperfection signal response exceeding certain thresholds or inspection levels. In a well-controlled inspection process, these decision features are clearly identified and the thresholds clearly established. Unfortunately, in many inspections the decision features are not clearly identified and the thresholds may be left to the judgment of the individual inspectors. Even in most well-controlled inspections, the thresholds have not been selected such that the total life cycle cost of the material units is a minimum.

Section IV

UNDERLYING CORRELATION FUNCTIONS

This section introduces the underlying correlation functions between the signal characteristics used in making the accept/reject decisions and the actual imperfection severity. The functional relationship is developed between these correlation functions, the level of the imperfection response which is acceptable or which requires further analysis, and the imperfection detection and conditional rejection probability.

An important point to note in this development is that the measure of the imperfection size "a" is selected to reflect the relative severity of the imperfection and that the response "s" of the NDI probing agent to an imperfection depends upon many factors, other than the imperfection severity. These factors usually vary from one imperfection to the next imperfection of the same severity and from one inspection to the next inspection of the same imperfection. Hence, there is not a one-to-one relationship between the response of an imperfection to a NDI probing agent and the imperfection severity but instead there exists only a correlation and sometimes only a weak correlation between the NDI response and the imperfection severity.

For example, if the failure mode is fatigue, the projected area of the imperfection on the plane penpendicular to the maximum principal stress may be a measure of the imperfection size that reflects the relative severity of the imperfection (28). The NDI response on the other hand may depend upon the area of the imperfection directed toward some other plane. Hence, as illustrated in Fig. 3, two imperfections of essentially the same severity can give markedly different responses. This variability of NDI response for imperfections of the same severity is characteristic of all NDI.

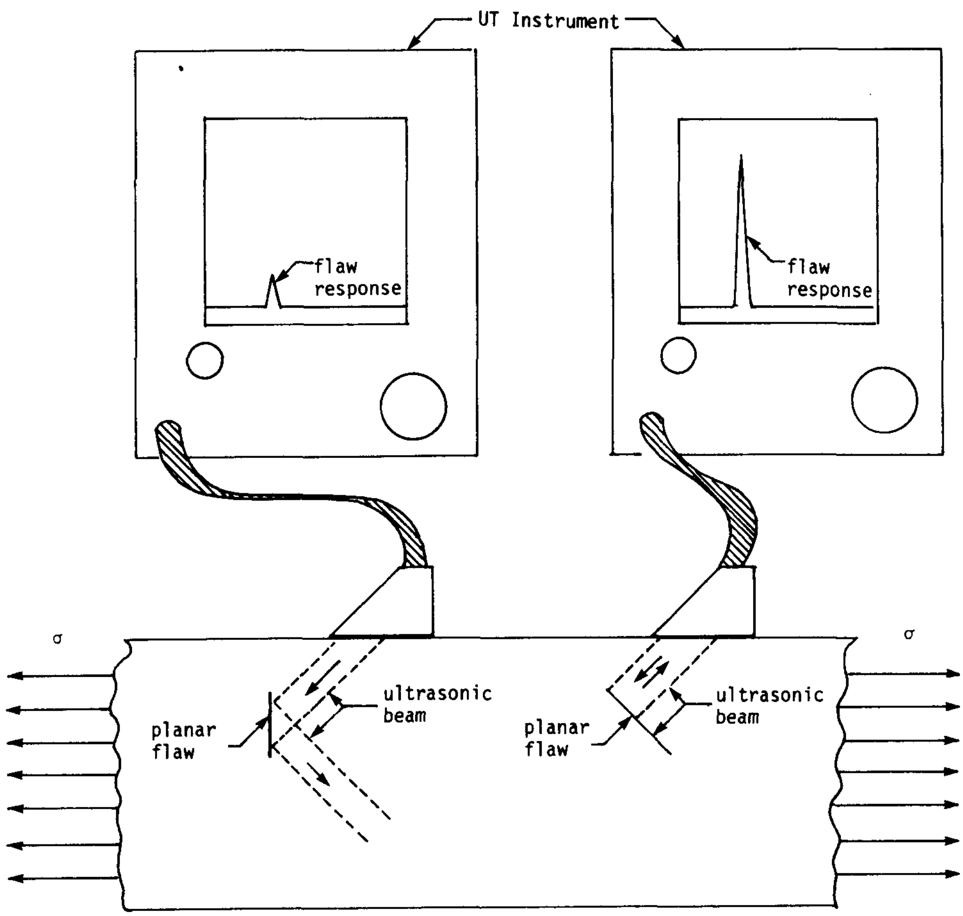

FIGURE 3 - Ultrasonic Response From Two Imperfections Of The Same Severity

The dependence of the various inspection event probabilities upon the decision thresholds can be determined if the correlation between the signal response used in the decision and the imperfection size (severity) is known. For example, the detection process is usually based on a signal amplitude exceeding some threshold s_D. The correlation between the signal amplitude and the imperfection size can be described by $pd(s|a)$. Here, $pd(s|a)\,ds$ is the probability the signal amplitude is between s and s + ds given an imperfection of size a.

The imperfection detection probability is related to this indication-amplitude imperfection-size correlation function and the detection threshold s_D as follows:

$$P(D|a) = \int_{s_D}^{\infty} pd(s|a)\,ds. \qquad \text{(IV-1)}$$

In an ASME Section XI Code, Ultrasonic Inspection Procedure, for example, s_D = 0.50 DAC of a standard side-drilled hole (29).

After detecting the imperfection, the decision to reject the material unit may be based on apparent size (severity) of the imperfection exceeding some threshold $\hat{a}_R$. The correlation between the apparent size and the actual size of the imperfection can be described by $pd(\hat{a}|D,a)$. Here, $pd(\hat{a}|D,a)\,d\hat{a}$ is the probability the apparent size is between $\hat{a}$ and $\hat{a} + d\hat{a}$, given the detection event has occurred and the actual imperfection size is a. This apparent-size correlation function and the rejection threshold $\hat{a}_R$ is related to the conditional rejection probability as follows,

$$P(R|D,a) = \int_{\hat{a}_R}^{\infty} pd(\hat{a}|D,a)\,d\hat{a}. \qquad \text{(IV-2)}$$

If the rejection or detection criteria is not carefully controlled, Eqs. IV-1 and IV-2 will have to be modified to account for the distribution of $\hat{a}_R$ and s_D. For example,

$$P(R|D,a) = \int_{0}^{\infty} d\hat{a}_R\, pd(\hat{a}_R) \int_{\hat{a}_R}^{\infty} pd(\hat{a}|D,a)\,d\hat{a} \qquad \text{(IV-3)}$$

where $pd(\hat{a}_R)$ describes the distribution of rejection levels.

In light of the above discussion it is evident that the determination of these underlying correlation functions, e.g., $pd(s|a)$ and $pd(\hat{a}|D,a)$, permits the determination of imperfection rejection probability and the dependence of the imperfection rejection probability upon the inspection levels or distribution in these levels. The next section outlines how the underlying correlation functions can be estimated from field or production inspection data and destructive examination of field or production material units containing selected NDI response amplitudes.

Section V

CORRELATION FUNCTIONS AND FIELD OR PRODUCTION DATA

The underlying correlation functions can be estimated directly with a flawed specimen program by recording the distribution of response amplitudes arising from imperfections of a given size range. Such an inspection uncertainty analysis using flawed specimens has two potential advantages over the conventional rejection-nonrejection counting analysis in determining the imperfection rejection probability (IRP).

- With some limited modeling of the inspection process, the IRP at a given inspection level can be estimated with greater confidence from fewer flawed specimens, and
- The dependence of the IRP upon the inspection level can be determined from a single inspection of the flawed specimens rather than requiring inspection of the flawed specimens at each inspection level of interest.

Unfortunately, the IRP estimated from a flawed specimen program by either uncertainty analysis or detection counting analysis applies rigorously only to the inspection performance on the geometry, imperfections and in the environment characteristic of the flawed specimen inspection, and it is very difficult to translate the IRP observed under these artificial conditions to the IRP that describes the actual field or production inspection performance on actual parts with normal production or field imperfections.

The remainder of this section outlines a procedure for determining the underlying correlation functions and subsequently the IRP from field or production inspection data rather than from flawed specimen data. This approach avoids the cost of manufacturing intentionally flawed specimens and also avoids the uncertainty associated with how well the imperfections, geometry, and environment in specimen inspections represent actual field or production inspections.

In attempting to use field or production data to generate the underlying correlation functions, the first thing to realize is that the underlying functions (e.g., pd(s|a)) cannot be estimated directly from field or production data because of the difficulty in obtaining an unbiased estimate of the NDI responses for a given imperfection size. What can be obtained from field or production data and destructive examination of selected indications in a given amplitude range is a function (e.g., pd(a|s)), which is closely related to the desired correlation function. The desired correlation function, for example, pd(s|a), describes the distribution in NDI response amplitudes "s" associated with a given imperfection size "a" while the field or production observable, for example, pd(a|s), describes the distribution of imperfection sizes "a" associated with a given NDI response "s".

To generate the desired underlying correlation function pd(s|a), both the function pd(a|s) and a second observable pn(s), which describes the frequency of NDI responses at each amplitude,must be determined. Using Eq. A-15, developed in Appendix A, the following relationship can be written between the potential field or production observables pd(a|s) and pn(s) and the desired indication-amplitude imperfection-size correlation function discussed in Section IV:

$$pd(s|a) = pd(a|s)\ pn(s) \left[\int_0^\infty pd(a|s)\ pn(s) ds \right]^{-1} . \qquad (V-1)$$

Here pd(a|s) da is the probability that given an indication of amplitude s, the actual flaw size is between "a" and a + da, and pn(s) ds is the probable number of indications per unit volume with amplitude between s and s + ds.

The observables can be obtained from field or production data. For example, pn(s) can be estimated from a record of the frequency with which indications of different magnitudes occur. Figure 4 illustrates the relationship between pn(s) and the number of indications found in production inspection of 1,000 meters of weld. This figure illustrates an ultrasonic inspection in which the ultrasonic signal amplitude s is expressed in a distance amplitude corrected (DAC) percent of a signal from a standard reference hole. A histogram of the number of indications in each 10% DAC interval

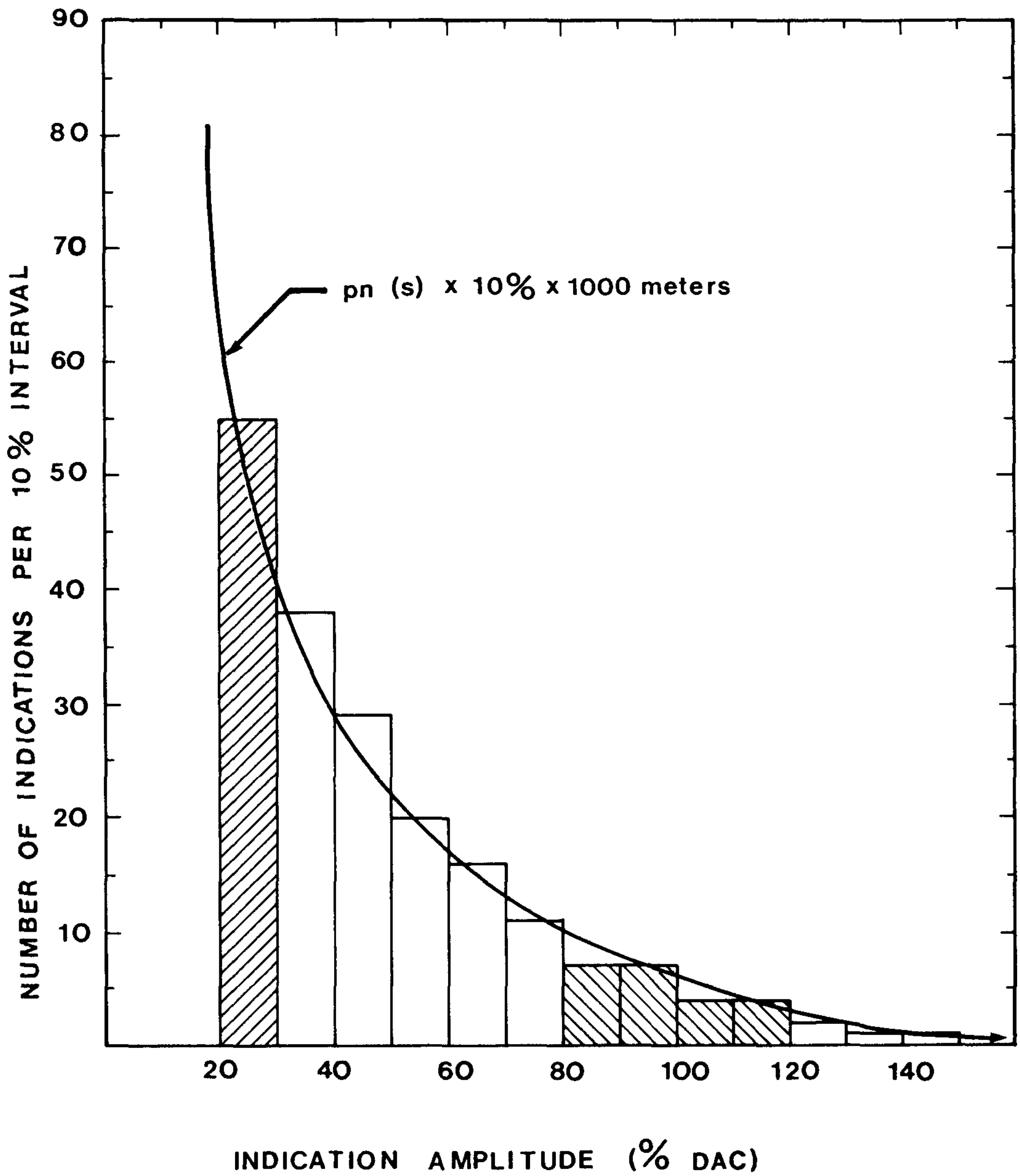

FIGURE 4 - Distribution of Imperfection Detection Response Amplitudes

is recorded. An estimate of pn(s) in units of $(\text{meter }\%)^{-1}$ is given by

$$pn(s) = \frac{\text{Number of indications per 10\% interval}}{\text{1000 meters x 10\%}} \qquad \text{(V-2)}$$

The second observable pd(a|s) can be estimated by destructive examination of selected indications to determine the actual flaw size that gave rise to the indication. For example, the indications in the vicinity of a 100% DAC and 25% DAC (illustrated on Fig. 4 by cross hatching) can be destructively examined to obtain an estimate of the distribution of imperfection sizes that correspond to a 100% DAC signal (pd(a|100%)) and the distribution of imperfection sizes that correspond to a 25% DAC signal (pd(a|25%)), respectively. Figure 5 illustrates the relationship between pd(a|100%) and the actual flaw sizes found in the destructive examination of the 22 indications with ultrasonic signal in the vicinty of 100% DAC; and between pd(a|25%) and the actual flaw sizes found in the destructive examination of 55 indications in the vicinity of 25% DAC. A histogram of the number of flaws in each 0.1 cm interval with ultrasonic signal amplitude in the vicinity of 100% DAC and 25% DAC are recorded. Estimates of pd(a|100%) and pd(a|25%) in units of cm^{-1} are given by

$$pd(a|100\%) = \frac{\text{Number of flaws in size interval a with 100\% DAC indications}}{\text{Total number of flaws with 100\% DAC indication x 0.1 cm}} \qquad \text{(V-3)}$$

and

$$pd(a|25\%) = \frac{\text{Number of flaws in size interval a with 25\% DAC indications}}{\text{Total number of flaws with 25\% DAC indication x 0.1 cm}} . \qquad \text{(V-4)}$$

In summary, pd(a|s) and pn(s) can be estimated from field or production data. Using these estimates of pd(a|s) and pn(s) in conjunction with Eq. V-1, gives an estimate of the indication-amplitude imperfection-size correlation function pd(s|a). Through Eq. IV-1, this can be used to estimate the imperfection detection probability and the dependence of the imperfection detection probability on the detection threshold.

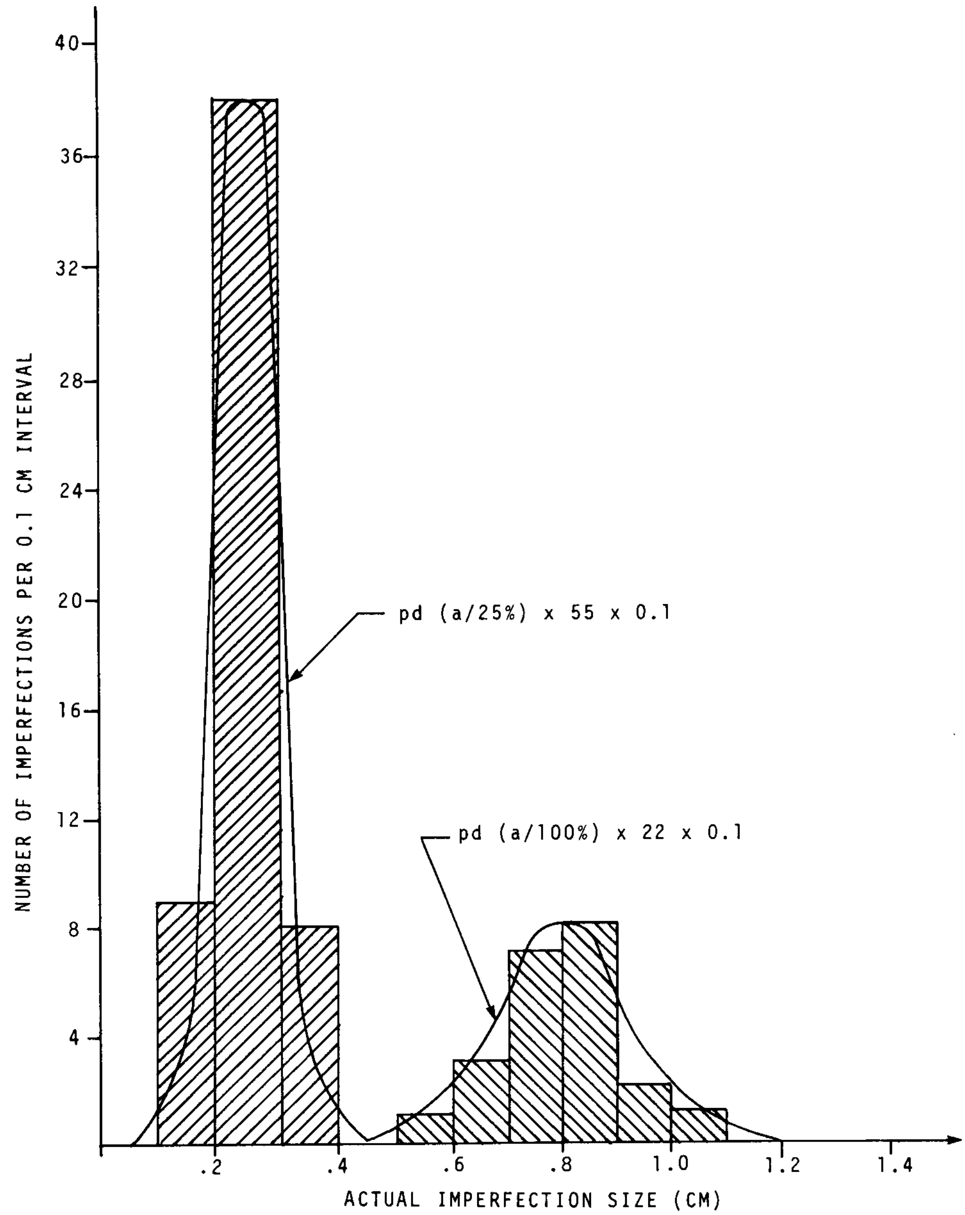

FIGURE 5 - Relationship Between pd(a|s) and the Actual Imperfection Sizes Found in Destructive Examination

In an analogous manner the apparent-size actual-size correlation function can be estimated.

$$pd(\hat{a}|D,a) = pd(a|D,\hat{a})\ pn(D,\hat{a}) \left[\int_0^\infty pd(a|D,\hat{a})\ pn(D,\hat{a})\ d\hat{a}\right]^{-1} . \quad (V-5)$$

Here $pd(a|(D,\hat{a}))\ da$ is the probability that a detected imperfection of apparent size $\hat{a}$ will have an actual imperfection size between "a" and $a + da$ and $pn(D,\hat{a})\ d\hat{a}$ is the probable number of detected imperfections per unit volume of apparent size between $\hat{a}$ and $\hat{a} + d\hat{a}$. These functions can also be estimated from field or production data in a manner analogous to that outlined above. Using these estimates in conjunction with Eq. V-5, gives an estimate of the actual-size apparent-size correlation function $pd(\hat{a}|D,a)$. This in turn through Eq. IV-2 can be used to estimate the conditional rejection probability.

In this manner the imperfection rejection probability can be estimated along with the dependence of this probability upon inspection levels without the introduction of a flawed specimen program. Inspection uncertainty analysis for evaluating the inspection reliability is presently being used by the Electric Power Research Institute in the development of a retirement-for-cause program for large steam turbines.

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APPENDIX A

This Appendix develops the basic equation that relates the potential inspection observables to the underlying correlation functions required to describe the inspection performance.

The conditional probability of event A given the event B, denoted by $P(A|B)$ is defined (30) as the ratio of the probability of the intersection of A and B to the probability of event B, *i.e.*;

$$P(A|B) = P(A \cap B)/P(B). \tag{A-1}$$

Correspondingly

$$P(B|A) = P(B \cap A)/P(A). \tag{A-2}$$

Since the probability of the intersection of A and B is equal to the probability of the intersection of B and A, one can combine Eqs. A-1 and A-2 and write

$$P(A|B)\ P(B) = P(A \cap B) = P(B \cap A) = P(B|A)\ P(A) \tag{A-3}$$

or

$$P(A|B)\ P(B) = P(B|A)\ P(A). \tag{A-4}$$

If event A is defined as the existence of a flaw of size between "a" and a + da and event B the existence of an indication of magnitude between i and i + di then

$$pd(a|i)\ da = P(A|B), \tag{A-5}$$

$$pd(i|a)\ di = P(B|A), \tag{A-6}$$

$$\frac{pn(a)\ da}{\int_0^\infty pn(a)\ da} = P(A), \tag{A-7}$$

and

$$\frac{pn(i)\ di}{\int_0^\infty pn(i)\ di} = P(B)\ . \tag{A-8}$$

Combining Eqs. A-5 through A-8 with Eq. A-4 gives

$$pd(a|i)\ pn(i) \int_0^\infty pn(a)\ da = pd(i|a)\ pn(a) \int_0^\infty pn(i)\ di\ . \tag{A-9}$$

If we assume that each indication corresponds to a flaw of some size (possibly zero) and each flaw corresponds to an indication of some size (possibly zero) then

$$\int_0^\infty pn(a)\ da = \int_0^\infty pn(i)\ di \tag{A-10}$$

and Eq. A-9 can be reduced to

$$pd(a|i)\ pn(i) = pd(i|a)\ pn(a). \tag{A-11}$$

Furthermore,

$$\int_0^\infty pd(a|i)\ da = \int_0^\infty pd(i|a)\ di = 1. \tag{A-12}$$

Integrating both sides of Eq. A-11 over i from zero to infinity gives

$$\int_0^\infty pd(a|i)\ pn(i)\ di = pn(a) \int_0^\infty pd(i|a)\ di. \tag{A-13}$$

From Eq. A-12, the integral on the right hand side of Eq. A-13 is one. Hence

$$pn(a) = \int_0^\infty pd(a|i)\ pn(i)\ di. \tag{A-14}$$

Substituting Eq. A-14 into Eq. A-11 and rearranging terms gives

$$pd(i|a) = pd(a|i)\ pn(i) \left[\int_0^\infty pd(a|i)\ pn(i)\ di \right]^{-1} . \tag{A-15}$$