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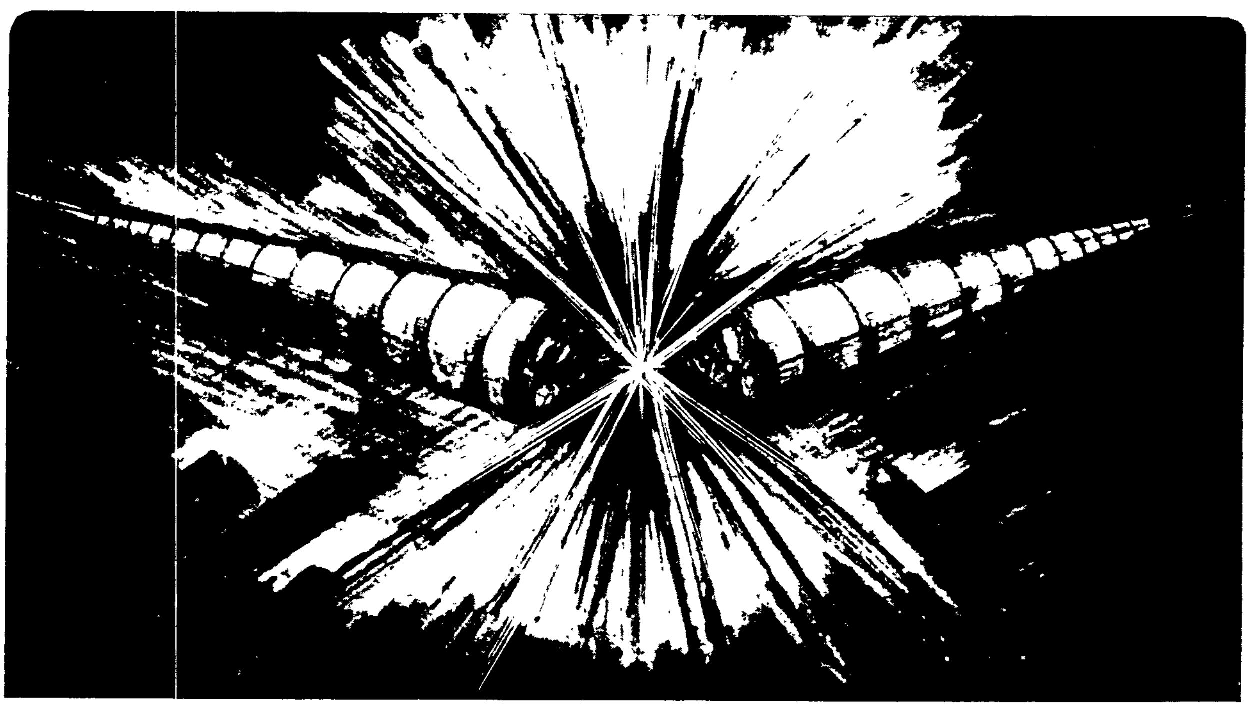

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ABSTRACT

Undulator radiation contains, in addition to the usual component with narrow spectral features, a broad-band component in the low frequency region emitted in the near forward direction, peaked at an angle $1/\gamma$, where γ is the relativistic factor. This component is referred to as the transition undulator radiation, as it is caused by the sudden change in the electron's longitudinal velocity as it enters and leaves the undulator. The characteristics of the transition undulator radiation are analyzed and compared with the infrared radiation from the usual undulator harmonics and from bending magnets.

1. INTRODUCTION

Undulators in modern synchrotron radiation facilities are normally optimized for bright soft x-ray and x-ray radiation in the forward direction. It is well known that the spectrum of the undulator radiation at a large angle, away from the forward direction, is red-shifted to a low frequency. It is probably less well known that low frequency radiation is also generated in the near forward direction, peaked at an angle $1/\gamma$, where γ is the relativistic factor. The origin of this radiation lies in the fact that when electrons enter or leave an undulator, their average longitudinal velocity changes suddenly. The situation here is similar to the case of the transition radiation [1,2] which is generated when a charged particle passes through an interface separating two optical media. Thus, the low frequency radiation in the near forward direction may be referred to as the transition undulator radiation (TUR). In this paper, we derive the characteristics of the transition undulator radiation in the infrared region, and compare with those of the bending magnet radiation and the red-shifted normal undulator radiation. It is shown that the TUR could be an intense, bright, infrared radiation source.

2. TRANSITION UNDULATOR RADIATION

The flux per unit solid angle into a frequency bandwidth $\Delta\omega$ due to a beam of relativistic electrons moving on a curved trajectory is given by (See, for example, Ref. [3])

$$\frac{d^2F}{d^2\Omega} = \alpha\frac{\Delta\omega}{\omega}\frac{I}{e}|A(\omega)|^2 \quad , \qquad (1)$$

where $\alpha \approx 1/137$ is the fine structure constant, ω the radiation frequency, I the beam current, e the electron charge and $A(\omega)$, the electric field amplitude given by

$$A(\omega)=\frac{1}{2\pi}\int_{-T/2}^{T/2} dt \frac{d}{dt}\left(\frac{\mathbf{n}\times(\mathbf{n}\times\beta)}{\kappa}\right)\exp\, i\omega\int_0^t \kappa(t')dt' \ . \tag{2}$$

Here T is the time interval during which the acceleration is non-zero, $\mathbf{n}$ is the observation direction, $\beta(t)$ the electron velocity divided by c (the speed of light) as a function of the emitter time t [3], and $\kappa(t)=1-\mathbf{n}\cdot\beta(t)$.

We apply the above formula for electrons passing through an undulator of length $L = N\lambda_u$ where N is the number of the periods and λ_u is the period length. We consider the case where the frequency ω is so small that

$$\omega\int_0^{T/2} \kappa(t')dt' = \omega\cdot\frac{T}{2}\cdot<\kappa>\approx \pi N\nu(\theta)<1 \ . \tag{3}$$

In the above, $<\kappa>$ is the average of κ in one period, θ is the angle of observation with respect to the forward direction, and

$$\nu(\theta) = \frac{\lambda_1(\theta)}{\lambda}=\frac{\omega}{\omega_1(\theta)} \ . \tag{4}$$

Here $\lambda_1(\theta)=\lambda_u<\kappa>$ is the fundamental wavelength of the undulator radiation as a function of θ, and λ is the radiation wavelength, and $\omega_1(\theta)=2\pi c/\lambda_1(\theta)$.

The integral in Eq. (2) can be evaluated by expanding the exponential function as

$$\exp\, i\omega\int_0^t \kappa(t')dt' = 1 + i\omega\int_0^t \kappa(t')dt' + ... \tag{5}$$

Keeping to the second term in the above expansion, Eq. (2) becomes

$$A(\omega)=\frac{i\omega}{2\pi}\left[\frac{\mathbf{n}\times(\mathbf{n}\times\beta_0)}{\kappa}\int_{-T/2}^{T/2} dt\ \kappa(t) - \int_{-T/2}^{T/2} dt\mathbf{n}\times(\mathbf{n}\times\beta(t))\right] \tag{6}$$

Here $\beta_0=\beta(T/2)=\beta(-T/2)$ is the value of β before and after the undulator. Assuming that there is no angular deflection in the undulator, we have

$$\beta_0=\beta\mathbf{e}_z \tag{7}$$

where e_z is the forward direction and βc is the electron speed (which is a constant for a motion through a static magnetic field). Assuming further that there is also no net displacement in the transverse coordinates through the undulator, Eq. (2) can be evaluated as follows:

$$A(\omega) = \frac{i\omega}{2\pi c} \mathbf{n} \times (\mathbf{n} \times \mathbf{e}_z) \frac{\beta c T - L}{1 - \beta \cos\theta} . \tag{8}$$

In the above, βcT is the arc length of the curved trajectory in the undulator, while L is the straight distance between the undulator ends. Thus, we have, in terms of the transverse velocity $\beta_\perp$,

$$\beta c T \approx \left(1 + \frac{1}{2} < \beta_\perp{}^2 >\right) \cdot L . \tag{9}$$

Here $< \ >$ implies taking the average over an undulator period. Equation (8) becomes

$$A(\omega) = \frac{i\omega}{4\pi c} \mathbf{n} \times (\mathbf{n} \times \mathbf{e}_z) \frac{L < \beta_\perp{}^2 >}{1 - \beta \cos\theta} . \tag{10}$$

Finally, we obtain, by inserting Eq. (10) into Eq. (1), and assuming $\theta << 1$,

$$\frac{d^2F}{d^2\Omega} = \alpha \frac{\Delta\omega}{\omega} \frac{I}{e} \left(\frac{L}{\lambda\gamma}\right)^2 < \gamma^2 \beta_\perp{}^2 >^2 \frac{\gamma^2\theta^2}{\left(1 + \gamma^2\theta^2\right)^2} . \tag{11}$$

According to Eq. (11), the angular distribution of the low frequency radiation vanishes in the forward direction, peaked at angle $\theta = 1/\gamma$, and decreases as $1/\theta^2$ for $\gamma^2\theta^2 >> 1$ .According to Eq. (10), the polarization is radial, i.e., directed away from the forward direction. These properties are very similar to those of the transition radiation [1,2]. The similarity is due to the fact that the low frequency radiation from undulator analyzed here arises from the fact that the electron's average longitudinal velocity $\overline{v}_z$ within the undulator is slower than that outside of the undulator, $v = \beta c$. In fact, a factor occuring in Eq. (8) can be written as

$$T - \frac{L}{v} = \frac{L}{\overline{v}_z} - \frac{L}{v} \tag{12}$$

If $\overline{v}_z = v$, then Eq. (8) vanishes and hence it is necessary to consider higher order terms in the expansion (5). We are therefore justified to refer the low frequency radiation from undulator as given by Eq. (11) as the TUR.

By integrating Eq. (11) over the solid angle, one obtains the total flux. Since the angle integration is logarithimically divergent, one must specify the upper limit of θ. Assuming that photons in $\theta \leq 3/\gamma$ are collected, one obtains

$$F_{TUR} = 1.4\pi\alpha\frac{\Delta\omega}{\omega}\frac{I}{e}\left[\frac{L}{\lambda\gamma^2} < \gamma^2\beta_\perp{}^2 >\right]^2 ; \quad \theta \le \frac{3}{\gamma} \; . \qquad (13)$$

When $\pi N\nu(\theta) > 1$, then the above formulae are not valid since the inequality (3) is not satisfied. However if

$$\pi\nu(\theta) < 1 \; , \qquad (14)$$

then one can still obtain similar results by rewriting Eq. (2) as follows.

$$A(\omega) = \frac{1}{2\pi}\left[\frac{\sin N\pi\nu(\theta)}{\sin \pi\nu(\theta)}\right]\int_{-\lambda_u/2c}^{\lambda_u/2c} dt \;\; \mathbf{a}(t)\exp\left(i\omega\int_0^t \kappa(t')dt'\right) . \qquad (15)$$

The exponential function in the integral of the above can now be expanded in a Taylor series when the inequality (14) is satisfied. Keeping to the second term as before, the final result is that Eq. (11) becomes modified as follows:

$$\frac{d^2F}{d^2\Omega} = \alpha\frac{\Delta\omega}{\omega}\frac{I}{e}\left(\frac{\sin N\pi\nu(\theta)}{N\pi\nu(\theta)}\right)^2\left(\frac{L}{\lambda\gamma}\right)^2 < \gamma^2\beta_\perp{}^2 >^2 \frac{\gamma^2\theta^2}{\left(1+\gamma^2\theta^2\right)^2} \; . \qquad (11.a)$$

Figure 1 shows the angular distribution as a function of the horizontal angle ϕ at $\lambda = 100 \;\mu$ from the SU2 undulator $(\lambda_u = 0.129\text{m},\; N = 24,\; K = 6.9)$ at Super Aco (beam current = 0.2A, electron energy = 0.8 GeV) calculated by C.X. Wang with his RADID code [4]. The double peaks near the center is due to the TUR. The curve in this region agrees very closely with Eq. (11.a). Note there is also a ring of radiation near $\phi \approx 40$ mrad. This is the red-shifted undulator radiation to be discussed in detail in Section 3.1. The TUR is bright because it has a small anglular width less than a few times $1/\gamma$. It can be collected in an optical arrangement schematically shown in Fig. (2), where a mirror intercepts and reflects the TUR, while the short wavelength undulator radiation is transmitted through an aperture in the mirror with an angular opening $\sqrt{\left(1+K^2/2\right)} \;/\; \gamma\sqrt{N}$.

3. COMPARISON WITH OTHER INFRARED SOURCES: LARGE-ANGLE UNDULATOR HARMONICS AND BENDING MAGNET RADIATION.

3.1 Large-Angle Undulator Harmonics.

The nth harmonic undulator radiation at angle θ has the frequency

$$\omega = \omega_n(\theta) = \frac{2\pi nc}{\lambda_u < \kappa >} \; . \qquad (16)$$

We are interested in frequencies much lower than that of the undulator harmonic in the forward direction:

$$\frac{\omega}{\omega_1(0)} << 1 \ . \tag{17}$$

Since

$$<\kappa>=<\kappa>_{\theta=0}+\frac{\theta^2}{2} \ , \tag{18}$$

The undulator harmonics are red-shifted to very low frequencies at a large angle. An infrared radiation beam line collecting the large angle undulator harmonics was constructed at Super Aco [5].

The angular density of the undulator flux at a large angle can be calculated starting from Eq. (15). The argument in the exponential function is decomposed into two terms:

$$\omega\int_0^t \kappa \ dt \ = \ \omega<\kappa>t+\omega\int_0^t \Delta\kappa dt' \ , \tag{19}$$

Here $\Delta\kappa$ is the deviation of κ from its time average value $<\kappa>$. In the low frequency limit defined by the inequality (17), it can be shown that the second term in eq. (19) is negligible. Thus the integral appearing in Eq. (15) simplifies as

$$\int_{-\lambda_u/2c}^{\lambda_u/2c} dt \ \ a(t)e^{i(2\pi nct/\lambda_u)} \ . \tag{20}$$

It can be shown that the acceleration **a** becomes, in the large angle limit,

$$\mathbf{a}=\left(a_x,a_y\right)\sim\frac{2}{\theta^2}\left(\dot{\beta}_x\cos 2\xi+\dot{\beta}_y\sin 2\xi,\ \dot{\beta}_x\sin 2\xi-\dot{\beta}_y\cos 2\xi\right) \ . \tag{21}$$

Here, we have decomposed the transverse direction into x and y components, ξ is such that $\theta\cos\xi$ and $\theta\sin\xi$ are respectively the x and y components of the observation direction, and the dot represents the derivative with respect to the time variable.

Inserting the resulting expression into Eq. (1), one obtains the angular density of flux. Integrating over the angular distribution, which is sharply peaked around θ determined by Eq. (16), and summing over the contributions from different harmonics n, it can be shown that the total flux in the undulator harmonics at frequency ω satisfying the condition (17) is given by

$$F_{UH} = \alpha\pi\frac{I}{e}\left[\frac{L}{\lambda\gamma^2} < \gamma^2\beta_\perp{}^2 >\right] . \qquad (22)$$

Here the subscript UH refers to the undulator harmonics. Note the close resemblance of this expression to that of the undulator transition radiation Eq. (13). Both expressions contain the factor

$$f = \frac{L}{\lambda\gamma^2} < \gamma^2\beta_\perp{}^2 > . \qquad (23)$$

However, Eq. (13) is quadratic in f while eq. (22) is linear.

3.2 Bending Magnet Radiation

The spectrum of the bending magnet radiation extends to very low frequencies. This fact has been utilized by several infrared radiation beam lines [6] .

The flux of the bending magnet radiation per unit horizontal angle ϕ and integrated over the vertical angle is given by

$$\frac{dF_B}{d\phi} = \frac{\sqrt{3}}{2\pi}\alpha\gamma\frac{\Delta\omega}{\omega}\frac{I}{e}y\int_y^\infty K_{5/3}(y')dy' . \qquad (24)$$

Here $y = \omega / \omega_c$, ω_c (the critical frequency) $= 3\gamma^3 c / 2\rho$, ρ = the bending radius, and $K_{5/3}$ is the modified Bessel function of the 2nd kind. In the limit $y \to 0,\ y\int_y^\infty K_{5/3}(y')dy' \to 2.15\ y^{1/3}$. The bending magnet flux with a horizontal collection angle $\Delta\phi$ in the infrared limit, then, becomes

$$F_B \sim 0.59\ \alpha\gamma\Delta\phi\frac{\Delta\omega}{\omega}\frac{I}{e}\cdot\left(\frac{\omega}{\omega_c}\right)^{1/3} ; \quad \frac{\omega}{\omega_c} \to 0 . \qquad (25)$$

3.3 Comparison of the Performances

In view of Eqs. (13), (22) and (25), we obtain the following ratios between the fluxes of the transition undulator radiation, the large angle undulator harmonics, and the bending magnet radiation:

$$\frac{F_{TUR}}{F_{UH}} = 1.4 f . \qquad (26)$$

$$\frac{F_B}{F_{UH}} = \frac{0.59\gamma\Delta\phi(\omega / \omega c)^{1/3}}{\pi f} . \qquad (27)$$

The factor f was defined in Eq. (23).

Let us now consider an explicit example for 30 μm infrared radiation at ALS (1.5 GeV). The undulator parameters are those of U8, a hybrid design with $\lambda_u = 8$ cm. At the pole gap of 2.4 cm, the deflection parameter K = 5.67. We will assume that the undulator field is sinusoidal. The fundamental wavelength in the direction θ is given by

$$\lambda_1(\theta) = \frac{\left(1+K^2\!/\!_2+\gamma^2\theta^2\right)\lambda_u}{2\gamma^2} = \left(1+\frac{\gamma^2\theta^2}{1+K2/2}\right)(0.079\mu m) \ . \quad (28)$$

With N = 61, the inequality (3) is satisfied for $\gamma\theta \le 3$. The ratio $F_{TUR} / F_{UH} = f$ becomes

$$f = \frac{L}{\lambda\gamma^2}\frac{K^2}{2} = 0.31 \ . \quad (29)$$

The ALS bending magnet radiation $(\rho = 4.8\text{m})$ has the critical wavelength $\lambda_c \approx 8\,\mathring{A}$. Assuming the collection angle $\Delta\phi = 100$ mrad , we have

$$\frac{F_B}{F_{UH}} = 5.5 \ . \quad (30)$$

From these results, we have $F_{TUR} : F_{UH} : F_B =$ 1: 3.2 : 17.7; Compared to the flux of the transition undulator radiation, the fluxes of the undulator harmonics and the bending magnet radiation are larger by a factor of 3.2 and 17.7, respectively.

However, it should be noted that the undulator harmonics and the bending magnet radiation require much larger collection angles; The transition undulator radiation is collected in an angular cone $\theta \le 3/\gamma$, which in the present case is about 1 mrad. The undulator harmonics is collected in a thin annular region centered at an angle θ such that $\lambda_1(\theta) = \lambda$. In the present case, this leads to $\theta \approx 78$ mrad. For the bending magnet, we have assumed the horizontal collection angle of 100 mrad.

The large collection angle places demanding requirements on the optical system. Even more important, it has implication on the transverse coherence. For the bending magnet radiation with the horizontal collection angle $\Delta\phi$, the rms phase space area is approximately $\rho(\Delta\phi/2)^3$. In the present case, this is about 6×10^{-4}m-rad, and is larger than the minimum coherent phase space area $(\lambda/4\pi)$ by a factor 250. This means that the radiation is incoherent transversely. The maximum horizontal collection angle $\Delta\phi$ that is transversely coherent is

$\Delta\phi \approx 2(\lambda / 4\pi\rho)^{1/3}$, which is about 16 mrad in the present case. The coherent flux is therefore smaller by a factor 6 compared to the 100 mrad collection.

For the undulator harmonics, the source size viewed at an angle 78 m-rad is large, about 20 cm. This large source size can, in principle, be compensated by the small width of the angular annulus. However, the angular divergence of the electron beam, the near field effect, etc., will rapidly destroy the transverse coherence.

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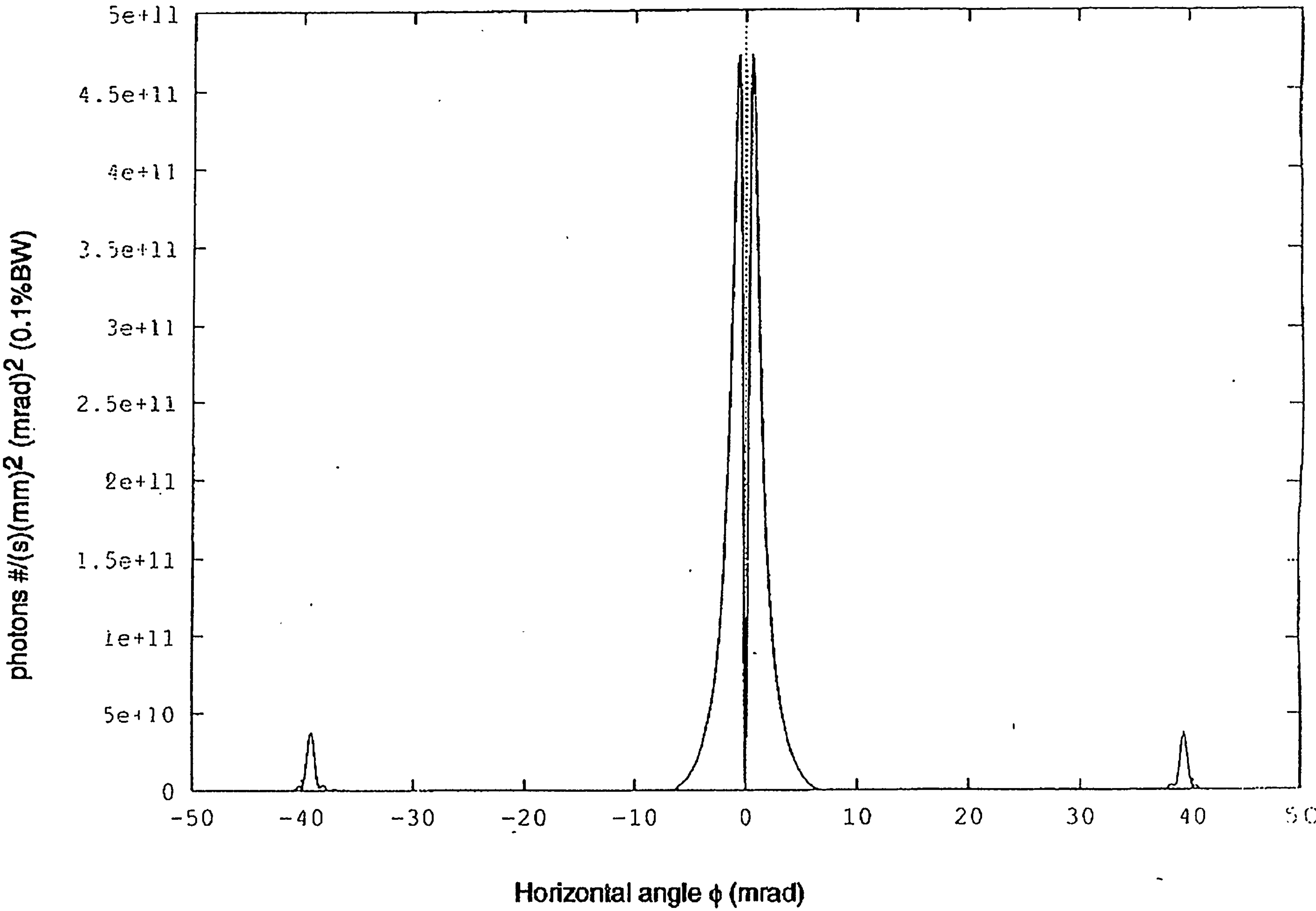
REFERENCES

[1] V.L. Ginzburg and I.M. Frank, Zh. Eksp & Ther. Fiz. 16, 15 (1946).
[2] L.D. Landau and E. M. Lifshitz, "Electrodynamics of Continuous Media", Chapter XV (Pergamon Press, 1984).
[3] K.-J. Kim, "Characteristics of Synchrotron Radiation", AIP Conference Proceeding 184 "Physics of Particle Accelerators", M. Month & M. Dienes ed., AIP 1989.
[4] C.X. Wang and D. Xian, Nucl. Instr. and Meth., A288, 649 (1990).
[5] P. Roy, et.al., Nucl. Instr. Meth., A325, 568 (1993).
[6] G.P. Williams, Nucl. Instr. Meth., A291, 8 (1990).

FIGURE CAPTIONS

Figure 1. Angular density of the flux from Super Aco SU2 undulator at λ=100 μ as a function of the longitudinal angle ϕ.

Figure 2. A schematic illustration of a beam line. The TUR is reflected by a mirror and the usual undulator radiation is transmitted through a narrow aperture in the mirror.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4e+11
3.5e+11
3e+11
2.5e+11
2e+11
1.5e+11
1e+11
5e+10
0
photons #/(s)(mm)2 (mrad)2 (0.1%BW)
-50
-40
-30
-20
-10
0
10
20
30
40
50
Horizontal angle ϕ (mrad)

Figure 1

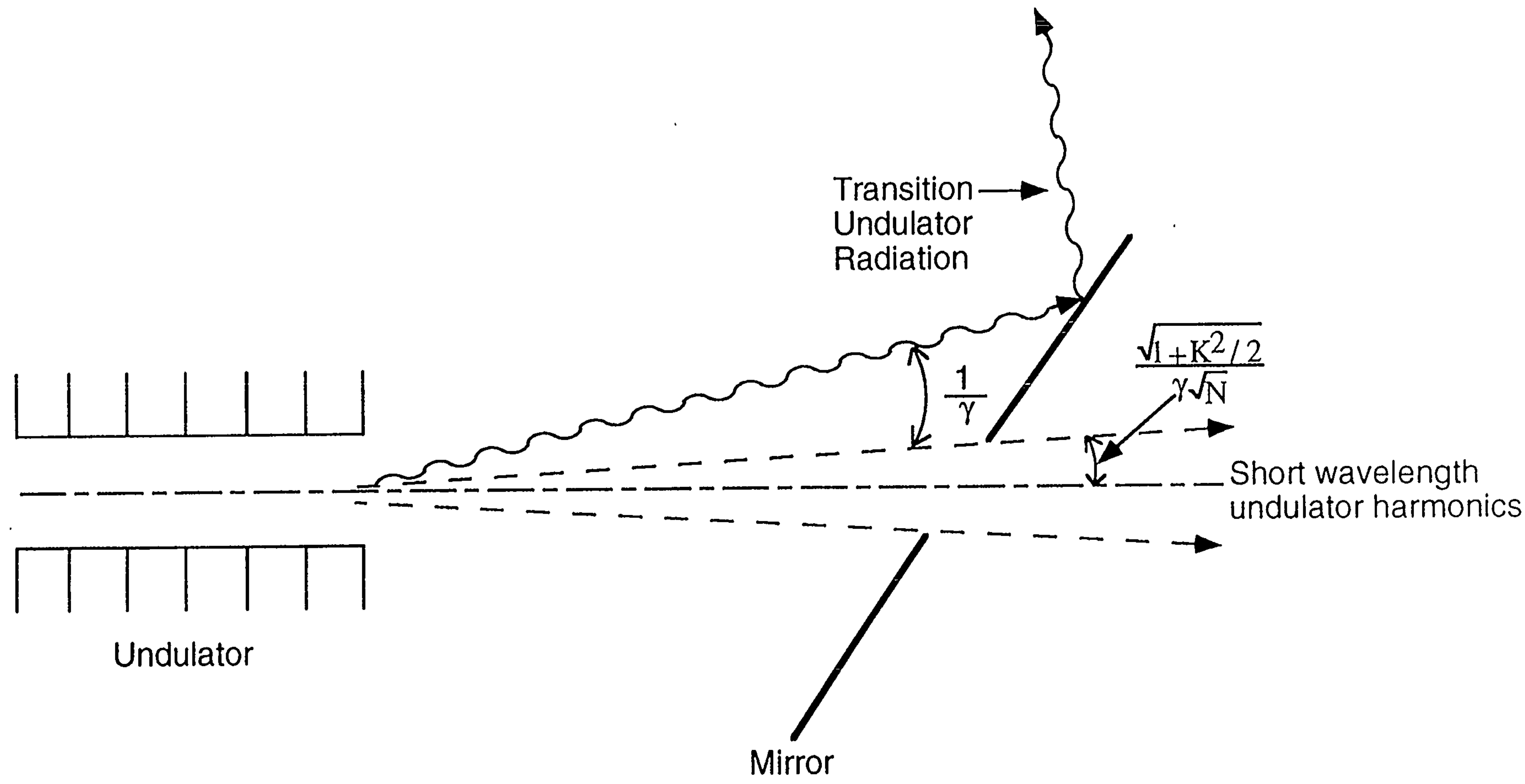

Figure 2:
A schmetic of an optical arrangement to intercept the transition undulator radiation.