

PROGRESS IN RESEARCH

January 1, 1976 - December 31, 1976

By the

Theoretical Nuclear Physics Group

January 1977

**Department of Physics
University of Texas
Austin, Texas**

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I. INTRODUCTION

This is a progress report, describing the accomplishments in basic research in nuclear physics carried out by the theoretical nuclear physics group of the Department of Physics of the University of Texas at Austin, during the period of January 1, 1976 to December 31, 1976.

The major part of the report is contained within subsections A, B, C, and D of Section II. The subsection A is a complete presentation of our research achievements and their significance. Both work completed and work in progress are covered, the coverage of the latter being given in somewhat more detail than that of the former. In subsection B, title pages of papers published during the year are reproduced, while in subsection C, first pages of papers submitted but not yet in print are reproduced. These papers are numbered as B-1 through B-18 and C-1 through C-7, according to the order in which they appear in these two subsections. In subsection D there are copied abstracts of talks presented at APS meetings.

The subsection A was written in such a way as to give an overview of our work, its interrelations, and its relation to work done by other groups. As is seen, a large variety of subjects are included as part of our work, and thus they are presented in seven separate parts; (i) through (vii). The overall subject discussed in each part can be found in the list of contents given at the beginning of this report. In the presentations of subsection A, frequent reference is made to papers in subsections B and C and also to many other papers. These papers are summarized in the list of references given at the end of subsection A.

II. THEORETICAL RESEARCH

II. A. OVERVIEW OF THE WORK PERFORMED

1. Nuclear Scattering

One of the subjects we engaged in during the past year was to investigate the effect of interference between nuclear and Coulomb scattering in inelastic processes. This is a fairly old subject, but interest was revived recently by an accumulation of accurate data, particularly for heavy-ion collisions. The new heavy-ion data do seem to require that one performs full quantum mechanical calculations, including the Coulomb excitation effects, which has been rather difficult because they were extremely time consuming.

Nevertheless, Feng in collaboration with Barnett and Goldfarb at Manchester performed a systematic full quantum mechanical calculation, not for very heavy ions but rather for (α, α') scattering from a number of target nuclei.¹ The calculations were made by using the computer code PATIWEN, developed by Feng and Barnett². One of the results of such calculations is reproduced in Fig. 1, which presents $d\sigma_{DWCE}$, $d\sigma_{nuc}$ and $d\sigma_{CN}$.^{*} These are DWBA inelastic cross sections calculated by including only the Coulomb form factor, only the nuclear form factor, and both, respectively, the distorting potential having nuclear and Coulomb parts all the time. Fig. 1 also presents the ratios of these cross sections to the pure Coulomb excitation cross section $d\sigma_{CE}$, as well as the ratio of the elastic to Rutherford scattering cross sections $d\sigma_{el}/d\sigma_{Ruth}$.

As is seen, $d\sigma_{CN}/d\sigma_{CE}$ deviates from unity more strongly than does $d\sigma_{el}/d\sigma_{Ruth}$, indicating that the inelastic scattering is more sensitive to the nuclear-Coulomb interference than is the elastic scattering. It is also clear that $d\sigma_{CN}$ agrees very well with experiment.

Beside PATIWEN, a code JPWKB which carries out coupled-channel (CC)

* Figures are given on p. 45-60. Note that they do not necessarily appear in the order as they are numbered.

calculations for the inelastic process was also developed by Kim et al.³ In order to speed up the calculation an approximate but rather accurate method proposed by Alder and Pauli⁴, was used in solving the CC equation outside the nuclear force region. Inside the nuclear force region JPWKB performs exact CC calculations taking into account both nuclear and Coulomb excitations. A user's manual has been written for JPWKB³ and this program can be obtained by request.

By using JPWKB and thus by taking into account the Coulomb excitations, Kim analyzed⁵ data for $^{144}\text{Nd}(^{12}\text{C}, ^{12}\text{C}')$ scattering⁶. This was done in order to prepare for reanalyzing the $^{144}\text{Nd}(^{12}\text{C}, ^{14}\text{C})^{142}\text{Nd}$ reaction data which we analyzed earlier⁷. The results of this reanalysis will be discussed below in part (iii). Here we show in Fig. 2 only the fit obtained for the inelastic cross section. The parameters deduced from this good fit were then used in the reanalysis of the transfer reaction⁵.

The work performed by Shamu et al.⁸ was to measure and interpret the energy averaged total cross section, i.e., the s-wave strength function for neutrons. It is well known that the giant resonance in the s-wave strength function observed in the mass region of $A=150-180$ (rare earth region) is split in two and further that this splitting can be well accounted for by a coupled channel calculation which couples the s-wave to the d-wave, the coupling being caused by large deformations which the nuclei in this mass region have. Shamu et al.⁸ measured the total cross section σ_A of neutrons from three Sm isotopes; $^{148}, ^{152}, ^{154}\text{Sm}$, for the incident energies, E_n , ranging from 0.75 to 14.5 MeV. They found that the excitation functions for $^{152}, ^{154}\text{Sm}$ which are both well deformed nuclei, are rather similar with one another, but are largely different from that of ^{148}Sm , which is known to be spherical. In Fig. 3, we duplicate a figure from ref. 8, where $(\sigma_{152} - \sigma_{148})/\sigma_{148}$ and $(\sigma_{154} - \sigma_{148})/\sigma_{148}$

are plotted against E_n , and it is seen that CC calculations fit these excitation functions rather well. Note that the CC prediction is rather sensitive to the deformation β_2 assumed for the nuclei involved and thus the above fit determines β_2 rather accurately. Shamu et al.⁸ found that the β_2 values for $^{152,154}\text{Sm}$ thus determined were about 10% lower than the β_2 for the nuclear charge distribution extracted from the Coulomb excitation, electron scattering and other methods.

Finally, Osterfeld, Udagawa and Wolter⁹ solved a long standing puzzle in the theory of the (h,t) reaction, which may most naively be considered a charge exchange scattering. With this understanding and with the simple DWBA, however, one could not fit data. Toyama¹⁰ was the first to go ahead to perform successive one nucleon transfer calculations, and analyzed particularly the $^{48}\text{Ca}(h,t)^{48}\text{Sc}$ reaction leading to the lowest 0^+ , 2^+ , 4^+ , and 6^+ states of the $(f_{7/2}^{-1}f_{7/2})$ configuration. He considered only the two-step $h \rightarrow \alpha \rightarrow t$ process and found that the two-step mechanism could explain all observed features of the reaction.

However, it was found that the fit to the data obtained in this way is essentially destroyed if the usual one-step process is also included in the calculations¹¹, because the interference between the one- and two-step processes is destructive. As a result, calculated cross sections, particularly those of the 0^+ and 2^+ states, become extremely small compared with experiment. Toyama and de Takacsy¹² thus arbitrarily reversed the sign of the interference to obtain a good fit to the data. Similarly disturbing results have also been found in some other cases.

We suggested earlier¹³, however, that this difficulty might be removed if important nonorthogonality corrections are included in the calculations, and what Osterfeld et al.⁹ did was to perform a calculation

along this line. In the calculation, the nonorthogonality corrections and the usual one- and two-step processes were all included. Tensor force effects in one-step charge exchange process was also included. It was found that such a calculation explains all the observed features of the data, not only of the normal parity 0^+ , 2^+ , 4^+ , and 6^+ states, but also of the unnatural parity 1^- , 3^- , 5^- , and 7^- states of the $(f_{7/2}^{-1}f_{7/2})$ configuration. One example of the result is reproduced in Fig. 4, where the calculated cross sections are compared with experiment. The agreement obtained is very good.

ii. One-Nucleon Transfer Reactions

Most of the work published or completed during the past year for one-nucleon transfer reactions concerned with EFR-CCBA analysis of heavy-ion induced reaction data. However, a successful analysis was also made by Ray and Coker¹⁴ of a light-ion induced reaction: $^{28}\text{Si}(^3\text{He},d)^{29}\text{P}$, leading to the lowest seven states of ^{29}P .

The highest four of these seven states are proton unbound, and thus were treated using complex energy Gamow states. Both CCBA and DWBA calculations were performed, and it was found that good fits were obtained only when CCBA was used. One example of the fit obtained with the CCBA is shown in Fig. 5 for the cross section of the 1.38 MeV, $3/2^+$ state, and for three different incident energies. It is seen in the figure that the results of CCBA calculations, given by solid lines, fit the experiment very well, but a DWBA result given by the dotted line for 35.3 MeV fits the data rather poorly.

Ray *et al.*¹⁵ analyzed the reaction $^{28}\text{Si}(^{13}\text{C},^{12}\text{C})^{29}\text{Si}$, leading to the ground and the first excited states of ^{29}Si , for which data were taken by Westfall and Zaidi¹⁶ at the University of Texas, and fairly extensive analyses were made using both the EFR-DWBA and EFR-CCBA methods. As in the case of the above light-ion-induced reaction, good fits to experiment were obtained only when a CCBA calculation was made, as is seen in the example presented in Fig. 6. In this figure, the solid, dashed and dash-dot curves are the calculated cross sections obtained, respectively, by including the inelastic coupling in the exit channel only, in the incident channel only, and in both. As is seen, the best fit was obtained when the coupling is included in both channels, although the quality of the fit is not so much deteriorated so long as one of the couplings is maintained.

The fit becomes very poor, however, if both couplings are ignored.

Another work that was published during the past year was the EFR-CCBA analysis of $^{19}\text{F}(^{16}\text{O}, ^{15}\text{N})^{20}\text{Ne}$ data, taken by a group at Orsay¹⁷.

A particular significance in the data was that ... 4^+ member, as well as the 0^+ and 2^+ members, of the ground band were strongly excited, whereas the transition is strongly forbidden through a one step mechanism. The observation of the strong excitation of this 4^+ state may thus be considered as strong evidence of multistep processes.

Low, Tamura and Udagawa¹⁸ thus performed EFR-CCBA calculations and obtained results that are shown in Fig. 7, where the calculated 0^+ , 2^+ , and 4^+ CCBA and the 0^+ and 2^+ DWBA cross sections are compared with experiment. It should be noted that CCBA fits the cross sections of all the three states, both in shape and magnitude, while the DWBA fit to the 0^+ and 2^+ cross sections is rather poor. This shows that even a transition which is not forbidden for a one-step process can be significantly affected by higher order processes.

An interesting heavy-ion study which could be analyzed also in terms of CCBA was reported by a group at Brookhaven¹⁹, who observed one-nucleon transfer reactions induced by ^{13}C on a series of even Ca-isotopes. The data showed that the angular distributions were out of phase between the $(^{13}\text{C}, ^{14}\text{N})$ and $(^{13}\text{C}, ^{12}\text{C})$ reactions. The DWBA calculations fit well the angular distributions of the $(^{13}\text{C}, ^{12}\text{C})$ reaction, but predicted very similar angular distributions for the $(^{13}\text{C}, ^{14}\text{N})$ reaction as well, thus disagreeing completely with experiment.

It is tempting to attribute the source of this anomaly to the contribution of the multistep processes via strongly collective 3^- states that are known in Ca targets. A possible pitfall in this approach, however, is

that the multistep contribution may at the same time modify the ($^{13}\text{C}, ^{12}\text{C}$) cross section, thus destroying the DWBA fit that has been obtained.

The authors of ref. 19 thus cast doubt on the possibility of explaining the observed anomaly in terms of multistep effects.

Low, Tamura and Udagawa²⁰, nevertheless carried out detailed CCBA calculations, particularly for the $^{40}\text{Ca}(^{13}\text{C}, ^{12}\text{C})^{41}\text{Ca}(7/2^-)$ and $^{40}\text{Ca}(^{13}\text{C}, ^{14}\text{N})^{39}\text{K}(3/2^+)$ reactions, and found that the calculations consistently fit both the normal and anomalous angular distributions. They found that such results came about because the effect of the indirect transition via the 3^- state was very strong for the ($^{13}\text{C}, ^{14}\text{N}$) reaction but was weak for the ($^{13}\text{C}, ^{12}\text{C}$) reaction.

One of the sources of this difference is the difference in the "Q-matching" in the two reactions. Take the ($^{13}\text{C}, ^{14}\text{N}$) reaction, for example. The Q-value for the direct transition to the ground $3/2^+$ state of ^{39}K is -0.77 MeV, which is rather far from the optimal Q-value; $Q_{\text{opt}} = +5$ MeV. On the other hand, the Q-value for the indirect process via the 3^- state is $+2$ MeV, which is much closer to the Q_{opt} . As a result, the indirect transition is relatively enhanced as compared with the direct transition. The situation is completely reversed, however, for the ($^{13}\text{C}, ^{12}\text{C}$) case, making the indirect contribution strongly inhibited.

There are two more sources which also give effects similar to those of the Q-values, i.e. the difference in the binding energies and the $j<$ and $j>$ nature of the transferred nucleons. Since these three effects work in combination, the indirect contribution becomes almost comparable to the direct contribution for the ($^{13}\text{C}, ^{14}\text{N}$) reaction, but is almost negligible for the ($^{13}\text{C}, ^{12}\text{C}$) reaction. In Fig. 8, the final results of the CCBA analysis are presented, and it is seen clearly that the anomalous behavior in the Brookhaven data is accounted for very satisfactorily.

iii. Two-Nucleon Transfer Reactions

About two years ago, we reported on an EFR-CCBA analysis of the $^{144}\text{Nd}(^{12}\text{C}, ^{14}\text{C})^{142}\text{Nd}$ reaction⁷ leading to several states in ^{142}Nd , in which it was shown that a CCBA calculation can give in a natural way a nice simultaneous fit to an "anomalous" 2_1^+ and to a normal 0_g^+ angular distribution, predicting their relative magnitudes also in agreement with experiment. It was noticed, however, that the 2_1^+ angular distribution deviated from experiment at large angles, the theoretical cross section decreasing too fast. It was further noticed later that the agreement obtained with experiment could not be maintained, had optical parameters⁶, extracted by fitting new actual scattering data between ^{144}Nd and ^{12}C , been used in the EFR-CCBA calculations of ref. 7.

It often happens in heavy-ion induced reactions that the partial waves that are most important in determining the scattering cross section are quite different from those that play the dominant role in determining reaction cross sections. Therefore, one can say that the above discrepancy need not be taken too seriously.

We nevertheless recall that in the CCBA calculations reported in ref. 7, the effects of the Coulomb excitations were entirely ignored, mostly because, with the technique available at that time, the inclusion of the Coulomb excitation made the calculation *much too time consuming*. As was discussed in part (i) above, however, it has now become possible for us to include the Coulomb excitation without lengthening computational time too much. Thus, Kim⁵ reanalyzed the $^{144}\text{Nd}(^{12}\text{C}, ^{14}\text{C})^{142}\text{Nd}$ data⁷ and obtained the results given in Fig. 9. As is seen, the agreement is much improved from that in ref. 7, particularly fitting the large-angle data for the 2_1^+ cross section. It should be noted that the optical parameters used were

those that fit the elastic scattering data of ref. 6, as was discussed in part (i) above; see Fig. 2.

In spite of the improvement thus achieved, there still remains a discrepancy between theory and experiment. The theoretical cross sections were multiplied by a factor $N=9$, before they were plotted in Fig. 9. We shall discuss below, in a somewhat systematic way, this problem of too small theoretical cross sections.

With the interest of seeing the difference in the behavior of the "removal" and "additional" type pairing states involved in the two-nucleon transfer reactions, a reaction $^{142}\text{Nd}(^{18}\text{O},^{16}\text{O})^{144}\text{Nd}$, which is inverse to the $^{144}\text{Nd}(^{12}\text{C},^{14}\text{C})^{142}\text{Nd}$ reaction, was also investigated²¹ and we show in Fig. 10 the cross sections of the 0_g^+ and 2_1^+ states of ^{142}Nd . It is seen that, contrary to the case of the $(^{12}\text{C},^{14}\text{C})$ reaction, both angular distributions are normal, a situation which was expected because of the fact that both states are of "additional" nature and the reaction was also "additional". In obtaining the fit to the angular distributions, optical parameters were used which fit the elastic scattering of ^{18}O by ^{142}Nd , and no Coulomb excitation was included in the CCBA calculations. The results do not depend strongly on the Coulomb excitation, because both the 0_g^+ and the 2_1^+ transitions are dominated by one-step amplitudes.

A somewhat systematic investigation of $(^{16}\text{O},^{14}\text{C})$ reactions with three Ge isotopes, $^{72,74,76}\text{Ge}$, as targets were performed in collaboration with a Saclay group²², and a portion of the results of this work is presented in Figs. 11 and 12 where EFR-CCBA calculations were performed with 0^+-2^+ and 0^+-3^- couplings, respectively. The optical parameters used were those that were obtained by fitting only the elastic scattering, and not the inelastic scattering, of ^{16}O by Ge, which explains why the fitting of the 0_g^+ cross section is much better in Fig. 12 than in Fig. 11. The 0^+-2^+

coupling is so strong that, when CCBA calculations are made, it modifies the wave function in the 0_g^+ channel a bit more than is realistic. Ideally, the optical parameters should have been derived by fitting inelastic scattering data as well, but unfortunately such data was not available.

The fit to 2^+ and 3^- cross sections is not always very good, but the theory does reproduce correctly the mass number dependence of the experimental cross sections, showing that CCBA calculations can be in fact a useful tool in explaining these data. Concerning the absolute magnitude, however, we encounter here a trouble which is more grave than it was in the above $^{144}\text{Nd}(^{12}\text{C}, ^{14}\text{C})^{142}\text{Nd}$ reaction. In plotting in Figs. 11 and 12, the theoretical cross sections had to be multiplied with a factor N which was about 700.

As we saw in a few examples given above of two-nucleon transfer reactions, a rather consistent trouble we encounter in these reactions is that the theoretical cross sections are always too small compared with experiment (and yet the relative cross sections as well as angular distributions are predicted rather satisfactorily for most cases). A factor N which is defined as the ratio of the experimental over the theoretical cross sections ranges from 10 to 1000.

Actually the same problem of too-small theoretical cross section has already been encountered in the light-ion induced two-nucleon transfer reactions, i.e., in the (p,t) and (t,p) reactions, although the discrepancy factor N was only of the order of 2 to 3. Feng *et al.* worked earlier on this problem²³ and showed that N could be reduced to become very close to unity, if the form factor used in the DWBA calculations were constructed by first carrying out shell-model calculations with a large basis of oscillator functions. They had demonstrated this²⁴ for a pair of nuclei, ^{16}O and ^{18}O , and recently extended²⁵ the calculation to another pair, ^{40}Ca and ^{42}Ca , finding that N for a $^{42}\text{Ca}(t,p)^{40}\text{Ca}$ reaction²⁶ can be made as small as 1.3.

They have not taken into account in their calculation the contribution of a successive transfer mode; $t \rightarrow d \rightarrow p$, however, had they added this contribution, they might have obtained $N=1$. This can be guessed from a very similar analyses we carried out for heavy-ion induced reactions, to which we now proceed.

During the past year, we have taken up as one of our major projects the problem of explaining the anomalously large experimental cross sections for two-nucleon transfer reactions between heavy ions, and made the first successful calculation²⁷ which obtained $N=1$. This was for the case of a $^{40}\text{Ca}(^{18}\text{O}, ^{16}\text{O})^{50}\text{Ca}$ reaction²⁸, and the comparison between theory and experiment is made in Fig. 13. In this figure, the 0_g^+ cross section σ_1 was obtained by considering only the one-step (simultaneous) transfer of two neutrons with the form factor obtained from a shell model calculation performed by using a fairly limited number of configurations. The cross section σ_2 was then obtained in the same way as was σ_1 , except that an extended shell model basis was used, as in ref. 25. It is seen that σ_2 is about twice as large as is σ_1 . The cross section σ_3 , on the other hand, was obtained by assuming only the two-step (successive) one-nucleon mode; $^{18}\text{O} \rightarrow ^{17}\text{O} \rightarrow ^{16}\text{O}$. As is seen, the magnitudes of σ_3 and σ_2 are very close to one another, and are both about a quarter of the experimental value. In the actual comparison with experiment, however, the one- and two-step modes should be considered simultaneously, with amplitudes that may interfere constructively. The cross section σ_4 was calculated this way and it is seen that it agrees almost perfectly with experiment. We thus get $N=1$.

Encouraged by this success, we undertook similar calculations for various two-nucleon transfer reactions, and the results together with the details of the calculation will be published very shortly.²⁹ We give

in Figs. 14 and 15 part of the results of this work²⁹. Note that in these figures the quantity N_3 has the same meaning as $\sigma_{\text{exp}}/\sigma_4$ in the sense of Fig. 13, i.e., it is the N value we obtain when both one- and two-step processes are taken into account, the former being calculated with an extended shell model basis.

As is seen in Fig. 14, which collected the results for ($^{18}\text{O}, ^{16}\text{O}$) and ($^{16}\text{O}, ^{18}\text{O}$) reactions, N_3 is very close to unity, the worst case being $N_3=2$. In Fig. 15 in which ($^{16}\text{O}, ^{14}\text{C}$) reactions are collected, however, the situation is not as good as it is in Fig. 14. The largest N_3 we have is still as large as 30, although it is worthwhile to note that N could have been as large as 300, had the effects of two-step process were not taken into account. It is also worthwhile to note that even with the ($^{16}\text{O}, ^{14}\text{C}$) reaction, we obtained $N_3=1$ in one case, i.e. in the $^{48}\text{Ca}(^{16}\text{O}, ^{14}\text{C})^{50}\text{Ti}(0^+; 7.19\text{MeV})$ reaction.

An important difference of this last reaction, compared with the other two listed in Fig. 15, is that its final state is an excited state, while in the others, the final states are ground states. In the former, the Q value is rather close to its optimum value, while in the latter, it is rather far off. We may then note further that the reactions that are listed in Fig. 14 all have Q values that are rather close to optimum values.

What the results of Figs. 14 and 15 show is now rather clear. When the reaction Q value is not very far away from an optimum value, we can get sufficiently large theoretical cross sections for two-nucleon transfer reactions, if one- and two-step processes are taken into account simultaneously. This analysis is still not sufficiently realistic, however, for cases in which the Q value is far away from the optimum value, i.e., for cases in which the "Q-matching" is very poor.

In the $^{144}\text{Nd}(^{12}\text{C}, ^{14}\text{C})^{142}\text{Nd}$ reaction we discussed above, the Q-matching is fairly good, and thus the above recipe may be used to reduce N from its present value $N=9$. In applying this remedy, however, the scheme of the calculation of Refs. 27 and 29, which added the successive mode to DWBA, has to be extended so that the successive mode is added to CCBA. This is to be one of our future projects. In the $\text{Ge}(^{16}\text{O}, ^{14}\text{C})$ reactions of Ref. 22, on the other hand, the Q-matching is very poor, and thus a new approach beyond what was used in Refs. 27 and 29 would have to be found.

We found above that the theory predicts a large cross section if the Q-matching is good. We also saw, from the comparison of cases (b) and (c) in Fig. 15, i.e., of two $^{48}\text{Ca}(^{16}\text{O}, ^{14}\text{C})^{50}\text{Ti}$ reactions, the one leading to the ground state and the other to an excited state, both of ^{50}Ti , that the processes with highly excited states have a better Q-matching. These facts would suggest that a possible approach is to carry out a CCBA type calculation, in which the intermediate states are a number of highly excited states, rather than a few low lying collective states, as has been the case in most of the CCBA calculations made so far. These highly excited states may overlap acquiring the character of "continuum" states. In the next part (iv), we discuss problems involved in treating such continuum states. Techniques presented in (iv) might also apply to the present problem, too.

A feature which may look rather surprising in Fig. 14 is that the cross sections with a purely one-step mode and with a purely two-step mode are very similar to one another. It is also rather surprising to find that the amplitudes corresponding to these two modes interfere in an almost perfect constructive way. An explanation why such features do take place was given in detail in Ref. 29, which is somewhat too lengthy to reproduce here. The

essence of the explanation, however, lies in the fact that for heavy ions the optical potentials are highly absorptive. Therefore, even when the transfer of two nucleons takes place in an successive way, the spacial points at which the first and the second transfers take place cannot be too far apart from one another. In other words, these two nucleons must lie together very closely all the time, so long as they are to contribute to the two-nucleon transfer reactions, and thus behave as if they form a single unit. In the one-step treatment, the two nucleons are treated as one unit all the time. It would thus not be too much surprising that the one- and two-step cross sections behave very similarly.

In the above Refs. 27 and 29, we emphasized the importance of constructing microscopic form factors based on multi-configuration shell-model wave functions. A computer program to be used for this purpose will be published shortly³⁰. This program is called SATTNT, standing for SATURN for Two-Nucleon-Transfer. For microscopic two-nucleon transfer calculations this SATTNT may replace the form factor program SATURN in our previous computer program³¹ for EFR-DWBA calculations. The MARS, i.e., the DWBA part of Ref. 31 can then be used as it stands.

During the past three years we carried out a number of EFR-CCBA calculations for one- and two-nucleon transfer reactions between heavy ions. A summary of these investigations was present in the form of an invited paper³² read at the Balatonfured Symposium in Sept., 1975. Another invited paper read by K. S. Low³³ at the EPS meeting at Kaen, France, in Sept., 1976, also summarizes the work we have done. Low was a member of our group until two years ago, and also participated in a number of cooperative work after he moved to Saclay.

iv. Reactions with Continuous Spectra

-- Deep Inelastic Reactions --

In the past few years we, the theoretical group at the University of Texas, have carried out a number of analyses of reactions that were induced both by light-ions and heavy-ions, by using techniques which may collectively be called the multi-step direct-reaction (MSDR) approach. Our analyses made so far, however, have been almost exclusively confined to reactions in which the residual nuclei were left in discrete states, the outgoing particles thus having discrete spectra. We may call such reactions "spectroscopic", because the corresponding analyses were directly or indirectly done to extract spectroscopic information concerning the discrete states involved in the reaction.

There have also been available, however, a good deal of data in which the residual nuclei were so highly excited that the outgoing particles had "continuous" spectra. In particular such reactions induced by heavy-ions are called with various names; quasi-elastic, deep inelastic, strongly damped and so forth. It is our belief that the MSDR techniques, which we are familiar with through our experience with the previous spectroscopic analyses, can be applied to the analysis of various "continuum" data as well. Thus a project to perform such analyses has been initiated recently, and we report in this subsection briefly the work which has been done along this line. See references 34 and 35 for more detail.

For the reactions induced by light-ions, it was customary in the past to analyze such continuum data in terms of the "pre-equilibrium model," which was first proposed by Griffin³⁶ and then extended by Blann³⁷. Unfortunately, however, in spite of its general success, this model cannot predict angular distributions and thus its prediction has been compared only with angle-integrated cross sections. Recently attempts³⁸ have been made to

extend the pre-equilibrium approach further so that it can be tested against experimental angular distributions. The extension was made by combining somewhat crude application of the concept of a direct reaction with those of the pre-equilibrium model. It is our point of view that to make such a combination is not necessary. We can show rather easily that it is sufficient to carry out the MSDR calculations throughout.

The spectroscopic MSDR calculations are known to be very involved in general. Extension to the "continuum", however, is not as involved as one might suspect. The reason is that one is only interested in obtaining multi-state summed or averaged cross sections, not the detail of the cross sections of individual states. This fact allows us to make a few rather drastic but nonetheless well justified simplifications of the calculation.

We first note that, for our purpose, it is quite legitimate to assume that any state in a nucleus is well described by the single-particle shell-model (SPSM), which precludes any configuration mixing. A state of an actual nucleus can of course consist of very complicated configurations of such SPSM states, and thus the cross section corresponding to the excitation of this state includes interference terms for the contributions of different configurations. However, if we sum over cross sections that correspond to excitation of a large number of such states that lie within a fairly wide range of excitation energies, the interference contributions involved in the individual cross sections average out to a large extent. The summed cross section is thus reduced to what is obtained by assuming a pure SPSM throughout.

Once the use of the SPSM is justified this way, it is a rather easy matter to carry out the rest of the formulation for the MSDR calculations for continuous spectra. A somewhat detailed account of this formulation has been given in Ref. 34, in which explicit analyses were also made of

(p,p') data of Ref. 39. It was shown there, cf. Fig. 16, that the sum of one- and two-step contributions was sufficient to fit the data, and that, in particular, the one-step cross section could be written in the form

$$\sigma^{(1)}(E'_p; \theta) \Delta E_p = \sum_I d_I^2(E_p - E'_p) \sigma_I^{(1)}(E'_p; \theta). \quad (1)$$

In Eq. (1), I is the spin of the residual nucleus, which would be the same as the transferred angular momentum if the target had a 0^+ spin. The significance of the form (1) is that the summand is written as a simple product of two factors. The factor d_I^2 , which is purely structural, contains all of the shell structure information, yet has a very simple form. The factor $\sigma_I^{(1)}$ is, on the other hand, purely dynamical in that it can be calculated without any information about nuclear structure. A factorization like that in (1) is usually impossible to make in the "spectroscopic" type MSDR calculations. At least in this regard the "continuum" calculation is much simpler than the "spectroscopic" calculation.

Once the one-step cross section (1) is obtained, it is easy to write down the two-step cross section as

$$\sigma^{(2)}(E'_p; \theta) \Delta E'_p = \sum_{I_1 I_2 I} \int d_{I_2}^2(E''_p - E'_p) d_{I_1}^2(E_p - E''_p) \sigma_{I_1 I_2 I}^{(2)}(E'_p; E''_p; \theta) dE''_p; \quad (2)$$

and similarly $\sigma^{(n)}(E'_p; \theta)$ with $n \geq 3$.

The deep inelastic type reactions induced by heavy ions quite often involve transfers of a large number of nucleons as well as large energies⁴⁰. Consider, for example, a ($^{20}\text{Ne}, ^{12}\text{C}$) reaction⁴¹, in which ^8Be is transferred. It is our belief, however, that the dominant process involved in this reaction is not a one-step transfer of ^8Be , but two-step successive transfers of two α -particles. We can calculate rather easily the cross sections for such two-

step processes, if it is possible to write the cross section in a form very much like the one given in eq. (2) for a (p,p') reaction.

We have found that such an expression can in fact be obtained, if the SPSM is used again. Of course the meaning and thus the numerical values of the d^2 and the $\sigma_{I_1 I_2 I}^{(2)}$ factors are very different for the transfer and inelastic scattering processes. Still, the possibility of factorization assures us that the MSDR analysis of continuous spectra of multi-nucleon transfer reactions is practicable.

We may think of a somewhat simpler reaction like $(^{20}\text{Ne}, ^{16}\text{O})^{41}$, which will proceed as a one-step α -particle transfer reaction, or a still simpler one-nucleon transfer reaction like $(^{14}\text{N}, ^{13}\text{C})^{42}$. In either case the cross section may be written in a form as given in eq. (1), and the calculation would seem as easy as it was for the one-step (p,p') process. Actually, however, the former is somewhat more involved than the latter, simply because any calculation of transfer reactions between heavy ions is more involved than that of a simple inelastic scattering of light ions. It is thus desirable to make a further simplification of the former type of calculation.

It is well known that most of the cross sections observed for transfer reactions between heavy ions can be well represented by theoretical cross sections parameterized in a way suggested by Strutinski⁴³. We thus intend to parametrize the cross section $\sigma_I^{(1)}(Q, \theta)$, which is the same as $\sigma_I^{(1)}(E'_p; \theta)$ of eq. (1) except that now the reaction Q value plays the role which E'_p did in eq. (1).

One can think of various different degrees of sophistications in the parametrization, but the following may be one of the simplest conceivable:

$$\sigma_I(Q; \theta) = N \exp \left[- \left\{ \frac{Q - Q_{\text{opt}}}{aI + b} \right\}^2 \right] \frac{1}{\sin \theta} \left[\exp \{ -\Gamma^2 (\theta - \psi)^2 / 2 \} + \exp \{ -\Gamma^2 (\theta + \psi)^2 / 2 \} \right]. \quad (3)$$

In (3) the factors, except for the first exponential factor, are well known in Strutinski's work. The parameters Γ and Ψ also depend on Q and I , but the major dependence of the cross section on these two quantities Q and I is through the first exponential factor. What this factor means is that the cross section gets smaller the more the Q value deviates from its optimum value, Q_{opt} . In other words, the factor describes the so-called Q -window. The I dependence of the denominator of the exponent, with $a > 0$, further means that the width of the Q window widens as I , i.e., the transferred angular momentum, increases.

It should be noted that the introduction of (3) does not mean that we have given up the aim to perform accurate MSDR calculations. We in fact carry out such calculations for a number of sample values of I and Q , and thus find the best specific parametrization of (3).

In order to get an idea of how eq. (1) combined with (3) works, we show in Fig. 17 a fit we obtained³⁵ for the $^{92,100}\text{Mo}(^{14}\text{N},^{13}\text{C})$ reactions⁴². As is seen, the fit is rather good. A slight discrepancy that still remains for low $E(^{13}\text{C})$ values, i.e., for high Q values, may mean that we should add the contribution of two-step processes in which one of the processes is an inelastic scattering, while the other is a transfer reaction.

Analyses along the lines described above are underway, and the results will be reported in the near future.

v. Medium Energy Nuclear Physics

For about a year and half we have had several projects underway in the general area of medium-energy nuclear physics, partially in collaboration with two groups of experimentalists, at UCLA and at LAMPF of Los Alamos Scientific Laboratory. Our work so far has been based heavily on two computer programs, a relativistic optical model program with great flexibility, called at present BIGMEP, and a relativistic distorted-wave-Born approximation program based on the earlier program DWBA-VENUS⁴⁴. A number of other programs have been assembled into usable form, but have not yet led to independently published results; these include programs which construct the nucleon-nucleus optical model potential from nucleon-nucleon phenomenology, using various techniques of nuclear matter theory, and a program which treats coupled-channel effects in inelastic scattering at medium energy, based on the earlier non-relativistic program JUPITER 1.⁴⁵

We will now discuss some of the published and unpublished work completed during the contract period for 1976. The UCLA experimental group has obtained elastic scattering data for ${}^4\text{He} + {}^4\text{He}$ at 0.65 and 0.85 GeV; a preliminary analysis of this data by our group was included in the first published report of this data⁴⁶. A more complete analysis was published later, in Physics Letters⁴⁷. Results of this analysis are summarized in Fig. 18. The scattering is strongly Coulomb-dominated, and a double-folding procedure was used to obtain a realistic ${}^4\text{He} + {}^4\text{He}$ Coulomb interaction, with Gaussian charge form factors. The nuclear scattering was found to be describable only if the real potential has a short-range repulsive core. In Fig. 18, the dashed curve is the typical result of a calculation in which the nuclear potential is purely attractive;

the solid and dot-dashed curves show results in which the real part of the nuclear potential is, respectively, purely short-range repulsive and a mixture of short-range repulsion and long-range attraction. This short-range repulsion is already a characteristic feature of most low-energy analyses of ${}^4\text{He} + {}^4\text{He}$ scattering, and comes about because of the "clipping" of the low partial wave functions, within the overlap region, as required by the Pauli principle. In future work of this sort, it will be interesting to attempt to construct the ${}^4\text{He} + {}^4\text{He}$ potential in a microscopic way, using adaptations of the familiar resonanting-group method.

In a more recent publication in Physics Letters⁴⁸ Ray, Coker and Hoffman analyze medium-energy elastic and inelastic scattering data for protons on ${}^{58}\text{Ni}$ and ${}^{208}\text{Pb}$ and ${}^4\text{He}$ on ${}^{40}\text{Ca}$, using a medium-energy DWBA formalism. Two types of optical potentials are used in solving the Schrödinger equation with relativistic kinematics; a phenomenological potential; and a microscopic potential constructed using nuclear matter techniques, within the framework of the Kerman-McManus-Thaler multiple-scattering approach to the nucleon-nucleus optical model⁴⁹. This latter approach is discussed in detail at the end of this section. Results are quite similar with both sorts of potentials--except for the large-angle inelastic ${}^{58}\text{Ni}$ cross sections, about which there is some experimental question, the inelastic data are beautifully reproduced in both shape and magnitude, with no adjustable parameters. The inelastic form factor is obtained in the conventional way, so that the collective parameter β_L has the same meaning as in low-energy nuclear physics. Thus β_L in each case was simply fixed at the value earlier determined by low energy analyses. An important feature of these analyses was the

inclusion of spin-orbit coupling, which has a drastic effect on the theoretical predictions. Earlier analyses of such data, in terms of the questionable Glauber approach, which omitted these spin-orbit effects, essentially have nothing to do with physical reality. Results of the DWBA calculations are summarized in Fig. 19a-c. The solid curves result from use of an empirical potential; the dashed curves result from use of a microscopic potential constructed from the Kallio-Kolltveit effective nucleon-nucleon interaction and Hartree-Fock neutron and proton matter densities for the targets under consideration. No microscopic potential was constructed for the ${}^4\text{He} + {}^{40}\text{Ca}$ case.

Recently, we have examined the old Saclay inelastic scattering data for $\alpha + {}^{12}\text{C}$ at 1040 MeV, in terms of the coupled-channel approach, an extension of our earlier DWBA analyses⁴⁸. We find that the 2^+ , 4.44 MeV state can be very well fit, and that effects of channel coupling are relatively unimportant: DWBA and CC give rather similar results. However, when in the CC calculations a full nonspherical potential is used for ${}^{12}\text{C}$, the fit is considerably improved by the "reorientation" coupling of various partial waves. For the 3^- , 9.64 MeV state, multi-step mechanisms are far more important, as is suggested by the anomalous "smeared" experimental angular distribution. So far, computer time limitations have prevented us from carrying out a realistic multi-step analysis of this transition, but we will complete the analysis during the coming year.

We now turn to our work on the microscopic nucleon-nucleus optical model potential.⁵⁰ Since this work is not yet in print, we will cover it in somewhat more detail than the published work discussed above.

The nucleon-nucleus interaction potential, however defined, is energy-dependent. Over the past 25 years a vast lore has built up concerning

the empirical energy-dependence of the local, phenomenological nucleon-nucleus optical model potential. This phenomenology now extends from a few MeV, up to and beyond a GeV. Over the years, there have been a number of efforts to understand and explain the energy dependence and magnitude, particularly of the real part of the phenomenological optical potential.

Out of necessity, most early studies covered low incident nucleon energies ($\lesssim 100$ MeV), where the data seem to show a linear dependence on energy. The literature abounds in theoretical analyses of one kind or another which successfully reproduce this empirical linear kinetic-energy dependence.⁵¹

To be more specific, let us define $U_{\text{opt}}(0)$ as the value of the real part of the optical model potential evaluated at the center-of-mass of the target nucleus. Various authors have calculated $U_{\text{opt}}(0)$ using various techniques and have obtained an adequate reproduction of the low-energy phenomenology⁵¹.

However, studies of the empirical real potential volume per nucleon, or of $U_{\text{opt}}(0)$, which have extended to 1 GeV and beyond⁵² show that the overall energy dependence of the real part of the optical potential is a logarithmic one. Extrapolation of the low-energy linear dependence indicates that $U_{\text{opt}}(0)$ should go to zero at about 250 MeV. Empirically, the zero is found to occur near 500 MeV, the potential becoming increasingly repulsive thereafter.

A formal theory of the nucleon-nucleus optical potential exists, which gives what might be called a "multiple-excitation series" for the nucleon-nucleus interaction⁴⁹. Only the first term of this series is generally used for calculations; in momentum-transfer space this first term has the approximate form $\tau(q^2) \times F(q^2)$, where $\tau(q^2)$ is the nucleon-nucleon T-matrix in nuclear matter, and $F(q^2)$ is the nuclear matter distribution.

In all numerical applications known to us to date, a further approximation is made: namely, $\tau(q^2)$ is replaced by $t(q^2)$, the empirical free nucleon-nucleon scattering t-matrix.

Calculations using this approximate potential again suffer from too much repulsion at medium energies, and in addition give an energy dependence which does not agree with results of phenomenology. One is thus left with a puzzle. Does the discrepancy arise from the approximation $\tau \approx t$, as one would naively expect, or is it due to higher-order terms in the "multiple excitation series"? In the present work, we are attempting to answer both of the questions just raised, by avoiding the use of an empirical nucleon-nucleon t-matrix and also investigating several types of higher-order correction terms.

Our calculations are based upon the Reid soft-core potential⁵³, which we use at energies up to 1 GeV. Following the techniques of nuclear-matter calculations, we are able to use instead of τ , the nucleon-nucleon t-matrix, the so-called nuclear matter G-matrix, G^N , which allows us to take into account many-body effects and off-shell behavior in a relatively straightforward manner.

If we are to use the accumulated lore of nuclear matter calculations, we are in effect treating the interaction of the incident nucleon with infinite nuclear matter. At medium energies (~ 1 GeV) the wavelength of the incident nucleon is quite small compared to the nuclear radius, particularly for the heavy nuclei we consider in this work and we are interested only in the value of the optical potential at the nuclear center of mass. Therefore, while the use of G^N involves approximations, they are reasonable ones, and far superior to the usual "impulse" approximation $\tau \approx t$.

For example, the corrections due to the third order terms in the expansion of G^N in terms of G^F , the corresponding free-space quantity,

are routinely considered in nuclear matter calculations, and have been shown to be significant for high relative momenta. Rajaraman⁵⁴ has found that these higher-order effects may be properly accounted for by the device of neglecting the nuclear interaction in relative odd states and by increasing the statistical weight of the even state forces by a factor of 4/3, as explained further in the next section. However, as pointed out by Sprung et al.⁵⁵ this correction procedure fails for relative momenta $k_0 \approx k_f$, the Fermi momentum; for the higher energies we consider, the Rajaraman correction should be a valid procedure, and as we can show, gives good agreement with the empirical results. The calculational procedure as given by Rajaraman applies only to central forces and hence is only approximately valid in our case, since we include tensor forces in our nucleon-nucleon interaction. However, the influence of the tensor force is diminished at high energies and so this approximation should be fairly good, although a more quantitative evaluation will very likely be necessary once the experimental data become more refined.

We need to emphasize repeatedly here the advantage of the G-matrix approach as opposed to the more familiar Rayleigh-Lax-type calculation in which a phenomenological nucleon-nucleon T-matrix is used, in terms of the ability to deal with the important many-body corrections we have discussed.

In order to compare our calculated potential depth with empirically determined values we must first eliminate the usual potential well depth and geometry ambiguities which always plague low-energy phenomenological analyses. As has been frequently demonstrated, a unique quantity determined by optical model fits to elastic scattering data is the volume of the real potential. Thus we have taken the volumes of several tabulated optical model potentials from different sources⁵⁶ for $p + {}^{40}\text{Ca}$ and for $p + {}^{208}\text{Pb}$ over a wide range

of energies from 30 to 1040 MeV. Given this empirically determined real potential volume and assuming a Woods-Saxon geometry as given by Becchetti and Greenlees⁵⁷, where in the usual notation $r_0 = 1.17$ fm and $a_0 = 0.75$ fm, we thus obtain a consistent set of experimental values for the depth of the real optical potential which we may compare to our calculated values.

In Fig. 20 we show predictions for the real optical potential for ^{208}Pb , where the solid curve represents the calculation using only the even state forces, and the dashed curve gives the result of using all angular momentum states. The data points were taken from the tabulation of Perey and Perey⁵⁶ and from previous microscopic and empirical analyses of the 1 GeV data⁵². They were renormalized as explained above.

The cross-hatched region of Fig. 20 gives the prediction of the more usual t-matrix or Raleigh-Lax potential, which depends upon the free-space nucleon-nucleon t-matrix conventionally parameterized as

$$t(q^2) = (ik_0\sigma_T/4)(1 - i\alpha)\exp(-\beta q^2/2), \quad (4)$$

where q is the 4-momentum transfer, σ_T is the total cross section and α and β are parameters obtained by fitting the differential nucleon-nucleon cross section data. In the T-matrix approach the optical potential is computed using $t(q^2)$ and the Fourier transform of the target nucleus charge distribution as explained in the literature⁵⁸. The large width of this shaded region is a reflection of the experimental uncertainty in the three parameters σ_T , α and β ; the largest uncertainty occurs in the α parameter for protons on protons⁵⁹.

We observe that the even-state calculation gives good agreement throughout the entire energy range considered, although it is a bit too attractive at low energies. The potential becomes repulsive at about 500 MeV. At low

energies where the Rajaraman correction is known to fail the complete calculation using even plus odd state forces does rather well. Notice that the T-matrix calculation is quite poor both in absolute magnitude and in the predicted slope of the energy dependence.

We find that the sum of the various second-order, three-body-correlation and impulse-approximation correction terms of nuclear-matter type amount approximately to +10 MeV at low energies and +2 MeV at 1 GeV incident proton energy. With the exchange force included, the net correction at low energy would be small since this force would contribute about -10 MeV so that our calculations shown in Fig. 20 should be fairly accurate and are consistent with the existing data at both low energies and at 1 GeV.

This puts us in a position to say something about why the T-matrix approach, which is overwhelmingly the most often used in various medium-energy nuclear reaction and scattering studies, fails as it does. The T-matrix approach has the great advantage of being straightforward. It is so straightforward that one can see in advance it will fail, as will any approach which relies on the empirical nucleon-nucleon t-matrix (Eq. 4)). Through the strong energy dependence of the empirical parameters α and β in Eq. (4), the real potential well depth and geometry acquire an unphysical energy dependence which does not agree at all with phenomenology but which cannot be avoided within the approach because of its very straightforwardness.

There thus seem to be two problems, perhaps related. The first is that the parameterization of Eq. (4) is for some reason physically unreliable below 1 GeV, even when the approximation $\tau \approx t$ is a reasonably good one, so that t needs to be constructed directly from a nucleon-nucleon interaction rather than via the uncertain phenomenology of Eq. (4). The second is, as

shown in this work, that the Rajaraman-type corrections cause a sizable difference between t and τ throughout the medium-energy range; these corrections affect more the magnitude than the slope of the logarithmic energy dependence.

The work just described will appear shortly in Physical Review C. A continuation of this work by Ray and Coker involves the calculation of the geometry of the intermediate-energy proton-nucleus optical potential, including the nuclear-matter corrections found important in the earlier work. That is, we must obtain $\tau(q^2)$ for the range of q^2 appropriate to the specific scattering case considered. The imaginary part of the proton-nucleus optical potential is, at least in the energy range 800 to 1100 MeV, about 5 times the strength of the real potential and roughly independent of energy. Because we cannot hope to obtain this imaginary potential by explicit consideration of the various reaction channels open to the proton-nucleus system, we are forced to determine it essentially as is done in the conventional application of KMT, which we have called above the T-matrix or Rayleigh-Lax method. However, this seems adequate, since the behavior of the imaginary potential as calculated in this way is not in disagreement with phenomenology, in contrast to the behavior of the real potential.

To obtain the correct geometrical form and energy dependence of either potential, however, we have found that the usual Fourier transforms must be taken with great care. In earlier work by various authors, the transforms were taken analytically integrating over q from zero to infinity. It is vital to use the physical range for q , zero to $2k_0$, where k_0 is the incident nucleon momentum in the two-nucleon center-of-momentum system. With this proper range, the Fourier transform of $t(q^2)$ gives an imaginary potential geometry and energy dependence in good agreement with phenomenology. The geometry of the real potential is quite different depending upon whether

$t(q^2)$ or $\tau(q^2)$ (as obtained via our G-matrix approach) is used. Thus we have found that it is necessary to construct $\tau(q^2)$ in terms of the usual phase-shift expansion, and the Rajaraman prescription (odd partial waves omitted-- this is also required by the Pauli principle for the $p + p$ part of the scattering amplitude). A number of analyses of the available $p + {}^4\text{He}$ data from 500 to 1200 MeV have been made with this approach, and we find remarkable agreement with an earlier phenomenological analysis, to the discussion of which we now turn. At least two publications will be forthcoming on the nuclear-matter approach for constructing both the energy-dependent well-depth and geometry of the proton-nucleus optical model potential.

One example of the many preliminary results of this work is shown in Fig. 21. Here, the solid curve is the result of a calculation with the nuclear-matter approach just discussed, while the dashed curve is the result of a single-folding calculation of T-matrix or Raleigh-Lax type, except that instead of using the simple Gaussian effective interaction resulting from Eq. (4), we used a Kallio-Kolltveit effective interaction. Finally, the dotted curve shows the effect of inclusion of one type of spin-orbit interaction; note the extreme importance of the spin-orbit effect, which has been generally ignored in most medium-energy elastic scattering analyses in the literature.

In last year's progress report, mention was made of an extensive phenomenological analysis of the available medium energy $p + {}^4\text{He}$ data, which now includes incident energies of 348, 578, 587, 600, 650, 720, 1000, 1050 and 1154 MeV. This is a vastly greater range of energies than is available for any other medium-energy elastic scattering process involving nucleons, and we have used it as a crucial test-bed for our present efforts to construct a microscopic nucleon-nucleon potential. Results of the un-

published phenomenological analysis mentioned in last year's report are shown in Figs. 22 and 23, for selected energies. In Fig. 22, the dashed curves are the results of a conventional T-matrix or Raleigh-Lax microscopic potential, using the best available nucleon-nucleon parameters, those of Bystricki and Lehar⁵⁹. The solid curves are the results of using a purely phenomenological potential of KMT form, namely $(V + iW)\rho(r)$, where $\rho(r)$ is the nuclear matter density constructed from the charge form factor $F(q^2)$ as usual⁵⁸. In Fig. 23, results are shown for inclusion of a spin-orbit term. The solid and dashed curves show phenomenological results with inclusion of two different sorts of spin-orbit terms. The solid curves use a Dirac-equation form of the spin-orbit term, involving a derivative of the real potential. The dashed curves are calculated with a complex spin-orbit term of the sort obtained by Lambert and Feshbach from a nucleon-nucleon phase-shift analysis.

Details of these phenomerological analyses will eventually be included in the appropriate paper on our microscopic G-matrix nuclear matter approach as applied to the $p + {}^4\text{He}$ data.

vi. Time-Dependent Treatment of Heavy-Ion Collisions

One of the recent trends of developments in heavy-ion physics is an increased emphasis of the dynamical aspects of nuclear matter flow. The dynamical aspects in nuclear fission have been explored and emphasized for several years particularly by Griffin⁶⁰. In the rapidly developing field of heavy-ion collisions on the other hand, one often observes large mass transfers between the colliding nuclei. Hence one can visualize a large nuclear matter flow to be taking place when the two colliding nuclei are in contact with each other.

To study the dynamics of matter flow, we start with the simple but fundamental problem, namely, the problem of deriving the matter-flow dynamics from the time-dependent Schrödinger equation for a single quantal particle. This study begins with the thesis work of Kan at the University of Maryland⁶¹. Publications⁶² are being generated basing on his thesis research. We now briefly summarize this work as follows.

The fluid-like properties of the single-particle time-dependent Schrödinger problem with a time-dependent potential were extracted following Madelung⁶³ by use of the ansatz that the solution is in polar form. The unique (irrotational) velocity field \vec{v} of the single-particle state and the fluid density ρ , were seen to emerge as a natural consequence. It also followed that \vec{v} is generally compressible and possesses line vortices (i.e., singularities of circulation distribution at certain lines in space). As pointed out by Dirac⁶⁴, the circulation around a line vortex is quantized. This fluid dynamical formulation was seen to be a generalization of that of Hill and Wheeler⁶⁵ which is restricted to incompressible and irrotational flows.

Alternative velocity fields were also considered which conveniently

summarize the continuity relation, and satisfy other specified conditions such as incompressibility and regularity, but in contrast to \vec{v} , they are not fixed uniquely by the state wave function. Physical implications in the adiabatic approximation for collective momenta and energy were studied. A variety of forms, which \vec{v} as well as the alternative velocity fields allow, were explored.

In the cases where a particular kind of alternative velocity field defined as a regular solution \vec{v}_R of the continuity relation can be obtained, we found for the collective kinetic energy a remarkably simple and symmetric quadratic form involving not the square of one single velocity, as in the classical case, but instead the scalar product between \vec{v} and \vec{v}_R .

The theory of the Schrödinger fluid thus formulated was applied to study collective rotation, especially for nuclei bound by the harmonic oscillator potential at the equilibrium deformation and the infinite nucleus bound by a cubical square well potential with a periodic boundary condition. It is known that the cranking model gives the rigid moment of inertia for these two cases^{66,67}. We found for the former case that neither the velocity field \vec{v} for each single-particle nor the velocity field \vec{v}_T for the total matter flow resemble the velocity field for rigid body rotation \vec{v}_{rig} . For the latter case, it was seen that although $\vec{v} \neq \vec{v}_{rig}$ for each single particle, yet $\vec{v}_T \equiv \vec{v}_{rig}$ due to the uniformity of the density and the infinite size of the system.

In the literature there are two requirements which are believed to be the sufficient condition for the cranking model to give the rigid moment of inertia. They are the requirements for equilibrium deformation⁶⁶ and for equi-distribution of energy⁶⁸. Bohr and Mottelson⁶⁶ have already pointed out that the first condition might not work for potentials other

than the harmonic oscillator. We found by using the Hill-Wheeler box as a counterexample that the second condition is not generally valid either.

In analogy to classical mechanics, a parallel axis theorem for the cranking moment of inertia may be useful especially for the calculation of moments of inertia in heavy ion collisions. However, we found no such a theorem in the literature. By utilizing the theory of the Schrödinger fluid, we proved such a parallel axis theorem.

The study of nuclear deformation was also made by emphasizing the dynamical compressibility of the Schrödinger fluid of the particle occupying the last level. Physically, this associates with the motion of density ripples in the nucleus. This phenomenon was illustrated with the case of crossing of two harmonic oscillator single-particle levels in a quadrupole deformation, in which the particle adiabatically adjusts itself in occupying the lower level. The corresponding single-particle density distribution was seen to change its node structure, and at the same time \vec{v} was seen to acquire compressible vortices as contrast to the case of noncrossing harmonic oscillator levels, which gives \vec{v} identical to the incompressible liquid drop velocity. The strength of these vortices was shown to be related to the large deformation inertia occurred in case of level crossing and estimated by Griffin⁶⁹.

A quantity, which we label the "dynamical rippling" was constructed to measure the deviation of the Schrödinger fluid from the incompressible liquid drop velocity field. It was seen that this "dynamical rippling" peaked at the crossing deformation and therefore is believed to be an appropriate index for the rearrangement of density ripples in the Schrödinger fluid.

Another time-dependent approach to heavy-ion collisions in which Kan and Tamura worked during the past year is the time dependent Hartree-Fock

(TDHF) calculations⁷⁰, for which several papers have been reported recently, intending to elucidate the mechanism of the collision between heavy-ions⁷¹. Their results looked rather exciting. For example, they have obtained for the two partners in the collision process a lowering of the translational kinetic energy after the collision. This seems resembling to some extent the phenomena of deep-inelastic processes observed in many recent experiments⁴⁰.

In spite of these exciting results however, certain aspects in the TDHF calculations, especially those concerning the limitation of the theory and the problem of extracting the cross section from the calculation, do not seem to be well stood yet. Our present investigation aims at these aspects.

A characteristic feature in the TDHF calculation is that the spacial dependence of the wave functions has to be obtained numerically at each time step. It follows that the total period of time and the range of space which appear explicitly in the calculation must both be finite. Therefore, the approach has no room for the concept of an asymptotic form of the wave function, which is utilized in all time-independent collision theories, and is vital in defining the concept of the theoretical cross sections. We would thus have to find out a way to extract cross sections from a theory which does not allow for the use of the asymptotic wave functions.

In the present investigation we have found that due to the restriction of the TDHF solutions to the one-Slater-determinant manifold (a restriction which is intrinsic to TDHF) the TDHF theory does not allow a quantum mechanical wave packet for the center of mass (c.m.) motion, thus requiring that at least the c.m. motion be treated classically. This

classical behavior is most clearly seen in the initial situation in a collision process, but can in fact be traced throughout the time evolution of the TDHF solution. Hence it follows that if the calculation is to be fully quantum mechanical, the one-Slater-determinant restriction, and hence the TDHF itself, has to be given up.

In lifting this classical restriction, we have proposed a time-dependent generator-coordinate (TDGC) approach as a possible extension or replacement of the TDHF approximation. It was seen that the initial wave function in TDGC can be constructed so as to have the correct quantum mechanical wave packet nature, thus satisfying every quantum mechanical requirement.

Just as the time-independent GC method⁷² was an approximation to the time-independent Schrödinger equation, the TDGC (as well as TDHF) is an approximation to the time-dependent Schrödinger equation. The time-independent GC method has been used in the nuclear structure problems, particularly in describing collective motions⁷³. Its application to reactions has also been made, successfully analyzing the data of α - α scattering⁷⁴. Its extension to TDGC is an interesting problem, and we made a few qualitative investigations of how such an extension may be made.

In whatever way the TDGC equation may be solved, it is very likely that the solution obtained has a wave packet nature, and is obtained with a restricted time-period and spacial-range, just as was the case in TDHF. We have thus studied the properties of such a small packet by taking, however, a time-dependent Schrödinger equation for a one-body problem, rather than the TDGC equation for a many-body problem. More precisely, what we have tried was to figure out what sort of initial wave packet should be chosen, how the packet would behave during a finite period of time, and

and how the cross section can be obtained out of such a packet. At the same time, the problem of constructing outgoing channel states for the TDHF solutions was also investigated, by incorporating collective vibrations to the particle excitations. This will help to interpret the TDGC solutions which would be obtained in the future.

vii. Nuclear Structure

With the purpose of providing a tool to describe the properties of various collective nuclei, Tamura has engaged, together with Kishimoto of Texas A & M University, in improving the Boson expansion theory⁷⁵. The first paper of this project was published some time ago⁷⁶, in which the formulation of the theory was made in such a way that the Hamiltonian including terms up to sixth order was given in a very compact form. This formulation was further developed later, and numerical calculations based on this improved formulation were also made. The results of this new work were published very recently⁷⁷.

The most important improvement in the reformulation made in Ref. 77 over that of Ref. 76 was that the effect of the coupling between the collective and noncollective branches of the excitation modes was taken into account explicitly. Another modification made was to add to the original Fermion Hamiltonian a quadrupole-pairing interaction term. Earlier⁷⁶ the Hamiltonian had only the monopole-pairing and the quadrupole-particle-hole interactions. In the course of the numerical calculations, it was found that these two modifications were vital in obtaining good agreement of our results with experiment.

Numerical calculations in Ref. 77 were made for twelve nuclei with mass ranging from about 100 to 200. These nuclei were chosen so that they represent different collectivities that are found in different regions of the periodic table. For example, it is known that nuclei with mass from about 100 to 120 have a fairly good vibrational character, and we picked ^{110}Pd , ^{114}Cd and ^{122}Te as typical examples of such nuclei. Two nuclei, ^{126}Xe and ^{128}Ba were then chosen to represent the γ -unstable type nuclei, while ^{192}Os , ^{194}Pt and ^{198}Hg were chosen as nuclei that are typical

of those that lie in the region in which the transition from oblate to prolate shape takes place as the mass number is increased. Finally, $^{148,150,152,154}\text{Sm}$ were taken so as to exemplify the power of our theory in reproducing the well known rapid transition from spherical to deformed regions in the course of adding only six neutrons.

We show in Fig. 24, as an example the comparison of our theoretical spectra with experiments for the pair of nuclei ^{126}Xe and ^{128}Ba , which as we remarked above, are expected to have a γ -unstable nature. A characteristic feature of a γ -unstable nucleus is that the 2_2^+ and 4_1^+ states are nearly degenerate, and also the 3_1^+ , 4_2^+ and 6_1^+ states are nearly degenerate. It is seen in Fig. 24 that this feature is very well embodied by the experimental spectrum, particularly of ^{128}Ba , and that the theory reproduces this spectrum rather well. The above near-degeneracy of the levels is less conspicuous in the experimental spectrum of ^{126}Xe , and it is seen that the theory again reproduces this fact.

The examples presented in Fig. 25 show the transition from prolate to oblate shape in the $A \approx 200$ region, and this feature is most clearly seen in the calculated potential-energy surface which is drawn on top of the theoretical spectrum (marked A) for each nucleus. It is seen that ^{192}Os is oblate, but the heavier elements, ^{194}Pt and ^{198}Hg are both prolate.

We shall not go into detail of the fits obtained for other nuclei, because the interested readers can find them in Ref. 77. One remark we wish to make here is that the nuclei chosen in Ref. 77 all lie very close to the stability line. It may then be of interest to see whether a similar good fit can be obtained as well for nuclei that lie somewhat away from the stability line. Having this interest in mind, Weeks and Tamura recently undertook an analysis of data for ^{102}Pd and ^{104}Pd and the results are

shown in Fig. 25. Note that an analysis of ^{110}Pd was made successfully in Ref. 77, and essentially the same parameters were used also in Fig. 25. As is seen, the agreement obtained is fairly good, in particular agreeing almost perfectly with the experimental information⁷⁸ concerning the electromagnetic properties.

Rigorously speaking, however, the result of Fig. 25 has a fairly serious difficulty; the $Q_2(2_1^+)$ calculated for ^{102}Pd is too large by about a factor of two, compared with experiment. The experiment⁷⁸ showed that $Q_2(2_1^+)$ for the Pd isotopes decreases gradually as the mass number decreases, becoming very small for ^{102}Pd . Our calculations do reproduce this trend, but the decrease is not as fast as it is in the experiment. It is our conjecture that for nuclei away from the stability line, a good choice of single-particle energies where our calculation begins gets more and more important. Since we plan to extend the calculation of Ref. 77 to a much larger number of known collective nuclei, we may have to find a better way in choosing the single-particle energies, than that we used in Ref. 77.

It has been planned to improve the calculation of Ref. 77 also by including the sixth order terms and also the explicit excitation of the noncollective modes. Necessary formulations and the corresponding programming of the computer code have been almost completed, and it is hoped that we can begin a very systematic calculation very shortly.

A short summary of our previous Boson expansion calculations in combination with reaction analyses was reported as an invited paper at the Balatonfüred Symposium⁷⁹.

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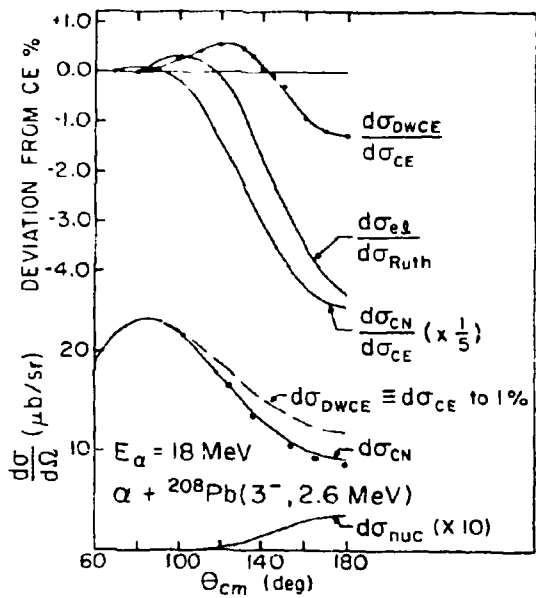


Fig. 1

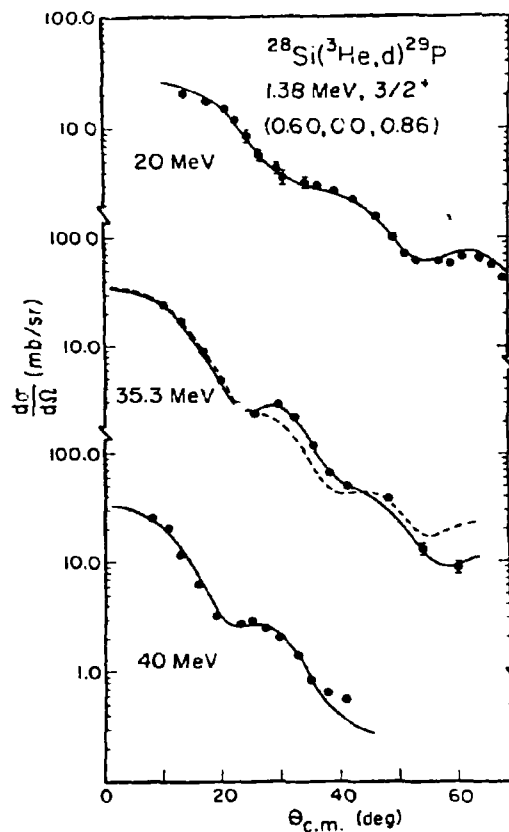


Fig. 5

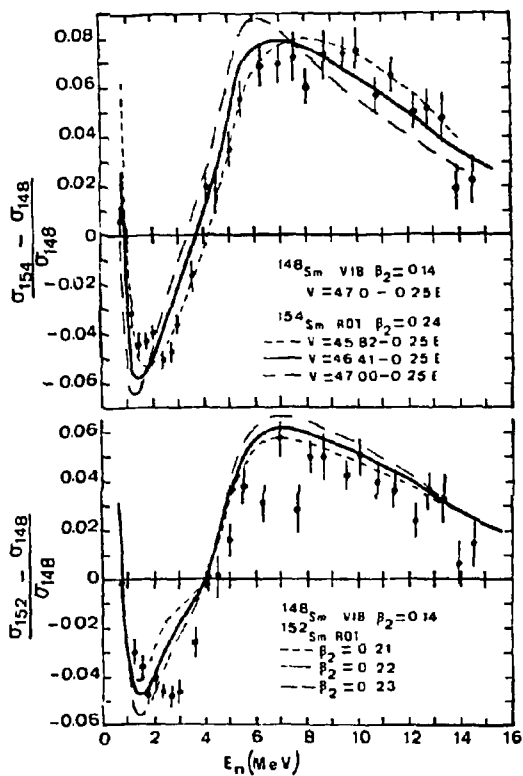


Fig. 3

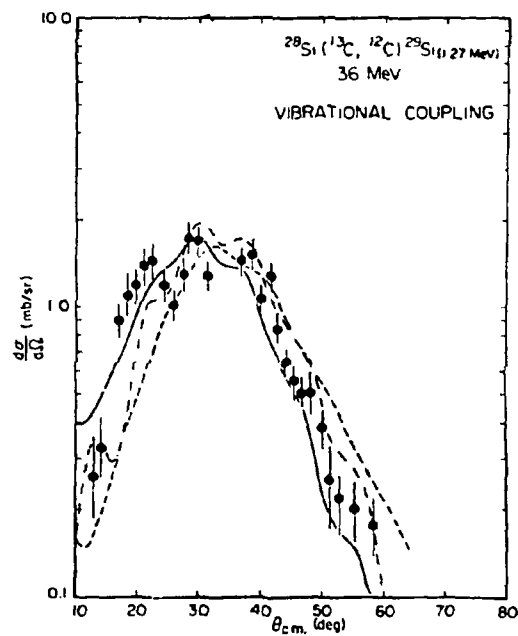


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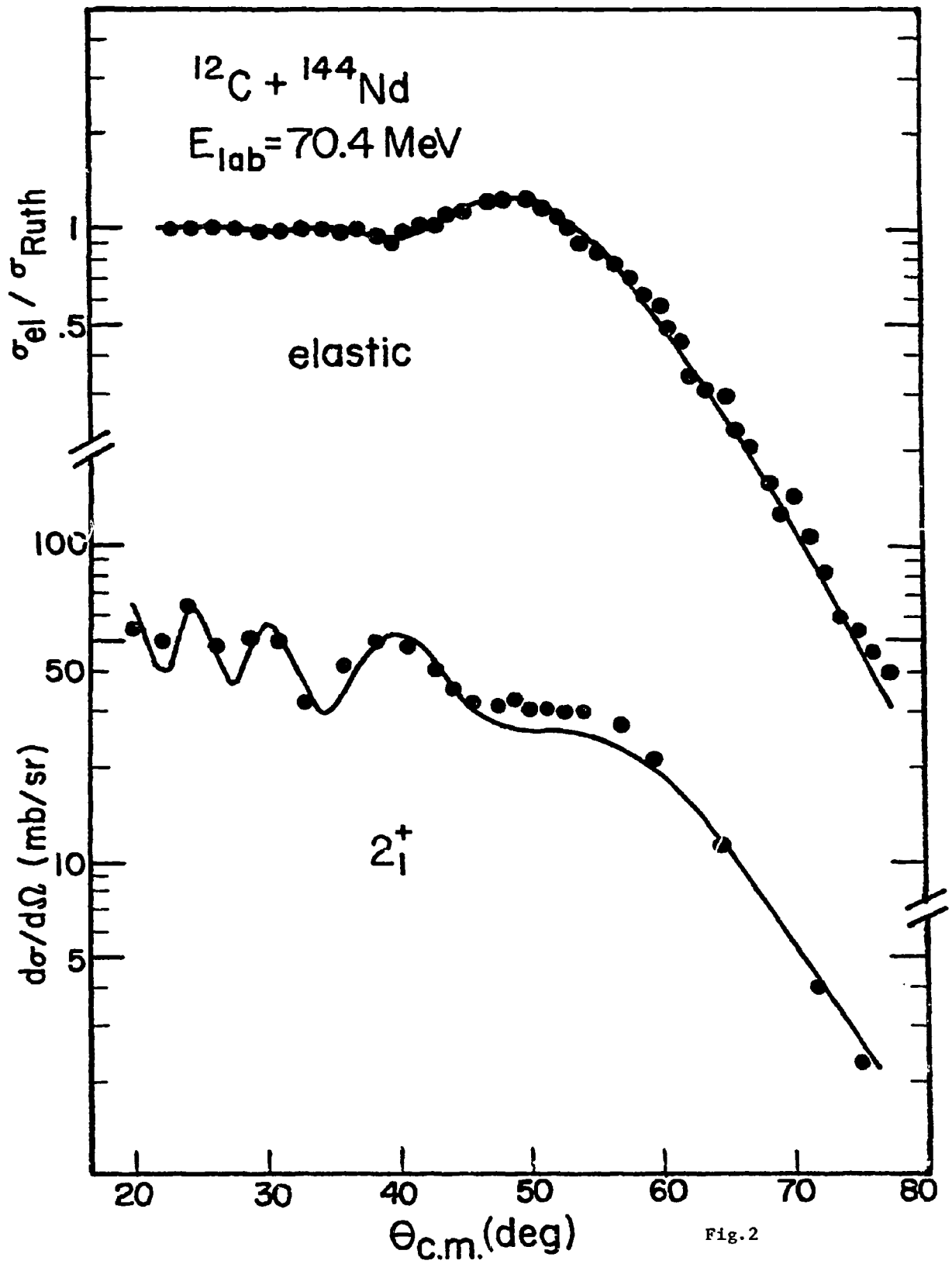


Fig. 2

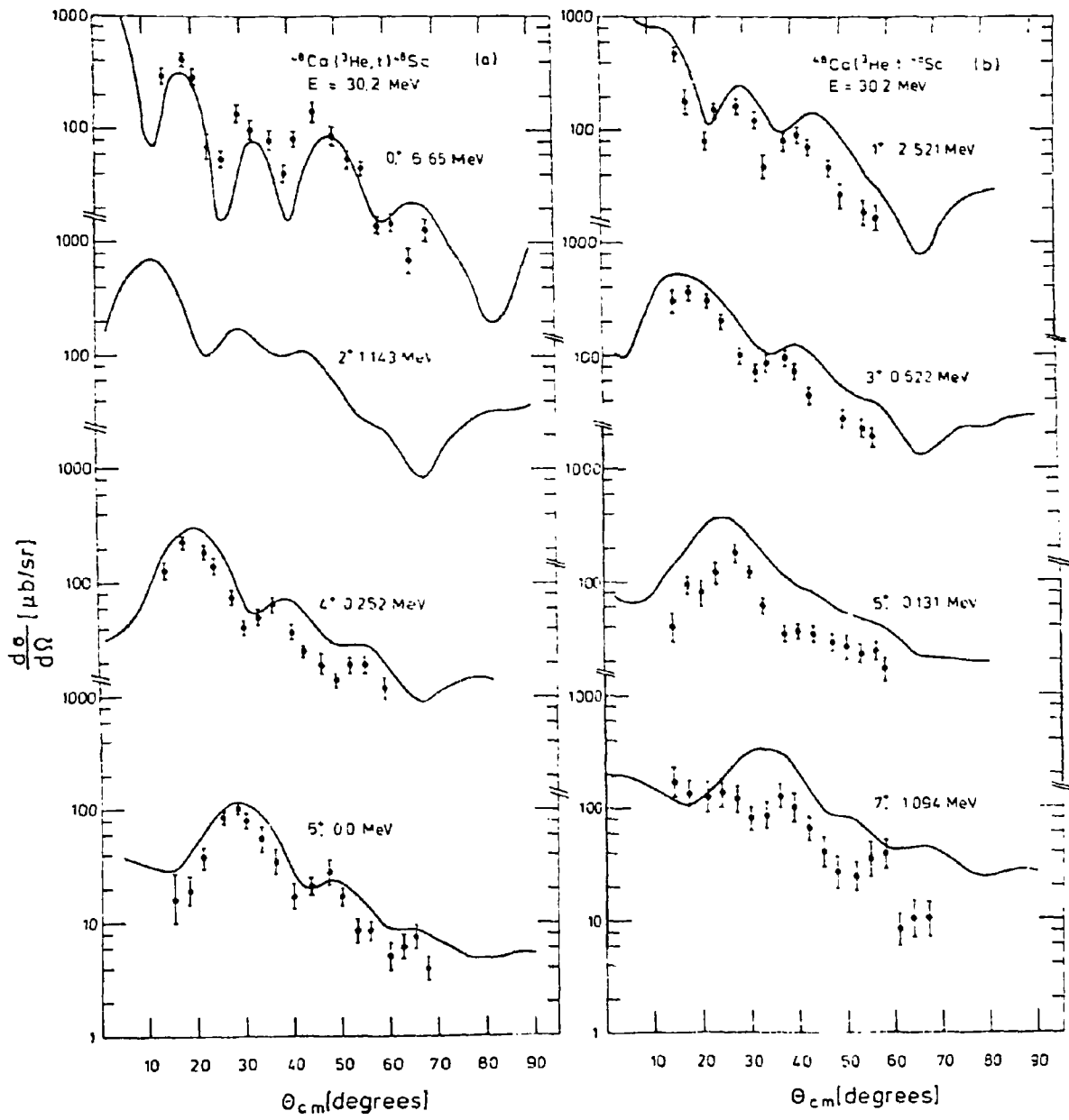


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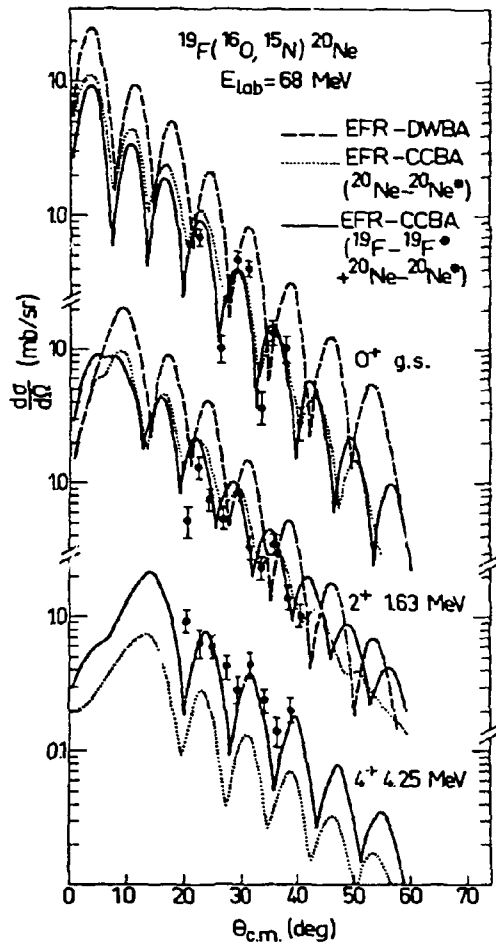


Fig. 7

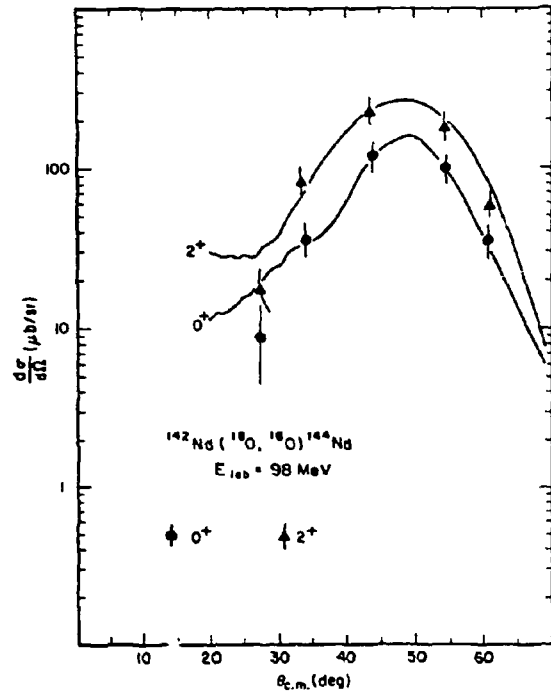
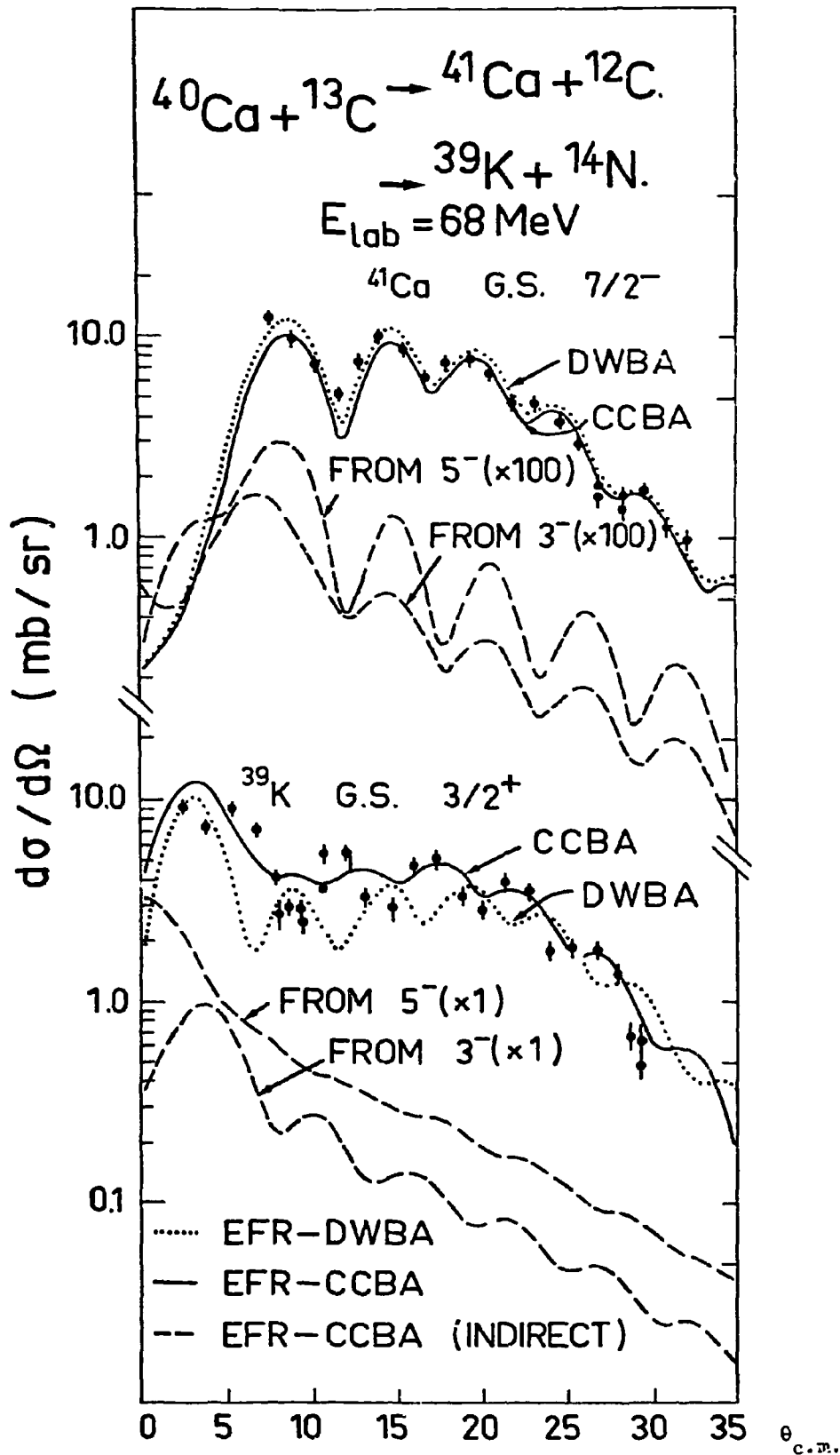


Fig. 10



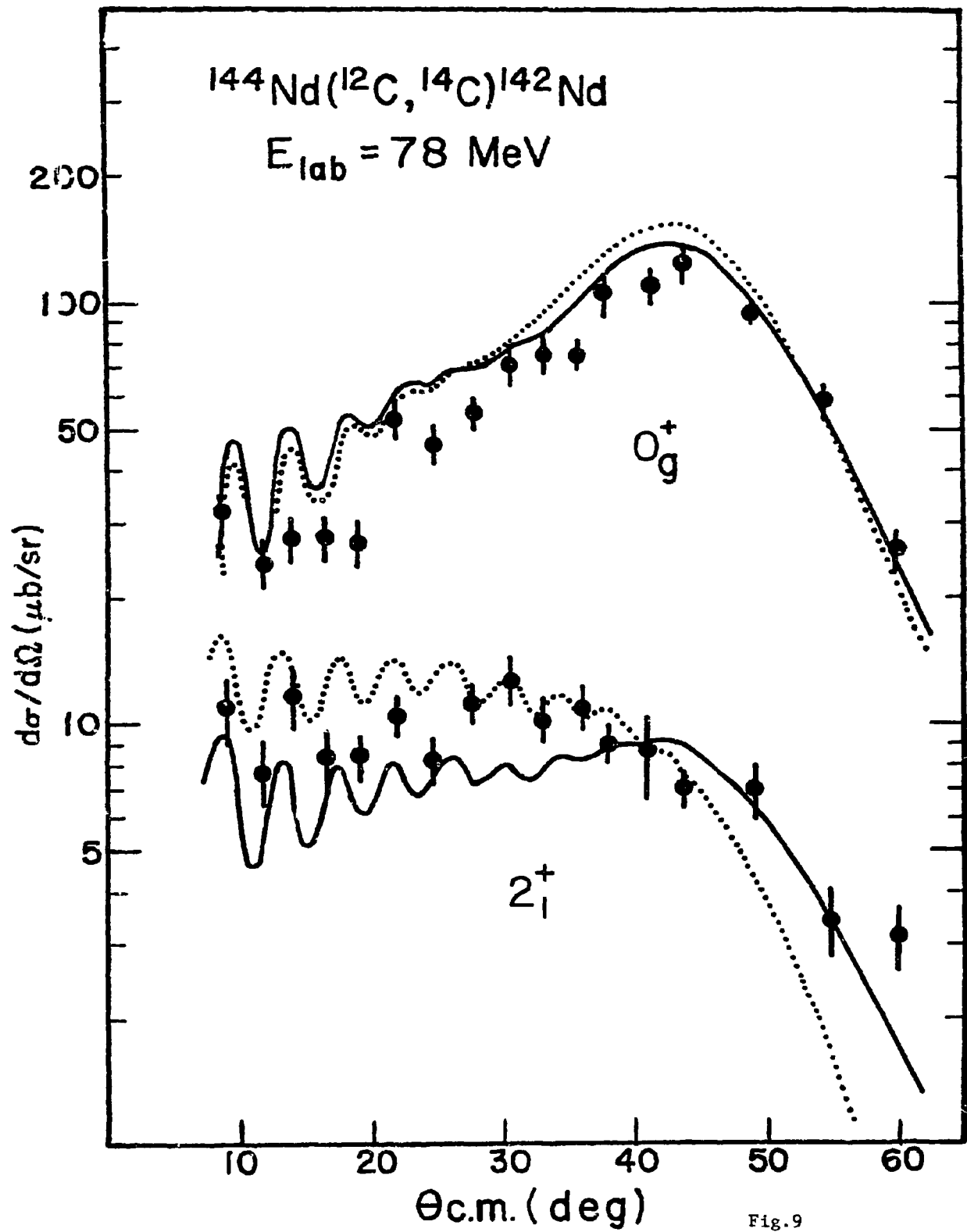


Fig. 9

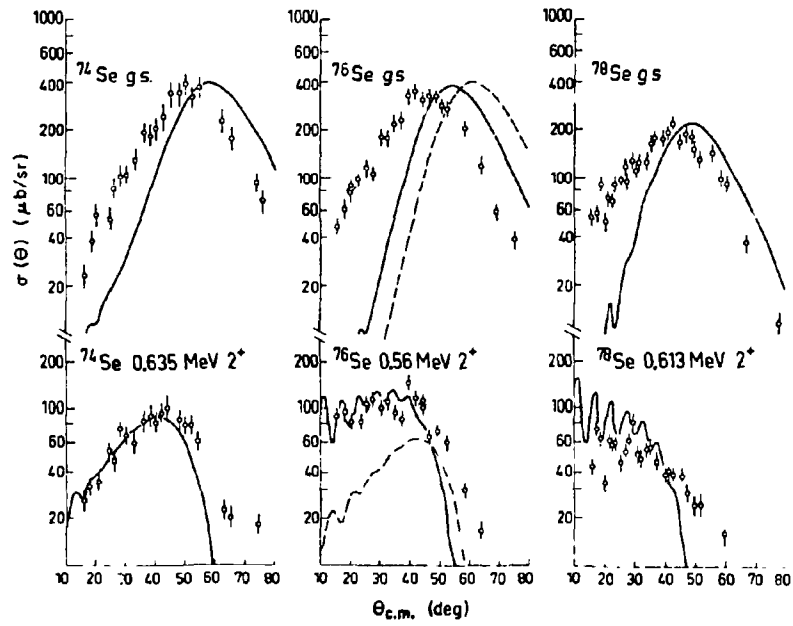


Fig. 11

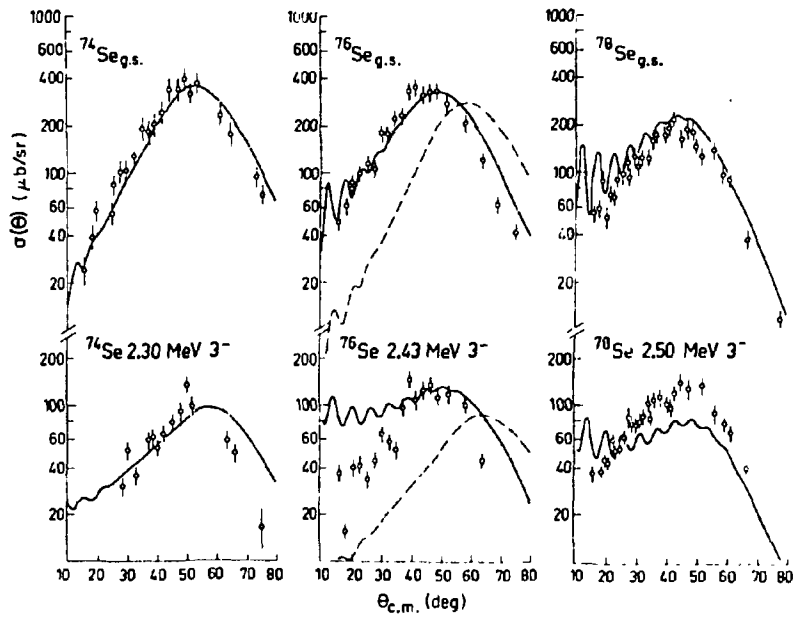


Fig. 12

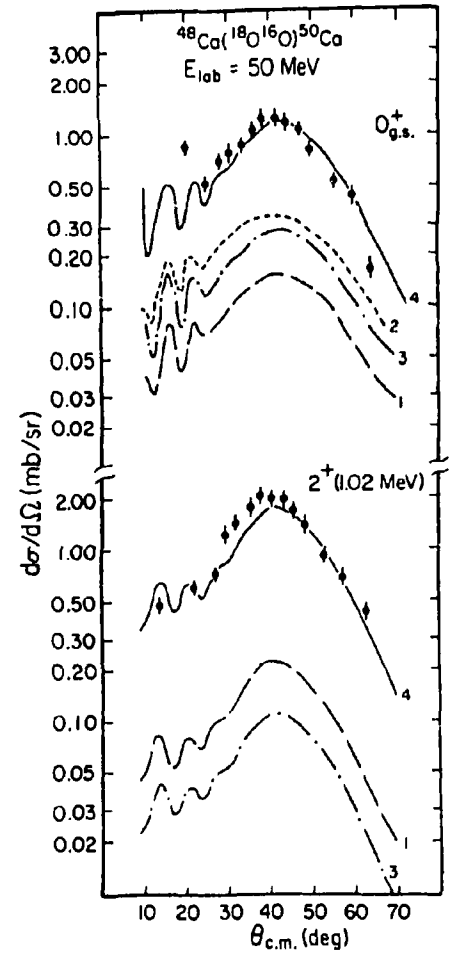


Fig. 13

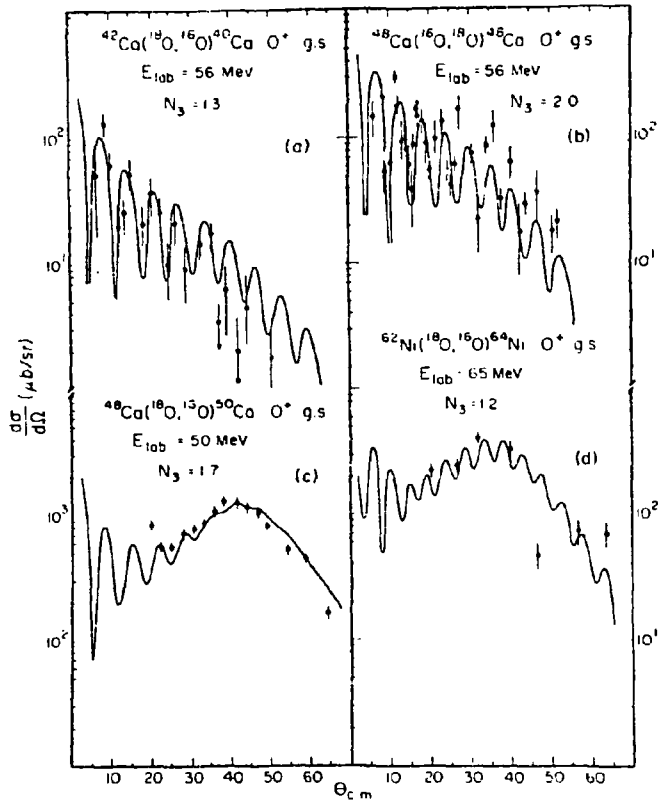


Fig. 14

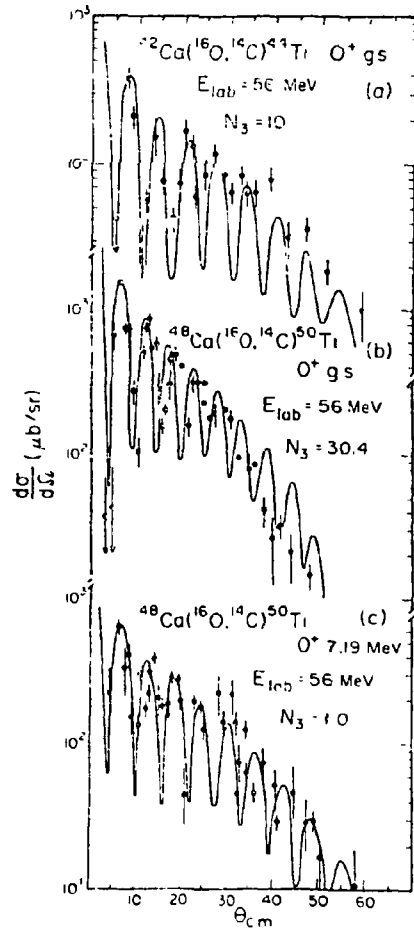


Fig. 15

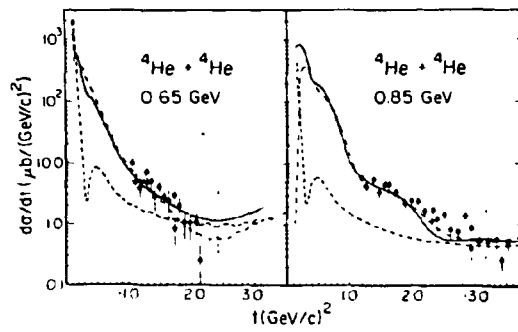


Fig. 18

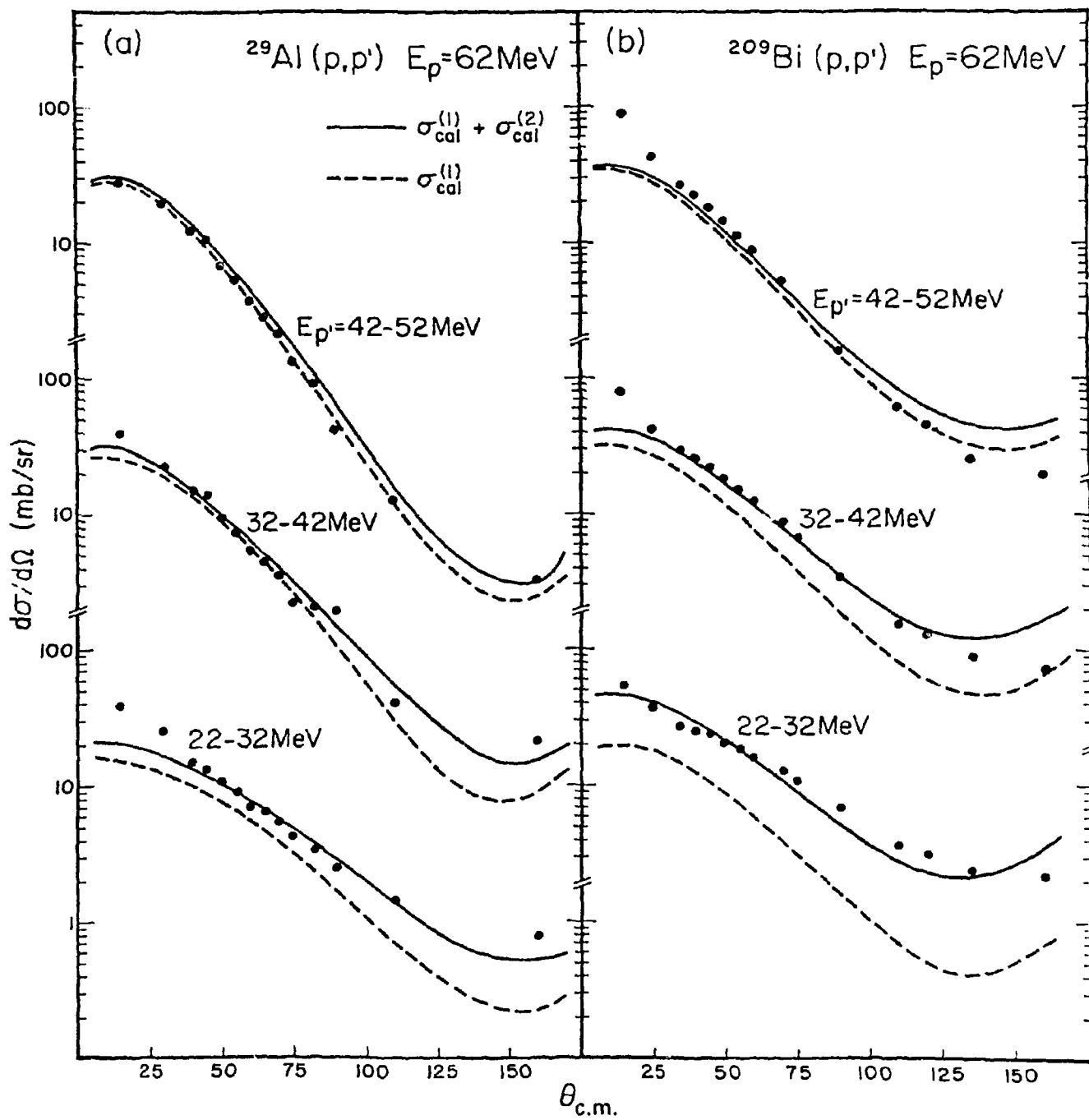


Fig.16

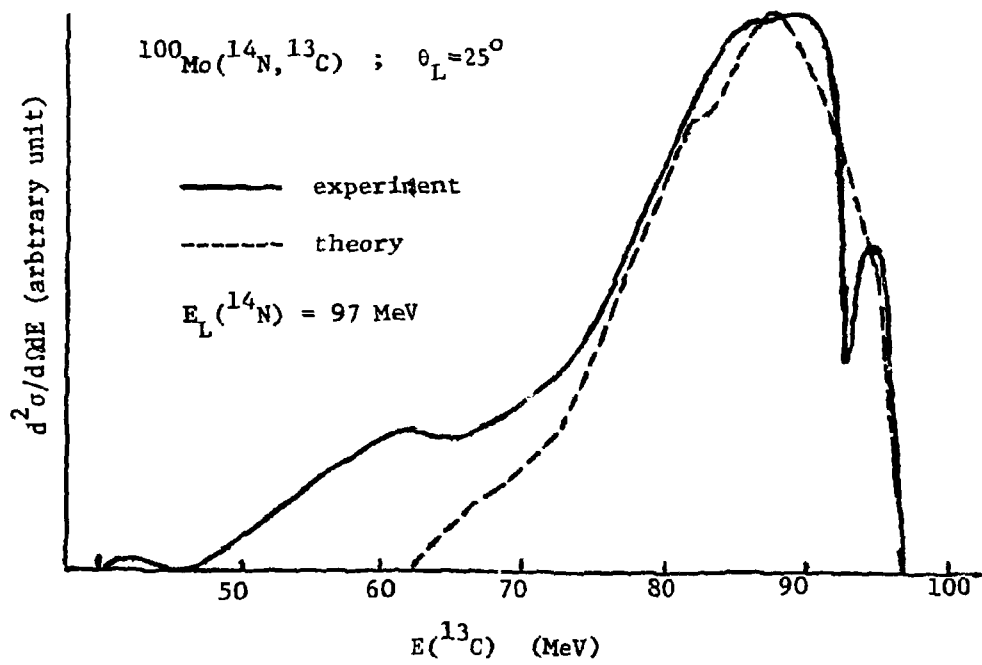
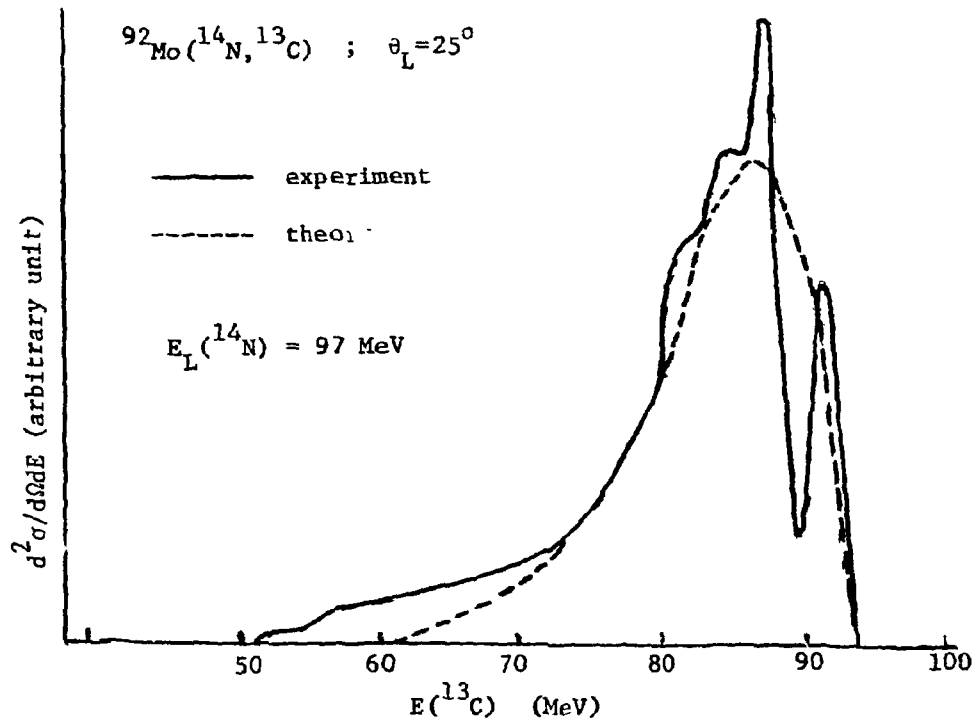


Fig. 17

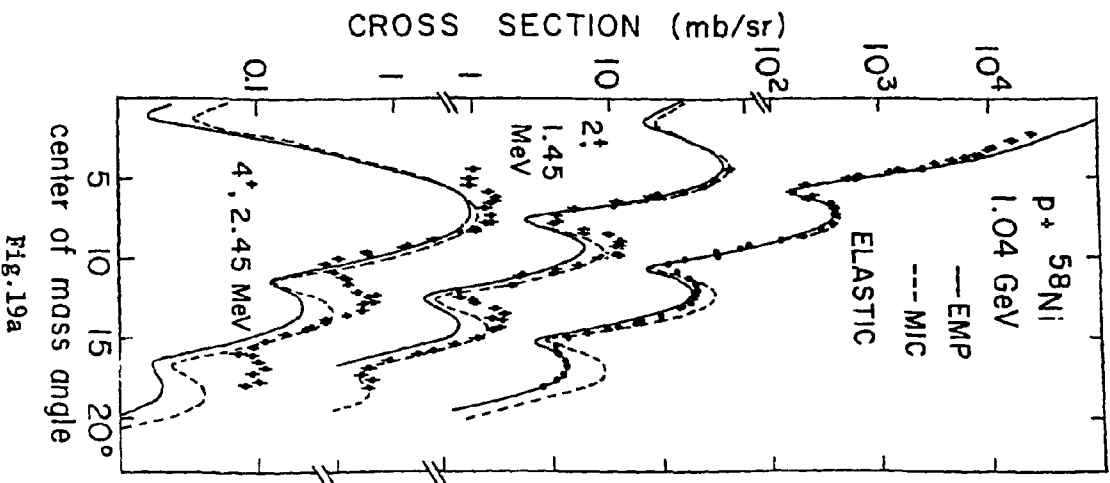


Fig. 19a

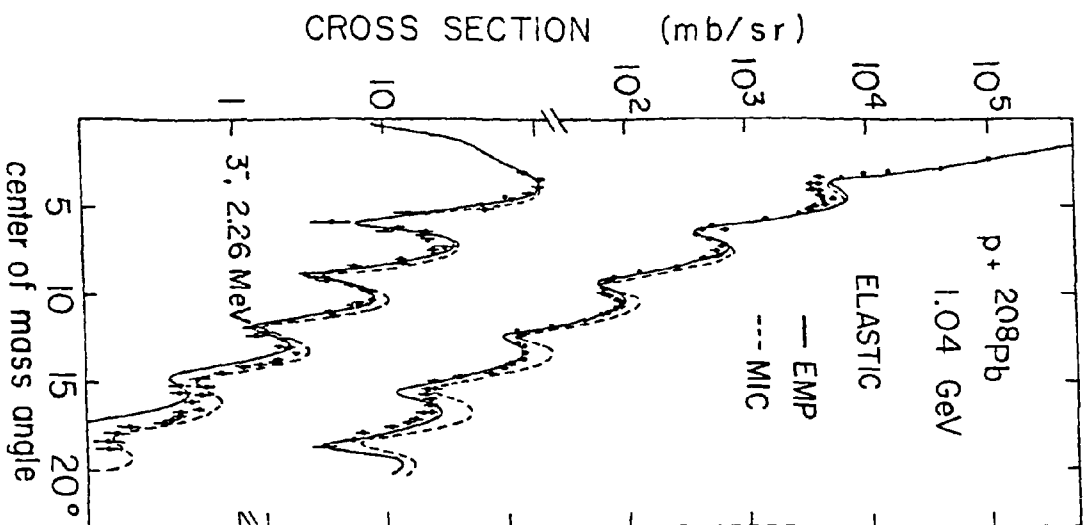


Fig. 19b

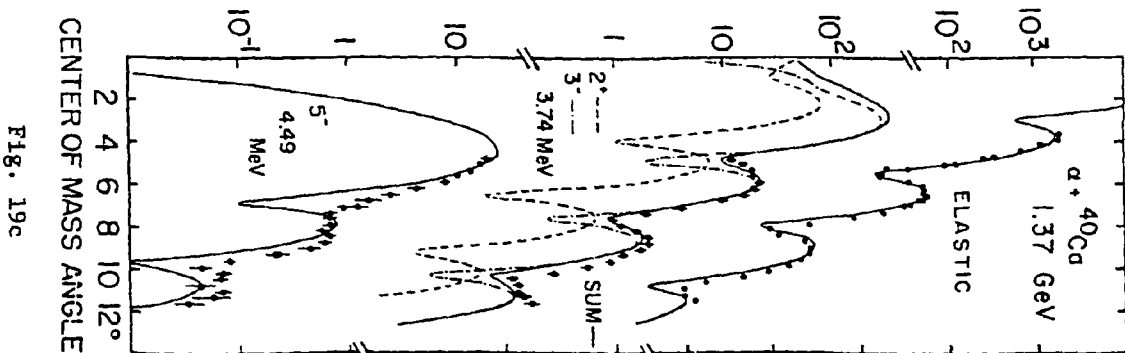


Fig. 19c

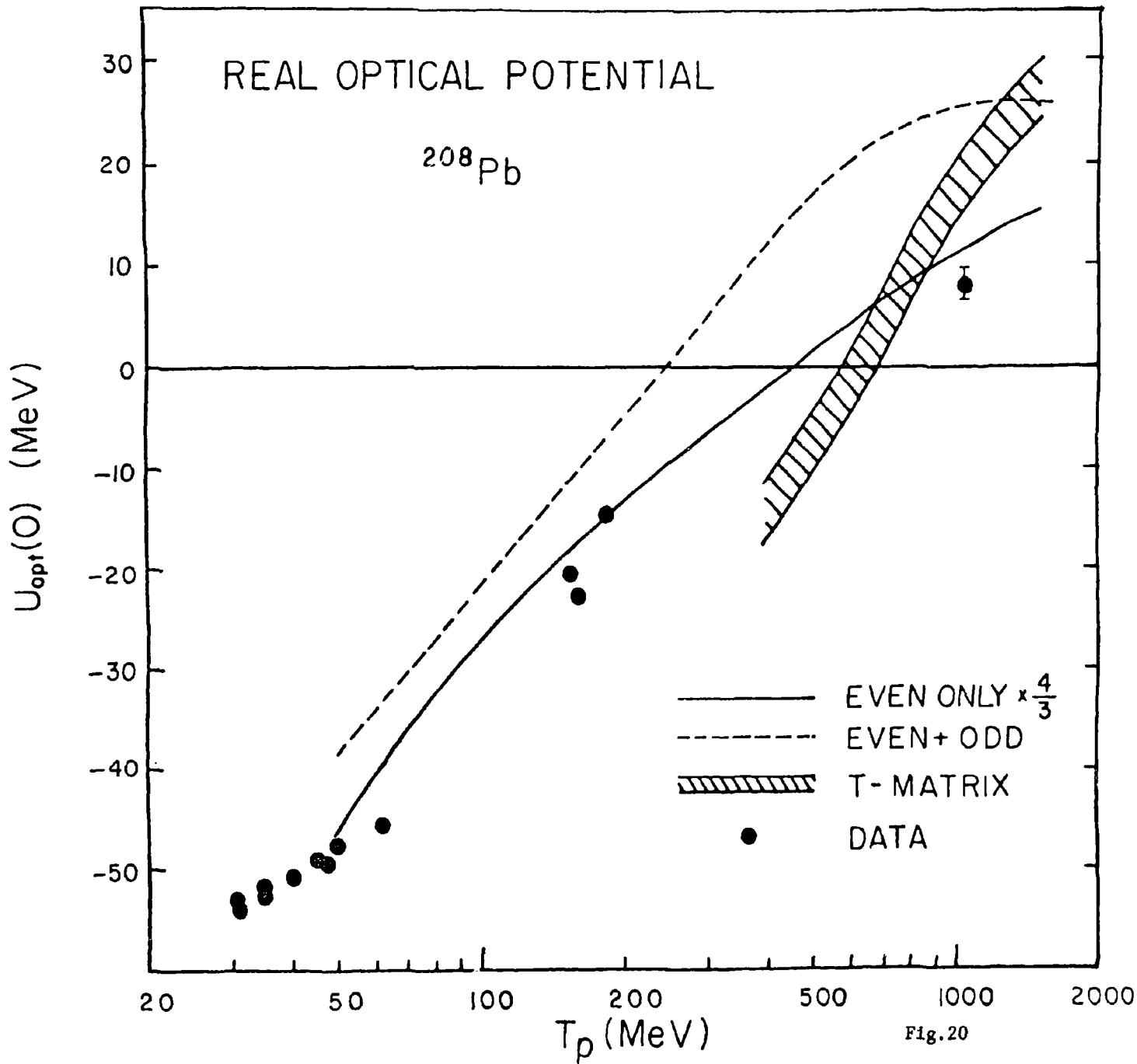
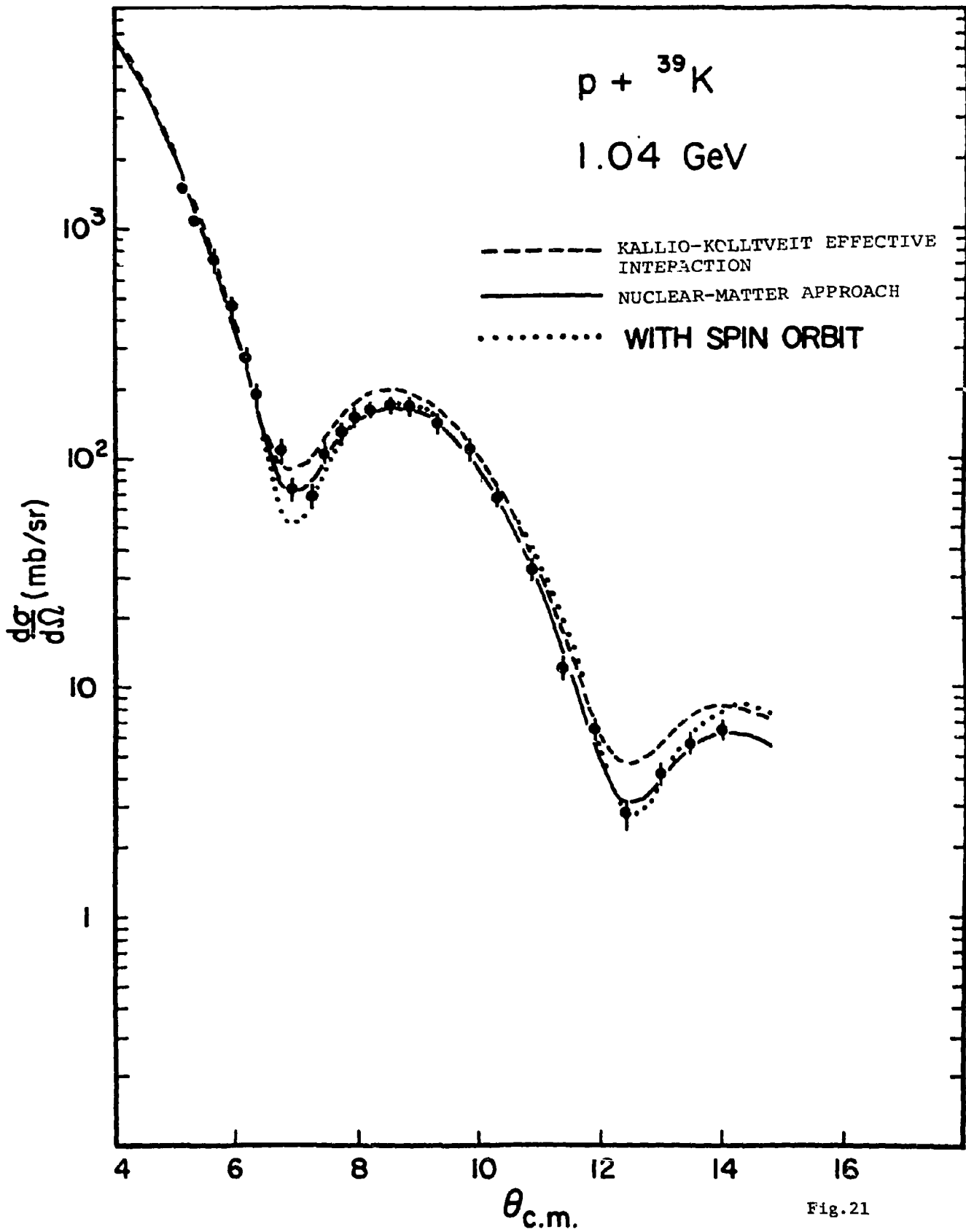


Fig.20



--- KALLIO-KOLLTVEIT EFFECTIVE INTERACTION
 — NUCLEAR-MATTER APPROACH
 WITH SPIN ORBIT

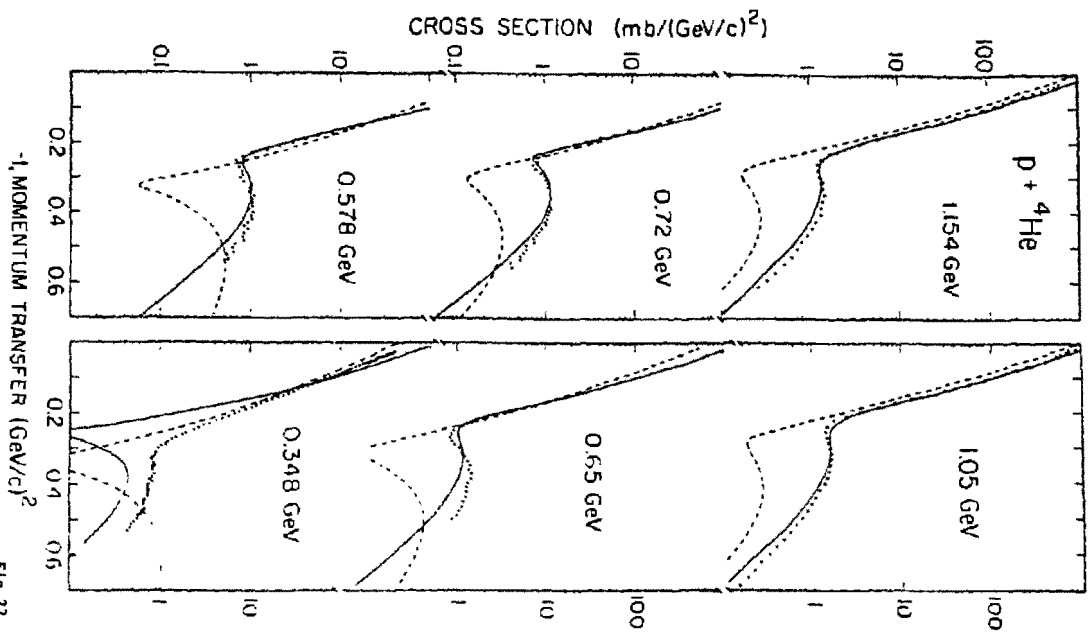


FIG. 22

FIG. 22

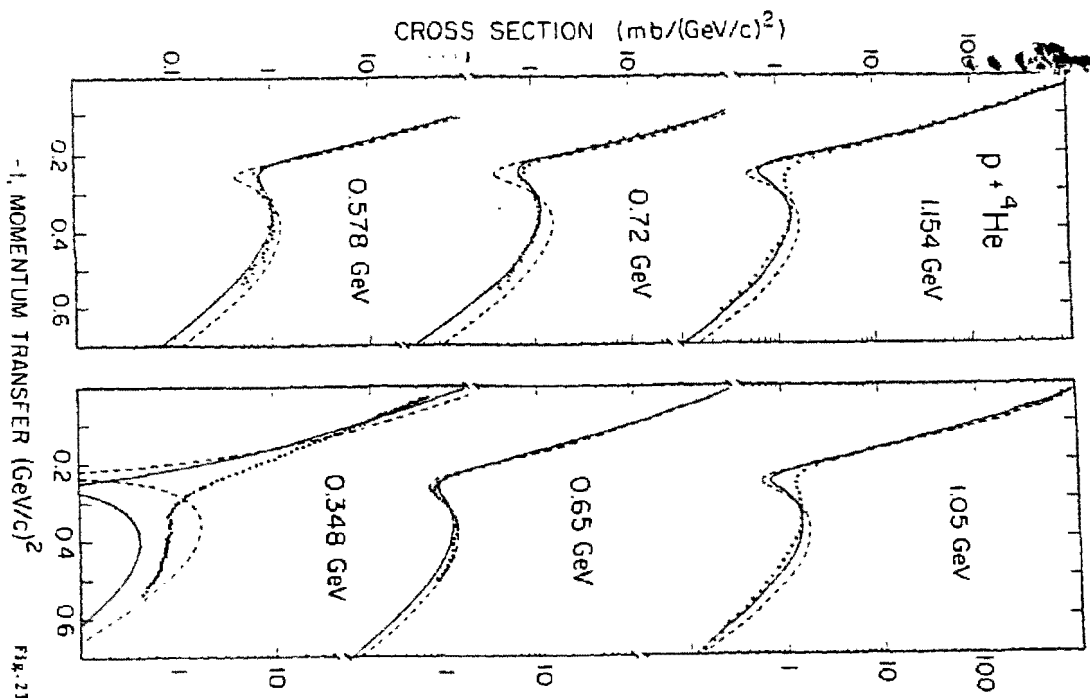


FIG. 23

FIG. 23

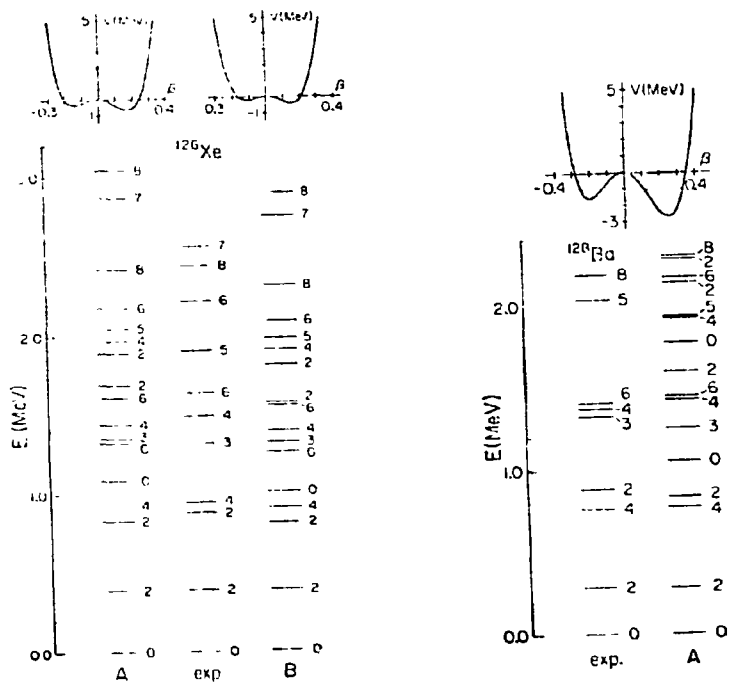


Fig. 24

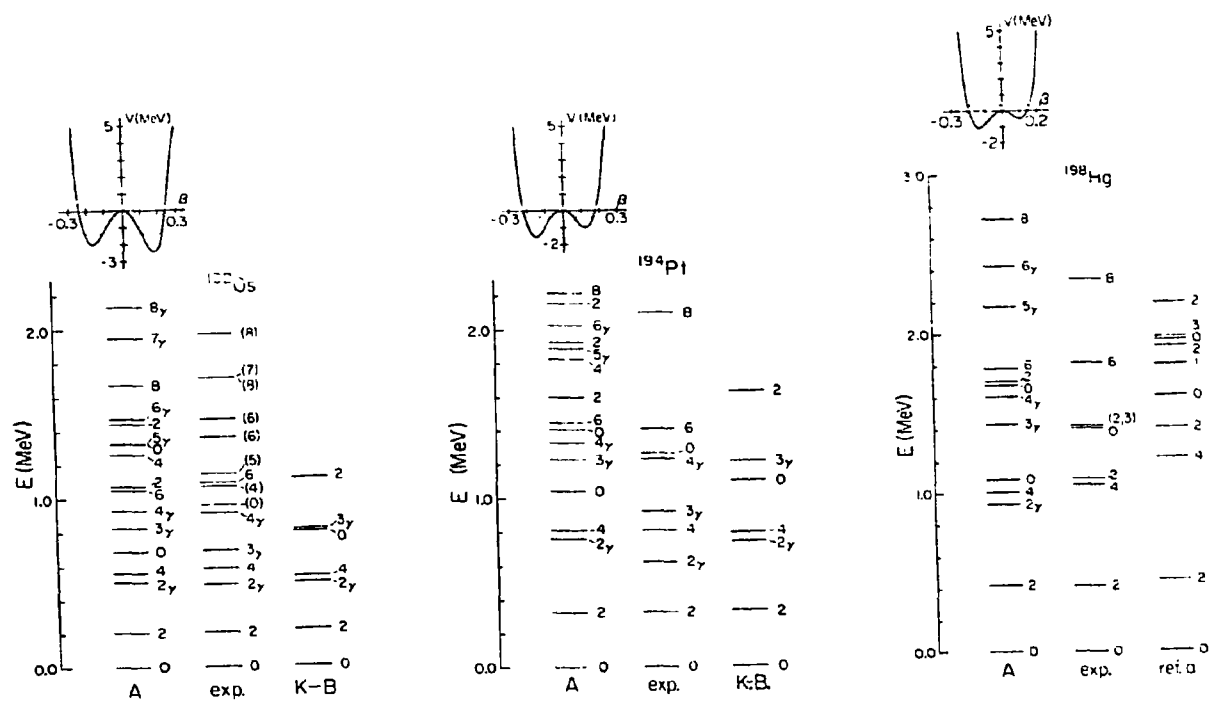


Fig. 25

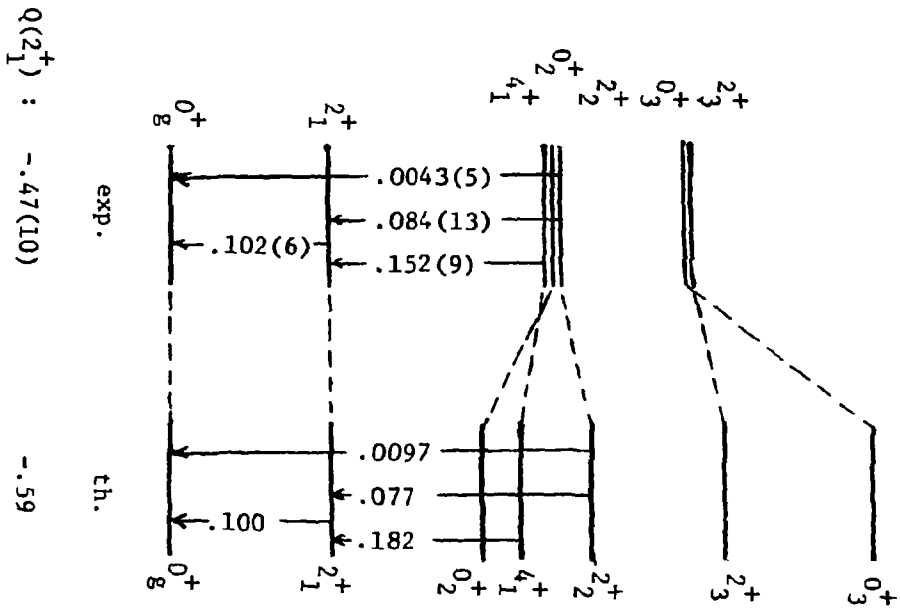
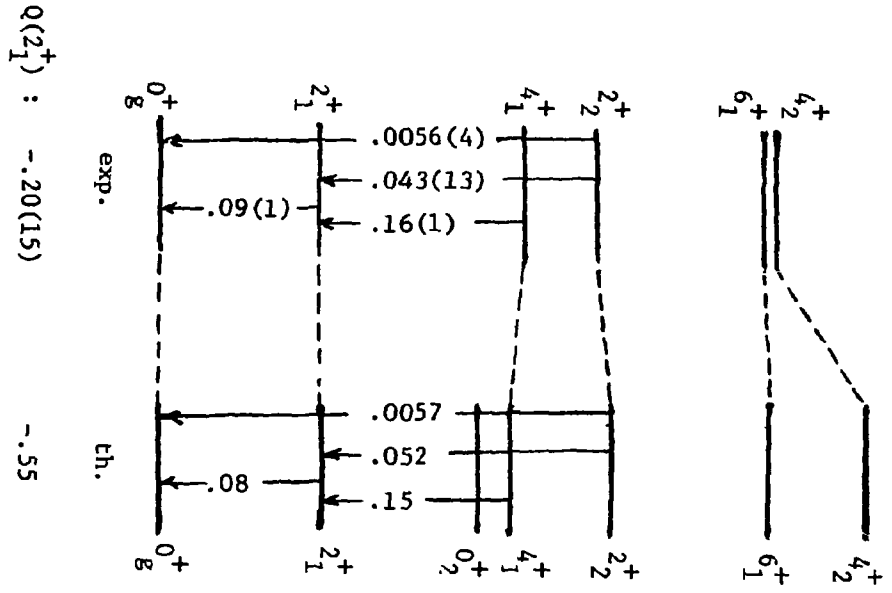


Fig. 26 : B(E2) in e^2b^2 , $Q(2_1^+)$ in eb ; $.043(16) = .043 \pm .016$ etc.

II. B. COPY OF TITLE PAGES OF PUBLISHED PAPERS

Systematics of Coulomb excitation with limited effects from nuclear distortion*

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A wide-ranging survey has been made of (α, α') scattering from various target nuclei. Particular emphasis was given to determination of the departures of the cross section from Coulomb excitation, which are due mainly from nuclear interaction. The energy range in this study covers the region from the Coulomb barrier and downwards. Calculations were performed to first order only via the distorted-wave Born approximation method and the usual collective model was assumed. It is found that the inelastic scattering is more sensitive to the nuclear distortion than the elastic scattering, which conforms with results of recent experiments. A discussion is given of the lgo ambiguity in elastic and inelastic scattering near the barrier.

[NUCLEAR REACTIONS Elastic and inelastic α -particle scattering on medium- and heavy-weight nuclei, Coulomb-nuclear interference, Coulomb barrier, safe distance, multipolarities, lgo ambiguities, strong absorption radius.]

I. INTRODUCTION

The influence of the nuclear interaction between the projectile and the target during inelastic scattering has not been studied systematically in the bombarding energy range below the Coulomb barrier. Coulomb excitation theories,¹⁻³ and especially those describing the reorientation effect (due to static quadrupole moments of the nuclei) or other higher order processes,⁴ specifically assume that there is no interaction between the two charges aside from the electromagnetic field; thus deviations from the first-order calculations are attributed to the effects of static quadrupole moment⁴ and of higher moments. It is consequently of great interest to establish quantitatively that nuclear effects do not upset the validity of the Coulomb-excitation approximations. We have completed a series of calculations designed to reveal the degree of importance of nuclear effects in some Coulomb-excitation experiments. These are the (α, α) and (α, α') reactions on the target nuclei ¹⁵²Sm, ²⁰⁹Pb, and ²³⁸U as a function of bombarding energy.

Recently a number of α -scattering experiments have been performed whose object has been to measure nuclear and Coulomb interference effects both in the Coulomb barrier region and at a sufficiently low energy that nuclear effects are negligible. The deformed targets ^{152, 154}Sm and ^{186 W,⁵ 154 Sm, 186 Er and 182 W,⁶ 188 Er and 184, 186 W,⁷}

and eight rare-earth isotopes from ¹⁵²Sm to ¹⁷⁴Yb⁸ have been studied by this method and in each case quantal coupled-channel calculations have been made. Nuclear and Coulomb effects are both included in such calculations.

The "safe energy," which is the energy below which nuclear effect is negligible, is defined in terms of a minimum distance between the two nuclei in a head-on collision

$$S = (Z_1 e)(Z_2 e)E^{-1} - r_0(A_1^{1/3} + A_2^{1/3}), \quad (1)$$

where Ze and A are the charge and mass number of the particles, E is their relative center-of-mass kinetic energy, and r_0 is a nuclear radius parameter. Various attempts have been made to estimate values for S for which nuclear effects are indeed negligible; de Boer and Eichler⁴ proposed $S > 3$ fm with $r_0 = 1.25$ fm, and it was soon recognized experimentally that this choice was not stringent enough; Cline *et al.* showed that the condition $S > 5.1$ fm⁹; later¹⁰ $S > 6$ fm, is necessary, while the Purdue group¹¹ demonstrated that with r_0 set equal to 1.6 fm, then $S > 3$ fm. It is important to realize that these experimental tests all refer to $L = 2$ excitation with projectiles of mass 16 to 32 incident on medium-mass targets. Thus the use of these criteria for widely different mass regions will lead to widely different predictions for values of $S(E)$.

The terms "safe" and "negligible" are clearly subjective in nature. We might arbitrarily adopt

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PATIWEN -- A CODE FOR COULOMB-NUCLEAR INTERFERENCE CALCULATIONS *

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Received 25 July 1975; revised manuscript received 19 December 1975

PROGRAM SUMMARY

Title of program: PATIWEN

Catalogue number: ABPD

Program obtainable from: CPC Program Library, Queen's University of Belfast, N. Ireland (see application form in this issue).

Computer: IBM 370/165
 CDC 7600
Installation: Daresbury Nuclear Physics Laboratory, Cheshire, U.K.
 UMRCC, Manchester

Operating system: OSRLESE 21

Program language used: FORTRAN IV

High speed storage required: 344 K words

No. of hits in a word: 32

Overlay structure: None

No. of magnetic tapes required: None

Other peripherals used: Line printer, card reader

No. of cards in combined program and test deck: 1256

Card punching code: EBCDIC

CPC Library subprograms used:

Cat. no.	Title	Ref in CPC
ABPC	RCWFN	8 (1974) 377
ABPC 0001 (optional)	RCWFF	to be published

* Work supported by the United Kingdom Science Research Council and the U.S. Energy Research and Development Administration.

** Present address.

Keywords: Nuclear, electric multipole, Coulomb excitation, differential, cross sections, inelastic, scattering, radial integrals, partial-wave expansion, EA Coulomb matrix elements, continued fractions.

Nature of the physical problem

The program PATIWEN calculates the differential and total cross sections for inelastic scattering of a structureless charged particle interacting via an electric multipole of order $E2-F6$ with a nuclear system. It is also designed to accept nuclear matrix elements (up to 150 partial waves) from any conventional DWBA program [1] and to complete a full Coulomb-nuclear interference calculation. A subroutine generates radial matrix elements of the type $\int_0^b dr u_l(kr) r^{-\lambda} u_l'(k'r)$, where $\lambda \geq 1$ and where $u_l(kr)$ is a combination of Coulomb wavefunctions, for positive, negative and zero values of the Sommerfeld parameter, η .

Method of solution

The numerical calculation closely follows the quantum-mechanical theory of nuclear and Coulomb excitations in the distorted-wave Born approximation [2]. The projectile is assumed to be a point charge. The radial matrix elements are integrated by the gaussian quadrature [3] technique. This represents an attractive procedure as the Coulomb functions needed for the integrand can be calculated independently at any radius using subroutine RCWFN [4]. An adaptation [5] of RCWFN which requires less array space can be used.

Restriction on the complexity of the problem

The calculation is non-relativistic and is dimensioned for 600 partial waves; more can be included if necessary. There is no inherent restriction to the scattering of like charges, although only this case has been tested.

QUADRUPOLE DEFORMATION PARAMETERS OF $^{148,152,154}\text{Sm}$ DETERMINED FROM NEUTRON TOTAL CROSS SECTIONS

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Quadrupole parameters of the nuclear potential were determined for $^{148,152,154}\text{Sm}$ using coupled-channel calculations. The β_2 values for $^{152,154}\text{Sm}$ are about 10 percent lower than β_2 values for the nuclear charge distribution obtained from Coulomb excitation, electron scattering, and μ -meson X ray experiments.

Neutrons are particularly suitable probes for investigating the shape of the nuclear potential since, unlike the case for α -particles and protons, electromagnetic effects can usually be neglected. It has been known for many years that certain low-energy neutron scattering parameters, e.g., the s -wave strength function, have a strong dependence on nuclear deformation [1]. Recent total cross-section measurements indicate that nuclear deformation has a substantial effect on neutron scattering in the MeV region as well [2, 3].

In the present work, neutron total cross sections for ^{148}Sm and total cross-section differences for $^{152,148}\text{Sm}$ and $^{154,148}\text{Sm}$ were measured from 0.75 to 14.5 MeV. These Sm isotopes are appropriate nuclei for deformation studies because they span the region near $N = 88$ where nuclear deformation changes rapidly with mass number. For each of the nuclei $^{148,152,154}\text{Sm}$ the quadrupole deformation parameter has been obtained from these data using non-spherical optical-model coupled-

channel calculations and is compared with values obtained from other types of experiments [4 - 8]. A preliminary report of the present work has been presented previously [9].

The ^{148}Sm total cross-section measurements and the total cross-section difference measurements were made at the Western Michigan University tandem accelerator facility. Each Sm sample consisted of about 40 g of powdered Sm_2O_3 with an isotopic enrichment $> 95\%$. The ^{148}Sm total cross section was measured in the usual way using as samples $^{148}\text{Sm}_2\text{O}_3$, BeO, Be, and a blank can. The total cross-section difference, e.g., between ^{152}Sm and ^{148}Sm , was determined directly by measuring the transmission ratio for the oxides of these two isotopes. The presence of oxygen in the samples did not affect this ratio, because each Sm sample contained essentially the same number of oxygen nuclei. Other experimental details are given elsewhere [3].

The coupled-channel calculations were similar to those reported previously [10]. The philosophy of the present work was to fit the data using a *minimum* of adjustable parameters. Except for the addition of a real isospin term, the non-spherical optical-potential

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² Work supported in part by the Research Corporation.

³ Work supported in part by the USERDA.

⁴ Work sponsored by the USERDA.

Multistep inelastic processes in the reaction $^{28}\text{Si}(^3\text{He}, d)^{29}\text{P}$ leading to bound and unbound states*

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(Received 8 December 1975)

We analyze the stripping reaction $^{28}\text{Si}(^3\text{He}, d)^{29}\text{P}$ at an incident ^3He energy of 35.3 MeV, considering seven residual states. These extend in excitation energy to 4.8 MeV with the highest four being unbound to proton emission and described via complex energy eigenstates. The results of a distorted-wave Born-approximation analysis are rather disappointing. Through a coupled-channel Born approximation analysis, inelastic excitation in both the entrance and exit channels is shown in several cases to be important in accounting for the detailed shape of the angular distributions. Additional data obtained at 20 and 40 MeV are included in the analysis as a check of the optical potentials and spectroscopic amplitudes adopted. The spectroscopic amplitudes used are consistent with results of large-basis shell-model calculations. Good fits to the $(^3\text{He}, d)$ data are obtained in both shape and magnitude, suggesting that the coupled-channel Born approximation should replace the distorted-wave Born approximation in the analysis of experimental data for single-nucleon transfer reactions in this mass region.

NUCLEAR REACTIONS $^{28}\text{Si}(^3\text{He}, d)$, $E = 20, 35.3, 40$ MeV; DWBA and CCBA calculations of $\sigma(E_d, \theta)$. Deduced multistep inelastic contributions and spectroscopic amplitudes.

I. INTRODUCTION

The use of $(^3\text{He}, d)$ and $(d, ^3\text{He})$ reactions for detailed studies of the nuclear spectroscopy of proton particle and hole states began in earnest about ten years ago.¹ Since then, a vast number of studies have been carried out using these reactions, at nuclear science facilities all over the world.

In recent years, it has been realized that excitation of inelastic channels during nucleon transfer reactions can have important consequences; analysis of direct reaction cross section data using the conventional distorted-wave Born approximation (DWBA) can lead to erroneous spectroscopic conclusions, and fails to account for many remarkable, systematic features of the angular distributions.²

The $(^3\text{He}, d)$ reaction has also been used frequently to explore the low-lying proton continuum. Data for $(^3\text{He}, d)$ to proton-unbound states has generally been analyzed in terms of ordinary DWBA with weakly bound states as residual nuclear form factors. Again, in recent years it has been realized that, even for proton resonances of width 10^{-5} MeV or less, use of a weakly bound state form factor in DWBA can lead to erroneous conclusions.^{3,4}

In the present work we discuss an analysis in which both of these difficulties are present. The reaction $^{28}\text{Si}(^3\text{He}, d)^{29}\text{P}$ has been studied at 35.3 MeV incident ^3He energy by Leleux *et al.*⁵ From previous coupled-channel Born approximation

(CCBA) analyses of the mirror reaction $^{28}\text{Si}(d, p)^{29}\text{Si}$, it is known that inelastic excitation in entrance and exit channels has an important effect on certain transitions to states with small single-particle spectroscopic factors.⁶ Further, a number of the states observed by Leleux *et al.* are unbound to proton emission.

Angular distributions are available covering center of mass angles from 10° to 70° for the following states in ^{29}P : $\frac{1}{2}^+$ ground, $\frac{1}{2}^+$ 1.38 MeV, $\frac{3}{2}^+$ 1.95 MeV, $\frac{5}{2}^+$ 3.1 MeV, $\frac{7}{2}^-$ 3.45 MeV, $\frac{3}{2}^-$ 4.34 MeV, and $\frac{1}{2}^-$ 4.76 MeV. The last four of these states are unbound to proton emission.⁵

Because of magnitude anomalies, to be discussed in the next section, we have also included in our analysis data for the ^{29}P ground state and the first excited state at 1.38 MeV obtained at incident ^3He energies of 20 MeV by Mertens, Mayer-Böricke, and Kattenborn⁶ and at 40 MeV by Stupin, Ristenen, and Schwandt.⁷

These data have been analyzed in terms of the CCBA² using complex-energy eigenstates⁷ to describe the unbound final nuclear states. To the best of our knowledge, this is the first instance of a CCBA description of direct reactions to unbound final states. In Sec. II we present the details of the analysis, and in Sec. III we draw conclusions.

In the CCBA analysis we have considered a number of alternate couplings in both entrance and exit channels, and have also made a study of the sensitivity of our results to variations in the relative phases of configurations contributing to parti-

Exact finite-range coupled-channels Born approximation analysis of ^{13}C -induced reactions*

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(Received 23 October 1975)

Different descriptions of the reaction $^{28}\text{Si}(^{13}\text{C}, ^{12}\text{C})^{29}\text{Si}$ at 36 MeV leading to the ground and first excited states of ^{29}Si are examined. Sensitivity of the reaction cross section to various families of optical parameters is investigated and the so-called "surface transparent potential" is used as an alternate treatment. Exact finite-range distorted-wave Born approximation and coupled-channels Born approximation analyses of the reaction are performed, showing that the coupled-channels Born approximation analysis, even in its presently restricted form, leads to an improvement over the distorted-wave Born approximation predictions.

[NUCLEAR REACTIONS $^{28}\text{Si}(^{13}\text{C}, ^{12}\text{C})$ calculated cross sections using exact finite-range CCBA with "surface transparent potential."]

I. INTRODUCTION

In recent years interest in ^{13}C as a projectile in heavy-ion-induced reactions has greatly increased, with a number of facilities now having ^{13}C beams available.^{1,2} However, the usefulness of ^{13}C as a means of revealing nuclear structure information remains somewhat dubious because of the collective nature and uncertain gross structure of this nucleus. In this paper we will present a study of the importance of multistep processes in the reaction data for $^{28}\text{Si}(^{13}\text{C}, ^{12}\text{C})^{29}\text{Si}$ at 36 MeV incident ^{13}C energy leading to the ground and 1.27 MeV $\frac{3}{2}^+$ first excited states of ^{29}Si , which should additionally reflect upon the usefulness of ^{13}C as a projectile.

The data for this study were obtained with the University of Texas EN tandem accelerator using a locally designed Middleton-type negative ion source. The experimental details and analyses can be found in greater detail elsewhere.¹

The theoretical aspects involved in describing such a reaction are manifold, and in this paper we have concentrated on the few particular aspects which we feel best illustrate the difficulties and advantages inherent, not only in the use of ^{13}C as a probe, but in treating such reactions in general.

The first facet of the problem that will be considered is that of the continuous ambiguity in heavy ion optical potentials. Often one ignores the fact that different sets of optical potentials which fit the elastic data equally well can lead to drastically different cross sections in both the distorted-wave Born approximation (DWBA) and coupled-channels Born approximation (CCBA) formalisms.³ Such sensitivities will be investigated and the so-called "surface transparent

potential"² will be examined as a possible alternative description.

Other considerations to be dealt with are the deformations of $^{12,13}\text{C}$ and the well-known collectivity of $^{28,29}\text{Si}$.^{4,5} In order to deal with these elements of the problem, multistep inelastic processes should be taken into account, which is fortunately now possible to do using our EFR-CCBA code (EFR denotes exact finite range) recently developed by Tamura, Low, and Udagawa called SATCCBA-MARSNEW.⁶ The results from this CCBA analysis will be contrasted with an EFR-DWBA analysis (using Tamura and Low's code SATURN-MARS⁷) to see if multistep inelastic processes are indeed significant in describing the reaction. To perhaps anticipate the discussion slightly, one notes that these states in ^{29}Si have been shown in light ion reactions to be populated mainly through direct processes^{4,5,8} and so DWBA can be expected to give a reasonably good fit to the data. However, to reproduce the finer details of the angular distribution one would expect that collective processes should also be accounted for, and this is actually found to be the case.

Other complications encountered in describing this reaction were the difficulty inherent in using a low energy beam^{9,10} and the uncertainty in the collective nature to be assumed for silicon, since it lies in a transition region between rotational and vibrational behavior. We now discuss the optical model ambiguity problem followed by the results of the CCBA analysis.

II. OPTICAL MODEL SENSITIVITY

It is quite often assumed that different families of optical potentials which fit the relevant elastic scattering data equally well will lead to essentially

Multistep processes in the $^{19}\text{F}(^{16}\text{O}, ^{15}\text{N})^{20}\text{Ne}$ reaction

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(Received 9 June 1975)

An exact-finite-range coupled-channels Born-approximation analysis, which takes into account multistep processes, was made of the reaction $^{19}\text{F}(^{16}\text{O}, ^{15}\text{N})^{20}\text{Ne}(0^+, 2^+, 4^+)$. Good fits to data were obtained not only for the 0^+ and 2^+ state cross sections, but also for the 4^+ state which is forbidden in a one-step process. Deviation of the distorted-wave Born-approximation result from the coupled-channel Born-approximation result was observed even for the allowed transition to the 0^+ and 2^+ states.

[NUCLEAR REACTIONS $^{19}\text{F}(^{16}\text{O}, ^{15}\text{N})$, $E=68$ MeV calculated $\sigma(\theta)$ with CCBA and DWBA.]

The importance of the multistep processes involving inelastic scattering has been known for a long time for light-ion-induced reactions, and there is no reason to believe that the situation differs for heavy-ion-induced reactions. In fact such evidence has been seen in many cases and analyses of the data were made in terms of coupled-channel Born-approximation (CCBA) calculations.¹ It is somewhat surprising to note, however, that all these cases reported so far have been restricted to two-neutron or two-proton transfer reactions between two even-even nuclei. It was found that in many cases the angular distribution, that left one of the outgoing partners in its first excited 2^+ state, had a rather anomalous shape; and this anomaly was explained by CCBA.^{1,2}

In spite of the successful fit by CCBA of the anomalous distributions, and also the magnitude of the 2^+ cross section relative to that of the ground state, the analysis of the two-nucleon transfer reactions has a common difficulty which has not yet been resolved satisfactorily. The predicted cross section was found to be too small by a factor of 1 to three orders of magnitude.² Although there is some indication that successive transfer reactions might provide the explanation of this difficulty,³ an unambiguous calculation has still to be made; particularly when anomalous angular distributions are involved.⁴

One may naturally expect that the nature of one-nucleon transfer reactions is much simpler, and in fact a large number of successful analyses have been reported for reactions in which one-step processes were (expected to be) dominant. On the other hand, very little has been done, either experimentally or theoretically, for one-nucleon transfer reactions in which the inelastic effect and

consequently the multistep contribution was expected to be important. The purpose of the present article is to fill this gap by analyzing the data of the $^{19}\text{F}(^{16}\text{O}, ^{15}\text{N})^{20}\text{Ne}$ reaction, recently reported by an Orsay group.⁵ In this experiment 4^+ as well as 0^+ and 2^+ states in the ground band of ^{20}Ne were strongly excited. Since ^{20}Ne is basically an s - d shell nucleus, with negligibly small admixture of the $N=4$ shell component in the intrinsic ground state, a distorted-wave Born-approximation (DWBA) calculation is expected to predict the 4^+ cross section to be orders of magnitude smaller than experiment. However, a CCBA calculation which takes into account the deformed nature of ^{19}F and ^{20}Ne may explain the experiment.

The exact-finite-range (EFR) CCBA and DWBA calculations were performed using the formalism and the technique developed in our previous publications.⁶ Since to consider the inelastic coupling, in both incident and exit channels simultaneously, required a very lengthy time and a large core storage in the computer, we first carried out calculations considering only the $0^+ - 2^+ - 4^+$ coupling in the ^{20}Ne channel. The deformation parameters used for ^{20}Ne were $\beta_2 = 0.45$ and $\beta_4 = 0.15$, an average set of values obtained through work using light-ion reactions.⁷ The $\frac{1}{2}^+ \rightarrow 0^+$ and $\frac{1}{2}^+ \rightarrow 2^+$ transitions were treated as the transfer of an $s_{1/2}$ and $d_{3/2}$ proton, respectively, bound in ^{20}Ne at the corresponding separation energies in a Woods-Saxon well with $r_0 = 1.2$ fm, $a = 0.65$ fm, and $V_w = 7$ MeV. For the projectile-ejectile system, it was assumed that the proton that is transferred comes from a pure single-particle state, namely the $p_{1/2}$ orbit in ^{16}O , in a Woods-Saxon well with the same parameters as used for ^{20}Ne .

If the Nilsson model⁸ is used, the spectroscopic

$^{142}\text{Nd}(^{18}\text{O}, ^{16}\text{O})^{144}\text{Nd}$ reaction and its contrast to $^{144}\text{Nd}(^{12}\text{C}, ^{14}\text{C})^{142}\text{Nd}$ reaction*

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(Received 15 March 1976)

An investigation is made of the $^{142}\text{Nd}(^{18}\text{O}, ^{16}\text{O})^{144}\text{Nd}$ reaction, in which it was found that the first excited 2^+ state was populated strongly with an angular distribution characteristic of one-step transfer. This is in contrast to an earlier result for the inverse-type reaction $^{144}\text{Nd}(^{12}\text{C}, ^{14}\text{C})^{142}\text{Nd}$, in which the first excited state was found to be populated dominantly by a two-step process and to have an anomalous angular distribution.

[NUCLEAR REACTIONS $^{142}\text{Nd}(^{18}\text{O}, ^{16}\text{O})$, $E = 98$ MeV; measured $\sigma(\theta)$, DWBA] analysis.

Investigation of two-nucleon transfer reactions included by light ions, such as (p, t) and (t, p) reactions, has been recognized as a powerful tool in obtaining information concerning the correlation of the nucleon pairs which is revealed, for example, in the excitation of the pairing vibrational states.¹ Similar information on this correlation can be obtained by carrying out two-nucleon transfer reactions induced by heavy ions. In the present article the $^{142}\text{Nd}(^{18}\text{O}, ^{16}\text{O})^{144}\text{Nd}$ reaction leading to the quadrupole-pairing vibrational states of removal (RQP) and addition (AQP) types is investigated, and compared with the inverse-type reaction $^{144}\text{Nd}(^{12}\text{C}, ^{14}\text{C})^{142}\text{Nd}$ which we reported earlier.²

A characteristic feature of an RQP (AQP) state is that it is excited strongly (weakly) by a pickup reaction, like (p, t) , while the situation is reversed for a stripping reaction. In fact, previous (p, t) work^{3,4} showed that the transitions to the RQP-type 2^+ states were strong and were of direct (one-step) nature. On the other hand, the transition to the AQP-type 2^+ state was much weaker and also had an anomalous angular distribution. A marked example of such an anomalous angular distribution was that of the $^{144}\text{Nd}(p, t)^{142}\text{Nd}(2^+)$ transition,⁵ which could be fitted well by coupled channel Born approximation (CCBA), but not by distorted wave Born approximation (DWBA). A very similar situation was encountered in the $^{144}\text{Nd}(^{12}\text{C}, ^{14}\text{C})^{142}\text{Nd}(2^+)$ reaction.²

It is interesting to perform the inverse stripping reaction, but to carry out a (t, p) reaction, particularly at high energy, is difficult. On the other hand, it is quite feasible to carry out two-nucleon stripping reactions induced by heavy ions, and it is for this reason that the present $(^{18}\text{O}, ^{16}\text{O})$ experiment was undertaken. A similar comparison between pickup and stripping reactions on Sn targets

was reported recently, showing that the interference between the direct and indirect transitions to the final 2^+ state was constructive (destructive) for pickup (stripping) reactions.⁶

The present experiment was performed by using a 98-MeV ^{18}O beam from the Berkeley 88-inch cyclotron. Reaction products were detected with the quadrupole-sextupole-dipole (QSD)-type magnetic spectrometer.⁷ Particle identification and energies of the reaction products were obtained by a combination of magnetic rigidity, a double dE/dx measurement, and the time-of-flight method.⁸ Figure 1 shows an energy spectrum of the ^{16}O ions from the $^{142}\text{Nd}(^{18}\text{O}, ^{16}\text{O})^{144}\text{Nd}$ reaction. A $350\text{-}\mu\text{g}/\text{cm}^2$ self-supporting, isotopically pure metallic ^{142}Nd target gave a resolution of 250 keV.

The differential cross sections for the $^{142}\text{Nd}(^{18}\text{O}, ^{16}\text{O})$ reaction leading to the ground state (0^+) and the first excited 0.69 MeV (2^+) state of ^{144}Nd are shown in Fig. 2. As is seen, both the 0^+ and the 2^+ states are strongly populated, and both display bell-shaped angular distributions characteristic of one-step transitions. The peak of the cross sections occurs at $\theta_{\text{c.m.}} \approx 50^\circ$.

The theoretical analysis was performed by using an exact finite range (EFR) microscopic version⁹ of the DWBA code SATURN-MARS-1.¹⁰ The wave functions used for ^{142}Nd and ^{144}Nd were constructed in the same manner as in Refs. 2 and 11. Both the pairing-type (of monopole and quadrupole nature) and the particle-hole-type (of quadrupole nature) interactions were used and thus the wave function for the two extra nucleons in ^{144}Nd had rather complicated configuration mixing. For the two extra nucleons in ^{18}O , we used the configuration mixing described by Bayman.¹² The radial part of the wave functions for the single neutrons

Multistep processes in transfer reactions induced by 56 MeV ^{16}O beam on $^{72,74,76}\text{Ge}$ isotopes

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Angular distributions of the $(^{16}\text{O}, ^{14}\text{C})$ reaction on $^{72,74,76}\text{Ge}$ targets were measured using a 56 MeV ^{16}O beam in 2° steps between 13° and 17° laboratory angles. The experiments were performed using three solid state detector telescopes. The observed difference between the shapes of the angular distributions of the ground states and the excited states indicates that an important role is played by multistep processes, involving vibrational excitation of the target and residual nuclei. It was found that the exact-finite-range coupled-channel-Born-approximation calculations were able to reproduce qualitatively the observed variation of the angular distributions.

[NUCLEAR REACTIONS $^{72,74,76}\text{Ge}(^{16}\text{O}, ^{14}\text{C})$, $E = 56$ MeV; measured $\sigma(\theta)$ and calculated $\sigma(\theta)$ with EFR-CCBA and DWBA.]

I. INTRODUCTION

It has long been known in light-ion-induced reactions, particularly in (p, t) reactions, that multistep processes involving core excitation of target and residual nuclei play an important role in transfer reactions. It has thus been expected that such might also be the case in reactions induced by heavy ions. It is anticipated that anomalies in the shape of the angular distributions, as well as in the absolute and/or relative magnitudes, could be used as a clear signature of the presence of such higher order processes. Indeed, it has already been found that in the reaction $^{40}\text{Ca}(^{16}\text{O}, ^{20}\text{Ne})^{36}\text{Ar}$ the first excited 2^+ state of ^{20}Ne has a drastically different (forward peaked) angular distribution compared with the bell-shaped angular distribution from the ground state. Various CCBA (coupled-channel Born approximation) analyses²⁻⁸ have since then been made for heavy-ion-induced transfer reactions, showing that the anomalous angular distributions can, in general, be rather well accounted for by such an approach. In the present paper, we report on our study of the effects of multistep processes in the $^{72,74,76}\text{Ge}(^{16}\text{O}, ^{14}\text{C})^{74,76,78}\text{Se}$ reactions. All these Ge and Se isotopes involved are known to be strongly collective and we expect to observe two-step effects. The present study is an extension of our previous work^{4,5} for the reaction $^{76}\text{Ge}(^{16}\text{O}, ^{14}\text{C})^{78}\text{Se}$, to include all the three Ge isotopes and also the transitions leading to the octupole vibrational state of 3^- . The theoretical analysis is made in terms of EFR (exact-finite-range)-CCBA, the coupled-channel calculations which included the Coulomb

excitations being made in both incident and exit channels.

In Sec. II we present the experimental data and discuss their qualitative features. Formulation of the EFR-CCBA calculations is presented in Sec. III and comparison of the theoretical result with experiment is made in Sec. IV. Discussions about the results are given in Sec. V.

II. EXPERIMENTAL PROCEDURE AND RESULTS

The experiments were performed in a scattering chamber with three counter telescopes each consisting of two solid state detectors and using a 56 MeV ^{16}O beam delivered by the Saclay FN Tandem Van de Graaff. The thickness of each silicon surface barrier ΔE counter was about $30 \mu\text{m}$. As discussed in our previous paper,⁴ the reaction products were identified by using a function $F(A, Z) = \Delta E(A, Z) + 0.32E(A, Z)$, where $\Delta E(A, Z)$ and $E(A, Z)$ are, respectively, the energy losses in the first and second counters. The constant 0.32 is determined by the thickness of the first detector, which was $30 \mu\text{m}$ in the present experiment. For a wide variety of reaction products and a large range of energy, the function $F(A, Z)$ has a unique value for each pair of A and Z . The electronics have also been described earlier.⁴ A typical energy spectrum for the reaction $^{76}\text{Ge}(^{16}\text{O}, ^{14}\text{C})^{78}\text{Se}$ is presented in Fig. 1. The energy resolution is about 200 keV. In these experiments use is made of thin targets of $50 \mu\text{g}$ Ge on a $20 \mu\text{g}$ backing in order to prevent the destruction of the targets by beam-energy-loss heating.

The two proton transfer angular distributions

NOTE ON THE HARMONIC OSCILLATOR CALCULATION OF TWO-NUCLEON TRANSFER FORM FACTORS

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(Revised 26 August 1975)

Abstract: In a recent publication, Iano and Pinkston showed that the zero-range form factor for two-nucleon transfer reactions obtained through their shell model calculation was well approximated in the asymptotic region by the one calculated using the standard well-depth procedure. We wish to show that such an agreement is merely due to the restricted space which they have used. It is found that by including a larger working basis, the form factor may increase in the asymptotic region by as much as 50% over the one obtained by the well-depth procedure; this in turn will bring the theoretical cross section to within $\frac{1}{2}$ of the experimental one for the reaction $^{40}\text{Ca}(t, p)^{42}\text{Ca}$.

1. Introduction

A number of calculations¹⁻⁵⁾ have recently been done to determine accurately the two-nucleon transfer overlaps. The sudden interest in this subject has arisen due to the apparent failure of the distorted-wave Born approximation (DWBA) theory using the well-depth (WD) ansatz⁶⁾ to predict correctly the magnitude of the experimental cross sections of two-nucleon transfer reactions^{7,8)}. There are now reasons¹⁻⁴⁾ to believe, at least in light-ion induced reactions⁶⁾, that this is largely due to the inadequacy of the WD procedure in calculating two-nucleon overlaps. In a recent publication by Iano and Pinkston (IP)⁵⁾, however, a contrary conclusion was reached. On the basis of their "truncated" shell model calculations²⁾ they concluded that the WD two-nucleon overlap approximates very well the "correct" overlap they obtained in their calculations.

† Work supported in part by the National Research Council of Canada.

†† Work supported in part by USERDA.

††† Work supported in part by UPNSRC.

‡ Strictly speaking, all shell model calculations are done within a truncated space but in this present problem it is possible to truncate the basis only when convergence of the wave function at some large radial distance is achieved. This is very similar to the partial wave expansion in elastic scattering.

Absolute cross section of the reaction $^{48}\text{Ca}(^{18}\text{O}, ^{16}\text{O})^{50}\text{Ca}$

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(Received 12 January 1976)

It is shown that the long standing difficulty of very small theoretical cross sections for heavy-ion-induced two-nucleon transfer reactions appears to have been solved, at least for the case of the $^{48}\text{Ca}(^{18}\text{O}, ^{16}\text{O})^{50}\text{Ca}$ reaction.

[NUCLEAR REACTIONS $^{48}\text{Ca}(^{18}\text{O}, ^{16}\text{O})^{50}\text{Ca}$, $E = 50$ MeV, 0^+ ground state and 2^+ excited state. DWBA and coupled-reaction-channel analysis, density-dependent correlations.]

Distorted wave Born approximation (DWBA) and/or coupled channel Born approximation (CCBA) analyses made in the past of two-(identical)-nucleon transfer reactions between heavy ions have consistently shown that not only the angular distributions but also the relative magnitudes of experimental cross sections for various final residual nuclear states were fitted rather well.¹ It has also been consistently noted, however, that the theory underestimates the absolute magnitude of the cross section by one to two orders of magnitude, contrary to results for one-nucleon transfer reactions² in which such a difficulty has not been encountered.

One source of this difficulty could have been the use of a relatively crude cluster approximation in some earlier calculations. Recent studies which used a detailed microscopic description for the two-nucleon bound states, rather than a cluster approximation, however, still found too small cross sections.³ On the other hand, there have also been reported calculations⁴ which showed that the contributions of the two-step successive-transfer mode dominated that of the simultaneous two-nucleon mode by one order of magnitude, i.e., that

$$\sigma(\text{successive})/\sigma(\text{simultaneous}) \approx 10. \quad (1)$$

However, the numerical details of these calculations may be suspected, because of the use either of a semiclassical or of a no-recoil (NR) approximation. An example which shows that NR calculation of $\sigma(\text{successive})$ is rather unreliable was presented recently by Kammuri, Yoshida, and Yamaji.⁵

Kammuri *et al.*⁵ also performed exact finite range (EFR) calculations, as we did sometime ago.⁶ One example we analyzed was for $^{48}\text{Ca}(^{18}\text{O}, ^{16}\text{O})^{50}\text{Ti}$ data,⁷ and it was found that $N = \sigma_{\text{exp}}/\sigma_{\text{th}}$

≈ 200 , if only the simultaneous process was taken into account. However, it was also found that N was reduced to about 20, if the successive process was added, thus showing that the rule Eq. (1) holds also in the EFR calculations. Almost exactly the same result was reported in Ref. 5 too. This is certainly a drastic improvement, but to have an N which is still as large as 20 means that the original problem has not yet been solved satisfactorily.

Insofar as we are aware, no case has yet been reported in which $N = 1$ was obtained. To be a little more precise, however, we should note that Kammuri *et al.*⁵ reported the case of the $^{12}\text{C}(^{18}\text{O}, ^{16}\text{O})^{14}\text{C}$ reaction⁸ in which they obtained a σ_{th} which even exceeded σ_{exp} , giving $N \approx 0.4$. The theoretical angular distribution which they obtained agreed, however, rather poorly with experiment, being completely out of phase. Also the optical potential they used had an unusually weak absorption. Furthermore, Schneider *et al.*⁹ pointed out the importance of taking into account the contribution of the $^{12}\text{C}(^{18}\text{O}, ^{14}\text{C})^{16}\text{O}$ mode of the reaction to the $^{12}\text{C}(^{18}\text{O}, ^{16}\text{O})^{14}\text{C}$ mode. Therefore, the achievement of $N \approx 0.4$ may very well be suspect. It should also be noted that Scott *et al.* (the third paper of Ref. 1) claimed that they obtained N as small as 2.5 for the reaction $^{120}\text{Sn}(^{18}\text{O}, ^{16}\text{O})^{122}\text{Sn}$, by considering only the simultaneous process. Their calculation was made, however, in the NR approximation. We reanalyzed the same data with EFR calculations and with the configurations as their's, we found that $N = 8$.

The purpose of the present article is to show that at least in one example of the $^{48}\text{Ca}(^{18}\text{O}, ^{16}\text{O})^{50}\text{Ca}$ reaction, we are able to obtain $N \approx 1$. The results of the EFR calculations are presented in Fig. 1, together with experiment.¹⁰ Since all the theoretical curves in Fig. 1 are presented without any

MULTI-STEP PROCESSES IN TRANSFER REACTIONS BETWEEN HEAVY IONS[†]

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A discussion is given of how nuclear structure information can be extracted or tested, by performing analyses of data of heavy ion transfer reactions. Some aspects of the reaction calculations, particularly pertaining to the heavy-ion induced reactions are also discussed.

1. Introduction.

In the past a few years rapid progress has been made in the understanding of the mechanisms of transfer reactions between heavy ions. This was achieved largely because it has become possible to carry out very fast exact-finite-range (EFR) calculations, not only with DWBA but also with CCBA and CRC approaches as well¹.

In the present paper, we focus our attention to the two-nucleon transfer reactions, and present as an example the analysis of the reaction $^{144}\text{Nd}(^{12}\text{C}, ^{14}\text{C})^{142}\text{Nd}$ leading to a few low-lying states of ^{142}Nd ². Since the quadrupole collective mode is involved in the reaction, particularly in the excitation of the first 2^+ state, the analysis has to be made including the effect of multi-step inelastic processes. Recoil effects³ and the problem of obtaining the correct absolute magnitude for the two-nucleon transfer reaction will also be discussed⁴.

[†] Work supported in part by the U.S.E.R.D.A.

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THE ELASTIC SCATTERING OF α -PARTICLES FROM HELIUM AT 0.85 AND 0.65 GeV †

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Abstract: Elastic scattering of α -particles from helium has been measured at bombarding energies of 0.85 and 0.65 GeV over the range of four momentum transfer from 0.9 to 3.8 (GeV/c)². The results are compared with the predictions of an α - α potential obtained by a folding procedure using a p-⁴He potential and ⁴He charge distribution.

NUCLEAR REACTION ⁴He(α , α), $E = 650, 850$ MeV; measured $\sigma(\theta)$. Results compared with the predictions of a ⁴He-⁴He potential obtained by a folding procedure which used a p-⁴He potential and ⁴He charge distribution.

Very little is known experimentally about α - α scattering above a bombarding energy of 140 MeV [ref. 1,2]). One measurement at 900 MeV [ref. 3)] exists but the statistical accuracy is low and the angular bin size is large. There are compelling reasons to study further the α - α reaction over broader ranges of total energy squared s and four-momentum transfer squared t . The reasons arise out of theoretical studies of the α - α interaction itself, and from the need for the s - and t -dependence of α - α scattering in order to understand results obtained from quasi-elastic knockout of α -particles from nuclei by α -particles. At least two different approaches have been taken to try to understand α - α elastic scattering. The first is based on a paper by Swan 4) who suggested that in elastic scattering where complex structures are involved, certain states are forbidden by the Pauli principle. Swan inferred that these

† Supported in part by the Energy Research Development Agency.

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**PHENOMENOLOGICAL ANALYSIS OF $^4\text{He} + ^4\text{He}$ SCATTERING
AT 0.65 AND 0.85 GeV^{*}**

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Received 20 February 1976

We present results of a phenomenological analysis of recently obtained data for $^4\text{He} + ^4\text{He}$ elastic scattering at 0.65 and 0.85 GeV incident laboratory energies. We find that the observed angular distributions cannot be fit unless the nuclear potential has a strong, short-range repulsive contribution, in agreement with the results of microscopic and phenomenological analyses of low energy $^4\text{He} + ^4\text{He}$ scattering.

Very recently, Fong et al. [1] have obtained data for elastic scattering of ^4He from ^4He at 0.85 and 0.65 GeV incident laboratory energies, covering center-of-mass angles from about 50 to 100°. This represents the first useful measurement of the $^4\text{He} + ^4\text{He}$ cross section above about 150 MeV lab energy [2]. These data are of interest because of previous speculation as to high-energy behavior of s and d phase shifts for $^4\text{He} + ^4\text{He}$ scattering, based on a generalisation of Levinson's theorem. For details and references, see the work of Neudatchin et al. [3]. The $^4\text{He} + ^4\text{He}$ interaction at low energies has of course been of considerable interest historically because of the existence of the "alpha-cluster" model of light nuclei, and there is an extensive literature which it would be irrelevant to reference here [4].

The data are also of interest simply because such data have not existed previously, and have not been analyzed theoretically. Because of the extreme importance of exchange effects and the Pauli principle in $^4\text{He} + ^4\text{He}$ scattering, low energy data (c.m. energies from 1 to 15 MeV) have been most meaningfully analyzed within the framework of the resonating group method [e.g. 5,6] although there have also been a number of useful and complementary phenomenological analyses, using l -dependent optical model potentials [7,8].

We have no reliable theoretical guide to the precise form of a phenomenological optical model for medium energy $^4\text{He} + ^4\text{He}$ scattering, and therefore some exploratory calculations seemed worthwhile. These

are reported here.

In our numerical calculations, we made use of an optical model program which solves the non-relativistic Schrödinger equation with exact relativistic kinematics. This program was written with the analysis of 1 GeV nucleon scattering from nuclei in mind. Since for the present case the ratio of lab kinetic energy to $M_\alpha c^2$ is closer to 1/4 than to 1, we took some care that the relativistic expressions for kinematic parameters appearing in the Schrödinger equation reduced accurately to their non-relativistic values.

The Schrödinger equation was solved in the form

$$\left[-\frac{\hbar^2}{2\mu} \frac{d^2}{dr^2} + U(r) + V_c(r) + \frac{\hbar^2 l(l+1)}{2\mu r^2} - \frac{\hbar^2 k^2}{2\mu} \right] u_l(r) = 0, \quad (1)$$

where $k = (M_\alpha/\hbar c) \{ K_L (K_L + 2M_\alpha) / (2M_\alpha(2M_\alpha + K_L)) \}^{1/2}$, K_L being the lab kinetic energy of the incoming ^4He and M_α the mass of ^4He in MeV. Similarly, μ is the "reduced energy", [9] given in this case by $\mu = \epsilon_\alpha/2$, with $\epsilon_\alpha^2 = (\hbar kc)^2 + M_\alpha^2$. The Coulomb potential V_c was obtained by a double-folding method, via the obvious expression

$$V_c(r) = Z_1 Z_2 e^2 \iint \frac{\rho_\alpha(r') \rho_\alpha(r'')}{|r + r' - r''|} d^3 r' d^3 r''. \quad (2)$$

Unless otherwise noted, a Gaussian charge form factor (normalized to unity) was chosen; that is, $\rho_\alpha(r) = [\nu/\pi]^{3/2} \exp(-\nu r^2)$, with $\nu = 0.533 \text{ fm}^{-2}$.

To get the calculations started, we constructed a nuclear potential using the familiar single-folding model, well-known in low-energy heavy-ion scattering

^{*} Research supported in part via contract with the USERDA.

DWBA APPROACH TO INELASTIC SCATTERING AT MEDIUM ENERGIES^{*}

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Received 22 July 1976

Medium energy proton elastic and inelastic scattering to states of ^{58}Ni and ^{208}Pb , and ^4He elastic and inelastic scattering to states of ^{40}Ca , are analyzed using the partial wave approach, by solving the Schrödinger equation with relativistic kinematics and using the distorted wave Born approximation. Our results can be compared with results of several previous analyses of the nucleon inelastic data using the Glauber approximation. Our calculations are absolute, using nuclear collective parameters obtained from a survey of a large number of low-energy analyses of inelastic scattering of electrons, nucleons and nuclei from ^{40}Ca , ^{58}Ni and ^{208}Pb .

In 1973, the Saclay Saturne group [1] obtained elastic and inelastic scattering data for protons incident on ^{12}C , ^{58}Ni and ^{208}Pb at 1040 MeV. We present here, for the first time insofar as we know, a full distorted wave Born approximation (DWBA) analysis of the inelastic scattering data for ^{58}Ni and ^{208}Pb , including effects of spin-orbit coupling.

Since 1973, several analyses have appeared in the literature of the elastic and inelastic angular distributions, but all of these analyses [2-4] have made use of the Glauber approximation, in which the scattering amplitude is obtained directly from an optical potential via a semi-classical multiple-scattering expansion [5]. A discussion of the limits of validity of the Glauber approximation is beyond the scope of this letter; suffice it to say that it is appropriate for rather high energy, and its use at intermediate energies requires a rather careful treatment of Coulomb [6] and spin-orbit [7] interactions, as well as nucleon overlap and motion in the target nucleus [8, 9]. Such corrections have generally *not* been made in the published analyses. Furthermore, as pointed out by Ahmad [4], some previous Glauber analyses of inelastic scattering [3] have omitted multiple-scattering terms larger than the ones they keep!

When the various effects mentioned above are included to a degree which insures reasonable accuracy in the final calculated elastic and inelastic cross sections

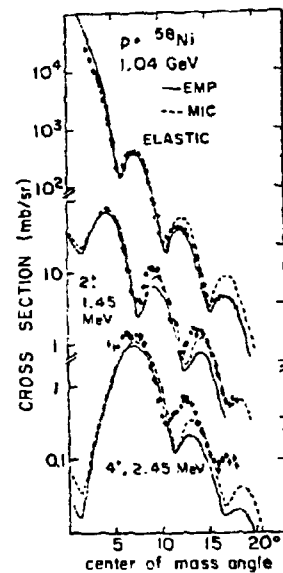


Fig. 1. Elastic and inelastic scattering of $p + ^{58}\text{Ni}$ at 1.04 GeV. The solid curve is the result of Schrödinger or DWBA calculations using an empirical optical potential, while the dashed curve is the result of calculations using a microscopic potential obtained from a realistic effective nucleon-nucleon interaction and Hartree-Fock proton and neutron matter densities. Both potentials include a spin-orbit term.

at the larger momentum transfers, the original analytic simplifications which made the Glauber approach attractive are almost completely lost. Finally, it is often difficult or unwise to relate the various parameters used

^{*} Research supported in part by the U.S.E.R.D.A.

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Colliding heavy ions: Nuclei as dynamical fluids*†

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Heavy ion experiments are already enriching nuclear science with a tapestry of new phenomena which require explanation. In response, theoretical nuclear physics is rapidly expanding its insights to encompass these new observations, especially those concerned with the macroscopic aspects. Preliminary theoretical studies already suggest that the dynamical nuclear fluid must sometimes be considered viscous,¹ compressible, and/or rotational, if its microscopic properties are to be encompassed. These and some threads already well placed in the picture will be discussed. Other reasons will be cited to support the expectation that theoretical nuclear macroscopists may more and more come to be fluid dynamicists who specialize in those few thousand fluids called nuclei. Three such reasons are (a) the promised richness of their structure as dynamical fluids, (b) their unique prospect, among all the objects of modern physical science, of allowing a complete microscopic, as well as a phenomenological macroscopic, description, and (c) the possible overflow of such nuclear implications into classical fluid theory, from the viewpoint of which the nuclear heavy ion data are a significant novelty.

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*This paper is adapted from an invited talk given by J. J. Griffin at the Pittsburgh Meeting of the Division of Nuclear Physics, American Physical Society, on October 31, 1974 (Bull. Am. Phys. Soc. II, 19, 996).

†Work supported by U. S. Energy Research and Development Administration.

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¹In retrospect, the word "dissipative" might better have been used here to avoid any suggestion that the microscopic structure of the dissipative process (so far unknown) is necessarily associated with the particular "hydrodynamical" form familiar from the Navier-Stokes equation.

I. DATA, PARAMETERS, AND THE HYDRODYNAMICAL ANALOGY

This paper attempts to communicate a few simple ideas on the general subject of nuclei as dynamical fluids. It is in no sense a "review" of the subject, which is probably too new anyway for cataloging, but rather an outline of one still changing viewpoint, and of some recent related research.

The first section of the paper attempts to demonstrate for those who are unfamiliar with recent heavy ion studies that heavy ion data is rich and complex. It also suggests the view that the utility of fluid dynamical parameters is already well established in organizing the phenomenology of heavy ion collisions. As an illustration, we emphasize the idea that macroscopic liquid droplets in collision are very interesting bases for gaining insight into nuclear collision.

A. Heavy ion data is rich and complex

Figure 1 is taken from Kratz *et al.* (1974). It presents the results of their radiochemical studies of the products from ⁸¹Kr accelerated against ²³⁸U at a laboratory energy of 605 MeV. (We emphasize that this data is thick target data so that reactions occur over a range of energies, and that radiochemistry measures only those products which live for a substantial time after the collision. Both of these limitations to the information available in this data, should be kept in mind.)

B. Some qualitative features still need explanation

Of richness, Figure 1 is a remarkable example. In the lower half of the figure the several components—one might more precisely say, "Components as interpreted"—in the distribution are labeled by letters A, B, C, D, E, F, G.

Component A is a broad distribution of the type which is expected to follow the formation of a compound nucleus (the "complete fusion" process) by the amalgamation of the krypton nucleus with a uranium nucleus into

1.D.2

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DESCRIPTION OF NUCLEAR COLLECTIVE MOTIONS IN TERMS OF THE BOSON EXPANSION TECHNIQUE

(II). Additional formulation and numerical calculations

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Received 26 January 1976

(Revised 24 May 1976)

Abstract: A formulation of the boson expansion technique given in our previous publication has been developed further by adding the contributions of the quadrupole-pairing interaction and the coupling between the collective and noncollective modes of excitations. Based on this formalism numerical calculations were carried out to describe the properties of a number of collective nuclei found in various regions of the periodic table. In general, very good agreement with experiment was obtained both for vibrational and rotational nuclei. Comparison with earlier theoretical work is also made.

1. Introduction

In our recent publication ¹⁾ which will be referred to henceforth as paper I, a detailed formulation of the boson expansion technique was given, such that it can be applied directly to the description of the properties of collective nuclei. The purpose of the present article is to report on the result of some numerical calculations based on the formalism of I. We shall not give here references to earlier work on the boson expansion technique, nor discuss the motivation of using it for actual numerical calculations. These references and discussions can be found in I, as well as in more recent review articles ²⁾.

The Hamiltonian derived in I included terms up to sixth order in purely collective bosons. In the course of numerical calculations, however, it became more and more clear that, in order to get a good fit to experiment, it was vital to include the effect of noncollective bosons in one way or another. It was also found that the fit to data was improved significantly by including the quadrupole-pairing interaction, in addition to the monopole-pairing and the quadrupole particle-hole interactions which were already considered explicitly in I.

[†] Work supported in part by the National Science Foundation.

^{**} Work supported in part by the USERDA.

INTERACTION BETWEEN COLLECTIVE AND NONCOLLECTIVE MODES IN THE
NUCLEAR EXCITATION*

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We shall begin this paper by showing to what extent one can describe the nature of so-called collective nuclei in terms of Boson expansion technique, in which the effect of noncollective excitation mode upon that of collective mode is taken into account. We shall then show how the theoretical predictions are modified if the Boson expansion technique is extended so as to treat the noncollective mode more explicitly.

The use of Boson expansion technique to describe nuclear collective motion was first proposed by Belyaev and Zelevinsky¹, and pioneering numerical work along this line was carried out by Sorensen². Marumori and his coworkers³ proposed another formulation of the Boson expansion technique, which apparently takes into account the Pauli principle more accurately than does the Belyaev-Zelevinsky approach. The Marumori approach, however, has a convergence problem and not much numerical work has been made along its line. Several other formulations were also proposed⁴, but again it appears that not much numerical work followed them.

Our present work follows basically the Belyaev-Zelevinsky approach and may be considered an extension of Sorensen's work. As will be seen below, our work differs from Sorensen's in two major respects. We have in the original (Fermion) Hamiltonian the quadrupole-pairing inter-

*Work supported in part by the U.S.E.R.D.A.

II. C. COPY OF TITLE PAGES OF SUBMITTED PAPERS

COULOMB EXCITATION EFFECT IN THE REACTION $^{144}\text{Nd}(^{12}\text{C}, ^{14}\text{C})^{142}\text{Nd}^*$

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ABSTRACT

The Coulomb excitation effect in the reaction $^{144}\text{Nd}(^{12}\text{C}, ^{14}\text{C})^{142}\text{Nd}$ at 78 Mev has been studied with a special emphasis on the transition to the first excited 2^+ state in ^{142}Nd .

* Work supported in part by the U.S. Energy Research and Development Administration.

Effects of Nonorthogonality Corrections in Two-Step
Processes in (h,t) Reactions

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On the Difference between One-nucleon Stripping and Pick-up
Reactions Induced by Heavy Ions

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ABSTRACT

It is shown that the phase difference observed between the angular distributions of one-nucleon stripping and pick-up reactions induced by ^{13}C on ^{40}Ca is resolved in an EFR-CCBA analysis. The reason why CCBA works is also discussed.

⁺Work supported in part by the U.S.E.R.D.A..

ABSOLUTE MAGNITUDE OF HEAVY-ION INDUCED TWO-NUCLEON TRANSFER REACTIONS[†]

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ABSTRACT

It is shown that the long standing difficulty of too small theoretical cross sections for heavy-ion induced two-nucleon transfer reaction has been solved, at least for a limited amount of experimental data. It was found that this is achieved chiefly by using sufficiently sophisticated wave functions for the nuclear states involved, and by including both the simultaneous and successive transfer amplitudes in the calculation. A somewhat detailed formulation for these amplitudes is given, presenting in particular the coupled-reaction-channel equation in a form suited for exact-finite-range calculations. In the course of the calculation, a surprising similarity was found between the behaviors of the successive and simultaneous amplitudes, and an explanation for this similarity is given.

[†]Work performed under auspices of the United States Energy Research and Development Administration.

EXACT-FINITE-RANGE MICROSCOPIC CALCULATIONS FOR HEAVY-ION INDUCED
TWO-NUCLEON TRANSFER REACTIONS*

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DEEP INELASTIC SCATTERING TREATED AS MULTI-STEP DIRECT REACTION PROCESSES*

-- Application to (p,p') Reaction --

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ABSTRACT

A possibility is investigated to treat reactions with continuous spectra as multi-step direct reaction processes. It is shown that a few rather drastic but very reasonable approximations can be introduced, so that the calculations can be done very easily. Application to the (p,p') reaction shows that this approach does work well.

* Work supported in part by the U.S.E.R.D.A.

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"Nuclear Matter" Approach to the Energy Dependence
of the Real Part of the Proton-Nucleus Optical
Potential at Intermediate Incident Energies*

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ABSTRACT

We present and discuss here the results of a calculation of the real part of the potential energy of a proton within nuclear matter, making use of the realistic Reid soft-core nucleon-nucleon interaction. The problem of evaluating the real part of the proton-nucleus optical model potential at the center of mass of the target nucleus, as a function of incident energy, is solved with the help of a number of techniques developed in the literature of nuclear-matter theory. We find good agreement with results of phenomenological studies over an energy range of 50 to 1050 MeV incident proton energy, for targets ^{40}Ca and ^{208}Pb . Furthermore, we consider a number of higher-order corrections to our initial result; all of these have minimal effect in the energy regime we have considered, except for the third-order correction proposed by Rajaraman. We also discuss and use a technique for handling overlap integrals containing potentials with strongly or infinitely repulsive cores, which avoids the customary separation method of nuclear matter calculations. Finally, we show that the conventional approach to the nucleon-nucleus optical potential at medium energy, the so-called Raleigh-

*Research supported in part under contract with the United States Energy Research and Development Administration.

ABSTRACT, continued

Lax potential, which makes use of a straightforward impulse approximation and an empirical nucleon-nucleon scattering amplitude, is in serious disagreement both with phenomenological energy dependence of the optical potential and with results of the present calculations. We make an effort to shed some light on the reasons for the failure of the Rayleigh-Lax approach, as opposed to the success of other approaches relying upon realistic nucleon-nucleon interactions rather than an empirical medium-energy nucleon-nucleon T-matrix.

THE SINGLE-PARTICLE SCHRÖDINGER FLUID

I: FORMULATION^{*†}

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January 1976

ABSTRACT

The problem of a single quantal particle moving in a time-dependent external potential well is formulated specifically to emphasize and develop the fluid dynamical aspects of the matter flow. This idealized problem, the Single-Particle Schrödinger Fluid, is shown to exhibit already a remarkably rich variety of fluid dynamical features, including compressible flow and line vortices. It provides also a sufficient framework to encompass

*Research supported in part by U. S. Energy Research & Development Administration.

† Much of this research is reported in more detail in a dissertation submitted to the Graduate School, University of Maryland, by K.-K. K., in partial fulfillment of the requirements of the Ph.D. degree.

†† Alexander von Humboldt Foundation U. S. Senior Scientist at Universität, Giessen, and Hahn Meitner Institute, Berlin, during 1975-76. The support of the Foundation during the final stages of this work is greatly acknowledged.

TIME-DEPENDENT HARTREE-FOCK AND TIME-DEPENDENT
GENERATOR COORDINATE METHOD*

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ABSTRACT

We point out that an intrinsic classical character in the relative center of mass motion is unavoidable in the usual time-dependent Hartree-Fock (TDHF) theory (with the one-Slater-determinant restriction) when it is applied to heavy ion collisions. Thus TDHF does not allow for extracting the cross section in the standard quantum mechanical sense. A time-dependent generator coordinate (TDGC) method is proposed as an extension of the TDHF approach with the purpose of lifting this classical limitation. The theory of measurement, which is well known for time-independent calculations and is used to extract cross sections, is extended to a time-dependent treatment which deals with a wave packet for the relative center of mass motion. The problem of defining channel states in TDHF calculation is also studied.

*Work supported in part by U.S.E.R.D.A.

II. D. ABSTRACTS OF TALKS PRESENTED AT MEETINGS

AB 2 Energy Dependence of the Real Part of the Proton-Nucleus Optical Potential via Nuclear Matter Techniques. L. RAY AND W. R. COKER, Univ. of Tex. Austin-- We have used a number of techniques borrowed from the theory of nuclear matter, and the Reid soft-core nucleon-nucleon interaction, to calculate the real part of the proton-nucleus optical model potential at the target nucleus center-of-mass, over an incident proton energy range from 50 to 1050 MeV. Specific results for targets of ^{40}Ca and ^{208}Pb are in good agreement with the trends known from phenomenological analyses. Various higher-order corrections to the simplest result are estimated, and all found to be insignificant except for the third-order correction suggested by Rajaraman.¹ We also show that the conventional approach to the nucleon-nucleus optical potential at medium energy, which uses the first term in the KMT² expansion and the impulse approximation, relying therefore upon an empirical nucleon-nucleon scattering amplitude, disagrees seriously both with phenomenological trends and with our results. *Supported in part under contract with the U.S.E.R.D.A. R. Rajaraman, Phys. Rev. **129**, 265 (1963).
²A.K. Kerman, H. McManus and R. M. Thaler, Ann. Phys. (N. Y.) **8**, 551 (1959).

AE 5 Description of Heavy-Ion Collision in Terms of Time-Dependent Slater Determinants. K. K. KAN and T. TAMURA, University of Texas.--Recently Bonche et al¹ reported on an interesting calculation based on the time-dependent Hartree-Fock (TDHF) theory, which described the time evolution of a single Slater determinant, the determinant being constructed first as the ground state of the time-independent HF (TIHF) theory for two systems before they collide. In the present study we include in the calculation other Slater determinants which evolve with time starting from those that correspond to excited states in the initial TIHF calculation. The way these new determinants are admixed into the original single determinant is described in terms of the time dependent Schrödinger equation, a problem which has some similarity to the work of Schütte and Wilers². It is hoped that this type of calculation gives some idea of how good the single determinant approximation can be, and, if so, what particular aspects of a supposedly very complicated process are indeed described by such an approximate treatment.

*Work supported by the U.S.E.R.D.A.
¹P. Bonche, S. Koonin and J. W. Negele, to be published.
²G. Schütte and L. Wilers, Phys. Rev. **C12**, 2100 (1975).

AE 11 Couple-Reaction-Channel (CRC) analysis of heavy-ion induced three-nucleon transfer reactions. T. Udagawa, T. Tamura and D. H. Feng, University of Texas.--Previously we fit the data of an exotic reaction, $^{48}\text{Ca}(^{16}\text{O},^{15}\text{C})^{49}\text{Ti}$, in terms of successive $^{16}\text{O}-^{14}\text{C}-^{15}\text{C}$ and $^{16}\text{O}-^{17}\text{O}-^{15}\text{C}$ transfer processes¹. A very similar description may very well be applied to three-nucleon transfer reactions, and in fact analysis of recent data of a few three nucleon transfer reactions² are underway. Results of such calculations will be presented.

*Work supported by the U.S.E.R.D.A.
¹T. Udagawa, T. Tamura and K. S. Low, Phys. Rev. Letts. **30**, 30 (1975).
²D. Kovar, private communication.

AE 12 Absolute magnitude of two-nucleon transfer reactions between heavy-ions. D. H. Feng, T. Udagawa and T. Tamura, University of Texas.--To obtain sufficiently large absolute magnitude of two-nucleon transfer reactions between heavy-ions had constituted a long standing problem. Recently¹, however, we succeeded in fitting the magnitude of $^{48}\text{Ca}(^{18}\text{O},^{16}\text{O})^{50}\text{Ca}$ reaction, and then further of $^{42,48}\text{Ca}(^{16}\text{O},^{18}\text{O})$ and $^{62}\text{Ni}(^{18}\text{O},^{16}\text{O})$ reactions. It was found that simultaneous and successive processes contributed about equally, and that to use a fairly sophisticated configuration mixing was vital. In these reactions, the effect of inelastic scattering was unimportant, but it becomes important for reactions like $^{144}\text{Nd}(^{12}\text{C},^{14}\text{C})$. In fact it was shown that CCBA fits the data very well² though not the overall magnitude. We are applying the recipe of ref.1 to this and similar reactions, hoping to fit now the data completely including the absolute magnitude.

*Work supported by the U.S.E.R.D.A.
¹D. H. Feng, T. Udagawa, T. Tamura, J. Lynch and K. S. Low, to be published.
²K. Yagi et al, Phys. Rev. Letts. **34**, 96 (1975).

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APS East Lansing Meeting October 1976

AB 12 DWBA Approach to Inelastic Scattering at Medium Energies. W. R. COKER, L. RAY and G. W. HOFFMANN, U. of Texas at Austin --Medium energy proton elastic and inelastic scattering to states of ^{58}Ni and ^{208}Pb , and ^4He elastic and inelastic scattering to states of ^{40}Ca , have been analyzed using the partial-wave approach by solving the Schrödinger equation with relativistic kinematics and using the distorted wave Born approximation. Our calculations are absolute, using nuclear collective parameters obtained from a survey of a large number of low-energy analyses of inelastic scattering of electrons, nucleons and nuclei from ^{40}Ca , ^{58}Ni and ^{208}Pb . If time permits, we will also discuss results of coupled-channel calculations for proton elastic and inelastic scattering on ^{12}C at medium energies.
*Research supported in part under contract with the U.S.E.R.D.A.

BB 8 Time-Dependent Generator Coordinate Method and TDHF Theory. K. K. Kan and T. Tamura, University of Texas.-- The time-dependent Hartree-Fock (TDHF) approach¹ seems promising in describing various aspects of reactions induced by heavy ions. However, we feel it desirable to improve the formalism¹ somewhat, before a full scale numerical calculation is undertaken. One such improvement is to avoid the classical treatment of the c.m. motion and hence to make the problem fully quantum mechanical. This can be done by describing the c.m. motion in terms of a wave packet. If this is done and then combine with the extension² of TDHF to time-dependent Schrödinger equation approach, it turns out that the resultant equation is that of the generator coordinate method, though now the generating function, i.e., the packet function, is time-dependent. Discussions will be made on features of this new equation, including a way to extract observables out of the solutions of this equation.

* Work supported by the U.S.E.R.D.A.

¹P. Bonche, et al., Phys. Rev. C 13, 1226(1976); R. Y. Cusson, et al., Phys. Rev. Lett. 36, 1166(1976).

²K. K. Kan and T. Tamura, Bull. Am. Phys. Soc. 21, 511 (1976).

DB 7 Analysis of Deep Inelastic Scattering Processes in Terms of Multi-Step Direct Reaction Theory. T. Tamura, T. Udagawa, D.H. Feng¹ and K.K. Kan, Univ. of Texas. --- Multi-step direct reaction (MSDR) theories such as CC, CCBA and CRC have enjoyed success in fitting various reaction data that leads to discrete states. We report here on an extension of MSDR analyses to deep inelastic scattering data i.e. to data of reactions leading to continuum states. By taking advantage of the high density of levels involved, one can introduce a rather drastic but well justified simplification into the calculation. As a consequence, the theoretical cross section is written as sum of terms, the summand being a product of two factors, one depending on nuclear structure and the other on reaction dynamics. So far a successful analysis has been completed of (p,p') data¹. Applications to other reactions involved by both light- and heavy-ions are underway.

* Work supported in part by U.S.E.R.D.A.

¹ Presently at Dept. of Physics, Drexel Univ., Phila., Penn. I. F.E. Bertrand and R.W. Peelle, Phys. Rev. 8C 1045(1973)

EB 3 Description of Nuclear Collective Motions in Terms of Boson Expansion Technique. T. Kishimoto, Texas A&M Univ.^{*}, T. Tamura, U. of Texas. ** Our previous formulation¹ of the boson expansion technique has been developed further by adding the contributions of quadrupole-pairing interaction and the coupling between collective and noncollective modes of excitations. Based on this formulation numerical calculations were carried out to describe properties of a number of collective nuclei found in various regions of the periodic table. In general very good agreement with experiment was obtained for not only vibrational but also transitional and rotational nuclei. Comparison with predictions of other theories, particularly with that of Kumar and Baranger will be made.

* Supported by National Science Foundation.

** Supported by U.S.E.R.D.A.

¹ T. Kishimoto and T. Tamura, Nucl. Phys. A192, 246(1972).

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