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THE ROLE OF THE PAULI PRINCIPLE
IN PION-NUCLEUS SCATTERING*

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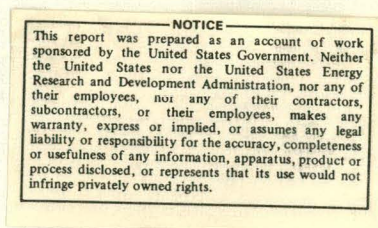
Notes for Invited Talk at the University of Maryland

Seminar Workshop on Pion-Nucleus Scattering --

April 22-23, 1977

College Park, Md.

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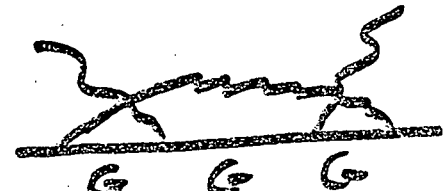
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FRAMEWORK: many body theory
with propagators

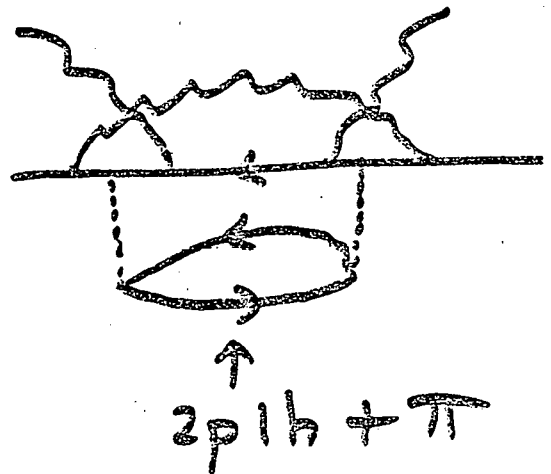
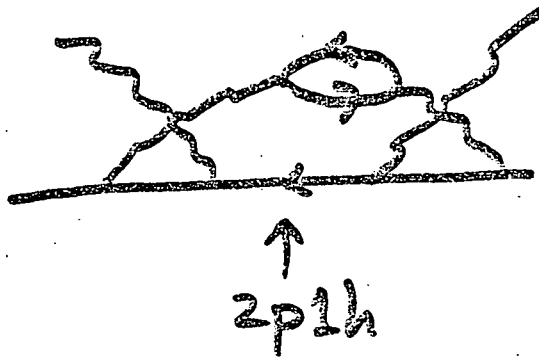
OBJECT: calculate the pion
self energy $\Pi(q)$ in the nucleus
from an effective πN amplitude
(density dependent)

NOTES: resum the set of πN
graphs which make up the
" Δ " in the nucleus; Δ not
treated as elementary particle
bathing in a complex Hartree
field

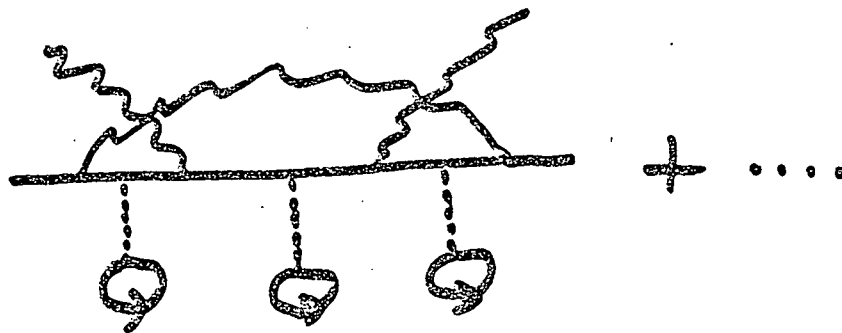
Nature of Many Body Corrections to Effective Amplitude $\hat{f}_{mn}$

→ 1) Pauli blocking effect 

2) "collision damping"



3) "off-shell effects"



4)

object: correct treatment
of Pauli effect
(pedagogical interest only)

Conclusions (Preview)

- 1) Pauli effect on V_{opt} is small
(5% level)
- 2) 1st order hole line expansion
for $\hat{f}_{nn}$ (or V_{opt}) is inappropriate
- 3) Particle and hole propagation
must be treated symmetrically
→ RPA-like processes
cancel out most of
Pauli effect in hole
line expansion

Reference: Phys. Rev. C14, 2211 (1976)

Propagators:

$$\text{pion: } D_0(q) = \frac{1}{(\omega_q - i\delta)^2 - \omega^2}$$

$$D(q) = \frac{D_0(q)}{1 - D_0(q)\Pi(q)}$$

$$\text{nucleon: } G(k) = G_<(k) + G_>(k)$$

$$G(k) = \frac{n_<(k)}{\epsilon_k - k_0 + i\delta} + \frac{n_>(k)}{\epsilon_k - k_0 - i\delta}$$

$$n_<(k) = n(k) = \begin{cases} 1 & k \leq k_F \\ 0 & k > k_F \end{cases}$$

$$n_>(k) = 1 - n(k)$$

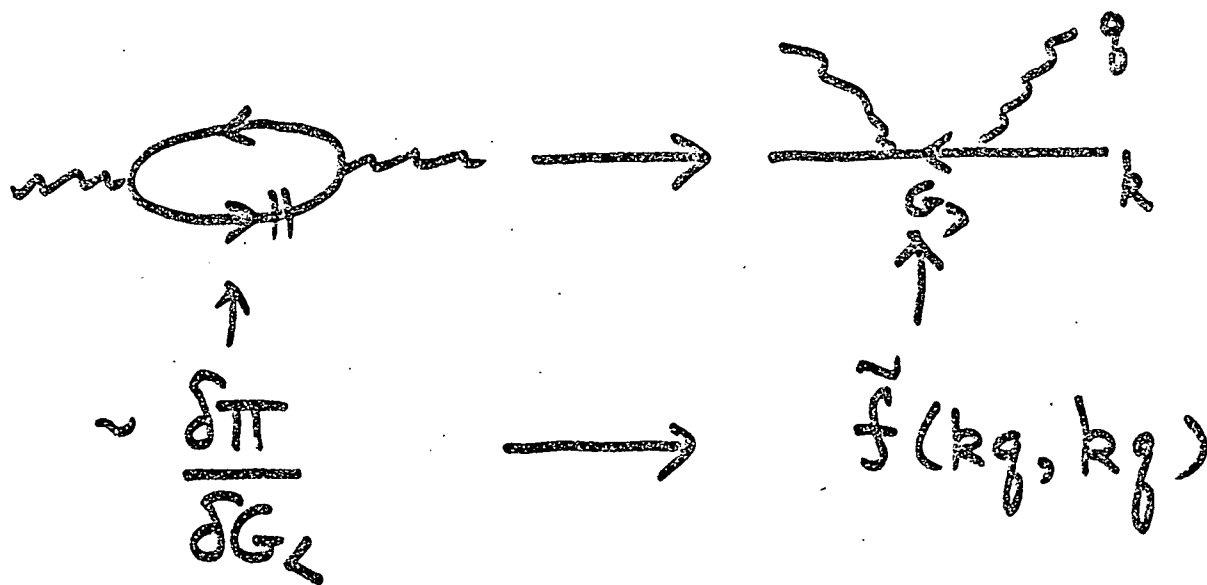
Pion scattering from an infinite Fermi gas

HOLE LINE EXPANSION:

$$\pi(q) = \sum_k n_k(k) \left. \frac{\delta \pi(q)}{\delta n_k(k)} \right|_{n_k=0} + \dots$$

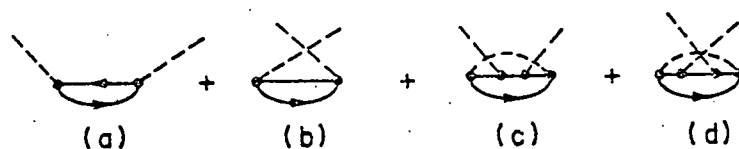
$$\pi(q) = 4\pi f \langle \tilde{f}(kq, kq) \rangle_{k_0 = E_k} + \dots$$

$$\left. \frac{\delta \pi(q)}{\delta n_k(k)} \right|_{n_k=0} = \frac{1}{i} \left. \frac{\delta \pi(q)}{\delta G_k(k)} \right|_{G_k=0}$$

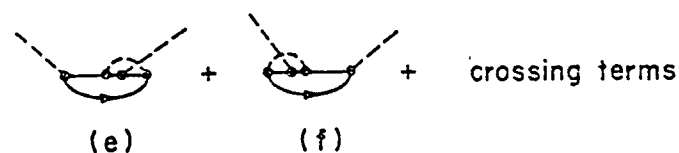


Equiv. to Weber & Eisenberg Phys. Rev. C10, 925 (1974), Bethe PRL 1975.

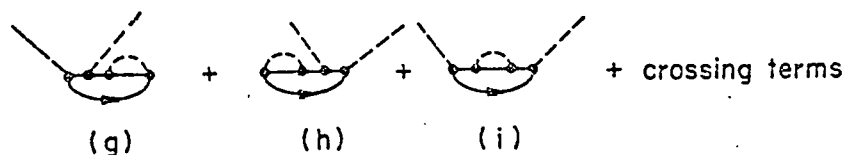
SELF ENERGY DIAGRAMS WITH ONE HOLE LINE



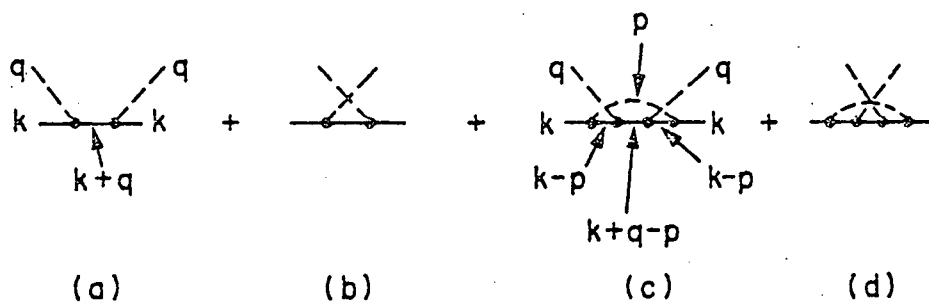
VERTEX $\rightarrow$



MASS $\rightarrow$



IRREDUCIBLE πN DIAGRAMS



HOLE LINE EXPANSION (cont.)

$$\tilde{f}(kq, kq) = \tilde{f}_B(kq, kq)$$

$$+ 2\pi \sum_p \frac{1}{\omega_p} \frac{n_>(k+q-p)}{\omega_p - \omega - i\delta} \tilde{f}^\dagger(kq; k+q-p, p) \tilde{f}(kq; k+q-p, p)$$

+ crossing terms

$$\text{Diagram} = \frac{\text{Diagram 1}}{n_>} + \frac{\text{Diagram 2}}{n_>} + \text{Diagram 3} + \dots$$

Born term:

$$4\pi \tilde{f}_B(kq, kq) = |W_q|^2 \left[\frac{n_>(k+q)}{\omega_k(q) - \omega} + \frac{n_>(k-q)}{\omega_k(-q) + \omega} \right]$$

$$\omega_k(q) = \epsilon_{k+q} - \epsilon_k$$

$$\langle \tilde{f}_B(kq, kq) \rangle = F(q) f_B^{\text{free}}$$

$$F(q) = \frac{1}{\rho} \sum_k n_<(k) n_>(k+q) \quad \text{Bethe}$$

$$F(q) = \begin{cases} \frac{3\rho/4\rho_F (1 - \frac{1}{12} q^2/\rho_F^2)}{1} & q < 2\rho_F \\ 1 & q > 2\rho_F \end{cases}$$

HOLE LINE EXPANSION (cont.)

$$\tilde{f} = \sum_{\alpha} P_{\alpha}(\vec{q}', \vec{q}) \tilde{h}_{\alpha}(\omega)$$

$$\tilde{h}_{\alpha}(\omega) = \frac{\lambda_{\alpha}}{\omega} F(q) + \frac{1}{\pi} \int_{\mu}^{\infty} d\omega_p p^3 F(p)$$

$$\times \left[\frac{|\tilde{h}_{\alpha}(\omega_p)|^2}{\omega_p - \omega - i\delta} + \sum_{\beta} A_{\alpha\beta} \frac{|\tilde{h}_{\beta}(\omega_p)|^2}{\omega_p + \omega} \right]$$

$$\lambda_{\alpha} = \frac{f^2}{3} (-8, -2, -2, 4) \text{ for } (11, 13, 31, 33) \pi N$$

to 4th order:

$$\tilde{h}_{\alpha}(\omega) = \frac{\lambda_{\alpha}}{\omega} F(q) + \frac{1}{\pi} \int \frac{d\omega_p}{\omega_p^2} p^3 F(p)$$

$$\times \left[\frac{\lambda_{\alpha}^2}{\omega_p - \omega - i\delta} + \sum_{\alpha\beta} A_{\alpha\beta} \frac{\lambda_{\beta}^2}{\omega_p + \omega} \right]$$

Consequences for optical potential

$$V_{\text{opt}}(r) \stackrel{q \rightarrow 0}{=} \frac{2\pi}{\mu} \left\{ -b_0 \rho(r) - 2b_1 \vec{\tau}_\pi \cdot \vec{T} \frac{\rho(r)}{A} \right. \\ \left. + \vec{\nabla} \cdot (C_0 \rho(r) + 2C_1 \vec{\tau}_\pi \cdot \vec{T} \frac{\rho(r)}{A}) \vec{\nabla} \right\}$$

Calculate C_0, C_1 (p-wave part)

$$C_0^{\text{free}} = \frac{1}{3} \sum_{\alpha} V_{\alpha} h_{\alpha}(\mu) \approx 0.22 \mu^{-3}; V_{\alpha} = 1, 2, 2, \frac{1}{2}$$

$$C_1^{\text{free}} = \frac{1}{3} \sum_{\alpha} \tilde{V}_{\alpha} h_{\alpha}(\mu) \approx 0.18 \mu^{-3}; \tilde{V}_{\alpha} = -1, -2, \frac{1}{2}, \frac{1}{2}$$

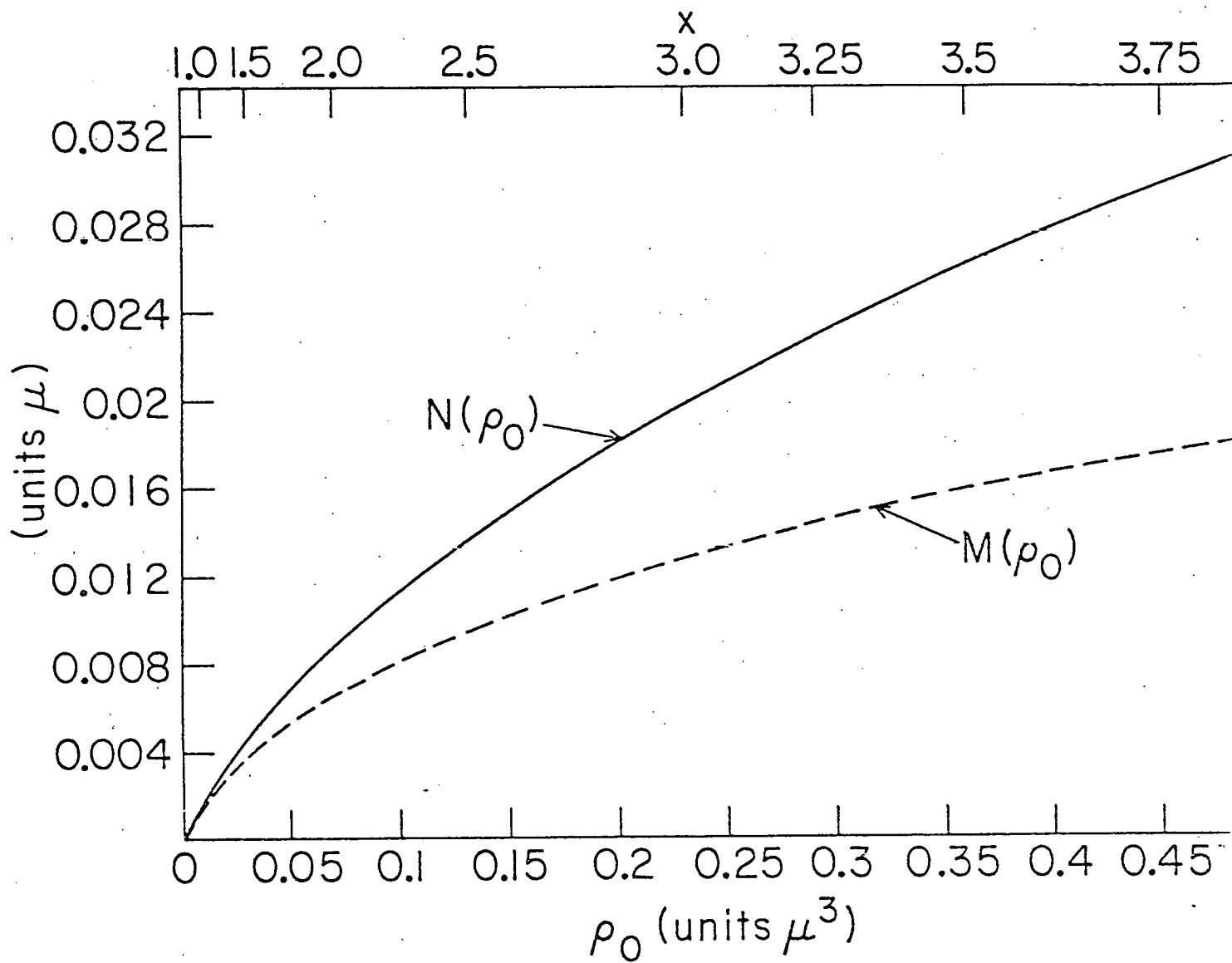
HOLE LINE EXPANSION GIVES

$$C_0 - C_0^{\text{free}} = -4\pi N(\rho) \frac{1}{3} \sum_{\alpha} V_{\alpha} \hat{\lambda}_{\alpha}^2$$

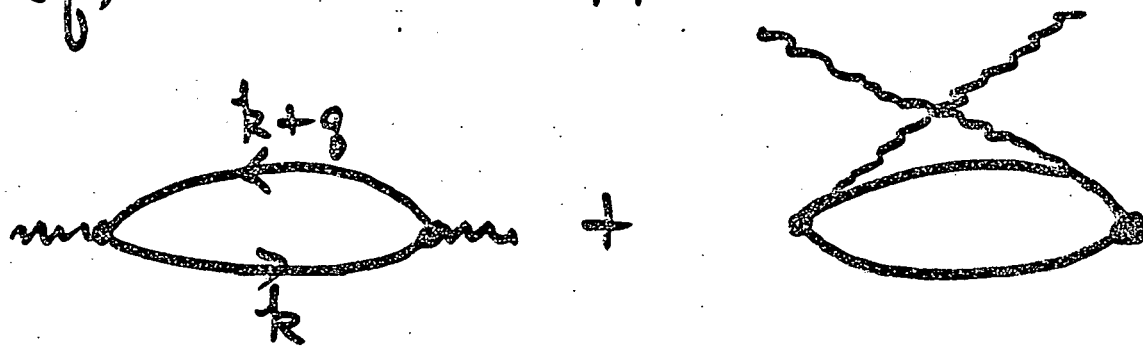
note $\sum_{\alpha} \lambda_{\alpha} V_{\alpha} = 0$, so Born Term gives C

$$C_1 - C_1^{\text{free}} = -C_1^B - 4\pi f^4 M(\rho)$$

$$\text{but } C_1^B = \frac{1}{3} \sum_{\alpha} \tilde{V}_{\alpha} h_{\alpha}^B(\mu) \approx 0.16 \mu^{-3} \\ \approx C_1^{\text{free}}$$



$\Pi(q)$ in Born Approximation:



$$= |W_q|^2 \sum_k n_c(k) \left[\frac{1 - n_c(k+q)}{\omega_k(q) - \omega} + \frac{1 - n_c(k-q)}{\omega_k(-q) - \omega} \right]$$

$$= |W_q|^2 \sum_k n_c(k) \left[\frac{1}{\omega_k(q) - \omega} + \frac{1}{\omega_k(-q) - \omega} \right]$$

$$\Pi = 4\pi \rho \langle f_S^{\text{free}} \rangle \quad \text{no Pauli effect!}$$

$$\text{Proof: } \sum_k n_c(k) \left[\frac{n_c(k+q)}{\omega_k(q) - \omega} + \frac{n_c(k-q)}{\omega_k(-q) - \omega} \right]$$

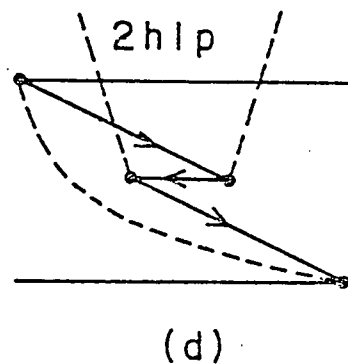
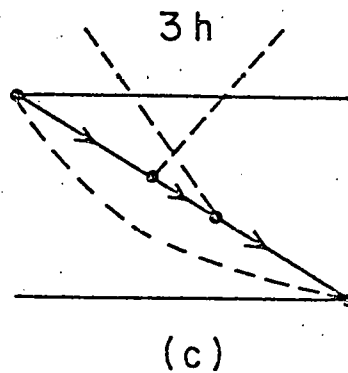
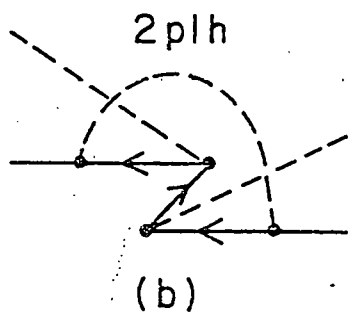
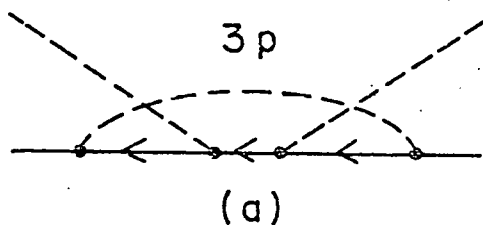
$$= \sum_k \left[\frac{n_c(-k-q) n_c(-k)}{-\omega_k(q) - \omega} + \frac{n_c(-k) n_c(-k-q)}{\omega_k(q) + \omega} \right]$$

$$= 0$$

$$\text{NOTE: } \Pi \neq 4\pi \rho \langle f_S^{\text{free}} \rangle F(q)$$

EFFECTIVE π N AMPLITUDE (ORDER f^4)

HOLE LINE EXP.



EXACT AMPLITUDE TO ORDER f^4

$$f_u^{(4)} = 4\pi g^2 f^4 \sum_p \frac{p^2}{2\omega_p} \left\{ \frac{\eta_3(k+q-p) \eta_3^2(k-p)}{(\omega_p - \omega - i\delta) \omega_p^2} \right. \quad 3p$$

$$+ \eta_4(k+q-p) \eta_3^2(k-p) \left[\frac{1}{(\omega_p - \omega - i\delta) \omega_p^2} - \frac{2\omega_p D_0(p)}{\omega^2} \right] \quad 2p11$$

$$- \eta_4(k+q-p) \eta_4^2(k-p) \frac{1}{(\omega_p + \omega) \omega_p^2} \quad 3h \equiv$$

$$- \eta_3(k+q-p) \eta_4^2(k-p) \left[\frac{1}{(\omega_p + \omega) \omega_p^2} - \frac{2\omega_p D_0(p)}{\omega^2} \right] \quad 2h1p$$

note cancellations of Pauli restrictions

$$\begin{aligned} \frac{1}{h_\alpha}^{(4)}(\omega) - h_{\alpha, \text{free}}^{(4)} &= -\frac{2}{\pi} \int d\omega_p \frac{p^3}{\omega_p} (1 - F(p)) D_0(\vec{p}, \omega) \\ &\quad \times \left[\lambda_\alpha^2 + \sum_\beta A_{\alpha\beta} \lambda_\beta^2 \right] \end{aligned}$$

COMPARISON OF EXACT AND HL RESULTS

$$\text{EXACT: } C_0 - C_0^{\text{free}} = -8\pi N(\rho) \Sigma,$$

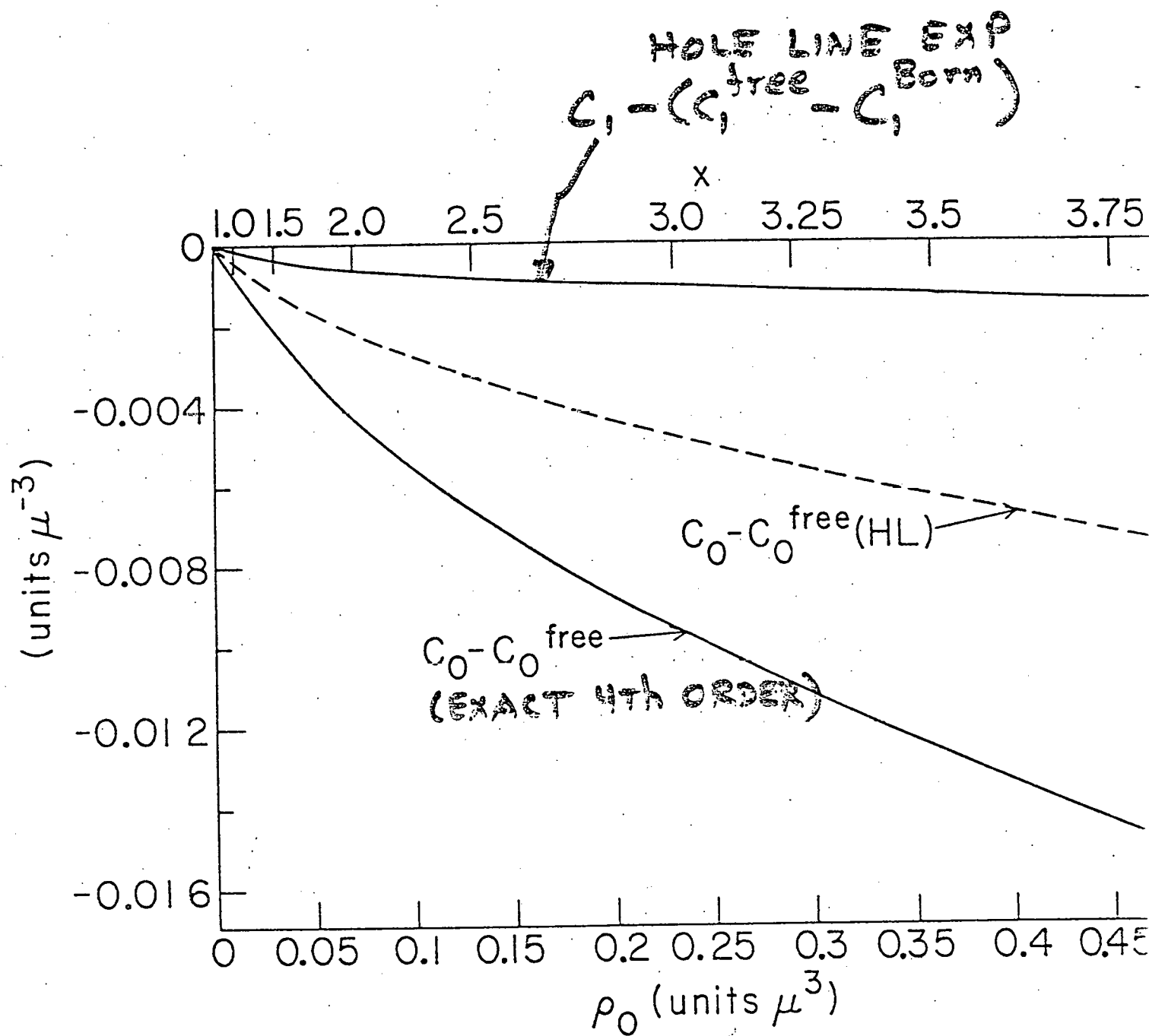
$$\text{HOLE LINE: } C_0 - C_0^{\text{free}} = -4\pi N(\rho) \Sigma, \quad \leftarrow C_0 \text{ part}$$

$$\text{EXACT: } C_1 = C_1^{\text{free}} \quad \leftarrow \vec{\tau} \cdot \vec{\tau} C_1 \text{ part}$$

$$\text{HOLE LINE: } C_1 = C_1^{\text{free}} - C_1^B - \frac{4\pi}{3} M(\rho) \Sigma,$$

Conclusions:

- 1) Pauli effects very small for both C_0 and C_1
- 2) Hole line expansion predicts large damping of $\vec{\tau} \cdot \vec{\tau}$ part of pion potential (no exper. indication for this)



$$C_0^{\text{free}} \approx 0.22 \mu^{-3}$$

$$C_1^{\text{free}} \approx 0.18 \mu^{-3}$$