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Fractal Modeling of Natural Fracture Networks

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Fractal Modeling of Natural Fracture Networks

Contract Information

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Schedule and Milestones **FY 94/95 Program Schedule**

	J	J	A	S	O	N	D	J	F	M	A	M
Outcrop Analysis												
Code Development												
Borehole Data Analysis												
Borehole Simulation												

Objectives **Background**

West Virginia University will implement procedures for a fractal analysis of fractures in reservoirs. This procedure will be applied to fracture networks in outcrops and to fractures intersecting horizontal boreholes. The parameters resulting from this analysis will be used to generate synthetic fracture networks with the same fractal characteristics as the real networks.

Recovery from naturally fractured, tight-gas reservoirs is controlled by the fracture network.¹ Reliable characterization of the actual fracture network in the reservoir is severely limited. The location and orientation of fractures intersecting the borehole can be determined, but the length of these fractures cannot be unambiguously determined. Fracture networks can be

determined for outcrops, but there is little reason to believe that the network in the reservoir should be identical because of the differences in stresses and history. Seismic techniques do provide some large scale (resolution of tens or hundreds of feet) information about the fracture density and average fracture orientation, although there is some controversy about interpretation of the multi-component surface seismic data, especially regarding which layer is being probed.

Furthermore, independent of the assumption of fractal behavior, it is known that typical fractures in the second set should begin and end at fractures of the first set.² This effect is commonly observed in real fracture networks from outcrop studies, for example 92% of the secondary fractures in the MWX outcrop (Fig. 1) satisfy this criterion.³ Imposing this constraint upon the secondary fractures increases the visual similarity between our networks and the real network over simulated networks from other fractal modeling schemes.⁴

Because of the lack of detailed information about the actual fracture network, modeling methods must represent the porosity and permeability associated with the fracture network, as accurately as possible with very little *a priori* information. Three rather different types of approaches have been used: i) dual porosity simulations, ii)'stochastic' modeling of fracture networks, and iii) fractal modeling of fracture networks. The dual porosity approach is a natural extension of the gridding schemes widely used in

describing reservoirs, however in assuming mesoscopic scale (tens or hundreds of feet) averages of fracture porosities and permeabilities, they may be smoothing the very heterogeneities which control the recovery. This may limit reliability for strongly anisotropic fracturing. That is, even if fractures are located randomly throughout the grid-block so that an average porosity may be sensible, the conductivity of similar fractures differ widely invalidating assumptions of an average permeability.

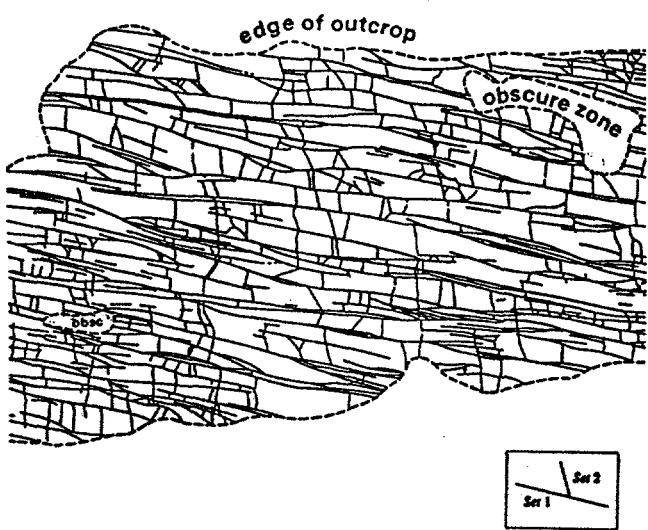


Figure 1 Outcrop Fractures at MWX site.
Shows the primary fractures (set 1) and the secondary fractures (set 2).

Stochastic models which assume a variety of probability distributions of fracture characteristics have been used with some success in modeling fracture networks.⁵⁻⁷ The advantage of these stochastic models over the dual porosity simulations is that real fracture heterogeneities are included in the modeling process. On the other hand these

stochastic models need information about all features of the actual fracture network to provide the most accurate modeling. In the highest level (most accurate) model for each set of fractures with a given orientation, one needs to determine the probability distribution of i) the location of independent fractures ii) the location of fracture clusters or swarms iii) locations of fractures within clusters, iv) cluster lengths, v) fracture lengths, vi) fracture apertures, and vii) fracture orientations. The less reliable the information determining these probability distributions; the less reliable the fracture network. Reliable information about many aspects of the real fracture network is impossible to determine; the assumption of self-similar fractal behavior (if valid) enables us to predict features of one aspect of the distribution from other aspects of the distribution; i.e. i), ii), and iii) result from the box-counting along the borehole which, in turn, predicts features of the distributions for iv), v), and vi) for self-similar fractal networks.

Aspects of fractal geometry have been applied to mimic the heterogeneity associated with layering in real reservoirs for a number of years. In these cases, the variation in permeability with height at the borehole was found to obey fractal statistics,⁸ and the correlations implicit in fractal geometries allowed them to interpolate between the known permeabilities at the borehole in such a way that results from flow models agreed with analyses of production logs and tracer breakthrough.⁹ Examples in the open literature reporting the use of fractal geostatistics to treat naturally fractured reservoirs are less common.^{4,10}

If a set of natural fractures is described by a self-similar fractal geometry, the self-similar, scale invariance of the fracture network implies relationships among the fracture distribution, and the various length scales: clustering or fracture correlation, fracture aperture, and fracture length. Therefore, if fracture networks obey a self-similar fractal geometry, borehole data locating orientational sets of fractures, will enable a determination of the fractal dimension and 'lacunarity'. This along with relatively generic information about the typical aperture size and length of fractures,¹ will allow us to produce a self-similar fractal network. The clustering occurs naturally in the fractal network because of the correlations inherent in fractal geometries. The fractal parts of the aperture size and length distributions (even the fracture shape distributions) should be the same as the fractal parts of the fracture location vs. scale distributions.

In the sections following this introduction, we will i) present 'fractal' analysis of the MWX site, using the box-counting procedure^{11,12}; ii) review evidence testing the fractal nature of fracture distributions and discuss the advantages of using our 'fractal' analysis over a stochastic analysis; iii) present an efficient algorithm for producing a self-similar fracture networks which mimic the real MWX outcrop fracture network.

Project Description-Fractal Analysis

Illustrative Example Before analyzing the MWX outcrop (Fig. 1), one must understand the box-counting

in our method for generating the fracture networks. As discussed later in this section, the box-counting procedure automatically reproduces the random aspects of the distribution of fractures in addition to reproducing the clustering obvious in Fig. 1.

For a simple example of the box-counting procedure consider the distribution of fractures intersecting a length of borehole. To determine the fractal dimension as well as the range of size scales over which the distribution is fractal, one covers the array of fractures by successively smaller and smaller rulers (one-dimensional 'boxes'), and then one counts the number of 'boxes' or rulers covering one or more fractures. If the distribution has a fractal dimension D_f over a range of sizes, then

$$N = A(\Delta)^{D_f}, \quad (1)$$

where N is the number of rulers which cover fractures, the constant A is called the lacunarity, and the scale Δ determines the length of the rulers (L/Δ). If one covers the 24 fractures in Fig. (2) by a ruler of length L , (shown at the bottom of Fig. 2) one ruler covers the fractures; with two ruler of length $L/2$ (near the bottom of Fig. 2) both cover fractures; with four rulers of length $L/4$ all 4 cover fractures, but with 8 rulers of length $L/8$ only 6 cover fractures. This is continued down to 128 rulers of length $L/128$ as shown in Table 1.



Figure 2a Fractures intersecting a borehole.

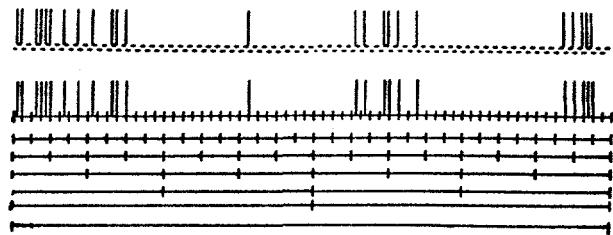


Figure 2b The lower half of the figure shows the fractures in Fig. 2a with the scale rulers 'covering' the set of fractures from a ruler of length L , proceeding upwards to rulers of length $L/64$ just below the fractures. The top half shows the same set of 'covering' rulers of length $L/128$. The rulers are left-justified so that the fractures at the right-end of the ruler are covered by the ruler.

Table I

Ruler Length	# of Rulers Covered
L/Δ	N
L	1
$L/2$	2
$L/4$	4
$L/8$	6
$L/16$	8
$L/32$	13
$L/64$	17
$L/128$	24
$L/256$	24
$L/1024$	24

Since there are only 24 fractures, at scales smaller than $L/128$, there will

only be 24 rulers covering fractures. A log-log plot of the box-counting for Fig. 2 is shown below

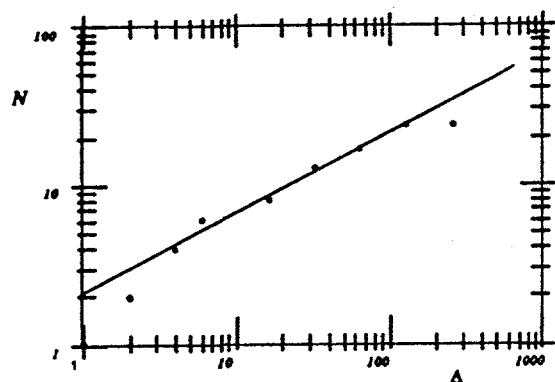


Figure 3 Fractal Plot for Fig. 2.

The fractal relationship is given by the solid line $N = 2.12 \Delta^{0.5}$, except at large and small scales for the reasons that follow. At small Δ , (coarse scales L , $L/2$ and $L/4$), N equals the number of rulers ($N = \Delta$) because all the rulers cover fractures; in later sections, we refer to this as the initial covering regime. At very large Δ , (very fine scales $L/256$ and $L/1024$), only 24 rulers are covered because there are only 24 fractures and there is no more detail in the fracture pattern, so that the box-counting 'cuts-off' or 'saturates' at 24. Therefore, for this fracture pattern, the pattern is fractal between the initial covering and cutoff regimes (over the range of scales $\Delta = 8$ to 128) with a fractal dimension of 1.5 and a lacunarity of 2.12. The ruler counting of this one-dimensional slice of the two-dimensional fracture network gives an exponent $D_f - 1$, i.e. the actual 2d fractal dimension minus one.

Before we continue, it should be pointed out that this fracture pattern was generated by our algorithm to have

a lacunarity of 2.12 and a fractal dimension of 0.5 over the range of scales from $L/8$ to $L/128$. The algorithm which generated this pattern is described and used in a following section.

It is important to realize that if the distribution of fractures in Fig. 2 were completely random (i.e., if there were no clustering of fractures), the points from the box-counting would obey a linear relationship ($N = \Delta$) up to cutoff. That is, on the average, each box would contain one fracture up to the total number of fractures (in this case $N_{\text{total}} = 24$); at finer scales, the one fracture would randomly occupy one of the smaller boxes. However, because of clustering, groups of fractures are closer together than average. Therefore, when box-counting, the linear regime ends before $N = N_{\text{total}}$; and one enters the 'fractal' or clustering regime where some boxes are empty and others have several fractures much closer together than average. The box-counting provides a routine procedure for characterizing (and, thus, for reproducing) this clustering.

Results

MWX Outcrop First the primary set of fractures in Fig. 1 was analyzed. A series of eight lines (boreholes) of length L were drawn through the set of primary fractures, and the box-counting procedure was used on each of these boreholes. The results for the number of boxes covering fractures vs. the scale Δ is shown in Fig. 4. The initial covering regime persisted until scale 16. The cutoff regime began at scale 80. In-between the data are well represented

by the fractal power law $N = 4.9 \Delta^{0.425}$, indicating a fractal dimension $D_f = 1.43$. It should be noted that scales intermediate to the simple doubling rule, $\Delta = 2^n$, (used in Figs. 2 & 3 and Table 1) were used to provide more data in the fractal regime.

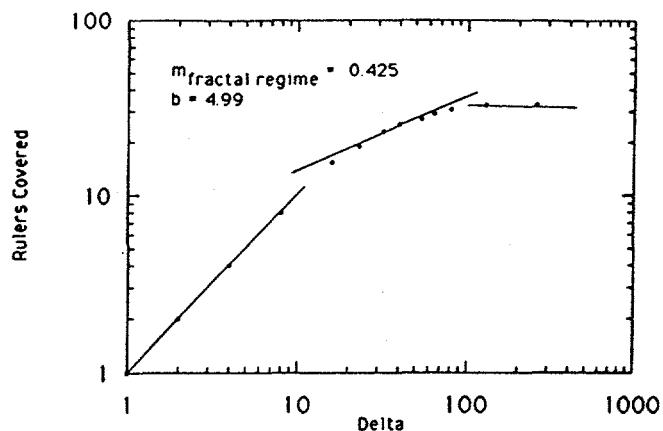


Figure 4. For the primary fractures, the box-counting from the 'boreholes' on the MWX outcrop (Fig. 1), shows the initial covering (the linear increase, $N = \Delta$, up to the clustering or fractal regime), the fractal regime, and the cutoff regime.

The secondary set of fractures in Fig. 1 were analyzed in the same way. A series lines of length L , perpendicular to these secondary fractures, were drawn through the secondary fractures, the box counting was performed and the values $N(\Delta)$ were averaged. Fig. 5 shows the plot of N vs. Δ and shows that for these secondary fractures the initial covering regime persists until scale 6 and that the cutoff regime begins at scale 40. In-between the number of rulers obeys the fractal power law $N = 3.47 \Delta^{0.343}$,

indicating a fractal dimension $D_f = 1.34$. Again, intermediate scales were used in the fractal regime to provide more data in the fractal regime.

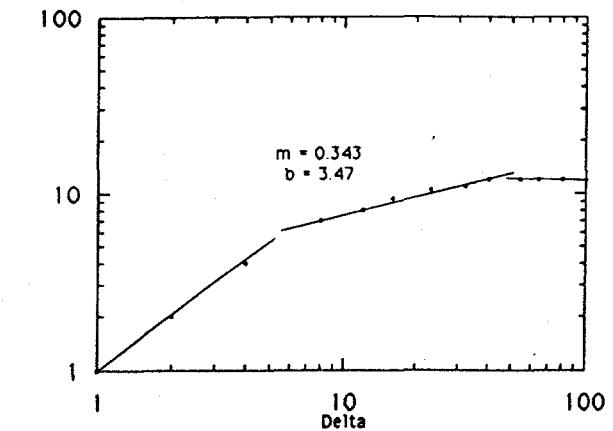


Figure 5. For the secondary fractures, the box-counting from the 'boreholes' on the MWX outcrop (Fig. 1), shows the initial covering (the characteristic linear regime), the fractal regime, and the cutoff regime.

To determine the length distribution from the data provided by M. McKoy,³ we plotted the total number of fractures with lengths greater than a given length L , $N(L)$, vs. L . It should be noted that this total number $N(L)$ with lengths $\ell > L$ is the integral of the number density of fractures $n(\ell)$ with length ℓ integrated from $\ell = L$ up to the one fracture of maximum length, i.e.

$$N(L) = \int_L^{L_{\max}} n(\ell) d\ell .$$

This graph of the data is shown in Fig. 6. It is convincingly fit by the characteristic exponential cutoff for the greatest lengths ($L > 14$), and by a fractal power law for the smallest lengths ($4 < L < 14$). For a self-similar fractal fracture network, the number density

should be given by $n(\ell) = \eta \ell^{-D_f}$ so that the total number should be given by $N(L) = \frac{\eta}{1-D_f} L^{1-D_f}$.¹¹ Therefore, the data are consistent with a fractal dimension $D_f = 1.48$.

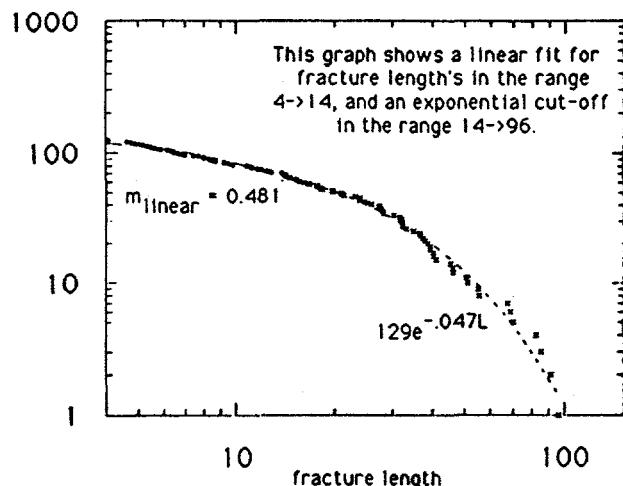


Figure 6. The number of fractures $N(L)$ with lengths greater than L plotted against L . This shows the exponential cutoff for the larger lengths and the fractal regime for the smaller lengths.

These data do not decide unambiguously whether or not the clustering regime is rigorously fractal. That is, these data do not unambiguously favor a strictly power law regime (i.e. fractal behavior) between the linear, initial covering regime and cutoff. However, the power law assumption used to draw the lines does represent a good fit to the box-counting data. Therefore, at worst, by assuming that the intermediate regime is fractal, we may be merely providing a good approximation to the data. If the assumption of fractal clustering only provides a good approximation to the true clustering, our

simulated fracture networks will represent a good approximation to the actual fracture network, which is all that is necessary.

On the other hand, it is encouraging that the power laws from the box counting and length distributions are all consistent with the same fractal dimension, $D_f = 1.4 \pm 0.1$, to within a realistic uncertainty from the data fitting. This equality of fractal dimensions from all length measures is the hallmark of self-similar fracture networks.

A program to carry out the box-counting procedure and return the fractal dimension and lacunarity has been developed in order to process multiple sets of data from various boreholes. To test these programs as well as the routines for simulating the fracture networks, numerous trial runs have been performed to analyze the "borehole fractures" from simulated networks.

Are Fractures Fractal? There is evidence that real fracture networks are fractal both in outcrops where Barton and others found a fractal dimension of $D_f = 1.55$, for different fracture systems,¹³ as well as from underground data in the Fanay-Augères uranium mine¹⁰ where they found a varying fractal dimension. The variation in their fractal dimension may result from use of too great a range of scales. As we saw for very large scales, all the rulers are covered so their finding a 'fractal dimension' of 2 at large scales is not surprising. Similarly, at very small scales one approaches a limit where the number of 'boxes' covered equals the number of fractures so the

fractal dimension approaches 1; this may be an artifact of the neglect of small aperture fractures (micro-cracks which may be significant in determining number at their 0.005 meter scale).

The length of the fractures has been found to be fractal,¹⁴ and the shape of the fractures has also been determined to be fractal¹⁵⁻¹⁷. This suggests that all features of the fractures may be fractal: distributions of i) centers, ii) lengths, iii) widths, and iv) shapes. The evidence that the shapes are fractal suggests that porosities and permeabilities may also obey fractal statistics. If all geometrical aspects of the fracture distribution are fractal with the same fractal dimension, the fracture distribution is self-similar. This may seem to be a very unusual occurrence, but in fact many examples of development (or growth) which occur in random media (like the development of fractures in stressed rock formations) have a self-similar geometry. The first level of our geostatistical modeling will assume a self-similar fractal geometry for the fracture distribution. Higher levels of our geostatistical modeling could use actual measurements to determine the fractal distribution of (e.g.) the fracture widths.

Fracture Generation Algorithm

Here, we describe the implementation and design of an algorithm developed to generate a fracture network in 2-dimensions. The primary assumption in our model is that the network geometry is fractal - i.e. has a self-similar or scale invariant geometry. Using this information we have developed a program to generate complete 2-d fracture outcrop

networks using only the lacunarity, fractal dimension, initial covering, and cutoff parameters obtained from MWX data.

In the broadest sense the program performs 2 tasks :

- (1) Generates a horizontal fracture set.
- (2) Generates a secondary fracture set consistent with the fracture set in (1)

To generate the primary fracture set the program first generates a 1-dimensional fracture set along a left-justified line

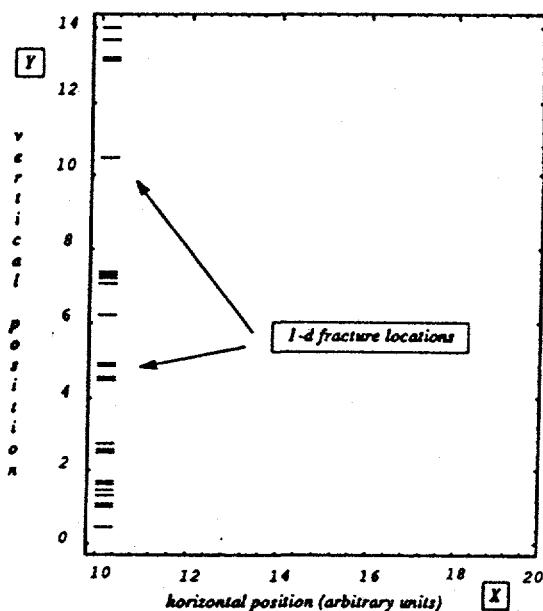


Figure 7. 1-d fracture generation output. The data are represented graphically by a series of small line segments in the x direction.

perpendicular to this fracture set, Fig. 7. This is accomplished by a procedure, the first step of which initializes the first row

in the 2-dimensional ruler array $L[j,k]$, where i labels the scale and k labels the ruler. If a fracture is covered by a ruler, then the value of the array for this specific ruler is 1. Conversely, an empty ruler site is given the value 0.

Having initialized the $L[1,i]$ array the procedure then divides each ruler into two new rulers and randomly chooses one of these 2 new rulers in $L[2,k]$ for each of the covered rulers in $L[1,i]$ and assigns this ruler a value of 1 while giving the other ruler a value of 0. The remaining rulers are randomly assigned fractures according to the distribution. This proceeds to finer and finer scale until saturation is reached.

Continuing with the generation of the primary fracture set, the program assigns a length to each fracture site according to the length distribution shown in Fig. 6. Once the length of a fracture is chosen, the exact location of the origin ($x=0$) along the fracture is chosen randomly. The result is shown in Fig. 8.

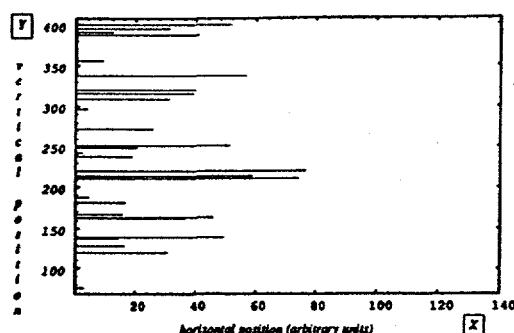


Figure 8. Lengths are assigned to the initial fracture set.

Having assigned lengths, the program will step forward by a specified amount

in the x -direction (along the fractures). If any fractures have ended during the step, new fracture assignments must be made to maintain the distribution. To guarantee that the new fracture assignments produce a fractal distribution, we must reverse the ruler doubling process and re-assign fractures that have crossed the specified grid point to half as many rulers used in the final step of the initial 1-d fracture generation process. The unoccupied fracture sites are then assigned new fractures following the same procedure described for the initial fracture generation. The procedure continues stepping along in the x -direction until the full region is characterized.

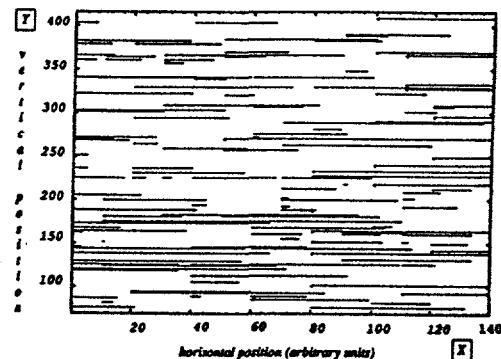


Figure 9. Primary Fractures-Simulated

The resulting output is given in Fig. 9 and can be compared with the MWX primary fractures shown in Fig. 1. The parameters used were those determined from the MWX outcrop using the box counting procedures described earlier.

To generate the secondary fracture set we first generate a fracture distribution along each of the horizontal fractures.

Starting in the upper left hand corner of Fig. 9 and proceeding downward, the program produces a fractal distribution (using a parameter set determined from the vertical fracture data) along the first fracture in the data set.

In our model we assume that vertical fractures can only begin or end along a horizontal fracture. In this case, we need only find the next horizontal fracture below each vertical fracture site to determine the fracture endpoint and therefore its length.

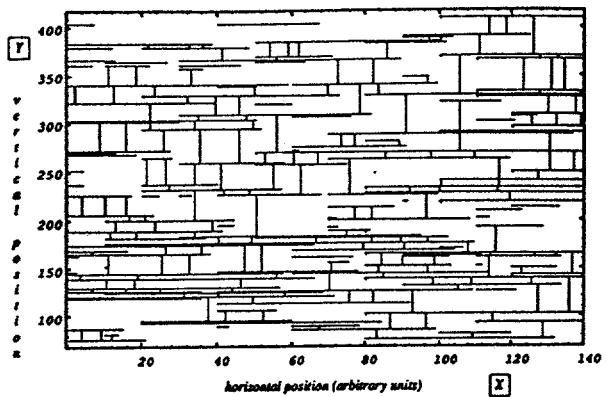


Fig. 10. MWX 2-d Fracture Outcrop Data

Conclusions & Future Work

To model the fracture outcrop networks occurring in naturally fractured tight-gas reservoirs we have taken an approach that incorporates:

A) Fractal Analysis of Available Data:

we characterized the MWX fracture data using four parameters (for the distribution of both horizontal and vertical fractures): i) Lacunarity, ii) Fractal Dimension, iii) Initial Covering Scale, and iv) Cutoff - determined from the distribution of fracture lengths.

B) Fracture Generation:

we generate self-similar fracture networks using data from I.) with an algorithm that incorporates fractal geostatistics.

From our work we have found that there are several advantages in an approach that uses fractal statistics:

- i) The networks produced by our model appear to be in agreement with actual fracture networks but do not require extensive a-priori knowledge of the network. Using data from isolated borehole sites we can generate entire networks with an algorithm that assumes a self-similar or scale invariant geometry.
- ii) We are able to generate horizontal and vertical fractures separately (although not independently) using distinct parameter sets in each case. The fractures can then be analyzed and combined later to produce complete self-consistent networks.
- iii) Since the data is generated using a statistical approach, the algorithms require relatively little computer time to produce complete networks
- iv) Evidence suggests that real fracture networks obey fractal statistics.

The characterization and analysis of the network data produced by our algorithms is not yet complete. By varying other parameters such as gridsize, fracture length, and the horizontal/vertical orientation of fractures, we believe that it will be

possible to generate fracture distribution patterns that are 'optimally similar' in the fractal/statistical sense - to real fracture networks occurring in nature.

We are in the process of analyzing the distribution of fractures along horizontal boreholes in the Austin Chalk and fracture lengths from nearby outcrops. The results from this analysis will be used to generate simulated fracture networks.

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