ORO-3992-301

THE UNIVERSITY OF TEXAS AT AUSTIN

Center for Particle Theory

THE HYDRODYNAMICAL QUARK MODEL*

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Abstract

A hydrodynamical approach to the quark model of extended hadrons is proposed. Quarks are identified with certain basic fluids of the Dirac type which carry definite amounts of the electric and baryonic charges and colors. The hadrons are viewed as droplets of suitable multicomponent liquids. The model implies some interesting possibilities for a drastic reduction of the number of independent basic fields which means, of course, some very essential simplification of the quark model.

*This work has been sponsored in part by the National Science Foundation, GF 42060.

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Introduction

Whenever in the last 100 years the physicists found a material object with some extension and structure, they tried to explain all its properties in terms of suitable constituent particles. Usually, the latter were first assumed to be extensionless, so the problem could be reduced to the classical or quantum mechanics of a system of interacting point particles. One of the most conclusive confirmations of the respective models consisted always in the isolation of the constituent particles. This general pattern for constructing more and more accurate models of reality worked very well in the case of macro-objects, molecules, ions, atoms and nuclei. However, the last of such point-mechanical models, namely the quark model of hadrons[1] scored not only several impressive successes but also led to many very serious troubles.[2] The most serious of these troubles is the apparent non-existence of free quarks. The confinement of point quarks to the interior of hadrons, the strange saturation properties of the interactions which allow formation of hadrons of only three types: $q\bar{q}$, $3q$ and $3\bar{q}$, can be explained only with the help of very strange forces. Moreover, in order to explain the observed properties of hadrons on the basis of the conventional point-quark model, the number of the fundamental particles (quarks and gluons) had to be gradually increased to several tens.

In view of this highly unsatisfactory situation it is tempting to ask the fundamental questions: Can the point-mechanical formulation of the quark model be saved at all? Should we not start from a continuum model of extended hadrons which may throw completely new light on the nature and basic properties of quarks? Can the models based on conventional field quantization provide an adequate framework for the description of extended hadrons?

It should be noted that in spite of their name, the quantum field theories are based on the point-particle concept and operate essentially only with global properties of particles like their total charges, masses, spins, energies, momenta, etc. The densities which can be calculated from such quantum field theories are interpreted as probability distributions of observables referring again to point particles and not to extended particles.

All these and many other difficulties and basic shortcomings of point-mechanical as well as quantum field-theoretical approach stimulated the search for other lines of approach in which the hadrons are identified with some extended but well confined states of suitable fields, e.g., solitons, strings, bags or droplets.[3)]

The present paper contains an outline of what may be called the hydrodynamical or droplet version of the quark model.[4)] Section 1 contains the basic formulae of relativistic hydrodynamics of an ideal, neutral fluid. It is shown that in the case of a barotropic fluid all the motions are described by one vector field $V_\alpha(x)$. The invariant energy density and pressure are then

expressible in terms of one scalar function which depends nonlinearly on the invariant $V_\alpha V^\alpha$. The equations of motion can be put in a quasiparticle form and the relation between the effective mass of the quasiparticle and the total mass of the droplet is discussed. Section 2 contains the respective formulae for an electrically charged Dirac fluid. In Section 3 a system of interacting charged Dirac fluids described by some basic vector currents is discussed. The properties of the basic currents implied by the conservation rules for the electric, baryonic, leptonic and color charges are studied. Creations and annihilations as well as decays of hadrons are related to some definite properties of the non-vanishing divergences of the basic currents. Charge conservation and charge quantization impose certain integral conditions on these divergences which resemble somewhat the Bohr-Sommerfeld quantization rules of the old quantum theory.

The hydrodynamical quark model of hadrons outlined in Section 3 offers several highly interesting possibilities. First, the problem of quark confinement and saturation of forces can be solved very simply by an algebraic condition that the density of the overall color current be zero in all points of space-time. Second, the number of independent basic quark currents can be drastically reduced. In fact, the mentioned condition that the density of the color current be zero implies certain simple equalities between component currents carrying different colors. Next, no gluons are necessary, because all the strong interactions are described by non-linear but local functions of the invariants of the current-current type. Furthermore, the hydrodynamical interpretation of quarks opens an interesting possibility that the c, s, t and b quark fluids may be just different phases of the u and d fluids. This could provide some interesting interpretation of such mysterious quantum numbers like strangeness, charm, etc.

The condition that the density of the color current be zero applied to single mesons or baryons implies that there is no relative motion of different component fluids in such simple systems. Thus, from the point of view of motion a mesonic or baryonic droplet behaves as if it consisted of only one fluid of constant composition prescribed by the conventional quark model. This would restrict very considerably the multitude of the otherwise possible excitation modes of the hadronic droplets.

1. One ideal, neutral, barotropic fluid

The motion of such a fluid can be described in terms of the following basic fields:

$$\begin{aligned} &u^{\alpha}(x) \text{ - four-velocity of the fluid} \\ &\varepsilon(x) \text{ - invariant mass density} \\ &p(x) \text{ - invariant pressure} \end{aligned} \qquad (1.1)$$

The energy-momentum tensor of the ideal, neutral fluid has the familiar form [5]:

$$T^{\alpha\beta} = (\varepsilon+p)u^{\alpha}u^{\beta} - g^{\alpha\beta}p \quad , \qquad (1.2)$$

where $g^{\alpha\beta}$ is the metric tensor ($g^{00} = -g^{ii} = +1$). The equations of motion

$$(\varepsilon+p)\frac{du^{\alpha}}{ds} = g^{\alpha\beta}p_{,\beta} \qquad (1.3)$$

together with one scalar equation of the form

$$(\varepsilon u^{\alpha})_{,\alpha} = -pu^{\alpha}_{,\alpha} \qquad (1.4)$$

imply the conservation laws for energy and momentum

$$T^{\alpha\beta}_{\ \ ,\beta} = 0 \qquad (1.5)$$

and vice versa: (1.2) and (1.5) imply (1.3) and (1.4). The conventions and notations used in the above formulae are

$$u^\alpha u_\alpha = 1, \qquad h^{\alpha\beta} = g^{\alpha\beta} - u^\alpha u^\beta, \qquad \frac{d\Phi}{ds} = \Phi_{,\beta} u^\beta \ . \tag{1.6}$$

None of the vector currents u^α, ϵu^α, pu^α, that one can construct directly from our basic fields, is conserved for arbitrary motions of the fluid. However, the situation simplifies considerably if one assumes that our fluid fulfils some sort of equation of state which can be regarded as the limit of a suitable conventional equation of state for temperature $T \to 0$. To be more specific we assume that both ϵ and p depend on x only through some invariant density $\kappa(x)$:

$$\epsilon = \epsilon(\kappa(x)) \ , \qquad p = p(\kappa(x)) \ . \tag{1.7}$$

Obviously, instead of κ we can use some function $\mu(\kappa(x))$, so the choice of the basic scalar density must be dictated by its physical meaning and by the mathematical convenience. We shall take $\mu(x)$ in the form:

$$\mu = A \exp\int \frac{d\epsilon(\kappa)}{\epsilon(\kappa) + p(\kappa)} \ , \tag{1.8}$$

which leads to the following expressions for ϵ and p

$$\epsilon = f(\mu) \ , \qquad p = \mu f' - f = \mu^2 \frac{d}{d\mu}(f/\mu) \ , \tag{1.9}$$

with

$$f'(\mu) = \frac{df(\mu)}{d\mu} \ . \tag{1.10}$$

Thus, we see that both ϵ and p are expressible in terms of one function $f(\mu)$ and its derivatives with respect to μ.

Inserting (1.9) into (1.3) we obtain

$$\mu \frac{d}{ds}(mu_\alpha) = \mu m_{,\alpha} \ , \tag{1.11}$$

with

$$m = f'(\mu) \ . \tag{1.12}$$

Dividing both sides of (1.11) by μ, one gets the quasiparticle form of the equations of motion

$$\frac{d}{ds}(mu_\alpha) = m_{,\alpha} \ . \tag{1.3'}$$

These equations look like equations of motion for a particle with the x-dependent effective mass, moving with four-velocity u_α in the scalar field of forces given again by the function $f'(\mu(x))$.

Inserting (1.9) into (1.4) one obtains a rigorous conservation equation

$$(\mu u^\alpha)_{,\alpha} = 0 \tag{1.4'}$$

for the vector current $V_\alpha = \mu u_\alpha$. It can easily be seen from (1.8) that the choice of the real constant A fixes the sign of μ which is then the same in all points x of space-time. In this section we shall assume that A is positive. The absolute value of A is then fixed by a suitable normalization condition for the total current. For a one-component fluid, in which we are now interested, the normalization condition takes the form

$$\int V^\alpha d\sigma_\alpha = \int \mu u^o d_3x = 1 \quad . \tag{1.13}$$

It follows from (1.13) that μ has the dimension cm^{-3} and, consequently, m has the dimension of mass. It is obvious from (1.4') that the conserved current μu^α should be related to some conserved charge and not to the mass of the fluid. The conservation of the total mass is secured by (1.5). We shall discuss this point later when dealing with electrically charged Dirac fluids.

Let us now discuss the form of the function $f(\mu)$. First we require the mass density $f(\mu)$ be zero in points where $\mu = 0$:

$$f(\mu = 0) = 0 \quad . \tag{1.14}$$

Further restrictions are implied by the structure of the energy-momentum density tensor expressed in terms of μ and $f(\mu)$:

$$T^{\alpha\beta} = \mu f' u^\alpha u^\beta + (f - \mu f') g^{\alpha\beta} \quad . \tag{1.2'}$$

The condition that the energy density be non-negative

$$T^{oo} = \mu f'((u^o)^2 - 1) + f \geq 0 \tag{1.15}$$

for all x and in all inertial frames implies that

$$f(\mu) \geq 0 \ , \qquad \mu f'(\mu) \geq 0 \tag{1.16}$$

For $\mu \geq 0$ this means that $f(\mu)$ must be not only non-negative but it must grow monotonically with increasing μ.

Furthermore, in contrast to the conventional, macroscopic fluids at $T \neq 0$, the function $f(\mu)$ should not allow the formation of the vapor phase. In fact, the transport of matter between the vapor and the liquid phases is responsible for the instability of the macroscopic droplets and for varying amount of matter contained in them. We shall require that the shape of $f(\mu)$ allows for the existence of liquid droplets in equilibrium with the vacuum. In this way we shall be able to regard our droplets as closed systems which may occur in the ground state or several excitation states but are always surrounded by vacuum corresponding to $\mu = 0$.

It follows from the equations of motion that the inside of the droplet will be in equilibrium if μ = const and consequently p = const. In order that the forces vanish on the surface of the droplet as well, we must require

$$p(\mu(x)) = \mu f'(\mu) - f(\mu) = 0 \tag{1.17}$$

for all x and for $\mu(x) \neq 0$ and satisfying the normalization condition

$$\int \mu d_3x = 1 \quad . \qquad (1.18)$$

Equation (1.17) must have at least one positive root $\mu = \mu_c$ to allow the existence of stable droplets. For a given value of μ_c satisfying the equation

$$\mu_c f'(\mu_c) = f(\mu_c) \qquad (1.19)$$

equation (1.18) fixes essentially only the volume τ_c of the droplet in the equilibrium state, because

$$\int \mu d_3x = \mu_c \tau_c = 1 \quad . \qquad (1.20)$$

We shall also require

$$\lim_{\mu \to 0} p = 0 \qquad (1.21)$$

and the inequality

$$\left(\frac{dp}{d\mu}\right)_{\mu=\mu_c} \geq 0 \quad . \qquad (1.22)$$

This inequality is the well known stability condition which ensures that the droplet doesn't collapse. We also expect the

function $p(\mu)$ to increase asymptotically with increasing μ either to $+\infty$ or to some positive constant.

Let us now calculate the total mass of the droplet. In the general case the formula for the total mass is given by the expression

$$M = \int T^{oo}(\text{rest}) d_3x = \int_{\text{rest}} (\mu f'(\mu)(u_o^2 - 1) + f(\mu)) d_3x \, , \qquad (1.23)$$

where the tensor element is to be calculated in the overall rest frame of the droplet. Obviously, M can be calculated only after inserting $\mu(x)$ and $u_o(x)$ satisfying the eqs. (1.3'), (1.4') and (1.13). The expression for M simplifies considerably for the equilibrium state in which $u_o = 1$ and $f = f(\mu_c) = \text{const}$ inside the droplet. We get then for the equilibrium droplet

$$M(eq) = \frac{f(\mu_c)}{\mu_c} = f'(\mu_c) \quad . \qquad (1.24)$$

When we compare the formulae (1.12) and (1.23) for m and M we see that in the general case there is no simple relation between the effective mass m(x) of the quasiparticle and the total mass of the droplet. The effective mass may be much smaller than M if $\mu(x)$ just happens to have the value for which $f'(\mu)$ has the minimum value. However, for the equilibrium droplet the effective mass inside the droplet is constant and has the same value as the total mass.

It can easily be seen that no simple power function $f(\mu) = \mu^k$ can satisfy all the requirements listed above. If one considers polynomials, then one can prove that the simplest polynomial which can fulfill the requirements is of 3rd degree. If we take $p(\mu)$ of the form

$$p = a\mu^2(\mu-\mu_c) \qquad (1.25)$$

with $a > 0$, we get for $f(\mu)$ and $f'(\mu)$ the following polynomial expressions

$$f(\mu) = \frac{a}{2}((\mu-\mu_o)^3 + \mu_o^3) + b\mu \quad ,$$
$$(1.26)$$
$$f'(\mu) = \frac{3}{2}a(\mu-\mu_o)^2 + b \quad ,$$

where $\mu_o = \frac{2}{3}\mu_c$ and the constant $b \geq 0$. Both $f'(\mu)$ and $p(\mu)$ have a minimum at $\mu = \mu_o$ where $f(\mu)$ has an inflection point. The occurrence of an inflection point in which $f'(\mu)$ has a local minimum implies many highly interesting physical effects.

It can easily be seen that the linear term in $f(\mu)$ does not contribute to the value of $p(\mu)$. However, it contributes to the values of M and m. For the particular case of the equilibrium state we get

$$M_c = m(eq) = f'(\mu_c) = \frac{1}{6}a\mu_c^2 + b \quad . \qquad (1.27)$$

Of course, the potential $f'(\mu)$ or the effective mass $m(\mu)$ have this constant value only inside the equilibrium droplet. At the

boundary the value of $f'(\mu)$ jumps up to $f'(\mu=0)$ which for our model function (1.26) is equal to

$$f'(\mu=0) = \frac{2}{3}a\mu_c^2 + b \qquad (1.28)$$

which is $\frac{2}{3}a\mu_c^2$ above the minimum value equal to b. This shape of $f'(\mu)$ implies the existence of some forces which tend to confine the droplet to a finite region. In order to see more clearly the action of these forces let us rewrite the equations of motion in the following fully equivalent but perhaps slightly more transparent form

$$\frac{du^\alpha}{ds} = \eta^{\alpha\beta}W_{,\beta} \quad , \qquad (1.29)$$

where

$$W = \ln \frac{f'(\mu(x))}{B} \qquad (1.30)$$

and B is some arbitrary constant of the same dimension as $f'(\mu)$. In a local rest frame of our fluid (1.29) reduces to

$$\frac{du^i}{ds} = -\frac{\partial W}{\partial x^i} \, , \qquad i = 1,2,3. \qquad (1.31)$$

Suppose now that for $\mu \to 0$ the function $f'(\mu)$ grows to infinity (or at least becomes very large). The same is then true for the function $W(\mu)$. If one assumes that at some fixed time t the function $\mu(x)$ is spherically symmetric and has the smooth form shown on Figure 1 and $f'(\mu)$ has the form shown on Figure 2, then the potential $W(r)$ has the form indicated on Figure 3, which prevents the fluid from diluting and from escaping to infinity in the form of vapor .

Let us now discuss the highly interesting possibility that the function $p(\mu)$ has $n > 1$ roots μ_i of (1.17) satisfying the stability condition (1.22). Consider, for example, the following polynomial

$$p(\mu) = a\mu^2 \left\{ \prod_{i=1}^{n} (\mu-\mu_i) \right\} \left\{ \prod_{k=1}^{n-1} (\mu-\tilde{\mu}_k) \right\} \qquad (1.32)$$

with $a > 0$ and

$$0 < \mu_1 < \tilde{\mu}_1 < \mu_2 < \tilde{\mu}_2 < \mu_3 \cdots \qquad (1.33)$$

It can easily be checked that the inequality (1.22) is indeed satisfied at $\mu_1, \mu_2 \ldots$. Thus the function $p(\mu)$ of this form describes a fluid which can appear in n liquid phases characterized by different densities μ_i. Because of the monotonic dependence of f on μ, the invariant mass density $f(\mu_i)$ increases with i:

$$f(\mu_1) < f(\mu_2) < \cdots \qquad (1.34)$$

On the other hand the sizes of the respective equilibrium droplets containing only one phase decrease:

$$\tau_1 > \tau_2 > \tau_3 > \cdots \qquad (1.35)$$

However, no such simple order can be established in the general case for the effective or total masses of one-phase equilibrium droplets. The order of $m(\mu_i) = M(\mu_i) = f'(\mu_i)$ depends on the choice of the particular function $p(\mu)$ or $f(\mu)$. Taking suitable model functions we can have e.g. $f'(\mu_1) < f'(\mu_2)$ or vice versa. Suppose that we have only two liquid phases and that

$$M_2 = f'(\mu_2) > M_1 = f'(\mu_1) \qquad (1.36)$$

The total mass of an equilibrium droplet containing both phases will be given by the formulae

$$M = M_1\tau_1\mu_1 + M_2\tau_2\mu_2 \ , \qquad (1.37)$$
$$\tau_1\mu_1 + \tau_2\mu_2 = 1 \ ,$$

where τ_i denotes the volume of the i-th phase. Depending on the value of τ_1 the total mass can take any value between M_1 and M_2

$$M_1 < M < M_2 \qquad (1.38)$$

Strictly speaking only the droplet with the lowest possible value of mass can be stable. The phases corresponding to higher values of masses may be, however, metastable. The phase transitions leading to the formation of metastable droplets with a higher value of the total mass should be regarded as a new excitation mode which can of course combine with the conventional excitation modes of a droplet: rotations, shape oscillations, radial oscillations, etc. For a spherical droplet the phase transition discussed here may be regarded as some new sort of radial excitation which is not accompanied by an increase of the kinetic energy but by a discontinuous change of the interaction energy of the fluid and of the self-consistent potential of forces. Fig. 4a shows a schematic plot of a function $f'(\mu)$ which implies the existence of two equilibrium densities μ_1 and μ_2. The respective shapes of $\mu_1(r)$ and $\mu_2(r)$ as well as of the potentials $f'(\mu_1(r))$ and $f'(\mu_2(r))$ are shown on Figure 4b,c.

The equilibrium condition of the form (1.17) does not distinguish any particular shape of the droplet. In order to ensure the shape stability of the spherical droplet in its equilibrium ground state, we must impose some additional boundary conditions, e.g., those provided by the surface tension. The condition for the minimum of the total mass of the droplet, including the surface energy, implies then the familiar relation between the pressures in two adjacent equilibrium phases i and k separated by a spherical surface of radius R_{ik}

$$p(\mu_i) - p(\mu_k) = \frac{2\alpha_{ik}}{R_{ik}} \quad , \qquad (1.39)$$

where α_{ik} is the surface tension constant. For a one-phase liquid droplet surrounded by a vacuum this reduces to

$$\mu_s f'(\mu_s) - f(\mu_s) = \frac{2\alpha}{R} \quad . \qquad (1.40)$$

It can easily be seen that $\mu_s > \mu_c$. One can show that for suitable $f(\mu)$ the surface tension ensures also the stability of the spherical ground state droplet with respect to spontaneous decays into two or more smaller droplets. However, one cannot forbid this kind of splitting processes if enough energy is supplied, e.g., in collisions between two droplets, unless one imposes suitable quantization rules for the amount of matter and the charges of the droplets.

2. One ideal, charged, Dirac fluid

We shall now consider a simple generalization of the relativistic hydrodynamics of a barotropic fluid to the case of a fluid carrying definite amounts of the electric and baryonic charges. We shall discuss the relatively simple but physically quite interesting situation when the fluid is fully described by only one vector field

$$V_\alpha(x) = \mu(x)u_\alpha(x) \quad , \qquad (2.1)$$

where $u_\alpha(x)$ is the four-velocity and the scalar density $\mu(x) \geq 0$. The field $V_\alpha(x)$ is assumed to satisfy the same conservation equation (1.4') as in the case of the neutral fluid. Moreover, for a single droplet we shall impose the same normalization condition (1.13) as before. Suppose now that the charge currents are of the Dirac type, i.e., strictly proportional to the basic current $V_\alpha(x)$:[6)]

$$J_\alpha(x) = eV_\alpha(x) \ , \qquad B_\alpha(x) = bV_\alpha(x) \qquad (2.2)$$

The conservation equations for the charge currents follow then immediately from the conservation of $V_\alpha(x)$. Because of the normalization condition (1.13), we have for a droplet

$$\int J^0(x)d_3x = e \ , \qquad \int B^0(x)d_3x = b \qquad (2.3)$$

Therefore, the constants e and b have the meaning of the total electric or baryonic charge, respectively.

The invariant mass density $f(\mu)$ can be now regarded as the result of non-linear but local self-interaction of the type $B_\alpha(x)B^\alpha(x) = b^2\mu^2(x)$ of the baryonic current. Since b is a constant we are left with the dependence on μ alone. However, the electromagnetic interactions are definitely not of this type, because they are certainly mediated by the long-range electromagnetic field. Therefore, in the case of charged barotropic Dirac fluid we can take the hydrodynamical part of the energy-momentum tensor in the same form (1.2') as for the neutral fluid but we must add to it the familiar energy-momentum tensor of the electromagnetic field:

$$T^{\alpha\beta} = \mu f' u^\alpha u^\beta + (f-\mu f')g^{\alpha\beta} + T^{\alpha\beta}(el) \quad . \qquad (2.4)$$

The equations of motion which together with the conservation law (1.4') for $V_\alpha(x)$ imply the conservation of energy-momentum can be again written in the quasiparticle form:

$$\frac{d}{ds}(f' u_\alpha) = \frac{\partial f'}{\partial x^\alpha} + eF_{\alpha\beta}u^\beta \quad , \qquad (2.5)$$

where $F_{\alpha\beta}(x)$ is the tensor of the electromagnetic field. The equation (2.5) looks like that for a particle of electric charge e and effective mass $f'(\mu(x))$ moving with four-velocity $u_\alpha(x)$ in the scalar field $f'(\mu(x))$ and the electromagnetic field $F_{\alpha\beta}(x)$.

The presence of the electric charge and of the resulting Coulomb repulsion makes the equilibrium conditions more complicated. Of course, in the equilibrium state the force on the right hand side of (2.5) must vanish inside the droplet, which implies

$$f(\mu(\mathbf{x})) + e\phi(\mathbf{x}) = \text{const} \quad , \qquad (2.6)$$

where

$$\phi(\mathbf{x}) = e\int \frac{\mu(\mathbf{x}')}{|\mathbf{x}-\mathbf{x}'|}\, d_3x' \qquad (2.7)$$

is the electrostatic potential. The value of the constant and the necessary and sufficient equilibrium condition can be obtained from the variational principle $\delta M = 0$ with subsidiary condition (1.13), where M is the total mass of the static droplet given by the formula

$$M = \int f(\mu(\mathbf{x}))d_3x + \frac{e}{2}\int \mu(\mathbf{x})\phi(\mathbf{x})d_3x + \alpha S \quad , \qquad (2.8)$$

with S denoting the area of the surface of the droplet. The equilibrium equation obtained in this way has the form

$$f'(\mu(r)) + e\phi(r) = \frac{f(\mu(R))}{\mu(R)} + e\phi(R) + \frac{2\alpha}{R\mu(R)} \quad , \qquad (2.9)$$

where R is the radius of the droplet. Since $\phi(R)$ is continuous at $r = R$ it follows that not only $\mu(r)$ but also $p(r)$ has a jump

at this point. In fact we find that

$$p(R) = \mu(R)f'(R) - f(\mu(R)) = \frac{2\alpha}{R} \qquad (2.10)$$

and, of course, $p(r) = 0$ for $r > R$. Thus, the discontinuity of the pressure on the surface of the equilibrium droplet has the same form like that for the neutral droplet, but the value will be, of course, different because of different R.

3. Hydrodynamical quark model of hadrons

In this section we shall outline a multicomponent hydrodynamical model of hadrons. The multicomponent fluid will be described by J basic vector currents

$$V_j^{\alpha}(x) = \mu_j(x)u_j^{\alpha}(x) \; ; \qquad \mu_j(x) \geq 0 \; ; \qquad j = 1,2,\ldots,J \quad . \tag{3.1}$$

We shall identify the quarks with these basic currents, or, perhaps, with the corresponding quasiparticles appearing in the respective equations of motion of our fluid. Thus our model can be regarded as a new version of the quark model.

Let us first discuss some general properties of the basic currents and in particular those which follow from the assumed quark concept and from the rigorous conservation laws of the charges. We shall assume that our basic hadronic or quark currents (3.1) are Dirac currents, which are carrying definite amounts of the electric and baryonic charges as well as the three basic colors $\mathbf{c}_1$, $\mathbf{c}_2$, $\mathbf{c}_3$. The colors may be regarded as some new kind of charges specified by two numbers. Thus the colors can be visualized as suitable vectors of the color plane, and, correspondingly, will be denoted by bold-face letters. Because of the particular role played by colors it is convenient to regard j as a double index: j = (ik) with k = 1,2,3 and i = 1,2,...,h, denoting respectively the three colors and the remaining properties necessary to specify a quark. The

value of h will be left open in the following formulae. In the original quark model we had h = 3, then after the discovery of charm it was raised to 4, but some models require h = 6. The general scheme presented here does not depend on the particular value of h.

To each basic quark current V_{ik}^{α} there corresponds an antiquark current which will be denoted by the symbol $\bar{V}_{ik}^{\alpha}$. The respective basic electric, baryonic and color charge currents are proportional to V_{ik}^{α} or $\bar{V}_{ik}^{\alpha}$:

$$\begin{aligned} J_{ik}^{\alpha}(x) &= e_i V_{ik}^{\alpha}(x) \; , \qquad & \bar{J}_{ik}^{\alpha}(x) &= -e_i \bar{V}_{ik}^{\alpha}(x) \; , \\ B_{ik}^{\alpha}(x) &= b_i V_{ik}^{\alpha}(x) \; , \qquad & \bar{B}_{ik}^{\alpha}(x) &= -b_i \bar{V}_{ik}^{\alpha}(x) \; , \\ \mathbf{C}_{ik}^{\alpha}(x) &= \mathbf{c}_k V_{ik}^{\alpha}(x) \; , \qquad & \bar{\mathbf{C}}_{ik}^{\alpha} &= -\mathbf{c}_k \bar{V}_{ik}^{\alpha}(x) \; , \end{aligned} \tag{3.2}$$

with the values of the proportionality constants e_i, b_i, $\mathbf{c}_k$ taken from the conventional quark model. Thus $b_i = 1/3$, $e_1 = e_3 = (e_5 =)\ 2/3\ e$, $e_2 = e_4 = (e_6 =)\ -1/3\ e$, where e is the value of the (positive) elementary charge. The constants $\mathbf{c}_k$ are three unit vectors of the color plane which intersect at 120°C, e.g.

$$\mathbf{c}_1 = (\sqrt{3}/2, 1/2) \; , \quad \mathbf{c}_2 = (-\sqrt{3}/2, 1/2) \; , \quad \mathbf{c}_3 = (0,-1) \quad . \tag{3.3}$$

In the same way we shall treat the four basic leptonic currents $L_\ell^\alpha(x)$ and the corresponding anticurrents $\bar{L}_\ell^\alpha(x)$ where $\ell = 1$ denotes electron, 2-electronic neutrino, 3-muon , 4-muonic neutrino. The currents L_1 and L_2 are carrying one kind of leptonic charge which is called electronic, and the currents L_3 and L_4 are carrying the second kind of leptonic charge which is called muonic. *We shall assume that these two leptonic charges are separately conserved.* Moreover, the leptonic currents 1 and 3 carry the electric charge as well. Assuming that the leptonic currents are also of the Dirac type we have

$$J_\ell^\alpha(x) = e_\ell L_\ell^\alpha(x) \ , \qquad \bar{J}_\ell^\alpha(x) = -e_\ell \bar{L}_\ell^\alpha(x) \ , \tag{3.4}$$

with $e_1 = e_3 = -e$, $e_2 = e_4 = 0$.

The expressions for the overall electric, baryonic, color and leptonic currents in any particular physical system are given *by the following sums.*

$$J^\alpha(x) = \sum_{ik} e_i(V_{ik}^\alpha - \bar{V}_{ik}^\alpha) + \sum_\ell e_\ell(L_\ell^\alpha - \bar{L}_\ell^\alpha) \ ,$$

$$B^\alpha(x) = \sum_{ik} b_i(V_{ik}^\alpha - \bar{V}_{ik}^\alpha) \ , \tag{3.5}$$

$$\mathbf{C}^\alpha(x) = \sum_{ik} \mathbf{c}_k(V_{ik}^\alpha - \bar{V}_{ik}^\alpha) \ ,$$

$$L^\alpha(x) = \sum_{\ell=1,2}(L_\ell^\alpha - \bar{L}_\ell^\alpha) \ , \qquad \Lambda^\alpha(x) = \sum_{\ell=3,4}(L_\ell^\alpha - \bar{L}_\ell^\alpha)$$

All these currents must *be strictly conserved*, i.e.,

$$J^\alpha(x)_{,\alpha} = 0 \ , \qquad B^\alpha(x)_{,\alpha} = 0 \ , \qquad \mathbf{C}^\alpha(x)_{,\alpha} = 0 \ ,$$
$$L^\alpha(x)_{,\alpha} = 0 \ , \qquad \Lambda^\alpha(x)_{,\alpha} = 0 \ . \tag{3.6}$$

It can easily be checked that the baryon and overall color conservation laws are equivalent to the three conservation laws for each color separately

$$\sum_i (V_{ik}^\alpha(x) - \bar{V}_{ik}^\alpha(x))_{,\alpha} = 0 \ , \qquad k = 1,2,3 \ . \tag{3.7}$$

These rigorous conservation equations can be satisfied in many ways depending on the choice of the system and its initial state. The simplest non-trivial solution is obtained, if we require the four-divergence of each basic current to vanish in all points of space-time

$$V_{ik,\alpha}^\alpha = \bar{V}_{ik,\alpha}^\alpha = L_{\ell,\alpha}^\alpha = \bar{L}_{\ell,\alpha}^\alpha = 0 \ . \tag{3.8}$$

Equations (3.7) imply that the values of the integrals

$$\int V_{ik}^0 d_3x = n_{ik} \ , \qquad \int \bar{V}_{ik}^0 d_3x = \bar{n}_{ik}$$
$$\int L_\ell^0 d_3x = n_\ell \ , \qquad \int \bar{L}_\ell^0 d_3x = \bar{n}_\ell \tag{3.9}$$

are constant. In the case of a one-component fluid we have imposed on the current describing one droplet the normalization condition (1.13). We generalize that condition requiring it to be valid for each basic current separately. However, since we may have 0,1,2,... droplets or quanta of each component fluid in the physical state and system of interest, we impose the condition that the constant n's appearing in (3.9) be non-negative integers.

$$n_{ik},\ \bar{n}_{ik},\ n_{\ell},\ \bar{n}_{\ell} = 0,\ 1,\ 2,\ \ldots \qquad (3.10)$$

This means that the equations (3.8) describe systems and processes in which the numbers of basic quarks and antiquarks as well as leptons and antileptons of each kind remain constant. Only some rearrangement collisions are allowed by (3.8) but any creation or annihilation processes are forbidden. This is usually regarded to be a necessary condition for the system to be describable in terms of particles and their mutual interactions both in classical as well as in quantum mechanics. It is generally believed that the description of creation and annihilation of particle-antiparticle pairs, decays, etc., requires the formalism of quantum field theory.

However, we shall show now that the hydrodynamical droplet picture of the elementary particles offers another possibility for the description of creation, annihilation and decay processes. In fact the conditions (3.8) are sufficient but by no means

necessary. One can satisfy (3.6) by imposing other, less restrictive, conditions. So, the strong and electromagnetic interactions are characterized by the following conservation laws for the basic currents

$$V^{\alpha}_{ik}(x)_{,\alpha} - \bar{V}^{\alpha}_{ik}(x)_{,\alpha} = 0\ , \quad L^{\alpha}_{\ell}(x)_{,\alpha} - \bar{L}^{\alpha}_{\ell}(x)_{,\alpha} = 0\ . \quad (3.11)$$

The common value of the four-divergence of each basic current and the corresponding anticurrent may be zero in all points of space-time, or it may be different from zero for some part of currents in some regions which we shall call briefly creation regions. The first case reduces to the already discussed above, so let us consider now the second case in which, e.g.,

$$V^{\alpha}_{ik}(x)_{,\alpha} = \bar{V}^{\alpha}_{ik}(x)_{,\alpha} = a_{ik}(x) \neq 0 \qquad (3.12)$$

in some region of x and for some definite values of the indices i, k. Equations (3.11) and (3.12) imply that only the differences

$$n_{ik}(t) - \bar{n}_{ik}(t) = N_{ik} = 0,\ \pm 1,\ \pm 2, \ldots \qquad (3.13)$$

are constant but the numbers n_{ik} and $\bar{n}_{ik}$ may be not. The numbers n_{ik} and $\bar{n}_{ik}$ are non-negative integers before the creation period with values specified by the initial conditions of the system. However, during the creation period they change continuously in time till they assume new integer values at the end of this period. Let us denote by K the three-dimensional region of space outside of which the currents V^{α}_{ik} and $\bar{V}^{\alpha}_{ik}$ are zero during all the creation period. Of course K contains the creation region in which $a_{ik}(t,\mathbf{x}) \neq 0$. Integration of the first equation of (3.12) over K gives

$$\int_K \frac{\partial}{\partial t} V^{0}_{ik} d_3x + \int_K \operatorname{div} \mathbf{V}_{ik} d_3x = \int_K a_{ik}(t,\mathbf{x}) d_3x = A_{ik}(t) \qquad (3.14)$$

The second integral is equal to zero because it can be transformed into a surface integral over the surface enclosing K, where $V^{\alpha}_{ik} = 0$. So we find

$$\frac{d}{dt}\int_K V^{0}_{ik} d_3x = \frac{d}{dt}\int_K \bar{V}^{0}_{ik} d_3x = A_{ik}(t) \qquad (3.15)$$

or

$$\frac{d}{dt} n_{ik}(t) = \frac{d}{dt}\bar{n}_{ik}(t) = A_{ik}(t) \ . \qquad (3.16)$$

Integrating (3.16) from a time t_1 before the creation period to a time t_2 after the creation period we get

$$n_{ik}(t_2) - n_{ik}(t_1) = \bar{n}_{ik}(t_1) - \bar{n}_{ik}(t_2) = \int_{t_1}^{t_2} A_{ik}(t)dt \ . \qquad (3.17)$$

It can easily be seen that the integrations involved in (3.14) and (3.17) can be extended over the whole space-time. Then (3.17) can be written in the form

$$n_{ik}(\infty) - n_{ik}(-\infty) = \bar{n}_{ik}(\infty) - \bar{n}_{ik}(-\infty) = \int a_{ik}(x) d_4x \qquad (3.18)$$

Because of (3.13) we must have

$$\int a_{ik}(x) d_4x = \int V^{\alpha}_{ik}(x)_{,\alpha} d_4x = 0, \pm 1, \pm 2, \dots \qquad (3.19)$$

with the integral being extended over the whole space-time.

The second set of equations (3.11) for the leptonic currents implies similar formulae

$$L^{\alpha}_{\ell}(x)_{,\alpha} = \bar{L}^{\alpha}_{\ell}(x)_{,\alpha} = b_{\ell}(x) \ . \qquad (3.20)$$

Hence

$$n_{\ell}(t) - \bar{n}_{\ell}(t) = N_{\ell} = 0, \pm 1, \pm 2, \dots = \text{const} \qquad (3.21)$$

$$n_{\ell}(\infty) - n_{\ell}(-\infty) = \bar{n}_{\ell}(\infty) - \bar{n}_{\ell}(-\infty) = \int b_{\ell}(x) d_4x = 0, \pm 1, \pm 2, \dots \qquad (3.22)$$

It is worthwhile stressing that *in the case of processes induced by the strong and electromagnetic interactions*, for which (3.11) is supposed to hold, only exactly matched pairs can be created or annihilated, e.g., one ik-quark together with one ik-antiquark, one ℓ-lepton together with one ℓ-antilepton, etc. This is no more true for most slow processes induced by the weak interactions, except very few processes like the elastic eν

scattering or the scattering of a neutrino by a hadron. In a very wide class of leptonic, semileptonic and non-leptonic processes either one quark is transformed into another one, or some unmatched quark and lepton pairs are created or annihilated. The divergencies of the respective currents must be then different from zero. However, their values can be always chosen so as to satisfy the conservation laws (3.6). The procedure to be applied is based on the fact that both the baryon and overall color are conserved if equations (3.7), which involve summation over i, are satisfied for each k. Similarly we can satisfy the lepton conservation laws requiring only the equality of the sums

$$\sum_{\ell=1,2} L^{\alpha}_{\ell,\alpha} = \sum_{\ell=1,2} \bar{L}^{\alpha}_{\ell,\alpha} \quad , \quad \sum_{\ell=3,4} L^{\alpha}_{\ell,\alpha} = \sum_{\ell=3,4} \bar{L}^{\alpha}_{\ell,\alpha} \quad . \tag{3.23}$$

Equations (3.7) and (3.23) can be satisfied in many ways depending on the particular process of interest. For example, one can satisfy (3.7) putting for $i \neq j$:

$$V^{\alpha}_{ik,\alpha} = \bar{V}^{\alpha}_{jk,\alpha} \neq 0 \ , \quad \text{or} \quad V^{\alpha}_{ik,\alpha} = -V^{\alpha}_{jk,\alpha} \neq 0 \ ,$$
$$\text{or} \quad \bar{V}^{\alpha}_{ik,\alpha} = -\bar{V}^{\alpha}_{jk,\alpha} \neq 0 \quad , \tag{3.24}$$

with all the remaining divergencies in the sum (3.7) equal to zero. If the electric charge is exchanged between hadrons and leptons, then similar cross relations must be postulated for the leptonic currents. Next step consists in inserting the

non-vanishing divergencies of the leptonic and quark currents into the conservation equation for the electric charge which imposes some relation between these divergencies. For example the decay

$$\pi^- \to e^- + \bar{\nu} \tag{3.25}$$

implies the following relation between the currents which appear in this decay

$$V^{\alpha}_{2k,\alpha} = \bar{V}^{\alpha}_{1k,\alpha} = - L^{\alpha}_{1,\alpha} = - \bar{L}^{\alpha}_{2,\alpha} = -h(x) \tag{3.26}$$

with the function h(x) satisfying the integral condition

$$\int h(x) d_4x = 1 \quad . \tag{3.27}$$

It is interesting to note that the conservation laws for the charges imply the equality (apart from sign) of the divergencies of our basic quark and leptonic currents.

Take a more complicated example of the non-leptonic decay

$$K^- \to 2\pi^- + \pi^+ \ . \tag{3.28}$$

In this case we find from (3.6) the following relations

$$\bar{V}^{\alpha}_{1k,\alpha} = V^{\alpha}_{1k,\alpha} = h(x) , \qquad \bar{V}^{\alpha}_{2k,\alpha} = V^{\alpha}_{2k,\alpha} + V^{\alpha}_{4k,\alpha} = i(x) \tag{3.29}$$

$$V^{\alpha}_{4k,\alpha} = -j(x)$$

with h(x), i (x) and j(x) satisfying

$$\int h(x)d_4x = \int i(x)d_4x = \int j(x)d_4x = 1 \quad . \tag{3.30}$$

Consider now the role of color in our hydrodynamical quark model. Because of the rigorous color conservation law we can define the total color of a system of quark currents

$$\mathbf{C} = \sum_{i,k} \mathbf{c}_k \int (V^o_{ik} - \bar{V}^o_{ik}) d_3x = \sum_k \mathbf{c}_k (n_k - \bar{n}_k) \quad , \tag{3.31}$$

where

$$n_k(t) = \sum_i n_{ik}(t) \quad , \quad \bar{n}_k(t) = \sum_i \bar{n}_{ik}(t) \tag{3.32}$$

denote respectively the total number of quarks and antiquarks of color k present in the system at time t. The numbers n_k and $\bar{n}_k$ may change in time during the creation period but their differences remain constant. In agreement with the conventional quark model we postulate that the value of $\mathbf{C}$ must be exactly zero:

$$\mathbf{C} = 0 \tag{3.33}$$

for all physical systems. There are three fundamental non-trivial solutions of (3.33) with non-negative integer values of n_k and $\bar{n}_k$:

$$a)\ n_k = \bar{n}_k = 1 \ , \quad b)\ n_k = 1,\ \bar{n}_k = 0 \ , \quad c)\ n_k = 0 \ ,\ \bar{n}_k = 1 \tag{3.34}$$

The first of these basic solutions describes one meson, the second one baryon and the third one antibaryon. All the other solutions of (3.33) can be reduced to these basic solutions and interpreted as describing systems with different numbers of mesons, baryons and antibaryons. In fact the integration in (3.31) may be performed over any finite region of space K and we should require $\mathbf{C}_K = 0$ if all the hadronic currents vanish on the surface enclosing K. Before, and after the creation period, one can always divide the whole space into non-overlapping regions each of which contains only one hadron or a nucleus, i.e. a cluster of strongly interacting hadrons. The color of each of these non-overlapping subsystems must be also zero: $\mathbf{C}_K = 0$, which of course implies

$$\mathbf{C} = \sum_K \mathbf{C}_K = 0 \quad . \tag{3.35}$$

In this way the condition (3.33) in fact ensures that no hadronic droplets with nonvanishing $\mathbf{C}$ can be produced. Thus the basic hadronic droplets or elementary particles are either two-component fluids of the type $q\,\bar{q}$ or three-component fluids of the type 3q or $3\bar{q}$, with all the three quarks bearing different colors.

In the conventional quark model which treats quarks as point particles (or at least as particles much smaller than the hadrons) the density of color cannot vanish in all points of space-time. Therefore, some dynamical justification of (3.33) in terms of suitable forces is necessary. Thus, several highly unconventional forces have been proposed to keep the quarks of different colors together and to prevent creation of colored particles. In the hydrodynamical model one can also follow this path: first to allow relative motions of different color fluids and then to restrict these motions by suitable forces so that no colored droplet can be created. We shall show below on a simple model how this can be done. However, it is interesting to note that the hydrodynamical approach offers another, much simpler and plausible possibility, which does not require any forces between different color quarks because they have no possibility to separate even locally within the hadrons. In fact, we may require that not only the total color $\mathbf{C}$ given by the integral (3.31) be zero but also the color density current be exactly zero.

$$\mathbf{C}^{\alpha}(x) = \sum_{ik} \epsilon_k (V^{\alpha}_{ik}(x) - \bar{V}^{\alpha}_{ik}(x)) = 0 \qquad (3.36)$$

in all points x of space-time, and for all the physical systems. It can easily be checked that (3.36) is equivalent to the slightly

more appealing equality of all three color currents:

$$\sum_i (V^{\alpha}_{i1} - \bar{V}^{\alpha}_{i1}) = \sum_i (V^{\alpha}_{i2} - \bar{V}^{\alpha}_{i2}) = \sum_i (V^{\alpha}_{i3} - \bar{V}^{\alpha}_{i3}) \quad . \qquad (3.37)$$

If (3.36) or (3.37) are rigorous and general conditions, then no local excess of any color can emerge and we don't need to worry about the nature of forces keeping different color quarks together. Obviously the hadronic matter is then a very peculiar multicomponent fluid with very restricted color composition given by (3.37).

Applied to a meson which is a $q\bar{q}$ system, equation (3.37) implies

$$V^{\alpha}_{ik}(x) = \bar{V}^{\alpha}_{jk}(x) \quad (= V^{\alpha}_{ijk}(x)) \quad . \qquad (3.38)$$

Applied to a baryon which is a 3q system it gives

$$V^{\alpha}_{h1}(x) = V^{\alpha}_{i2}(x) = V^{\alpha}_{j3}(x) \quad (= V^{\alpha}_{hij}(x)) \quad . \qquad (3.39)$$

Thus, both in a mesonic and a baryonic droplet there is only one independent current which describes all the motions of the component fluids. For any given composition of the droplet we shall have only one vector equation of motion of the form discussed in Sec. 2 for one charged fluid, instead of a set of two or three coupled equations of motion for each component fluid.

This is, of course, an immense simplification of the problem of motion. However, not only the value of the total electric charge of the droplet but also the equation of state of the fluid and the invariant energy density may depend in general on the quark content, i.e., on the indices ij or hij. Some additional symmetry requirements can be easily imposed on the interaction functions, if necessary.

It follows from (3.38) and (3.39) that the different component fluids completely penetrate, i.e., occupy the same region of space-time. They form some kind of a perfect fluid solution which is in many respects similar to a solution of two or three macroscopic liquids (for example: water, alcohol and glycerine). Such a multifluid solution will have equal composition in all points x and will move like one pure liquid. However, the equation of state, the energy density, etc., depend in general on the composition, and the velocity, pressure, density, etc., may vary from point to point. Of course, the general validity of equations (3.37) must be tested more carefully. It seems that they can be generally valid at all energies and for all kinds of physical processes including creation, annihilation, and transmutation processes.

If (3.36) or (3.37) is only an approximation valid for certain processes and states (e.g. for equilibrium states of the hadronic droplets and lowest excitation states), then the problem of forces keeping the different color fluids together comes up

again. We shall present now a relatively simple version of such forces which should be applicable to isolated hadronic droplets and to the purely elastic scatterings. These restrictions stem from the fact that we shall assume that each of the basic currents present in the initial state is conserved, so no creation or annihilation processes will be allowed. Each of the component fluids, which we shall again denote by one index j, is held together by some non-linear local interaction fully described by the invariant energy density expressed as a function of the invariant density of the respective current: $f_j(\mu_j)$. We shall assume that the functions $f_j(\mu_j)$ have the same general properties as those discussed for one fluid in Section 1. We shall now discard the condition (3.36) and allow the different component fluids to move with respect to each other. In order to prevent the separation of any one color droplet we must introduce suitable interaction between different fluids. The simplest way to achieve it is to add to the invariant energy density a term $g(\mu)$ where

$$\boldsymbol{\mu} = \sum_j \boldsymbol{\epsilon}_j \mu_j \ , \qquad |\boldsymbol{\mu}| = \mu \tag{3.40}$$

is a color vector but a Lorentz scalar. It can easily be seen that

$$\boldsymbol{\mu}^2 = \mu^2 = (\mu_1 - \mu_2)^2 \tag{3.41}$$

for a meson, and

$$\mu^2 = \mu^2 = (\mu_1^2 + \mu_2^2 + \mu_3^2 - \mu_1\mu_2 - \mu_2\mu_3 - \mu_3\mu_1) \quad (3.42)$$

for a baryon. The color vector $\boldsymbol{\mu}$ *vanishes if all the invariant* densities are equal, i.e. $\mu_j(x) = \mu(x)$ for all x. In order to prevent separation of colors we assume that the function $g(\mu)$ and its derivative $g'(\mu)$ with respect to μ vanish at $\mu = 0$ but grow rapidly with growing μ. A schematic plot of $g(\mu)$ and $g'(\mu)$ is indicated on Fig. 5.

The equations of motion for a system of interacting fluids written in the quasiparticle form are

$$\frac{d}{ds_j}(m_j u_{j\alpha}) = m_{j,\alpha} + e_j F_{\alpha\beta} u_j^\beta \quad (3.43)$$

very much resembling the equation (2.5) for one charged fluid but for a different expression for m_j which have now the form

$$m_j = f_j'(\mu_j) + \boldsymbol{\epsilon}_j \frac{\partial g}{\partial \boldsymbol{\mu}} \quad . \quad (3.44)$$

The gradient of $g(\mu)$ appearing in (3.44) refers, of course, to the color plane and is given by

$$\frac{\partial g(\mu)}{\partial \boldsymbol{\mu}} = \frac{dg}{d\mu} \frac{\boldsymbol{\mu}}{\mu} \quad . \quad (3.45)$$

The second term in (3.44) gives the contribution of the initial

interaction between different component fluids to the effective mass and the scalar potential of forces acting on j-th fluid.

The corresponding energy momentum tensor has the form

$$T^{\alpha\beta} = \sum_j \mu_j (f_j' + \boldsymbol{\epsilon}_j \frac{\partial g}{\partial \boldsymbol{\mu}}) u_j^\alpha u_j^\beta + g^{\alpha\beta} (\sum_j (f_j - \mu_j f_j') + g - \mu g') + T^{\alpha\beta}(el) \quad . \quad (3.46)$$

It can be easily verified that the equations of motion (3.43) and the conservation equation for the component currents $(\mu_j u_j^\alpha)_{,\alpha} = 0$ imply the energy-momentum conservation

$$T^{\alpha\beta}{}_{,\alpha} = 0 \quad .$$

For an equilibrium droplet at rest we can assume no motions, so $u_j^0 = 1$ and the energy density assumes the simple form

$$T^{00}(eq) = \sum_j f_j(\mu_j) + g(\mu) + T^{00}(el) \quad . \quad (3.47)$$

If we add the surface energy $\alpha_j S_j$ for each component fluid we obtain for the total mass of the droplet the following expression

$$M(eq) = \sum_j \int f_j(\mu_j) d_3x + \int g(\mu) d_3x + \sum_j \alpha_j S_j + \frac{1}{2} \sum_j e_j \int \mu_j \phi d_3x \quad . \quad (3.48)$$

The radius of the spherical equilibrium droplet and the functions

$\mu_j(r)$ can be obtained from the variational principle

$$\delta M[\mu_j(r)] = 0 \qquad (3.49)$$

with the subsidiary conditions

$$4\pi\int_0^{R_j} \mu_j(r)r^2dr = 1 \quad . \qquad (3.50)$$

The general structure of the mass formula (3.48) is relatively simple. The first term gives the contributions of the self-interaction energies of each component fluid, the second is due to the initial interaction between different fluids, the third is the sum of the surface energies of the component droplets, and the last is the Coulomb energy.

So far we have not imposed any symmetry requirements upon the functions $f_j(\mu_j)$ and the surface coefficients α_j. Suppose now that all α_j are equal and also the invariant energy densities

$$f_j(\mu_j) = f(\nu_j) \qquad (3.51)$$

are given by the same function. If we then neglect the Coulomb interaction, we find that these symmetry properties of the self-interactions imply that in the equilibrium state all densities $\mu_j(r)$ are equal. Consequently, $\mu = 0$ and the interaction term vanishes due to $g(0) = 0$. In such a symmetric droplet the total

mass is simply the sum of masses of all the component droplets which are approximately equal to their effective masses given by $f'(\mu_j)$.

This cannot be true for hadrons containing both non-strange and strange quarks because the experimental data seem to indicate that the strange quark must be heavier than the non-strange ones. In order to achieve this in our model, we must either break the symmetry (3.51) of the self-interaction functions or to keep this symmetry but assume that the strange quark is described by another phase of the d-liquid. The first possibility can be made compatible with the requirement (3.37) if all $\mu_j(x)$ in an equilibrium droplet are equal in spite of different f_j. The second case is rather incompatible with (3.37) unless the phase transitions between strange and non-strange quark is not of the first kind as described in Sec. 1, but of the second kind which is not accompanied by a change of density.

Obviously the same reasoning can be applied to the c, t, b quarks. Within the proposed hydrodynamical quark model there is definitely some chance for reducing the number of independent basic fluids by regarding the c and t fluids as different phases of u, and similarly s and b fluids as different phases of the d fluid. In this way we would be left with only two basic fluids u and d for each color. If (3.37) holds, then from the point of view of the equations of motion the number of independent fluids is indeed quite small. The possibility of such reduction of the number of independent fields is one of the most attractive features of the hydrodynamical approach to the quark model. One should also note that we have used no gluons, which means another essential reduction of the number of different fields in comparison to that used in other quark models.

Conclusion and Outlook

The hydrodynamical quark model outlined in this paper is a specific non-linear theory of interacting vector fields which are interpreted as basic currents of the hadronic matter. All the short-range interactions are described in terms of suitable local but non-linear functions of the current-current invariants. It turns out that some plausible physical requirements impose several essential restrictions upon the form of these functions.

Although the paper uses the formalism of classical hydrodynamics some integral conditions have been imposed on the currents which have the meaning of the quantization rules related to the charges. The next step should consist in the quantization of motions of the hadronic droplets. However, the presented version of the hydrodynamical model cannot be regarded as fully realistic because it neglects the spins of the quark droplets (unless the spin dependence can be factorized out). Nevertheless, many of the basic motions and relations discussed in this paper remain valid also in the more realistic version based on multi-component Dirac spinors that will be presented in another paper.

Obviously the conditions (3.38-39) and the corresponding equations of motion for a one-component fluid presented in Sec. 1 and 2, or--for the second version of our model--the equations (3.43-47) are valid only if the numbers of constituent quark droplets are constant. This means that the degrees of freedom related to the creation or annihilation of quark pairs are frozen. This may be a good approximation only for a restricted class of states and processes, e.g., for the ground states and lowest excited states of single hadrons, when the divergence of each basic current can be assumed to vanish. The description of more involved systems in which pairs can be created or annihilated requires more general forms of the equations of motion and of the energy-momentum tensors which allow the divergences of the basic currents to be different from zero. It is interesting to note that the conditions (3.36) or (3.37) are free from such restrictions and thus can be assumed to be generally valid for all physical systems. We shall study the possible more general forms of the equations of motion which are free of the mentioned restrictions in a separate paper.

The author expresses his thanks to the Department of Physics of the University of Texas at Austin and in particular to the Professors A. Bohm, T. Griffy and G. Sudarshan for all the hospitality extended to him during his visit to Austin where the major part of this work has been done.

Figure Captions

Fig. 1. *Tentative smooth shape of the invariant mass distribution as a function of r* (non-equilibrium case).

Fig. 2. Schematic behaviour of the function $f'(\mu)$.

Fig. 3. The shape of the potential $W(r)$ resulting from Fig. 1 and 2.

Fig. 4. The case of two phases corresponding to two different densities.

4a. Schematic shape of $f'(\mu)$ with two minima.

4b. Two functions $\mu_1(r)$ and $\mu_2(r)$ satisfying eq. (1.17).

4c. The corresponding shapes of the potentials $W_1(r)$ and $W_2(r)$.

Fig. 5. Schematical shapes of $g(\mu)$ and $g'(\mu)$.

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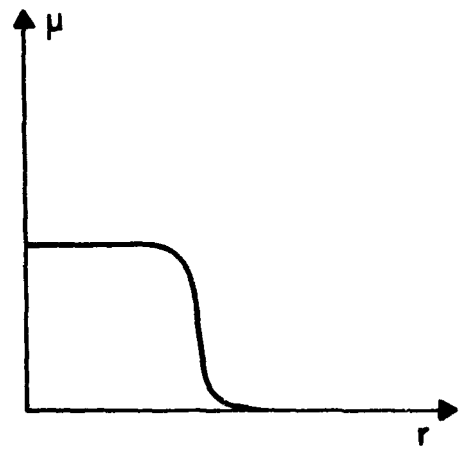

Fig. 1

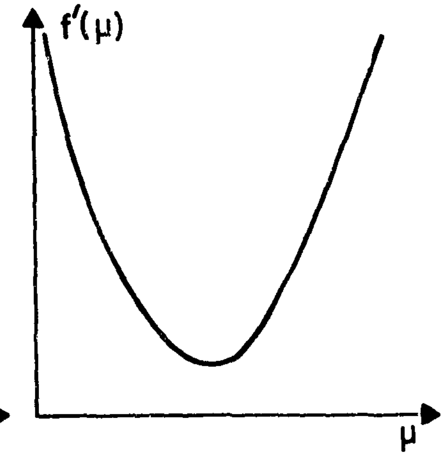

Fig. 2

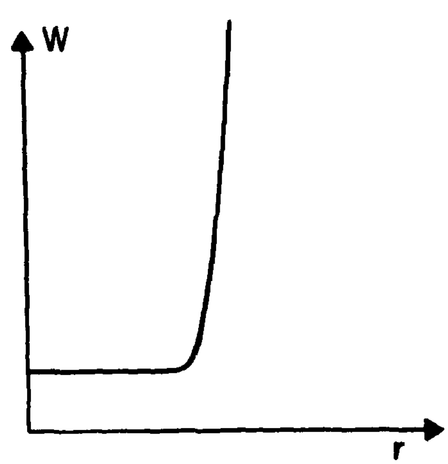

Fig. 3

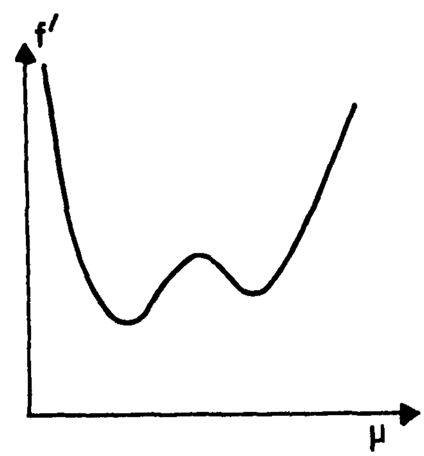

Fig. 4a

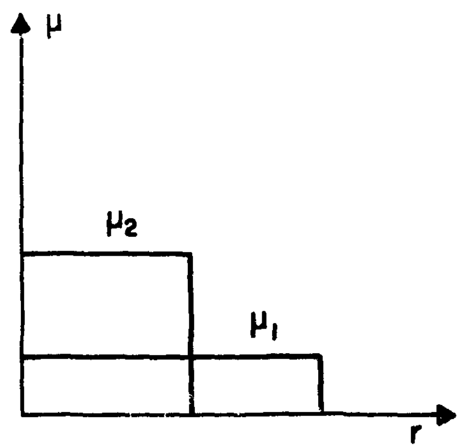

Fig. 4b

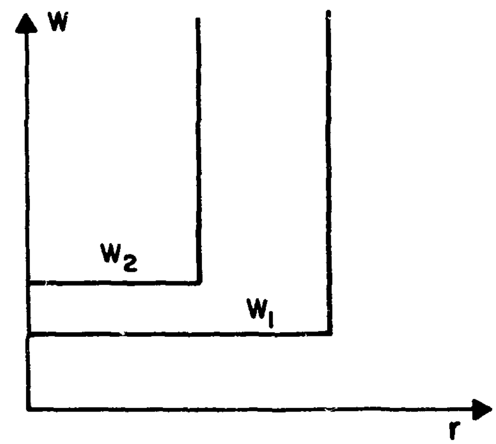

Fig. 4c

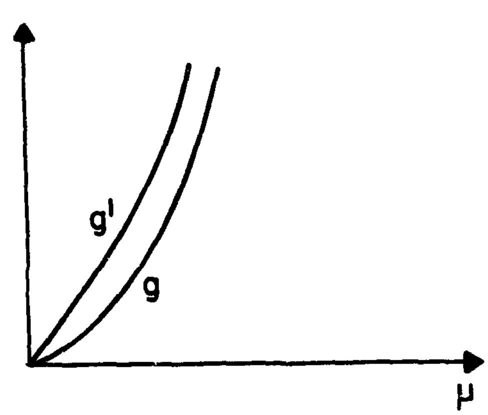

Fig. 5