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Abstract

A NEW RECOIL EFFECT IN NUCLEON TRANSFER REACTIONS BETWEEN HEAVY IONS

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ABSTRACT

In the last few years it has become clear that the effect of the shifting centers of mass (recoil effect) have to be properly incorporated into the theory of single nucleon transfer between nuclei, especially at higher energies. A re-examination of the present theory reveals, however, that there remains a serious problem, which is associated with the so-called spurious center of mass motion in nuclear structure models. In principle the present calculations that purport to treat recoil exactly, none the less ought to be corrected for the above defect in the nuclear wave functions that they employ. We have not obtained an exact resolution of the long outstanding problem of spurious center of mass motion in nuclear models, but we have formulated an approximate procedure for handling the correction to reaction calculations arising from this source. The correction has two components. There is a scalar one which corresponds merely to a scaling of the radial coordinate and applies to all reactions. There is also a vector correction which can be cast into a form in which a particle picked up or removed from a definite shell model state appears to occupy a distribution of states having the same parity but differing in angular momentum. This component of the correction applies only to certain reactions. Because of the dispersion in the apparent angular momentum of the transferred nucleon, the reaction can proceed through the transfer of larger units of angular momentum than the normal recoil calculations allow, and the correction therefore is expected to grow with increasing bombarding energy. The scheme we have developed for handling this effect, which might be referred to as bound state recoil to distinguish it from the recoil effect that the current theories focus on, will be presented together with preliminary estimates of its importance.

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Although present formulations of transfer reactions and the computer codes based upon them purport to treat recoil correctly,[1] there is nonetheless a possibly important recoil effect that has been overlooked. It has to do with the fact that the nuclear wave functions employed in the calculations have a spurious center-of-mass motion. This arises quite simply because in the shell model picture in which particles are conceived of as moving in a central potential, representing the effect of the bulk of nucleons on the particular ones considered explicitly, the center of mass of the latter is not fixed in space but moves. The best that can be said is that on the average, their center of mass coincides with the center of the potential. This problem, when it is faced in structure calculations aimed at computing energy levels, is only partially solved, usually by a procedure which assures that each state has the <u>same</u> spurious center of mass motion. In that case although the computed energies do have a spurious contribution, their differences do not. Not even this resolution of the problem is satisfactory in the case of reactions, for unless the spurious motion can be eliminated altogether, it can produce an unknown effect on the cross-section.

We have not obtained an exact resolution of the problem. However there is at least a simple way in which an inconsistency in the present formulation can be removed. At the next level, we have a prescription which we believe approximates the rejection of the spurious center of mass motion. We deal with these in order.

Resolution of an Inconsistency

First we point out the inconsistency in the present formulation that can be removed easily. Consider first the reaction

$$A(a,b)B \quad (a = b+1) \tag{1}$$

the amplitude for which in DWBA is currently computed from

$$T = \int \psi^{(-)*}(\underset{\sim}{R}_\beta) \langle \phi_B \phi_b | V | \phi_A \phi_a \rangle \ \psi^{(+)}(\underset{\sim}{R}_\alpha) \ d\underset{\sim}{R}_\alpha \ d\underset{\sim}{R}_\beta \tag{2}$$

where the ϕ's denote the nuclear wave functions and the overlap $\langle \phi_b | \phi_a \rangle$ integrated over the b coordinates is claimed to be within coupling and parentage factors, the single-particle wave function of the transferred nucleon in the nucleus a which is <u>said</u> to be bound to the "core" b, i.e.,

$$\langle \phi_b | \phi_a \rangle = \phi_{j_1} (\underset{\sim}{r}_{b1}) \tag{3}$$

A similar postulate is made for the heavy nuclei.

To expose the inconsistency in the above formulation which is standardly used, we consider another possible outcome of the reaction of a with A, namely a two nucleon transfer.

$$A(a,c)C \quad (a = c+2) \tag{4}$$

In this case the "core" is c and the standard formulation implies that the two nucleons are bound to it.

$$\langle \phi_c | \phi_a \rangle = \phi_{j_1}(\underset{\sim}{r}_{c1})\, \phi_{j_2}(\underset{\sim}{r}_{c2}) \tag{5}$$

Now although the same projectile is incident on the same target in reactions (1) and (4), the current formulation of the theory states through (3) and (5) that particle 1 is at one and the same time, bound to b with angular momentum j_1 and bound to c (= b-1) with the same angular momentum. Since this contradiction is arrived at by the same prescription, the prescription must be wrong.

The resolution of this contradiction is achieved by making the obvious observation that particle 1 is bound to neither core b nor c when it forms part of the projectile nucleus a, but rather is bound in a. Without solving the spurious center-of-mass motion this corresponds to saying that 1 is bound to the center of mass of a. Then the overlaps (3) and (5) are, respectively

$$\langle \phi_b | \phi_a \rangle = \phi_{j_1}(\underset{\sim}{r}_{a1}) \tag{6}$$

$$\langle \phi_c | \phi_a \rangle = \phi_{j_1}(\underset{\sim}{r}_{a1})\, \phi_{j_2}(\underset{\sim}{r}_{a2}) \tag{7}$$

and are consistent (though not corrected for spurious center of mass motion).

The inconsistency in the existent formulations of the theory can thus be easily removed, since it amounts only to a scaling factor, i.e.,

$$\underset{\sim}{r}_{b1} = \frac{a}{b}\, \underset{\sim}{r}_{a1} \tag{8}$$

Thus at the present level of discussion, (2) should be written as

$$T = \int \psi^{(-)*}(R_\beta) \left\langle \phi_{j_2}\!\left(\frac{A}{B}\, \underset{\sim}{r}_{A1}\right) \middle| V\!\left(\frac{b}{a}\, r_{b1}\right) \middle| \phi_{j_1}\!\left(\frac{b}{a}\, \underset{\sim}{r}_{b1}\right) \right\rangle \psi^{(+)}(\underset{\sim}{R}_\alpha)\, d\underset{\sim}{R}_\alpha\, d\underset{\sim}{R}_\beta \tag{9}$$

rather than as in the existent formulation

$$T' = \int \psi^{(-)*}(\underset{\sim}{R}_\beta)\, \langle \phi_{j_2}(\underset{\sim}{r}_{A1})\, | V(r_{b1}) | \phi_{j_1}(\underset{\sim}{r}_{b1}) \rangle\, \psi^{(+)}(\underset{\sim}{R}_\alpha)\, d\underset{\sim}{R}_\alpha\, d\underset{\sim}{R}_\beta \tag{10}$$

The main difference between (9) and (10) is one of magnitude, in as much as transfer takes place predominantly at distances corresponding to the overlap of the tails of the bound wave functions. The new amplitude (9) will be larger than the old (10). We can gain an impression of the ratio of magnitudes by considering the tail of the bound state in the nucleus a. According to (9) and (10) the relevant ratio is

$$\frac{e^{-k(b/a)r}}{e^{-kr}} = e^{+kr/a}$$

where k is given in terms of the binding energy B of particle 1 in a as $k \cong .22\sqrt{B}$. As an example, a neutron is bound in ^{16}O with $B \cong 15$ MeV, and yields for the above ratio at the edge of ^{16}O, $r \cong 3$ Fm, the value 1.17. This corresponds to a correction of about 40% in cross section coming from this one factor. The scaling in the heavy nucleus produces a smaller effect. The scaling in the potential V goes in the same direction. Using a Woods-Saxon potential with diffuseness 0.5 Fm we find a correction of more than 40%. The two factors together yield an estimate that the cross section computed from the new amplitude is about 90% larger than the old. This would imply that past analyses have over estimated single-particle spectroscopic factors by a factor approaching two.

Approximate Correction for Spurious Motion

In the foregoing, we followed the usual practice of identifying the overlap $\langle\phi_b|\phi_a\rangle$ as a single-particle wave function, to within coupling and parentage factors. However we have already noted that a particle has its center-of-mass located on the average at the center of the nucleus in which it is bound, and not at the center of the "core" of this nucleus. Implementing this observation implies

$$\langle\phi_b|\phi_a\rangle = \langle\phi_b(r_{b2},r_{b3},\cdots)|\phi_a(r_{a1},r_{a2},r_{a3},\cdots)\rangle \tag{11}$$

We have not removed the spurious center of mass motion since ϕ_a depends on the 3a nucleon coordinates instead of 3(a-1) internal coordinates. We do not attempt a calculation of the reaction amplitude with such proper internal wave functions in this paper. Instead we outline the nature of the corrections that arise by demanding simply that on the average the nucleons have their centers at the center of the nucleus in which they are bound, as is expressed in (11). Denoting the distance between the mass centers of a and b by d we note that the two sets of coordinates in (11) are related by

$$\underset{\sim}{r}_{bi} = \underset{\sim}{r}_{ai} + \underset{\sim}{d} \quad , \quad \underset{\sim}{d} \equiv \underset{\sim}{r}_{ab} = \frac{1}{b}\underset{\sim}{r}_{a1}$$

Thus after integration of the b coordinates $r_{a2}, r_{a3}\cdots$, the overlap remains in general a function of $\underset{\sim}{d}$ as well as the nucleon coordinate of the transferred particle $\underset{\sim}{r}_{a1}$.

$$\langle\phi_b|\phi_a\rangle = \Phi(\underset{\sim}{d},\underset{\sim}{r}_{a1})$$

Now we show more explicitly the form that this takes. To do so we express ϕ_b on a zero-spin parent, which if necessary can be done by referring to the last closed shell. Suppose for simplicity of illustration that all but two of the nucleons are coupled to spin zero: [denote $r_{ai} \equiv r_i$]

$$\phi_a = [\phi_j(r_1)\ \phi_{j_b}(r_2)]_{J_a}\ \ \phi_0(r_3, r_4, \cdots) \tag{11}$$

$$\phi_b = \phi_{j_b}(r_2+d)\ \phi_0(r_3+d,\ r_4+d, \cdots) \tag{12}$$

Then the overlap has two factors

$$\langle\phi_b|\phi_a\rangle = \langle\phi_0(r_3+d,\ r_4+d, \cdots)|\phi_0(r_3, r_4, \cdots)\rangle$$
$$\langle\phi_{j_b}^{m_b}(r_2+b)|[\phi_j(r_1)\ \phi_{j_b}(r_2)]_{J_a}^{M_a}\rangle \tag{13}$$

The first factor is a scalar correction in the length of d. We estimate that this factor is close to unity within order $1/a^2$. The second factor is more interesting since it can be a vector correction, depending on the spins involved. We shall not exhibit an explicit calculation of it here, but show its form, namely

$$\langle\phi_b|\phi_a\rangle = [1+O(\tfrac{1}{a^2})]\ \langle\phi_{j_b}^{m_b}(r_2+d)|[\phi_j(r_1)\phi_{j_b}(r_2)]_{J_a}^{M_a}$$

$$= \sum_{\Lambda=0,2} f_\Lambda\left(\frac{|r_1|}{b}\right)\ Y_\Lambda(\hat{r}_1)\ \phi_j(\underset{\sim}{r}_1) \tag{14}$$

where the sum on Λ extends to the minimum of $2\ell_b$ or $2j_b$. The correction is therefore vector when $j_b > 1/2$ and $\ell_b > 0$.

When the correction arising from the spurious center of mass motion is a vector correction, it has possibly very important consequences. For we can write the above result as

$$\langle\phi_b|\phi_a\rangle = \sum_{\mathcal{L}J} \phi'_{\mathcal{L}J}\ (\underset{\sim}{r}_1) \tag{15}$$

This result shows that although the particle 1 is transferred from the state ℓj in nucleus a, because of the center of mass correction it appears to occupy a superposition of states $\mathcal{L}J$ with the same parity as the original. Because J can be larger than j then the angular momentum transfer can be even larger or smaller than the new values introduced by the existent recoil treatments over those which appear in the no-recoil approximation. The larger values are probably more significant, for with increasing bombarding energy, they can dominate for kinematic reasons. Thus the high energy dependence of the cross section may be very significantly modified by the correction.

Finally we generalize the discussion to more than one particle transfer. Then the relevant mass ratio determining this recoil correction is x/b where x is the transferred mass. There is an entirely analogous correction from the heavy nucleus overlap $\langle\phi_B|\phi_A\rangle$ but it is controlled by x/A which is much smaller than x/b in most reactions. When it is not, then obviously it should be taken into account in the same way as described above.

Discussion and Range of Applicability

We have discussed two corrections to the existent treatments of particle transfer reactions between heavy ions. The first one is an inconsistency that arises because of the spurious center-of-mass motion in the nuclear wave functions. The inconsistency can be removed easily and its removal will be felt mainly in the normalization of the computed cross section which we estimate can be as much as a factor two larger than the existent calculations. This correction applies to all cases.

The second part of the paper attempts to approximate the effect of removal of spurious center-of-mass motion, but falls short of an exact rejection of the spuriousity, for we still do not use proper internal wave functions which can depend only on relative coordinates between nucleons, not on the nucleon coordinates themselves. We find that under certain conditions, a particle that occupies a definite shell model state ℓj, nonetheless appears to be transferred to (or from) a state having a range of angular momenta, $\mathcal{L}\mathcal{J}$ but of the same parity ($\mathcal{L}+\ell=$ even). This correction unlike the first, does not apply to most reactions in its most severe form, which would be the case when $\mathcal{J}$ can be different from j. To clarify the conditions under which this new situation obtains, we just state the selection rules that determine $\mathcal{L}$ and $\mathcal{J}$. They are:

$$J_a + j_b + \mathcal{J} = 0$$

$$\mathcal{L} + \mathcal{J} + 1/2 = 0$$

$$j + \mathcal{J} + \Lambda = 0$$

$$\ell + \mathcal{L} + \Lambda = 0$$

Λ is even and $\quad 0 \leq \Lambda \leq 2\ell_b,\ 2j_b$

Thus if $\Lambda \equiv 0$, as follows if $j_b = 1/2$, then only the radial shape of $\phi_{\ell j}$ is slightly modified by our second recoil correction, by the factor f_Λ in (14). However if

$$j_b > 1/2\ ,\quad \ell_b > 0\ ,\quad J_a > 0$$

then Λ can be non-zero and $\mathcal{J}$ can differ from j. In this situation, new angular momenta are introduced, and as mentioned above, the second correction can then possibly introduce a different energy dependence of the cross section, and a different angular distribution, depending numerically on how large the effect is.

FOOTNOTES AND REFERENCES

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1. P. J. A. Buttle and L. J. B. Goldfarb, Nucl. Phys. 78, 409 (1966); F. Schmittroth, W. Tobocman and A. A. Golestaneh, Phys. Rev. C1, 377 (1970);
L. A. Charlton, Phys. Rev. C8, 146 (1973);
R. M. Devries, Phys. Rev. C8, 951 (1973);
B. F. Bayman, Phys. Rev. Letters 32, 71 (1974);
K. S. Low and T. Tamura, Phys. Letters 48B, 285 (1974);
A. J. Baltz and S. Kahana, Phys. Rev. C9, 2243 (1974);
N. K. Glendenning and M. A. Nagarajan, Nucl. Phys. A236, 13 (1974);
G. Delic, Phys. Rev. Letters 36, 569 (1976).

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