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THE ROLE OF THE PAULI PRINCIPLE

IN PION-NUCLEUS SCATTERING*

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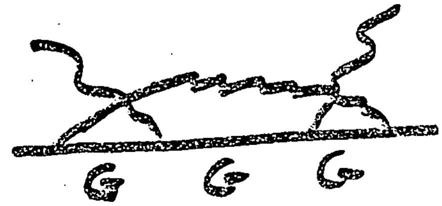
FRAMEWORK: many body theory with propagators

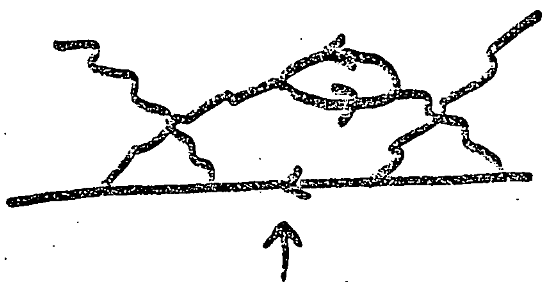
OBJECT: calculate the pion self energy $\Pi(q)$ in the nucleus from an effective πN amplitude (density dependent)

NOTES: resum the set of πN graphs which make up the "Δ" in the nucleus ; Δ not treated as elementary particle bathing in a complex Hartree field

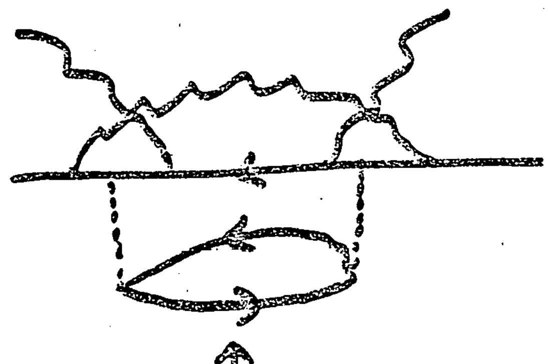
Nature of Many Body Corrections to Effective Amplitude $\hat{f}_{\pi N}$

→ 1) Pauli blocking effect 

2) "collision damping"

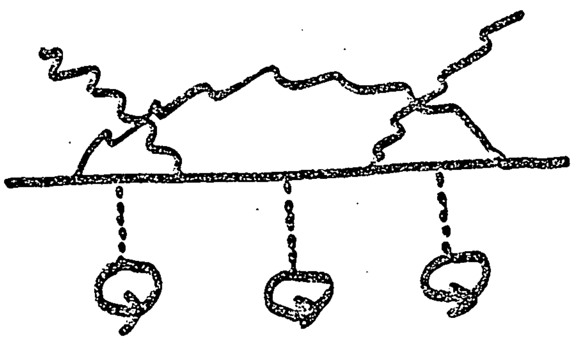


2p1h



2p1h + π

3) "off-shell effects"

 + ...

4) ....

object: correct treatment of Pauli effect (pedagogical interest only)

Conclusions (Preview)

1) Pauli effect on V_{opt} is small (5% level)

2) 1st order hole line expansion for $\hat{f}_{\pi N}$ (or V_{opt}) is inappropriate

3) Particle and hole propagation must be treated symmetrically
→ RPA-like processes cancel out most of Pauli effect in hole line expansion

Reference: Phys. Rev. C14, 2211 (1976)

Propagators:

pion: $$D_0(q) = \frac{1}{(\omega_q - i\delta)^2 - \omega^2}$$

$$D(q) = \frac{D_0(q)}{1 - D_0(q)\,\Pi(q)}$$

nucleon: $$G(k) = G_<(k) + G_>(k)$$

$$G(k) = \frac{n_<(\underset{\sim}{k})}{\epsilon_{\underset{\sim}{k}} - k_0 + i\delta} + \frac{n_>(\underset{\sim}{k})}{\epsilon_{\underset{\sim}{k}} - k_0 - i\delta}$$

$$n_<(\underset{\sim}{k}) = n(k) = \begin{cases} 1 & k \le k_F \\ 0 & k > k_F \end{cases}$$

$$n_>(\underset{\sim}{k}) = 1 - n(k)$$

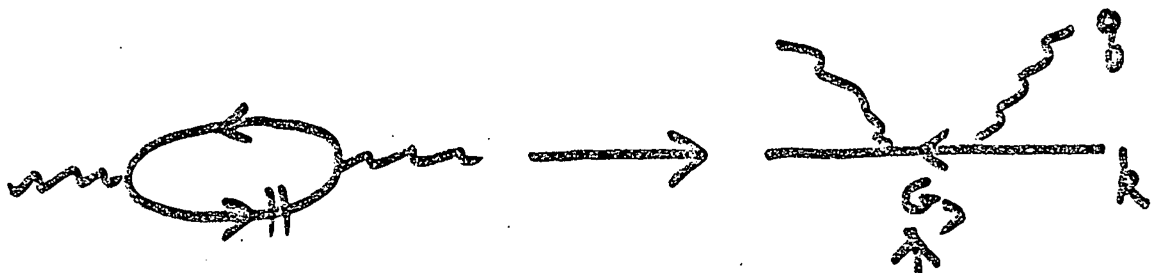
Pion scattering from an infinite Fermi gas

HOLE LINE EXPANSION:

$$\pi(q) = \sum_k n_<(k) \left. \frac{\delta \pi(q)}{\delta n_<(k)} \right|_{n_< = 0} + \cdots$$

$$\pi(q) = 4\pi\rho \left\langle \tilde{f}(kq, kq) \right\rangle_{k_0 = \epsilon_k} + \cdots$$

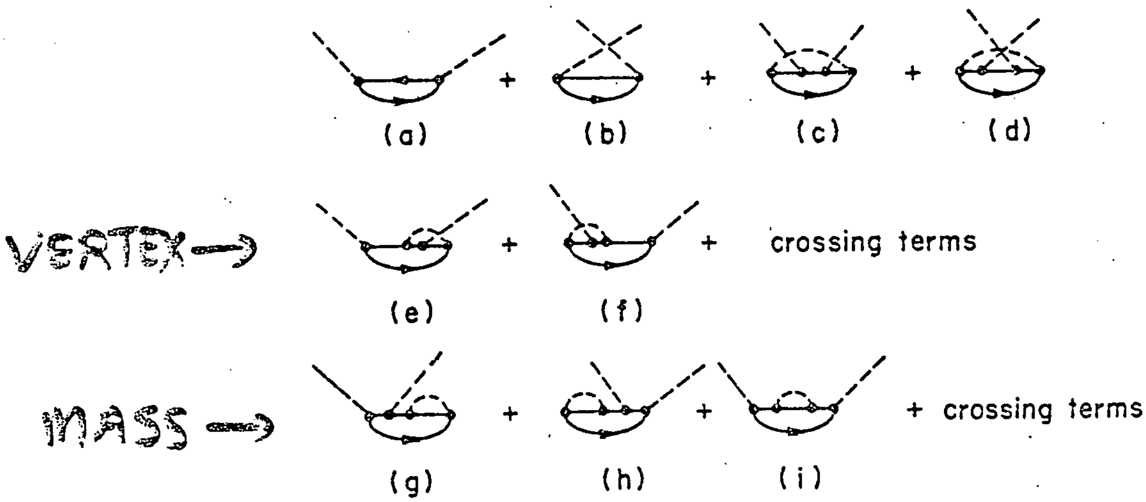
$$\left. \frac{\delta \pi(q)}{\delta n_<(k)} \right|_{n_< = 0} = \frac{1}{i} \left. \frac{\delta \pi(q)}{\delta G_<(k)} \right|_{G_< = 0}$$



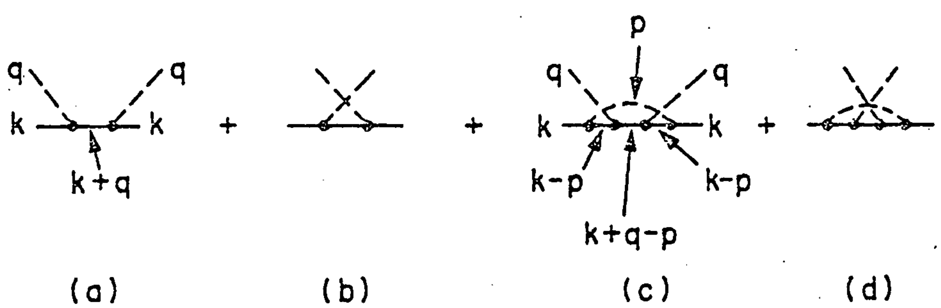
$$\sim \frac{\delta \pi}{\delta G_<} \longrightarrow \tilde{f}(kq, kq)$$

Equiv. to Weber & Eisenberg Phys. Rev. C10, 925 (1974), Bethe PRL (1973)

SELF ENERGY DIAGRAMS WITH ONE HOLE LINE



IRREDUCIBLE πN DIAGRAMS



HOLE LINE EXPANSION (cont.)

$$\tilde{f}(kq,kq) = \tilde{f}_B(kq,kq)$$

$$+ 2\pi \sum_{\underset{\sim}{p}} \frac{1}{\omega_p} \frac{n_>(k+q-p)}{\omega_p - \omega - i\delta} \tilde{f}^\dagger(kq; k+q-p, p)\tilde{f}(kq; k+q-p, p)$$

+ crossing terms

Born Term:

$$4\pi \tilde{f}_B(kq,kq) = |W_q|^2 \left[\frac{n_>(k+q)}{\omega_k(q) - \omega} + \frac{n_>(k-q)}{\omega_k(-q) + \omega} \right]$$

$$\omega_{\underset{\sim}{k}}(\underset{\sim}{q}) = \epsilon_{\underset{\sim}{k}+\underset{\sim}{q}} - \epsilon_{\underset{\sim}{k}}$$

$$\langle \tilde{f}_B(kq,kq) \rangle = F(q)\, f_B^{\text{free}}$$

$$F(q) = \frac{1}{\rho} \sum_{k} n_<(k)\, n_>(k+q)$$ Bethe

$$F(q) = \begin{cases} 3q/4p_F \left(1 - \frac{1}{12} q^2/p_F^2\right) & q < 2p_F \\ 1 & q > 2p_F \end{cases}$$

HOLE LINE EXPANSION (cont.)

$$\hat{f} = \sum_{\alpha} P_{\alpha}(\vec{q}', \vec{q}) \, \tilde{h}_{\alpha}(\omega)$$

$$\hat{h}_{\alpha}(\omega) = \frac{\lambda_{\alpha}}{\omega} F(q) + \frac{1}{\pi} \int_{\mu}^{\infty} d\omega_p \, p^3 F(p)$$

$$\times \left[\frac{|\tilde{h}_{\alpha}(\omega_p)|^2}{\omega_p - \omega - i\delta} + \sum_{\beta} A_{\alpha\beta} \frac{|\hat{\tilde{h}}_{\beta}(\omega_p)|^2}{\omega_p + \omega} \right]$$

$$\lambda_{\alpha} = \frac{f^2}{3}(-8, -2, -2, 4) \quad \text{for } (11, 13, 31, 33) \, \pi N$$

To 4th order:

$$\tilde{h}_{\alpha}(\omega) = \frac{\lambda_{\alpha}}{\omega} F(q) + \frac{1}{\pi} \int \frac{d\omega_p}{\omega_p^2} p^3 F(p)$$

$$\times \left[\frac{\hat{\lambda}_{\alpha}^2}{\omega_p - \omega - i\delta} + \sum_{\alpha\beta} A_{\alpha\beta} \frac{\hat{\lambda}_{\beta}^2}{\omega_p + \omega} \right]$$

Consequences for optical potential

$$V_{opt}(r) \overset{q\to 0}{=} \frac{2\pi}{\mu}\Big\{-b_0\,\rho(r) - 2b_1\,\vec{t}_\pi\cdot\vec{T}\,\frac{\rho(r)}{A}$$
$$+\vec{\nabla}\cdot\Big(C_0\,\rho(r) + 2C_1\,\vec{t}_\pi\cdot\vec{T}\,\frac{\rho(r)}{A}\Big)\vec{\nabla}\Big\}$$

Calculate C_0, C_1 (p-wave part)

$$c_0^{free} = \frac{1}{3}\sum_\alpha V_\alpha h_\alpha(\mu) \approx 0.22\,\mu^{-3}\ ;\ V_\alpha = 1, 2, 2, 4$$

$$c_1^{free} = \frac{1}{3}\sum_\alpha \tilde{V}_\alpha h_\alpha(\mu) \approx 0.18\,\mu^{-3}\ ;\ \tilde{V}_\alpha = -1, -2, 1, 2$$

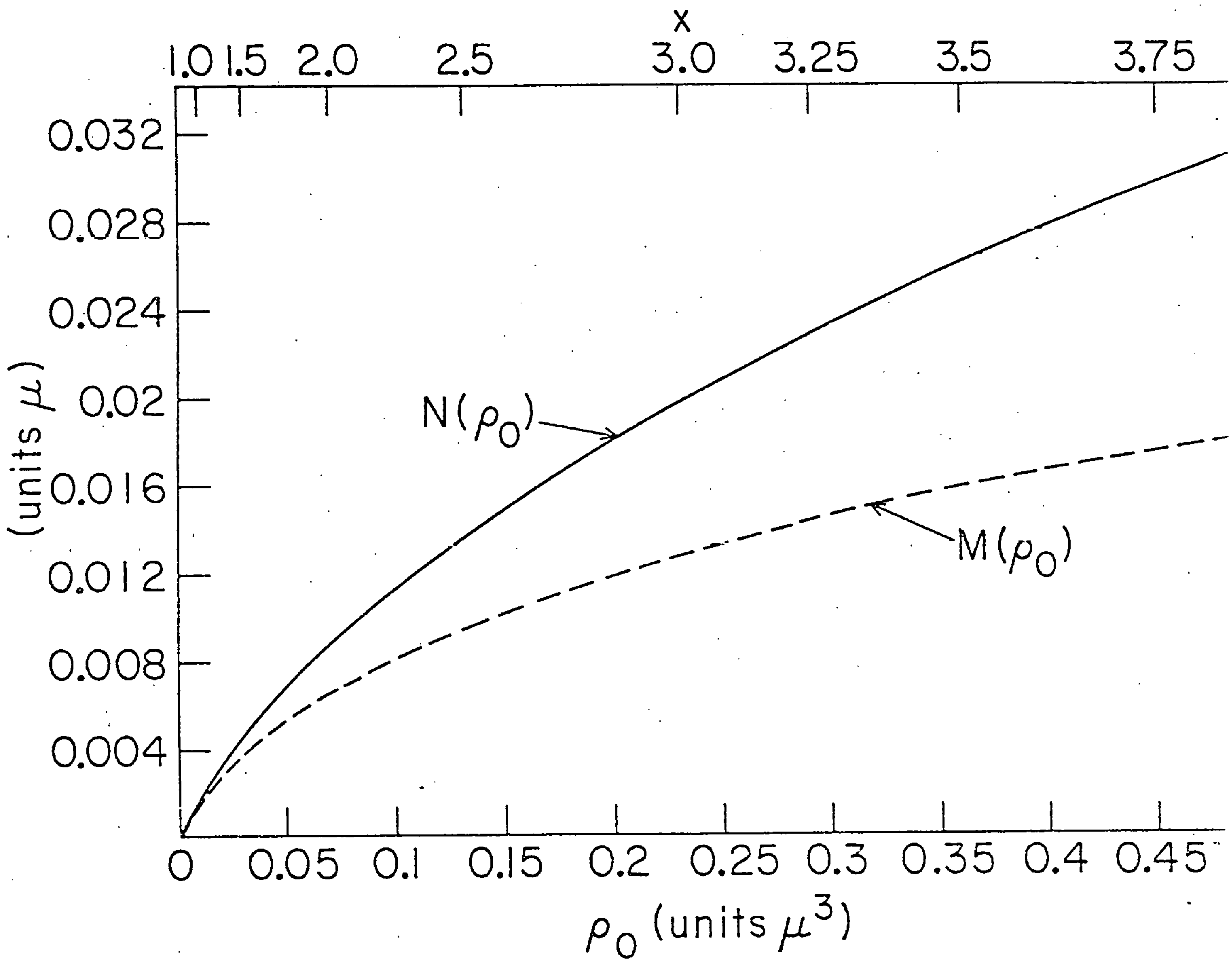
HOLE LINE EXPANSION GIVES

$$\boxed{C_0 - C_0^{free} = -4\pi N(\rho)\,\frac{1}{3}\sum_\alpha V_\alpha \hat{\lambda}_\alpha^2}$$

note $\sum_\alpha \lambda_\alpha V_\alpha = 0$, so Born Term gives C

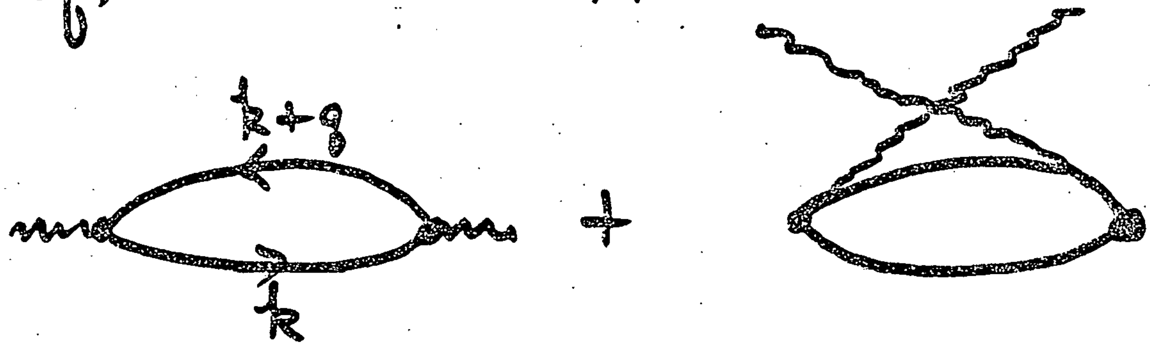
$$\boxed{c_1 - c_1^{free} = -c_1^B - 4\pi f^4 M(\rho)}$$

but $c_1^B = \frac{1}{3}\sum_\alpha \hat{\tilde{V}}_\alpha h_\alpha^B(\mu) \approx 0.16\,\mu^{-3} \approx c_1^{free}$



X
1.0
1.5
2.0
2.5
3.0
3.25
3.5
3.75
0.032
0.028
0.024
0.02
0.016
0.012
0.008
0.004
(units μ)
$N(\rho_0)$
$M(\rho_0)$
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35
0.4
0.45
ρ_0 (units μ^3)

$\Pi(q)$ in Born Approximation:



$$= |W_q|^2 \sum_k n_c(k) \left[\frac{1 - n_c(k+q)}{\omega_k(q) - \omega} + \frac{1 - n_c(k-q)}{\omega_k(-q) - \omega} \right]$$

$$= |W_q|^2 \sum_k n_c(k) \left[\frac{1}{\omega_k(q) - \omega} + \frac{1}{\omega_k(-q) - \omega} \right]$$

$$\boxed{\Pi = 4\pi\rho \langle f_B^{free} \rangle}$$ no Pauli effect!

Proof: $$\sum_k n_c(k) \left[\frac{n_c(k+q)}{\omega_k(q) - \omega} + \frac{n_c(k-q)}{\omega_k(-q) - \omega} \right]$$

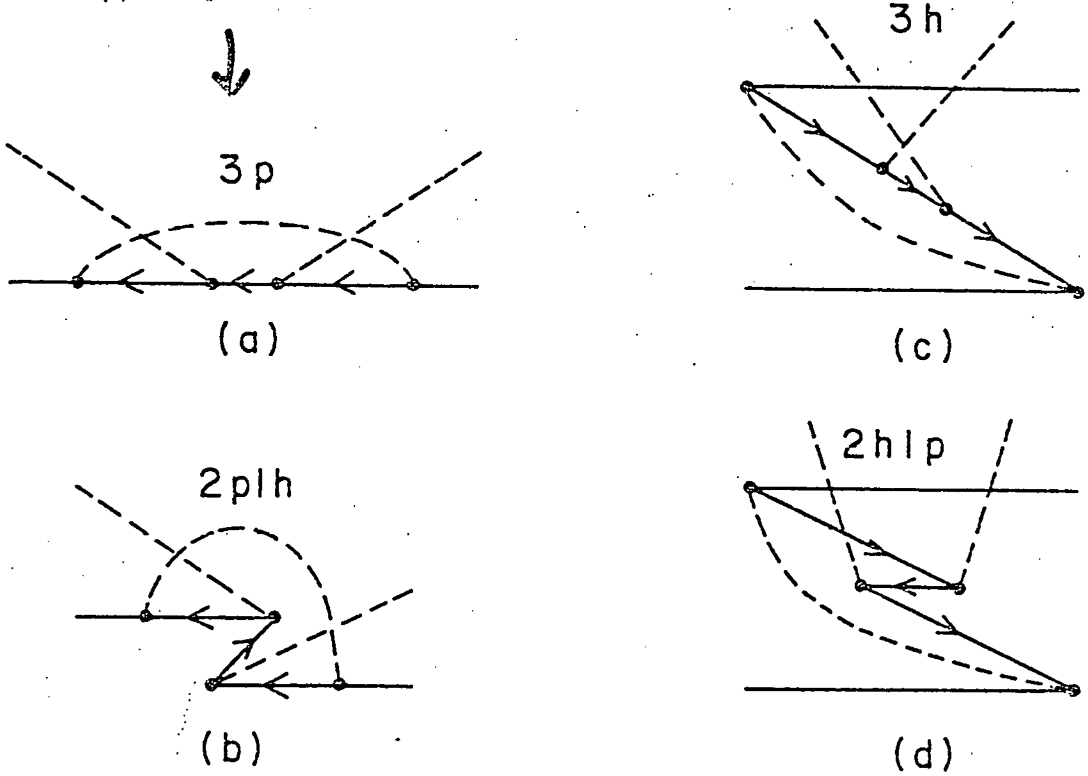
$$= \sum_k \left[\frac{n_c(-k-q)\, n_c(-k)}{-\omega_k(q) - \omega} + \frac{n_c(-k)\, n_c(-k-q)}{\omega_k(q) + \omega} \right]$$

$$= 0$$

NOTE: $$\Pi \neq 4\pi\rho \langle f_B^{free} \rangle F(q)$$

EFFECTIVE πN AMPLITUDE (ORDER f^4)

HOLE LINE EXP.



EXACT AMPLITUDE TO ORDER f^4

$$f_u^{(4)} = 4\pi q^2 f^4 \sum_{p} \frac{p^2}{2\omega_p} \left\{ \frac{n_>(k+q-p)\, n_>^2(k-p)}{(\omega_p-\omega-i\delta)\,\omega_p^2} \right. \qquad 3p$$

$$+\, n_<(k+q-p)\, n_>^2(k-p) \left[\frac{1}{(\omega_p-\omega-i\delta)\,\omega_p^2} - \frac{2\omega_p D_0(p)}{\omega^2} \right] \qquad 2p1h$$

$$-\, n_<(k+q-p)\, n_<^2(k-p)\, \frac{1}{(\omega_p+\omega)\,\omega_p^2} \qquad 3h$$

$$\left. -\, n_>(k+q-p)\, n_<^2(k-p) \left[\frac{1}{(\omega_p+\omega)\,\omega_p^2} - \frac{2\omega_p D_0(p)}{\omega^2} \right] \right\} \qquad 2h1p$$

note cancellations of Pauli restrictions

$$h_\alpha^{(4)}(\omega) - h_{\alpha,\mathrm{free}}^{(4)} = -\frac{2}{\pi} \int d\omega_p \, \frac{p^3}{\omega_p} (1-F(p))\, D_0(\vec{p},\omega)$$

$$\times \left[\hat{\lambda}_\alpha^2 + \sum_\beta A_{\alpha\beta} \hat{\lambda}_\beta^2 \right]$$

COMPARISON OF EXACT AND HL RESULTS

EXACT: $C_0 - C_0^{free} = -8\pi N(\rho)\Sigma_1$

HOLE LINE: $C_0 - C_0^{free} = -4\pi N(\rho)\Sigma_1$ $\leftarrow C_0\rho$ part

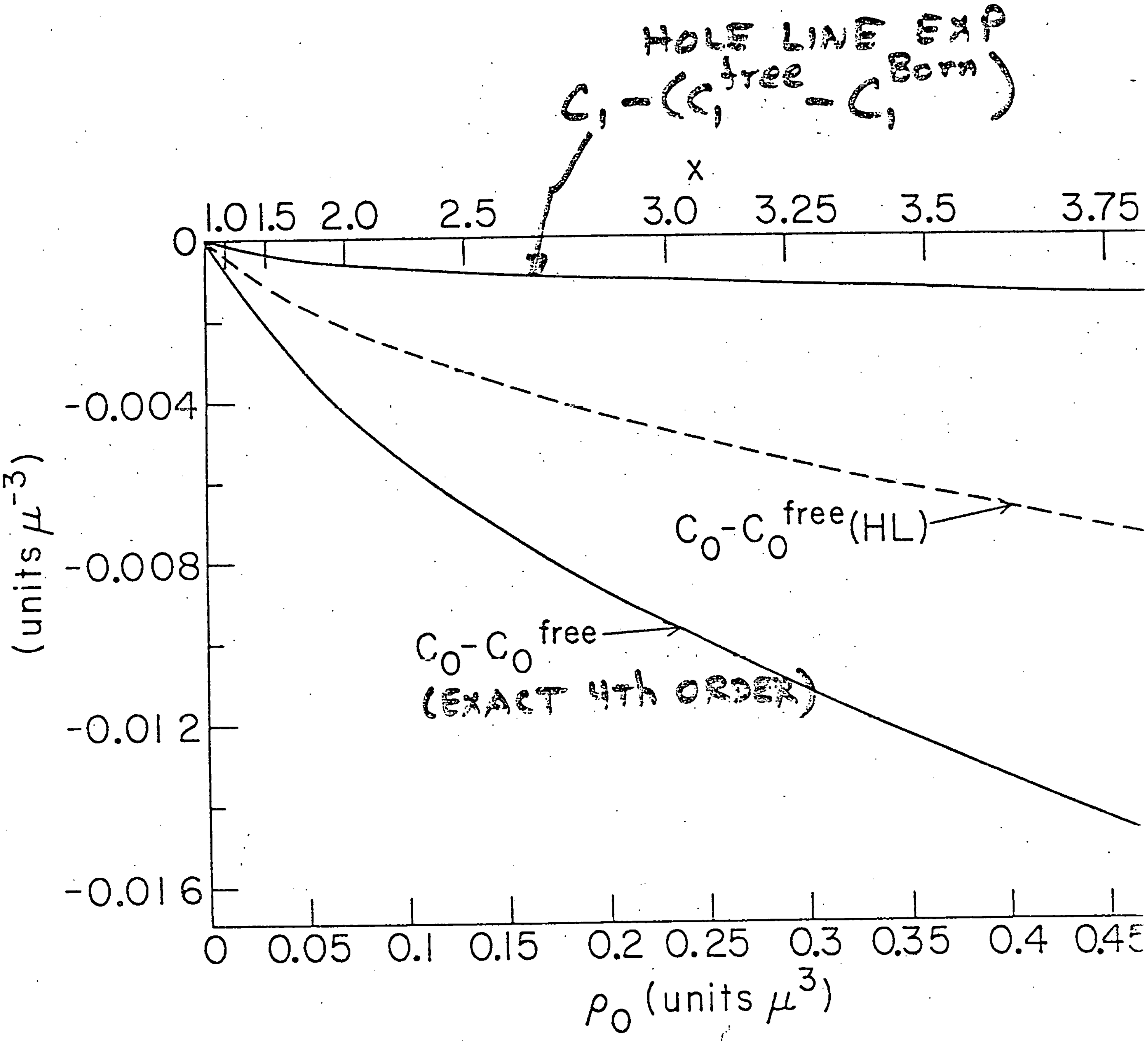
EXACT: $C_1 = C_1^{free}$ $\leftarrow \vec{t}\cdot\vec{T}\, C_1\rho$ part

HOLE LINE: $C_1 = C_1^{free} - C_1^{B} - \frac{4\pi}{3} M(\rho)\Sigma_1$

Conclusions:

1) Pauli effects very small for both C_0 and C_1

2) Hole line expansion predicts large damping of $\vec{t}\cdot\vec{T}$ part of pion potential (no exper. indication for this)



$C_0^{free} \simeq 0.22\,\mu^{-3}$

$C_1^{free} \simeq 0.18\,\mu^{-3}$