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THE KINETIC DESCRIPTION OF PONDEROMOTIVE EFFECTS IN A PLASMA*

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The Kinetic Description of Ponderomotive Effects in a Plasma

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The kinetic treatment of the ponderomotive force concept is found from nonresonant quasilinear theory for waves with spatially dependent amplitude. In general, the ponderomotive effect appears as a velocity space diffusion term, not as just a force. For an unmagnetized plasma, the quasilinear equations are solved directly, and the correct density modification exhibited explicitly. Examples are considered for both a homogeneous and an inhomogeneous magnetic field, and aspects of rf end plugging are discussed.

The effects of electromagnetic waves on a plasma are relevant to problems in both laser fusion and magnetic confinement. In the former case, self-focusing density modifications, parametric instabilities, and magnetic field generation are of interest, and, in the latter, wave heating, end plugging of open systems and impurity control. For many of these problems, the collisionless regime is appropriate, and, in nonlinear single particle and fluid treatments, a ponderomotive force often plays a role in nonresonant phenomena. In this paper, the inadequacy of a kinetic approach using only a ponderomotive force is demonstrated, and the proper collisionless, kinetic treatment of nonresonant wave effects on a weakly inhomogeneous plasma is presented.

When autocorrelation times are short compared with diffusion times, the lowest order wave modification of the particle distribution is given by quasilinear theory.[1] Temporal damping of waves is easily shown to contribute only fake thermal broadening to nonresonant particles. The self-consistent aspect of spatially damped modes has been understood since the beginning of quasilinear theory;[2] this problem is reconsidered here in a different context. Let E denote a first order electrostatic wave with a weak, nonpropagating spatial dependence, $E = \sum_\omega E_\omega(x) e^{i\omega t}$, and let $\bar{f}$ (x,v,t) denote the slowly varying part of the distribution function. If the first order kinetic equation is obtained in the usual way, by integrating along unperturbed orbit trajectories, the slowly varying part of the nonresonant second order equation is found,

$$(\partial_t + v\partial_x)\bar{f} + 1/m\,(F_{0x} - \partial_x \psi)\,\partial_v \bar{f} - v/m\,\partial_x \psi\,\partial_v^2 \bar{f} - 2\psi/m\,\partial_v(v\partial_x + F_{0x}/m\,\partial_v)\,\partial_v \bar{f} = 0. \tag{1}$$

As usual, the ponderomotive potential is denoted by,

$$\psi(x) = q^2/m \sum_{\omega} |E_\omega|^2/\omega^2.$$

The slowly varying force, F_{ox}, may include an external force, or the self-consistent electric field. In the time asymptotic limit, the nature of the various terms in (1) can be understood by seeking solutions which are Maxwellian in the limit, $\psi \to 0$; depending on which terms are kept, the following solutions are obtained:

$$\bar{f} = (n_o/\sqrt{\pi}\,\bar{v}) \exp\left[-(mv^2/2 + \psi + q\phi)/(m\bar{v}^2/2)\right]; \tag{2a}$$

$$\bar{f} = \left[n_o m\bar{v}/2\sqrt{\pi}\,(m\bar{v}^2/2 + 2\psi)\right] \exp\left[-(mv^2/2 + q\phi)/(m\bar{v}^2/2 + 2\psi)\right]; \tag{2b}$$

$$\bar{f} = (n_o\bar{v}/2\sqrt{\pi}) \int_0^\infty ds \frac{s J_o(sv) \exp(-\bar{v}^2 s^2/4)}{(1+2s^2\psi/m)^{1/2}} . \tag{2c}$$

The ponderomotive force term alone, $1/m\, \partial_x \psi \partial_v \bar{f}$, has been used as a basis for a kinetic treatment of parametric instabilities of an electromagnetic pump wave.[3] It gives, (2a), the usual exponential modification of the density, $n(x)=n_o \exp\left[-\psi/(m\bar{v}^2/2)\right]$. If the effective mean square oscillation energy[4] is included by also considering the term, $(v/m)\, \partial_x\psi\, \partial_v^2 \bar{f}$, a new power law density dependence appears in the approximate solution, (2b), i.e., $n(x)=n_o/(1+4\psi/m\bar{v}^2)^{1/2}$. Thus, for this example, the simple ponderomotive force problem exhibits neither the oscillation energy (i.e., fake heating), nor the correct density dependence (except to order $\delta = \psi/m\bar{v}^2$). However, an exponential dependence on Ψ is recovered by self-consistently solving for the electric field, $-\partial_x(\varphi)$, associated with the ponderomotive perturbation of the electron density.[2] The last term in (1) is higher order in δ, and usually neglected. For the case, $\phi = 0$, the solution, (2c), is found to have an interesting property in the strong field limit; for $\delta >> 1$ and $v > \bar{v}$,

$$\bar{f} \simeq n_0/2\,(2\pi\delta)^{1/2}\,|v|, \tag{3}$$

which illustrates that, for large amplitude waves, even nonresonant wave-particle interactions can produce high energy non-Maxwellian tails. The wide variety of velocity-space functionals obtained in this simple example indicates a need for considering the properly posed problem in more complex geometries.

For transverse electromagnetic waves with weak spatial dependence in the direction of propagation, $\vec{E} = \sum_{\omega} \vec{y}\, E_\omega(x)\, e^{i\omega t}$, neglecting the self-consistent field, the quasilinear equation is found to be

$$(\partial_t + v_x\partial_x)\bar{f} - 1/m\,\partial_x\psi\,\partial_{v_x}\bar{f} = (q/m)^2 \sum_{\omega} (|E_\omega|/\omega^2)\, v_x\partial_x\, |E_\omega|\,\partial^2_{v_y}\bar{f}. \tag{4}$$

Although identical in appearance to the electrostatic case, the ponderomotive force term here is new and arises from the time average of those terms with a product of the fluctuating electric and magnetic fields. This can be seen by considering purely propagating modes; the electrostatic case has no ponderomotive force, while the ponderomotive force for a propagating electromagnetic wave, is given by, $\partial_x\psi \rightarrow k\psi$. As before, the time asymptotic solution to (4) is obtained with a Maxwellian boundary condition,

$$\bar{f} = (n_o/\pi\,\bar{v}_x\bar{v}_z)\exp(-v_x^2/\bar{v}_x^2 - v_z^2/\bar{v}_z^2)\int_{-\infty}^{\infty} ds/2\pi \exp\Big[isv_y - s^2\bar{v}_y^2/4 - (1/2 + 1/s^2\bar{v}_x^2)\ln(1+2s^2\psi/m)\Big]. \quad (5)$$

For $\delta << 1$, this becomes

$$\bar{f} = (n_o/\pi^{3/2}\,\bar{v}_x\bar{v}_z)\left[\frac{m/2}{m\bar{v}_y^2/2+2\psi}\right]^{1/2} \times \exp\left[-\left(\frac{mv_x^2/2+\psi}{m\bar{v}_x^2/2}\right) - \frac{mv_y^2/2}{m\bar{v}_y^2/2+2\psi} - v_z^2/\bar{v}_z^2\right], \quad (6)$$

which is similar to the electrostatic result, (2b), except that it has the usual exponential modification of the density in the direction of the ponderomotive force. As before, fake heating occurs in the direction of the oscillating electric field. For a highly anisotropic temperature, $\bar{v}_x << \bar{v}_y$, equation (5) gives an apparent thermal broadening in the y- direction, $m\bar{v}_y^2/2 \rightarrow m\bar{v}_y^2/2 + 2\psi(1-2\psi/m\bar{v}_x^2)$ which is slightly reduced compared to the usual fluctuation broadening.

The quasilinear equation for an arbitrary electromagnetic wave in a homogeneous external magnetic field, $\vec{B}_o$, has been obtained, and an example relevant to rf plugging is presented here. Let an electromagnetic field, $\vec{E} = \sum_{k,\omega}\vec{E}_{k,\omega}(s) \times e^{i(\vec{k}\cdot\vec{x}-\omega t)}$, be parallel to $\vec{B}_o$, with $\vec{k}$ perpendicular to $\vec{B}_o$ and a nonpropagating spatial dependence in the parallel direction, where $s = \vec{e}_{//}\cdot\vec{x}$. The nonresonant quasilinear equation is found to be

$$(\partial_t + v_{\parallel}\partial_s)\bar{f} - 1/m\,\partial_s\psi\,\partial_{v_{\parallel}}\bar{f} =$$

$$(q/m)^2 v_{\perp}^{-1}\partial_{v_{\perp}}\left\{v_{\perp}\sum_{k,\omega>0}|E_{k,\omega}|\;v_{\parallel}\partial_s\left[|E_{k,\omega}|\sum_n \frac{(J_n')^2}{(\omega-n\Omega)^2}\partial_{v_{\perp}}\bar{f}\right]\right\}, \qquad (7)$$

where the ponderomotive potential is

$$\psi = (q^2/m)\sum_{k,\omega}(|E_{k,\omega}|^2/2v_{\perp})\,\partial_{v_{\perp}}\left[v_{\perp}^2\sum_n (J_n')^2/\omega(\omega-n\Omega)\right], \qquad (8)$$

the Bessel function argument is $k_{\perp}v_{\perp}/\Omega$, and Ω denotes the cyclotron frequency. In view of the previous examples, the qualitative features of (7) are evident, i.e., an effective force and nonresonant thermal broadening. In the cold limit, $(k_{\perp}v_{\perp}/\Omega) >> 1$, the usual single particle potential is found,[5]

$$\psi = q^2/m\sum_{k,\omega}|E_{k,\omega}|^2/2(\omega^2-\Omega^2). \qquad (9)$$

In the opposite limit, $(k_{\perp}v_{\perp}/\Omega) << 1$, the potential, (9), is reduced by a factor $(k_{\perp}v_{\perp}/\Omega)$,$^{-1}$ suggesting that nonresonant rf plugging is likely to be most effective in the long wavelength, cold particle limit.

For a plasma confined in an inhomogeneous magnetic field, the kinetic equation can be obtained by the usual techniques, which include averaging over the $\vec{v}_{\perp}$ phase angle.[6] However, in general, the analysis is extremely complicated, and a single illustrative example is presented here. Consider left circularly polarized waves, with weak variation in the parallel direction, and $(k_\perp v_\perp/\Omega) \ll 1$. To first order in all spatial gradients, except $(\vec{B}_0 \times \vec{\nabla})\vec{B}_0$ the nonresonant quasilinear equation is

$$(\partial_t + v_{//}\partial_s)\bar{f} + v_{//}F_{//}\partial_\epsilon \bar{f} = (q/m)^2 (D + v_t^{-1}) \sum_\omega |E_\omega| v_{//}\partial_s \left[E_{//}/(\omega-\Omega)^2\right] D\bar{f}, \tag{10}$$

where $\bar{f} = \bar{f}(\epsilon, \mu, s, t)$, $\epsilon = 1/2\, m v^2$, $\mu = m v_\perp^2/2 B_0$,

$v_{//} = \pm\left[2(\epsilon - \mu B_0)/m\right]^{1/2}$, and $F_{//} = -q^2/m \sum_{\omega>0} \left[B_0/2\omega(\omega-\Omega)\right] \partial_s (B_0^{-1} |E_\omega|^2)$,

Operator D arises from $\partial_{v_\perp}|_{v_{//}}$ on a function of ϵ and μ and is given by,

$$D = (2B_0 m\mu)^{1/2} \ (\partial_\epsilon + B_0^{-1}\partial_\mu).$$

Equation (10) is the inhomogeneous analog of (7), for left circular waves in the cold limit. However, the effect of the waves on magnetic confinement is given by the ponderomotive force, which, in general, will no longer even be the gradient of a scalar and depends on the sign of $B_0\partial_s(B_0^{-1}|E_\omega|^2) = \vec{\nabla}\cdot(\vec{e}_{//}\,|E_\omega|^2)$. The apparent thermal broadening can now be estimated as, $\Delta\mu \simeq (q^2/m B_0) \sum_\omega |E_\omega|^2/2(\omega-\Omega)^2$, from this model, it is apparent that some care is required in evaluating wave plugging of open systems.

In conclusion, the ponderomotive effect in kinetic theory has been analyzed by using nonresonant quasilinear theory for spatially dependent modes. For an unmagnetized plasma, explicit time asymptotic solutions of the quasilinear equation have been obtained; typically, density modification due to the ponderomotive force and thermal broadening were exhibited. Since the interaction is nonresonant, these effects are not cumulative i.e., no irreversible changes occur, and, e.g., local fake heating depends on the local diffusion operator, not on the history of the particles. "Real" irreversible heating is, of course, only produced by resonant interactions, or collisions. This understanding has been applied to the magnetic plasma. Wave plugging of open systems has been shown to be most effective for cold particles, and, for an inhomogeneous magnetic field, the ponderomotive force has been shown, in general, to no longer be derivable as a simple gradient. The apparent thermal broadening which has been obtained here can now be used to correctly estimate the wave energy losses to both cold and warm particles on a collisional time scale. Finally, in comparing velocity moments of the kinetic equations with fluid theories, it should be recalled that moments of $\bar{f}$ and the time average of the fluctuating fluid quantities must both be considered.

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