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NEOCLASSICAL CONDUCTIVITY OF A TOKAMAK PLASMA

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ABSTRACT

The parallel neoclassical conductivity of a fully ionized tokamak plasma is determined for all values of the effective charge and electron collisionality parameter and for arbitrary aspect ratios. A simple analytic expression for the conductivity is obtained and shown to agree accurately with numerical results over a wide range of discharge parameters.

I. FORMULATION

An accurate calculation of the neoclassical parallel conductivity $\sigma_{||}$ is essential for obtaining an estimate of the effective charge

$$\bar{Z} = \sum_{i \neq e} n_i Z_i^2 / n_e \quad , \tag{1}$$

and hence, the impurity content of a tokamak plasma [1]. Previous calculations of $\sigma_{||}$ have been deficient in several respects. In some calculations [2,3], $\sigma_{||}$ is tabulated only for certain integral values of $\bar{Z}$, whereas non-integral $\bar{Z}$ are clearly relevant in a multispecies plasma. Rutherford and Düchs [4] have given a formula for $\sigma_{||}$, which incorrectly neglects the $\bar{Z}$ dependence of the trapping correction, arising from collisions between trapped and circulating electrons, and gives an inaccurate dependence on $\bar{Z}$ of the electron collisionality parameter. Finally, the tabulated calculations of Hinton and Hazeltine [3] are restricted to very small values of the inverse aspect ratio

$$\delta = r/R_0 \quad . \tag{2}$$

To generalize and improve these previous values for the conductivity, we have integrated numerically an expression for $\sigma_{||}$, derived by Hirshman and Sigmar [5], which is valid for all $\bar{Z}$ and throughout the three neoclassical regimes. Furthermore, straightforward application of the method discussed in Refs. [6,7] extends the present result for $\sigma_{||}$ to include finite aspect ratio effects (which are most significant in the low-collision frequency regime).

The neoclassical conductivity relates the parallel current density to the electric field averaged over a magnetic flux surface:

$$j_{||} = \sigma_{||} \langle \underset{\sim}{E} \cdot \underset{\sim}{B} \rangle B / \langle B^2 \rangle \tag{3}$$

where <A> denotes the flux surface average of A. Following Refs. [5,7], $\sigma_{||}$ may be expressed in terms of the classical Spitzer function $F_s(\bar{Z}, x_e)$, where $x_e = v/\alpha_e$ and $\alpha_e^2 = 2T_e/m_e$, as follows:

$$\sigma_{||}/\sigma_o = \frac{8}{3\sqrt{\pi}} \int_o^\infty x_e^4 e^{-x_e^2} dx_e \Big((1 - f_T^*) F_s [1 - f_T^* (\nu_D^e \tau_{ee} F_s - 1)]\Big) \tag{4}$$

Here, $\sigma_o = n_e e^2 \tau_{ee}/m_e$, $\tau_{ee} = 3m_e^2 \alpha_e^3/(16\sqrt{\pi} n_e e^4 \ln\Lambda)$ is the electron Braginskii time,

$$\begin{aligned} \nu_D^e(x_e) &= \nu_D^{ee} + \sum_{i \neq e} \nu_D^{ei} \\ &= \left(\frac{4}{3\sqrt{\pi}} \tau_{ee}\right)^{-1} \left(\frac{\Phi(x_e) - G(x_e) + \bar{Z}}{x_e^3}\right) \end{aligned} \tag{5}$$

is the total pitch angle diffusion frequency (Φ and G are the error function and Chandrasekhar function, respectively), and

$$f_T^*(x_e) = f_T\left(1 + 1.75\nu_{*e}[\nu_D^e(x_e)\tau_{ee}]x_e^{-1}\right)^{-1} \qquad (6)$$

is the effective trapped fraction as a function of electron energy $T_e x_e^2$. In Eq. (6),

$$\nu_{*e} = \sqrt{2}\,\delta^{-3/2}\,\frac{R_o q}{\alpha_e}\,\tau_{ee}^{-1} \qquad (7)$$

is the electron collisionality parameter (chosen independent of $\bar{Z}$), where $q = rB_T/(R_o B_p)$ is the safety factor, and f_T is the fraction of trapped particles in the banana regime ($\nu_{*e} \to 0$), which is given as a function of the aspect ratio [7] as follows:

$$f_T(\delta) = 1 - (1 - \delta)^2(1 - \delta^2)^{-1/2}(1 + 1.46\delta^{1/2})^{-1}\ . \qquad (8)$$

For $\nu_{*e} \to 0$, Eq. (4) agrees with the value for the banana regime conductivity of an arbitrary aspect ratio plasma [6]. When ν_{*e} is finite and $\delta << 1$, Eq. (4) reproduces the results of Ref. [5].

To complete the conductivity formula Eq. (4), the expresion for the Spitzer function F_s, obtained in Ref. [8] by a three-term Sonine polynomial expansion in x_e^2, was used:

$$F_s(\bar{Z},x_e) = \Lambda_E(\bar{Z}) - \Lambda_T(\bar{Z})L_1(x_e^2) + \frac{8}{15}(\bar{Z}^{-1} - \Lambda_E + \frac{3}{2}\Lambda_T)L_2(x_e^2) \qquad (9a)$$

where

$$\Lambda_E(\bar{Z}) = \frac{3.40}{\bar{Z}}\left(\frac{1.13 + \bar{Z}}{2.67 + \bar{Z}}\right) \qquad (9b)$$

$$\Lambda_T(\bar{Z}) = 2.06\left(\frac{1.38 + \bar{Z}}{3.23 + 4.68\bar{Z}^2 + \bar{Z}}\right) \tag{9c}$$

$$L_1(x_e^2) = \frac{5}{2} - x_e^2 \tag{9d}$$

$$L_2(x_e^2) = \frac{35}{8} - \frac{7}{2}x_e^2 + \frac{1}{2}x_e^4 \tag{9e}$$

II. COMPARISON WITH PREVIOUS CALCULATIONS

The present conductivity expression, Eq. (4), is plotted in Fig. 1 for each of two values of the effective charge $\bar{Z} = 1, 4$ and for small and finite values of $\delta = 0.05$ and $\delta = 0.2$. For comparison, the results of Ref. [3] and Ref. [4] are also plotted. For $\delta = 0.05$, the present curve agrees well with the numerical results of Ref. [3], whereas for $\delta = 0.2$, finite aspect ratio corrections produce a significant discrepancy at small to intermediate values of ν_{*e}. The failure of the formula of Ref. [4] to scale correctly with increasing $\bar{Z}$, noted previously, is only marginally discernable for the relatively small values of $\bar{Z}$ depicted in the figure.

III. APPROXIMATE ANALYTIC CONDUCTIVITY FORMULA

The expression Eq. (4) for the conductivity suggests an analytic approximation of the form

$$\sigma_{||}/\sigma_o = \Lambda_E(\bar{Z})\left(1 - \frac{f_T}{1 + \xi\nu_{*e}}\right)\left(1 - \frac{c_R(\bar{Z}) f_T}{1 + \xi\nu_{*e}}\right) \qquad (10)$$

where

$$c_R(\bar{Z}) \equiv \frac{8}{3\sqrt{\pi}} \int_0^\infty x_e^4 e^{-x_e^2} dx_e F_s(\nu_D^e \tau_{ee} F_s - 1)/\Lambda_E(\bar{Z}) \qquad (11a)$$

is the conductivity reduction due to electron-electron collisions which vanishes as $1/\bar{Z} \to 0$. c_R was computed numerically and is accurately fit by the formula

$$c_R(\bar{Z}) = \frac{0.56}{\bar{Z}}\left(\frac{3.0 - \bar{Z}}{3.0 + \bar{Z}}\right) \qquad (11b)$$

Note that c_R is nearly zero for $\bar{Z} > 3$, thus decreasing more rapidly than indicated by the dependence $c_R(\bar{Z}) = c_R(1)/\bar{Z}$ previously assumed [3]. The fact that c_R is small and negative for $\bar{Z} > 3$ is probably due to the approximate nature of F_s used here. Physically, collisions between trapped and circulating electrons should decrease $\sigma_{||}$, implying positive-definite c_R.

The parameter ξ was determined by a least squares fit of Eqs. (4) and (10), for fixed δ and $\bar{Z}$, over the range $10^{-2} < \nu_{*e} < 10^2$. The extremizing value of ξ, for the range of tokamak parameters $0 \leq \delta \leq 0.4$, $1 \leq \bar{Z} \leq 20$, was found to be (almost) independent of δ and given by the relation

$$\xi(Z) = 0.58 + 0.20\bar{Z} \quad . \tag{12}$$

[Actually, there is a small δ dependence of ξ, but since $(\delta/\xi)|d\xi/d\delta| \lesssim 0.08$, it has a negligible effect on the conductivity given in Eq. (10).]

The approximate conductivity formula Eq. (10), with $\Lambda_E(Z)$ defined in Eq. (9b), $f_T(\delta)$ in Eq. (8), $c_R(Z)$ in Eq. (11b), and $\xi(Z)$ in Eq. (12), yields a maximum error (for all values of ν_{*e} and $\bar{Z}$ in the range studied) which increases from 4.3% for $\delta = 0.1$ to 5.5% for $\delta = 0.3$ to a peak of 7.3% for $\delta = 0.4$. To illustrate the accuracy of the present approximation, Fig. 2 compares the two conductivity relations, Eqs. (4) and (10), for $\bar{Z} = 1$ (the value of effective charge for which the maximum error occurs for fixed δ) and $\bar{Z} = 4$, and the two values $\delta = 0.1$ and $\delta = 0.3$.

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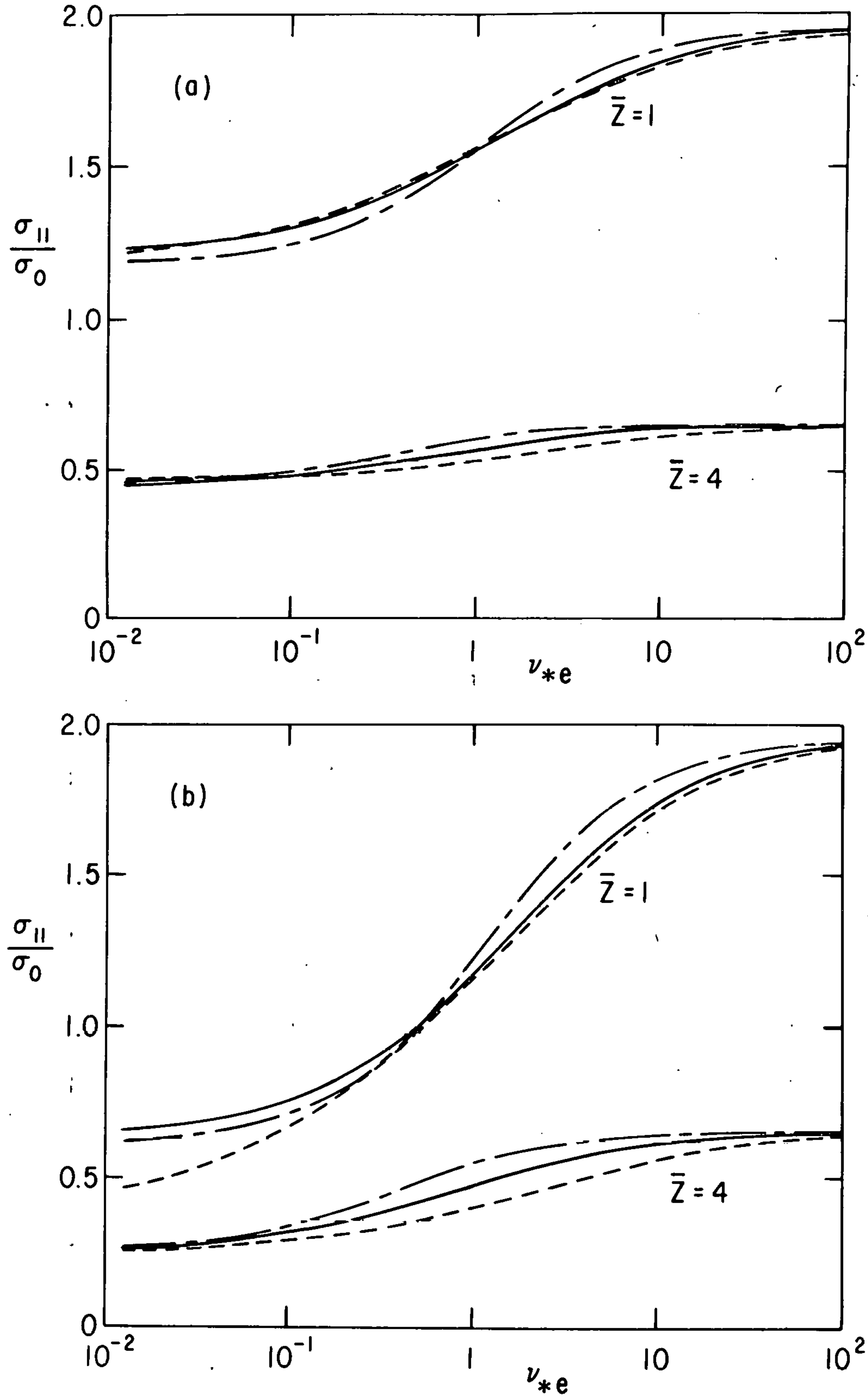

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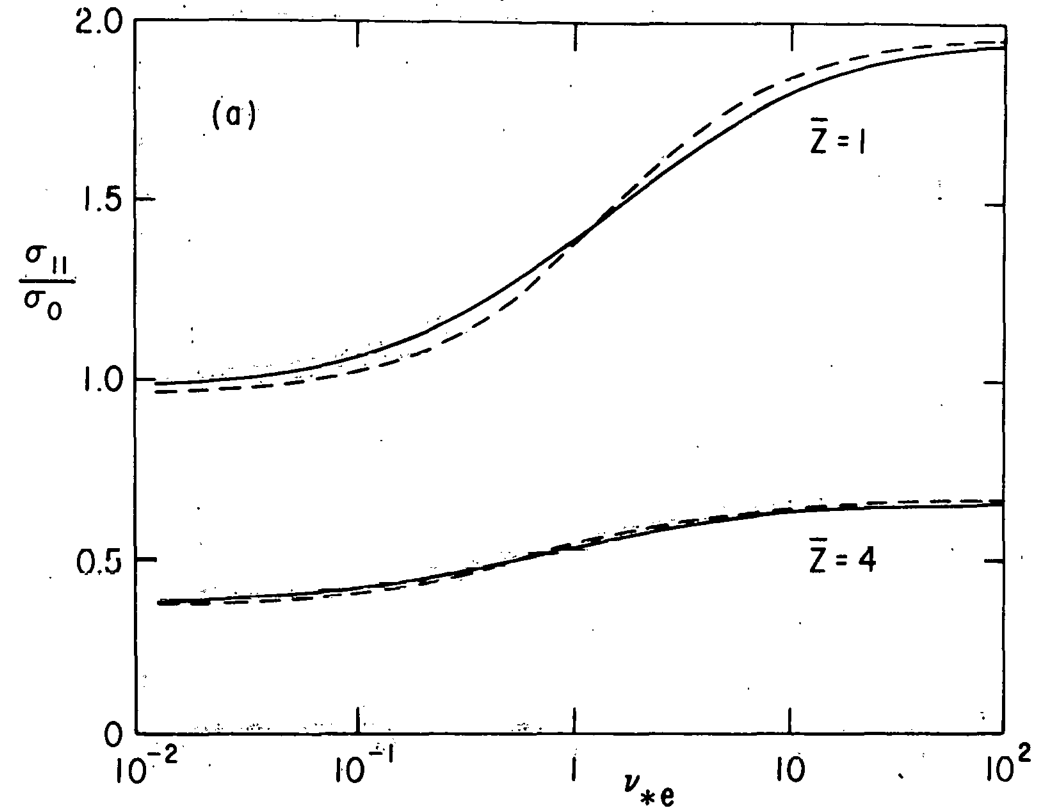
Fig. 1: Values of the conductivity for $\bar{Z}$ = 1, 4 vs. ν_{*e} computed from Eq. (4) (solid), Ref. [3] (dashed), and Ref. [4] (dashed-dotted), for (A) δ = 0.05 and (B) δ = 0.2.



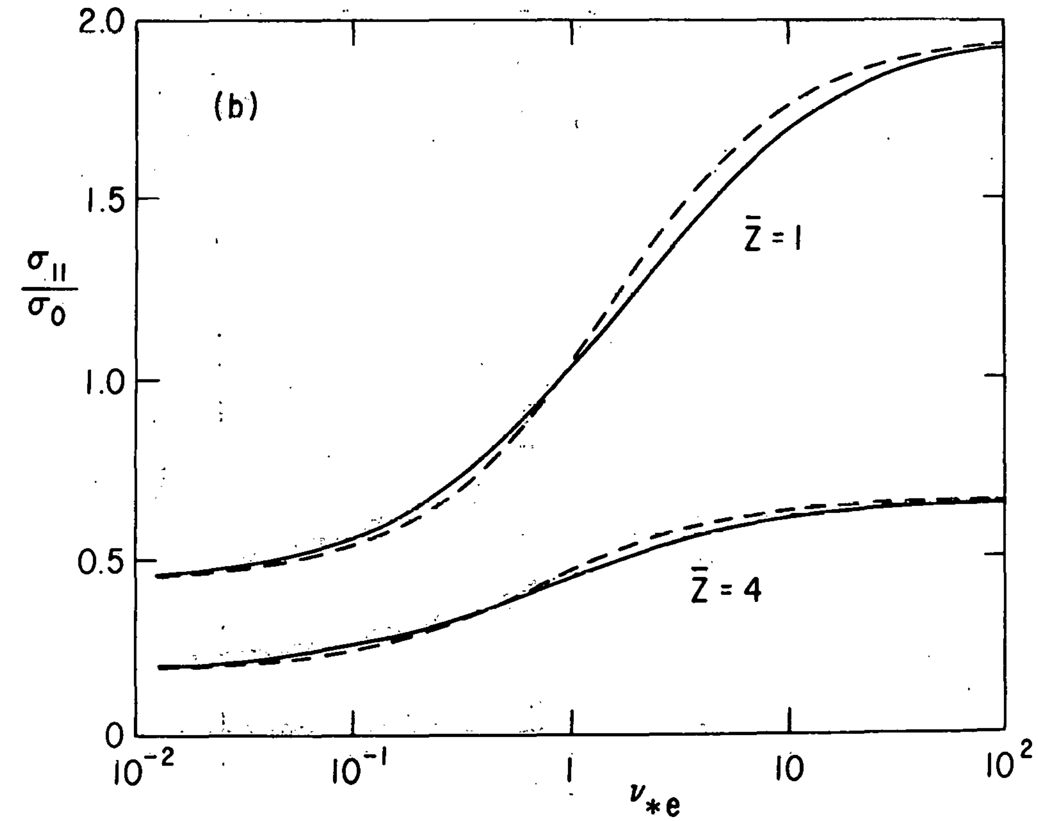


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Fig. 2: Values of the conductivity for $\bar{Z} = 1, 4$ vs. ν_{*e} computed from Eq. (4) (solid) and Eq. (10) (dashed), for (A) $\delta = 0.1$ and (B) $\delta = 0.3$.